

## WTNV109 - Hydrous and mechanical loading of a saturated porous environment

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### Summary:

One considers a three-dimensional problem of coupling thermo-hydro-mechanics of a saturated porous environment.

This test consists in studying the effect of mechanics and hydraulics on thermics. One stretches the element by imposing a displacement in the direction to him  $z$ , one imposes a water pressure on the whole of the field and one studies the effect of these loadings on the temperature of the model. It is a question of looking at the very weak coupling of poromechanic towards thermics. One limits oneself to the first step of time.

The studied models are 2D plans (DPQ8 and DPTR6) and 3D voluminal (HEXA20) with a linear behavior for thermal hydraulics and it.

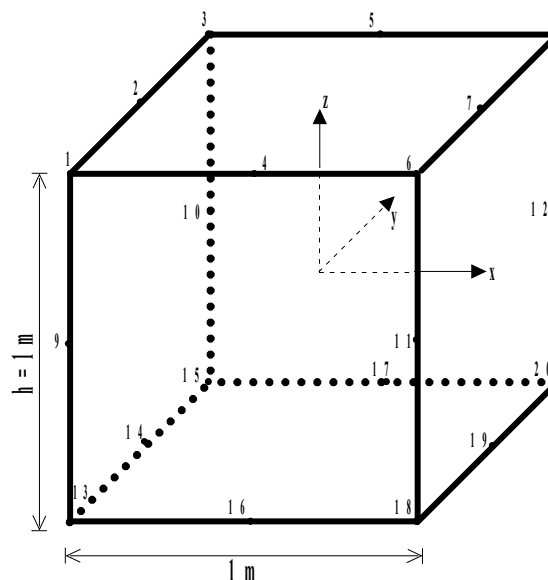
## 1 Problem of reference

### 1.1 Presentation

One studies in this case test the behavior thermo-hydro-mechanics of a saturated porous environment consisted only one fluid: water in its liquid phase. It acts in Code\_Aster of a modeling THM. The associated law of behavior of the fluid is of type LIQU\_SATU.

### 1.2 Geometry

One considers a cube of 1m of with dimensions centered on the center of the axis  $(-0,5 \leq x \leq 0,5; -0,5 \leq y \leq 0,5; -0,5 \leq z \leq 0,5; )$ .



### 1.3 Properties of material

solid	Density ( $kg.m^{-3}$ )	$2. \times 10^3$
	Drained Young modulus $E(Pa)$	$225. \times 10^6$
	Poisson's ratio	0.
	Thermal dilation coefficient of the solid ( $K^{-1}$ )	$8. \times 10^{-6}$
Fluid	Density ( $kg.m^{-3}$ )	$10^3$
	Heat with constant pressure ( $J.K^{-1}$ )	$2.85 \times 10^6$
	Thermal dilation coefficient of the liquid ( $K^{-1}$ )	$10^{-4}$
	Derived from the conductivity of the fluid compared to the temperature	0.
Thermics	Homogenized conductivity ( $W.K^{-1} m^{-1}$ )	1.7
	Derived from the conductivity homogenized compared to the temperature	0.

Coefficients of homogenisation	Coefficient of <i>Biot</i>	$10^{-12}$
	Porosity	0.4
Homogenized coefficients	Density ( $kg.m^{-3}$ )	$1.6 \times 10^3$
	Heat with constant constraint ( $J.K^{-1}$ )	$2.85 \times 10^6$

## 1.4 Boundary conditions and loadings

- Complete element:  
pressure of the fluid  $PRE1 = 500.0 Pa$  (not of flow nor of variation of the water mass)
- Lower face:  
displacements  $u_x = 0.0 m, u_y = 0.0 m, u_z = 0.0 m$ .
- Higher face:  
displacement  $u_z = 10^{-3} m$

## 1.5 Initial conditions

The fields of displacement, pressure, temperature are initially all worthless, the temperature of reference is worth  $T_0 = 273^\circ K$ .

## 2 Reference solution

### 2.1 Method of calculating

The reference solution is unidimensional because it depends only on the vertical coordinate. One will thus reason on a linear axis according to Z.

