WTNV139 – Modeling of a well dug in a transverse isotropic formation saturated with water

Summary:

This case test resulting from the literature (Abousleiman and al. [1]) presents the case of a well dug tilted in a medium saturated with water. It is about a coupled hydraulic modeling which has an analytical solution. The purpose of this case test is to test the good taking into account of the transverse isotropy of the porous environment.

1 Problem of reference

The objective of this case test is to compare the solution obtained with Code_Aster, by using a modeling with the finite elements, with an analytical solution. This case test and its analytical solution was developed and detailed by Abousleiman and al. [1]. We present of them here the principal lines as well as the acceptable analogies with our model.

1.1 Geometry of the problem

One considers a volume of rock height $h=1\text{m}$ and basic square of $l=15\text{m}$ of with dimensions. In the centre, a cylindrical cavity of is dug $R=0.1\text{m}$ and of infinite extension according to its axis z . It is noticed that a field height here is modelled 1m what is a restrictive assumption (ideally, one would need a field much larger but which would involve long computing times). This cavity is used to model the well dug in the solid mass of rock. This solid mass in addition is characterized by plans of isotropy perpendicular to the axis of the cylindrical cavity.

The well in addition is characterized by two angles compared to the reference mark of the principal constraints:

- its azimuth $\varphi_y=30°$
- its slope $\varphi_z=60°$

What corresponds to the nautical angles:

- $\alpha=30°$
- $\beta=-60°$

The Cartesian reference mark associated with the solid mass and the reference mark of the constraints is noted $(O x' y' z')$ and that associated with the cavity is noted $(Oxyz)$. A cylindrical frame of reference is also defined (r, θ, z) to characterize the cylindrical cavity.

On the Figure 1.1-1 one summarizes information described above:

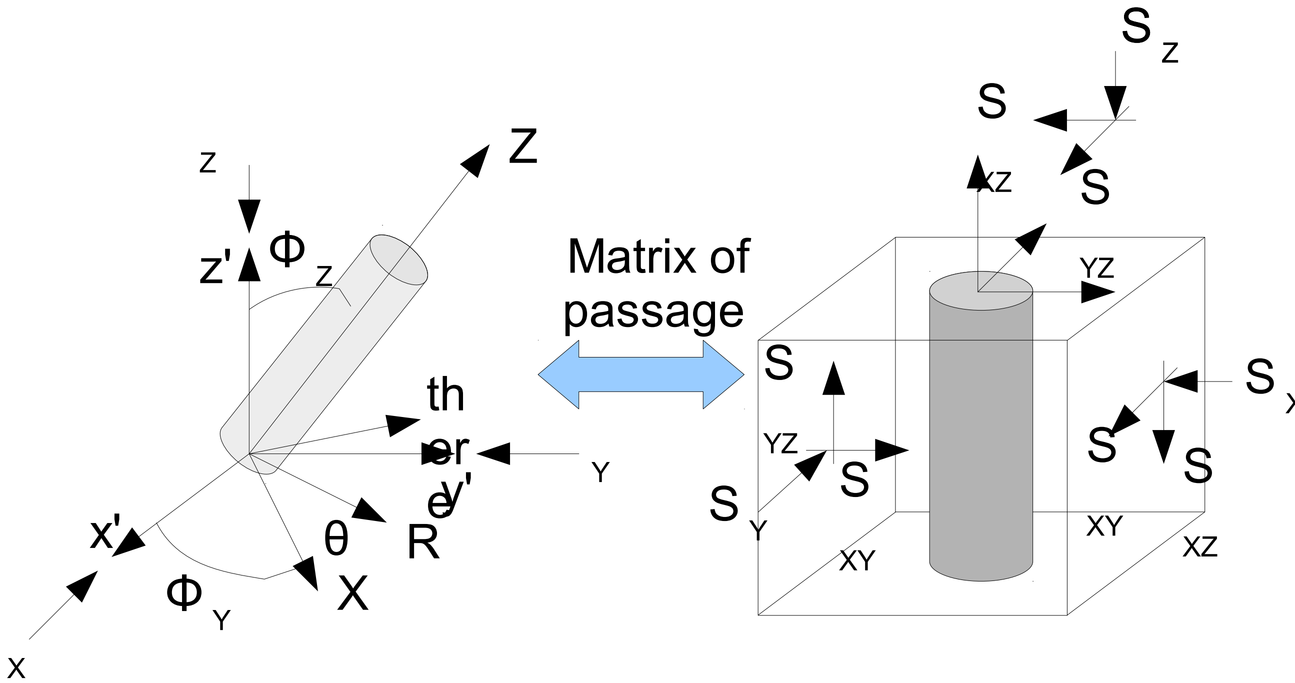

Figure 1.1-1 : Representation of the geometry of the problem of the tilted well

It is important to notice that the two geometries described above are strictly equivalent. However one chooses to adopt the geometry for which the well is vertical, in order to facilitate the modeling and the design of the grid. The selected reference mark is thus the transverse reference mark of isotropy and not that of the principal constraints.

With final, one will represent a part only of the model such as:

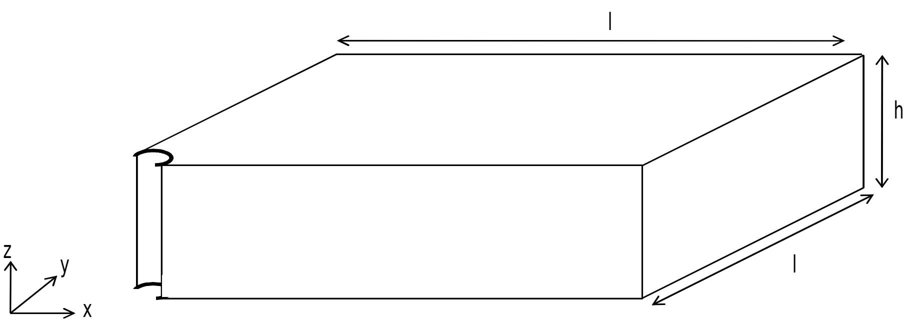

Figure 1.1-2 : Modelled geometry

Conditions of symmetries are thus applied.

1.2 Properties materials

One refers to the parameters preset in the article of Abou Sleiman 1.

One leaves the report of anisotropy $\frac{E_L}{E_N}=2$ and $\frac{\nu_{TL}}{\nu_{NL}}=2$. It is pointed out that the assumptions of transverse isotropy imply that $\nu_{TL}=\nu_{LT}$; $\nu_{NT}=\nu_{NL}$; $\nu_{LN}=\nu_{TN}$ and $\frac{\nu_{NT}}{\nu_{LN}}=\frac{E_N}{E_L}$.

In addition and by writing the terms of the matrix of Hook, such as:

$$M_{11}=\frac{E_{L.}(E_N-E_L\nu_{NL}^2)}{(1+\nu_{TL})(E_N-E_{N.}\nu_{TL}-2.\mathrm{E}_{L.}\nu_{NL}^2)}$$

$$M_{12}=\frac{E_L(E_N\nu_{TL}+E_L\nu_{NL}^2)}{(1+\nu_{TL})(E_N-E_{N.}\nu_{TL}-2.\mathrm{E}_{L.}\nu_{NL}^2)}$$

$$M_{13}=\frac{E_L E_N\nu_{NL}}{E_N-E_N\nu_{TL}-2.\mathrm{E}_L\nu_{NL}^2}$$

$$M_{33}=\frac{E_N^2(1-\nu_{TL})}{E_N-E_N\nu_{TL}-2.\mathrm{E}_L\nu_{NL}^2}$$

The coefficients of Biot are then deduced by the following relation 1:

$$b_L=1-\frac{M_{11}+M_{12}+M_{13}}{3\mathrm{K}_S}$$

and

$$b_N=1-\frac{2\mathrm{M}_{13}+M_{33}}{3\mathrm{K}_S}$$

Here, $K_S=27{,}5\,GPa$.

From that, one indicates here the properties materials indicated in Code_Aster:

Liquid	Viscosity μ_w (in $Pa.s$)	10^{-3}
	Module of compressibility $\frac{1}{K_w}$ (in Pa^{-1})	5.10^{-10}
	Density of the liquid ρ_w (in kg/m^3)	1
	Specific heat with constant pressure	4180
Elastic parameters	Young modulus E_L (in MPa)	9474
	Young modulus E_N (in MPa)	0.5×9474
	Poisson's ratio ν_{LT}	0,24
	Poisson's ratio ν_{LN}	0,24
	Modulus of Rigidity G_{ln} (in MPa)	8880

Parameters of coupling	Coefficient of Biot b_L	0,81689
	Coefficient of Biot b_N	0,89864
	Initial homogenized density r_0 (in kg/m^3)	2410
	Intrinsic permeability K_L^{int} (in m^2/s)	5.10^{-20}
	Intrinsic permeability K_N^{int} (in m^2/s)	5.10^{-20}

Table 1.2-1 : Properties of materials

1.3 Boundary conditions and initial of the model

In the continuation we consider the initial conditions and in extreme cases in the reference mark related to the well.