If gravity is neglected and that it is considered that there is no external source of heat, the equation of thermics is given by the following expression:

$$h_l^m \frac{d m_l}{dt} + \frac{\delta Q'}{dt} \text{Div } M_l + \text{div} q = 0 \quad (1)$$

Where  $h_l^m$  is the mass enthalpy of water,  $m_l$  its mass,  $M_l$  its flow, Q heat flow and  $Q'$  not convectée heat.

The conservation equation of the mass is the following one:

$$\frac{d m_l}{dt} + \text{Div } M_l = 0 \quad (2)$$

In the absence of hydrous flow, the equation (1) is simplified to become:

$$\frac{\delta Q'}{dt} + \text{div} q = 0 \quad (3)$$

In this equation, quantity  $Q'$  represent the heat received by the system in a transformation for which there are no contributions of heat per entry of fluid. One chooses a weak variation of pressure and parameters so that the temperature varies little and thus that:

$$\frac{d Q'}{T} = \frac{d Q'}{T_0}$$

$dQ'$  consequently this expression has:

$$dQ' = 3\alpha_0 K_0 T_0 d\varepsilon_v + C_0^\varepsilon dT - 3\alpha_l^m T_0 dp_l$$

with:

- $\alpha_0$  the thermal dilation coefficient homogenized equivalent with that of the solid  $\alpha_s$ .
- $K_0$  the drained coefficient of elasticity.
- $C_0^\varepsilon$  heat has constant deformation which has as an expression  $C_0^\varepsilon = C_0^\sigma - 9T_0 K_0 \alpha_0^2$  and  $C_0^\sigma$  heat with constant constraint.
- $\alpha_l^m$  The relative thermal dilation coefficient of the liquid compared to the skeleton, it has as an expression:  $\alpha_l^m = (b - \phi)\alpha_s + \phi\alpha_l$  with B the coefficient of Biot,  $\phi$  porosity and  $\alpha_l$  the dilation coefficient of the liquid.
- $d\varepsilon_v$  Voluminal variation of deformation.
- $dp_l$  Variation of pressure of liquid.

In addition, the heat flow has the following expression:  $q = -\lambda_T \partial \frac{T}{\partial z}$  where  $\lambda_T$  is the thermal coefficient of conductivity.

While replacing  $dQ'$  and Q by their value in the equation (3) and limiting itself to only step of time  $\Delta t$ , one obtains:

$$3 \alpha_0 K_0 T_0 \Delta \varepsilon_v - 3 \alpha_l^m T_0 \Delta p_l + C_0^\varepsilon \Delta T = -\Delta t \operatorname{div} q = \lambda_l \Delta t \frac{\partial^2 T}{\partial z^2}$$

By considering initially worthless temperatures and displacements, one can write:

$$\Delta T = T(t) - T(0) = T \text{ and } \Delta \varepsilon = \varepsilon(t) - \varepsilon(0) = \varepsilon \text{ and } \Delta p_l = p_l(t) - p_l(0) = p_l$$

Thus with the first step of time, one will have:

$$T - a \frac{\partial^2 T}{\partial z^2} = b$$

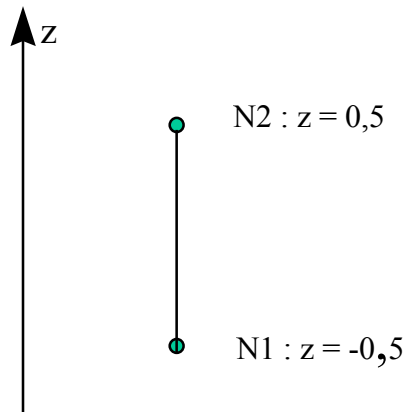
$$\text{with } a = \frac{\lambda_l \Delta T}{C_0^\varepsilon} \text{ and } b = -\frac{3 \alpha_0 K_0 T_0 \varepsilon - 3 \alpha_l^m T_0 p_l}{C_0^\varepsilon}$$

The variational formulation of this expression (in the unidimensional case) is then the following one:

$$\int_{\Omega} T \cdot \hat{T} dz + a \int_{\Omega} \frac{\partial T}{\partial z} \frac{\partial \hat{T}}{\partial z} dz - a \int_{\partial \Omega} \frac{\partial T}{\partial z} \hat{T} dz = \int_{\Omega} b \cdot \hat{T} dz \quad (4)$$

To establish the analytical solution a single element of degree 1 is considered since in modeling THM the thermohydraulic part is treated by linear elements.

That is to say a linear element:



One points out the boundary conditions:  $\frac{\partial T}{\partial z} = 0$  at 2 ends ( $z=0,5$  and  $Z = -0,5$ )

The temperature on the basis as of functions of forms is written in the following way:

$$T(z, t) = \sum_{i=1}^{i=2} T^i(t) \lambda_i(z)$$

with

$$\lambda_1(z) = 0,5(1 + 2z)$$

and

$$\lambda_2(z) = 0,5(1 - 2z)$$

The following matrices then are introduced:

$$[A] = [A_{ij}]; A_{ij} = \int_{-0,5}^{0,5} \lambda_i \lambda_j dz$$

$$[B] = [B_{ij}]; B_{ij} = \int_{-0,5}^{0,5} \frac{\partial \lambda_i}{\partial z} \frac{\partial \lambda_j}{\partial z} dz$$

what leads to:

$$\text{and}$$

$$[A] = \frac{1}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad [B] = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

One notes then classically:

$$\{T\} = \begin{Bmatrix} T^1 \\ T^2 \end{Bmatrix}$$

and

$$\left\{ \frac{\partial T}{\partial t} \right\} = \begin{Bmatrix} \frac{\partial T^1}{\partial t} \\ \frac{\partial T^2}{\partial t} \end{Bmatrix}$$

The equation (4) becomes then

$$[A]\{T\} + a[B]\{T\} - a[A]\left\{ \frac{\partial T}{\partial t} \right\} = [A]\{b\}$$

with the boundary conditions imposed (null displacement in bottom and worthless flows of temperatures at the two ends), one a:

$$\{b\} = \begin{Bmatrix} 0 \\ b \end{Bmatrix} \text{ et } \left\{ \frac{\partial T}{\partial t} \right\} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

What with final gives us

$$\begin{Bmatrix} T^1 \\ T^2 \end{Bmatrix} + a[A]^{-1}[B]\begin{Bmatrix} T^1 \\ T^2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ b \end{Bmatrix}$$

Finally one obtains:

$$\begin{bmatrix} 1+6a & -6a \\ -6a & 1+6a \end{bmatrix} \{T\} = \begin{Bmatrix} T^1 \\ T^2 \end{Bmatrix}$$

## 2.2 Sizes and results of reference

For a time court of 100s, one will have

$$\begin{Bmatrix} T^1 \\ T^2 \end{Bmatrix} = \begin{Bmatrix} 2.10^{-14} \\ 1,045.10^{-7} \end{Bmatrix}$$

One will consider  $T^1$  no one.

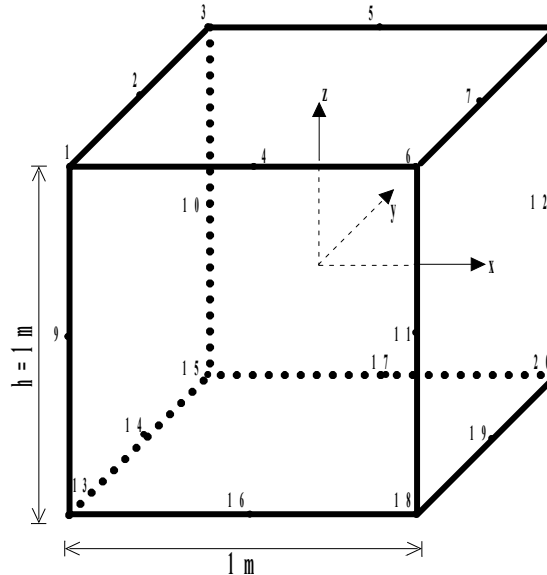
## 2.3 Uncertainty on the solution

None

## 3 Modeling A

### 3.1 Characteristics of modeling

Voluminal modeling 3D\_THM



### 3.2 Characteristics of the grid

1 mesh HEXA20.