1.3.1 Case of displacements

Taking into account the symmetry of the model, one will block on 3 faces corresponding respectively to the 3 plans $x=0\,;\,y=0$ and $z=0$.

1.3.2 Case of the pressure

Initial pressure of pore in the solid mass and for $r\rightarrow+\infty$ is fixed at $p_0=9{,}8\,MPa$. One will thus apply this pressure to the 3 faces corresponding respectively to the 3 plans $x=l\,;\,y=l$ and $z=h$. Following the setting in water of the well, it develops in wall a uniform pressure which one fixes at $p_w=12\,MPa$ on the edge of the cavity.

1.3.3 Case of the constraints

In the solid mass, L'state of stress *in situ* associated with the reference mark $(O\,x'\,y'\,z')$ before the digging of the well is characterized by the following initial state:

$$\begin{cases} S'_x=-25\,MPa \\ S'_y=-22\,MPa \\ S'_z=-29\,MPa \end{cases}$$

Moreover the digging of the well a disturbance of the state of stress implies *in situ* initial in the solid mass. In order to take into account this new state of stress in the vicinity of the well, it is necessary to consider two successive rotations to pass from the reference mark $(O\,x'\,y'\,z')$ associated with the solid mass, the reference mark $(O\,xyz)$ associated with the well (cf Appears 1.1-1).

Are $\boldsymbol{P_1}$ and $\boldsymbol{P_2}$ matrices of passage describing this change of reference mark:

$$P_1=\begin{pmatrix}\cos(\alpha) & -\sin(\alpha) & 0\\ \sin(\alpha) & \cos(\alpha) & 0\\ 0 & 0 & 1\end{pmatrix} \qquad P_2=\begin{pmatrix}\cos(\beta) & 0 & -\sin(\beta)\\ 0 & 1 & 0\\ \sin(\beta) & 0 & \cos(\beta)\end{pmatrix}$$

After change of reference mark, the state of initial stress in the reference mark $(Oxyz)$ and when $r\rightarrow+\infty$ to apply to the model is the following:

$$\begin{cases} S_{xx}=-27{,}81\,MPa\\ S_{yy}=-22{,}75\,MPa\\ S_{zz}=-25{,}44\,MPa\\ S_{xy}=0{,}65\,MPa\\ S_{xz}=2{,}06\,MPa\\ S_{yz}=1{,}125\,MPa\end{cases}$$

For the boundary conditions, one neglects shearing and one applies the conditions in pressure to the 3 faces $x=l\,;\,y=l$ and $z=h$.
In wall of the well $(r=R)$ and following, the radial constraint is applied: $\sigma_{rr}(r=R\,,t)=-p_w.H(t)$ where $H(t)$ is the function level of Heaviside.

1.4 Reference solution

In this part we recall the useful theoretical equations for the calculation of the radial constraint, and the pressure of pore.

One summarizes the boundary conditions of the problem general:

- for $r\rightarrow+\infty$

$$\begin{cases} \sigma_{xx}=-S_{xx}\\ \sigma_{yy}=-S_{yy}\\ \sigma_{zz}=-S_{zz}\\ \sigma_{xy}=-S_{xy}\\ \sigma_{xz}=-S_{xz}\\ \sigma_{yz}=-S_{yz}\end{cases} \quad \text{and} \quad p=p_0$$

- for $r=R$

$$\sigma_{rr}=-p_w.H(t)$$
$$\sigma_{r\theta}=\sigma_{rz}=0$$
$$p=p_w.H(t)$$

Taking into account the linearity of the problem, that Ci is in fact the result of the superposition of 3 pennies problems which allow a three-dimensional study 1:

- problem 1: one considers a problem poro-rubber band in plane deformations (one takes into account only the principal constraints according to the axes x and y as well as shearing in the plan (xy) and disturbances related to the pressure of pore). The boundary conditions of this problem are written then:

- for $r \to +\infty$

$$\sigma_{xx} = -S_{xx}$$
$$\sigma_{yy} = -S_{yy}$$
$$\sigma_{xy} = -S_{xy}$$

$$\sigma_{zz} = -\nu_{\mathrm{LN}}(S_{xx}+S_{yy}) - (\alpha_{\mathrm{LN}} - 2\nu_{\mathrm{LN}}\alpha_{\mathrm{LT}})p_0 - (\beta_3^s - 2\nu_{\mathrm{LN}}\beta_1^s)T_0$$

$$\sigma_{yz} = \sigma_{xz} = 0$$

$$p = p_0$$

- for $r = R$

$$\sigma_{rr} = -p_w . H(t)$$

$$p = p_w . H(t)$$

- problem 2: one considers only the vertical constraint directed according to z . The boundary conditions of this problem are written then:

- for $r \to +\infty$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{xy} = \sigma_{xz} = \sigma_{yz} = p = 0$$

$$\sigma_{zz} = -S_{zz} + (\nu_{\mathrm{LN}}(S_{xx}+S_{yy}) + (\alpha_{\mathrm{LN}} - 2\nu_{\mathrm{LN}}\alpha_{\mathrm{LT}})p_0)$$

β_3^s and β_1^s are defined in the continuation of this document.

- for $r = R$

$$\sigma_{rr} = \sigma_{r\theta} = \sigma_{rz} = p = 0$$

- problem 3 : one considers only the shearings contained in the plans (xz) and (yz) . The boundary conditions of this problem are written then:

- for $r \to +\infty$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = p = 0$$
$$\sigma_{xz} = -S_{xz}$$
$$\sigma_{yz} = -S_{yz}$$

- for $r = R$

$$\sigma_{rr} = \sigma_{r\theta} = \sigma_{rz} = p = 0$$

In the continuation of this document one will develop the solution of problem 1.

1.4.1 Presentation of the analytical problem

The expression of the radial constraint, the pressure of pore and the temperature, for the problem in plane deformations (problem 1 above) is also the result of the superposition of 3 load patterns whose boundary conditions are given hereafter:

- Mode 1: taking into account of the hydrostatic part of the constraints in extreme cases (in $r=R$)

$$\sigma_{rr}^{(1)}=\sigma_0-p_w$$

$$p^{(1)}=0$$

- Mode 2: taking into account of the disturbances related on the pressure of pore and the temperature (in $r=R$)

$$\sigma_{rr}^{(2)}=0$$

$$p^{(2)}=p_w-p_0$$

- Mode 3: taking into account of the deviatoric part of the constraints in extreme cases (in $r=R$)

$$\sigma_{rr}^{(3)}=-S_0\cos(2(\theta-\theta_r))$$

$$p^{(3)}=0$$

with:

$\sigma_0=\dfrac{(S_{xx}+S_{yy})}{2}$ the average constraint, $S_0=0.5\sqrt{((S_{xx}-S_{yy})^2+4S_{xy}^2)}$ the deviatoric constraint and

$$\theta_r=0.5\arctan\left(\frac{2S_{xy}}{S_{xx}-S_{yy}}\right)$$

- The general solution for the calculation of the radial constraint is thus given by the following expression:

$$\sigma_{rr}=-\sigma_0+S_0\cos(2(\theta-\theta_r))+\sigma_{rr}^{(1)}+\sigma_{rr}^{(2)}+\sigma_{rr}^{(3)}$$

with:

$$\sigma_{rr}^{(1)}=(\sigma_0-p_w)\frac{R^2}{r^2}$$

$$\sigma_{rr}^{(2)}=L^{-1}\left[\frac{1}{s}\left(b_L\left(1-\frac{M_{12}}{M_{11}}\right)\left[F_1\psi(\xi)+F_2\psi(\omega)\right]+\beta_1^s\left(1-\frac{M_{12}}{M_{11}}\right)\left[(T_w-T_0)\right]\psi(\omega)\right)\right]$$

$$\sigma_{rr}^{(3)}=L^{-1}\left[\frac{S_0}{s}\cos(2(\theta-\theta_r))\left(A_1C_1\left(\frac{1}{\xi r}K_1(\xi r)+\frac{1}{(\xi r)^2}K_2(\xi r)\right)-A_2C_2\frac{R^2}{r^2}-3A_3C_3\frac{R^4}{r^4}\right)\right]$$

The quantities used are defined at the end of this section.