### 3.3 Sizes tested and results

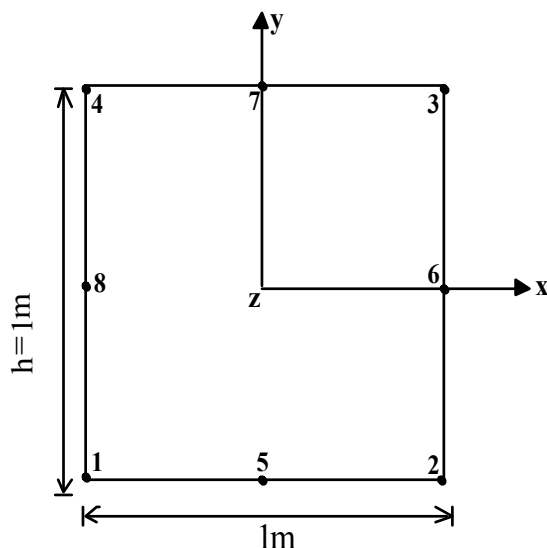
Discretization in time: only one step of time of 100 s . The diagram in time is implicit ( $\theta=1$ ) . The results correspond perfectly to the analytical solution.

Node	Type of value	Moment (s)	Reference (analytical)	Tolerance (%)
N1, N3	TEMP	10 <sup>2</sup>	$-1.045 \times 10^{-7}$	0,1
N6, N8	TEMP	10 <sup>2</sup>	$-1.045 \times 10^{-7}$	0,1

## 4 Modeling B

### 4.1 Characteristics of modeling

Plane modeling: D\_PLAN\_THM



### 4.2 Characteristics of the grid

1 mesh QUAD8.

### 4.3 Sizes tested and results

Discretization in time: only one step of time of 100s. The diagram in time is implicit ( $\theta = 1$ ). The results correspond perfectly to the analytical solution.

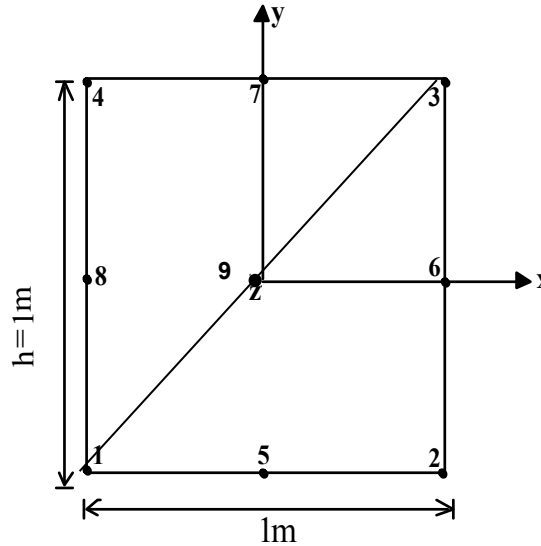
Node	Type of value	Moment (S)	Reference (analytical)	Tolerance (%)
N3	TEMP	$10^2$	$-1.045 \times 10^{-7}$	0,1
N4	TEMP	$10^2$	$-1.045 \times 10^{-7}$	0,1



## 5 Modeling C

### 5.1 Characteristics of modeling

Plane modeling: D\_PLAN\_THM



### 5.2 Characteristics of the grid

2 meshes TRIA6.

### 5.3 Sizes tested and results

Discretization in time: only one step of time:  $10^2 \text{ s}$ . The diagram in time is implicit ( $\vartheta = 1$ ). The results correspond perfectly to the analytical solution.

Node	Type of value	Moment (S)	Reference (analytical)	Tolerance (%)
N3	TEMP	$10^2$	$-1.045 \times 10^{-7}$	0,1
N4	TEMP	$10^2$	$-1.045 \times 10^{-7}$	0,1

## 6 Modeling D

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### 6.1 Characteristics of modeling

Voluminal modeling 3D\_THM.

### 6.2 Characteristics of the grid

2 mgoes PENTA15.

### 6.3 Sizes tested and results

Discretization in time: only one step of time:  $10^2$  s . The diagram in time is implicit ( $\theta=1$ ) . The results correspond perfectly to the analytical solution.

Node	Type of value	Moment (S)	Reference (analytical)	Tolerance (%)
N3	TEMP	$10^2$	$-1.045 \times 10^{-7}$	0,1
N4	TEMP	$10^2$	$-1.045 \times 10^{-7}$	0,1

## 7 Summary of the results

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The results are in coherence with the analytical solution.