- The general solution for the calculation of the pressure of pore is thus given by the following expression:

$$p=p_0+p^{(2)}+p^{(3)}$$

with:

$$p^{(2)}=L^{-1}\left[\frac{1}{s}\left[F_1\phi(\xi)+F_2\phi(\omega)\right]\right]$$

$$p^{(3)}=L^{-1}\left[\frac{S_0}{s}\cos(2(\theta-\theta_r))\left(A_1C_2\frac{R^2}{r^2}+\frac{c_f}{2G_{\mathrm{LT}}\kappa}C_1K_2(\xi r)\right)\right]$$

The quantities used are defined at the end of this section.

In addition one defines K_n as being modified functions of Bessel of second species of order n (where n is an entirety) such as:

$$K_n(x)=\begin{array}{l}\frac{1}{2}\left(\frac{1}{2}x\right)^{-n}\sum_{k=0}^{n-1}\left[\frac{(n-k-1)!}{k!}\left(\frac{-1}{4}x^2\right)^k\right]+(-1)^{n+1}\ln\left[\frac{1}{2}x\right]I_n(x)\\+(-1)^n\frac{1}{2}\left(\frac{1}{2}x\right)^n\sum_{k=0}^{\infty}\left[(\chi(k+1)+\chi(n+k+1))\frac{\left[\frac{1}{4}x^2\right]^k}{k!(n+k)!}\right]\end{array}$$

where $I_n(x)=\frac{1}{\pi}\int_0^{\pi}e^{x\cos\theta}\cos(n\theta)d\theta$ are the modified functions of Bessel of first species of order n (where n is an entirety) and χ the function digamma.

One can thus define the following quantities:

$$\phi(x)=\frac{K_0(xr)}{K_0(xR)}$$

$$\psi(x)=\frac{K_1(xr)}{xrK_0(xR)}-\frac{RK_1(xr)}{xr^2K_0(xR)}$$

$$F_1=(p_w-p_0)-\frac{c_{hf}}{1-\frac{c_f}{c_h}}(T_w-T_0)$$

$$F_2=\frac{c_{hf}}{1-\dfrac{c_f}{c_h}}(T_w-T_0)$$

$$c_f=\frac{\kappa M M_{11}}{M_{11}+b_L^2 M}$$

$$\kappa=\frac{K_L^{\text{int}}(\varphi)}{\mu_w}$$

$$c_{hf}=\frac{c_f}{\kappa}\left[\beta^{sf}-\frac{b_L\beta_1^S}{M_{11}}\right]$$

$$C_1=\frac{4}{2\mathrm{A}_1(B_3-B_2)-A_2B_1}$$

$$C_1=\frac{4}{2\mathrm{A}_1(B_3-B_2)-A_2B_1}$$

$$C_3=\frac{2\mathrm{A}_1(B_3+B_2)+3\mathrm{A}_2B_1}{3(2\mathrm{A}_1(B_3-B_2)-A_2B_1)}$$

$$A_1=\frac{b_L M}{M_{11}+b_L^2 M}$$

$$A_2=\frac{M_{11}+M_{12}+2b_L^2 M}{M_{11}+b_L^2 M}$$

$$B_1=\frac{M_{11}}{2G_{\mathrm{LT}}b_L}K_2(\xi R)$$

$$B_2=\frac{1}{\xi R}K_1(\xi R)+\frac{6}{(\xi R)^2}K_2(\xi R)$$

$$B_3=2\left(\frac{1}{\xi R}K_1(\xi R)+\frac{3}{(\xi R)^2}K_2(\xi R)\right)$$

$\xi=\sqrt{\dfrac{s}{c_f}}$ and $\omega=\sqrt{\dfrac{s}{c_h}}$ where s represent the variable of Laplace.

1.4.2 Method of resolution of the analytical solution

As we could note it previously, the solutions for modes 2 and 3 are expressed within the space of Laplace. Taking into account the complexity of the inversion of the solution towards temporal space, we resort to an algorithm of inversion known as of *Stehfest-to engrave.* This method is recognized for its digital stability.

The principle of the method consists in approximating the function to be reversed $\tilde{f}(s)$ using a delta-series by using the following formula:

$$f(t) \simeq \frac{\ln(2)}{t} \sum_{i=1}^{N} V_i \tilde{f}\left(i \frac{\ln(2)}{t}\right)$$

where $V_i = (-1)^{i+(N/2)} \dfrac{\sum_{k=E((i+1)/2)}^{min(i,N/2)} k^{N/2} (2\mathrm{k})!}{\left(\frac{N}{2}-k\right)! \, k! \, (k-1)! \, (i-k)! \, (2\mathrm{k}-i)!}$

with $E(x)$ the whole left function and N the number of terms of the series. It in general lies between 10 and 20. In our case we chose $N=18$

2 Modeling A

2.1 Characteristics of modeling A

Modeling `3D_HMS`. The grid is composed of 1806 HEXA8 and 204 PENTA6.
The time of simulation is of $t=86{,}4\,s$ (either $t=0{,}001\ jours$) carried out in 10 pas de time.

2.2 Results

2.2-1represent the radial constraints for an angle $\theta=90°$, that is to say along the axis y. The results of modeling and analytical are represented. If the total pace is the same one there are however differences which are logical: first of all for reasons of computing time the grid is coarse here in particular in the thickness h. In addition we are enough far from an infinite geometry and countered themintes of shearing at the edge is not applied. Finally the formalism in effective/forced constraint total proposed by Abousleiman is slightly different from our and the comparison requires certain adjustments.

Knowing that, one can consider that this agreement is completely acceptable.

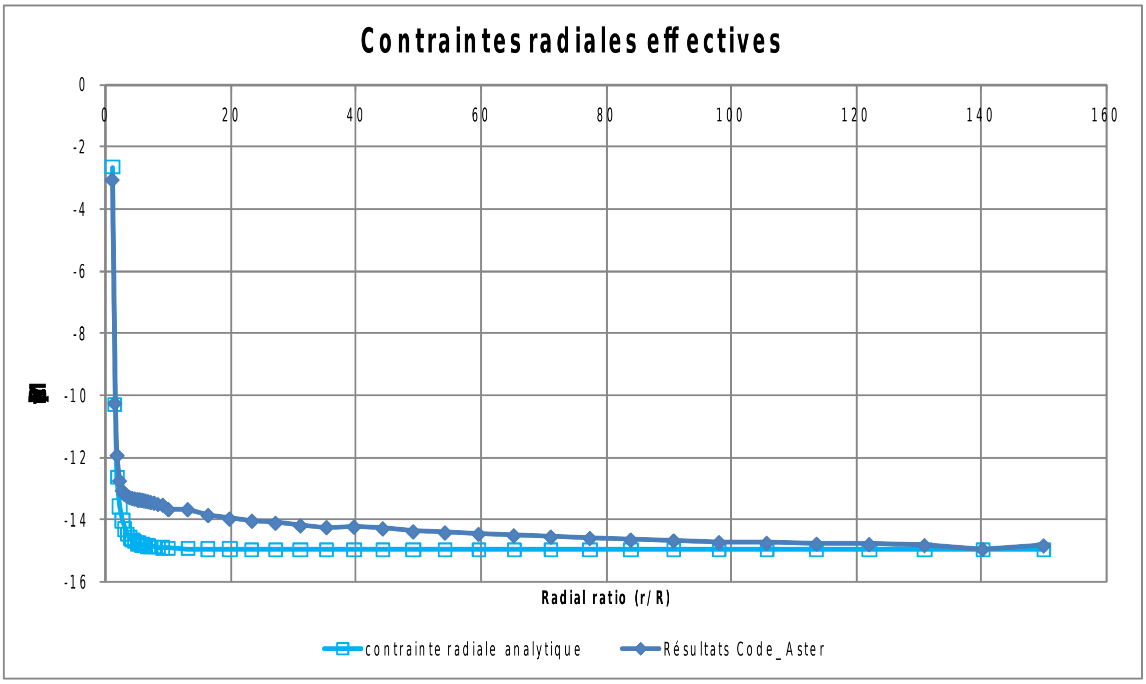

Figure 2.2-1 : Modelled geometry

2.3 Values tested

In accordance with 1, one looks at the radial constraints at time $t=86{,}4\,s$ (either $t=0{,}001\ jours$).

$N°$ **NODE**	$COOR\_X$	$COOR\_Y$	$COOR\_Z$	**Reference** $PRE1(MPa)$	**Aster** $PRE1(MPa)$	**Allowed tolerance** $(\%)$
2286	0	0.14	0.5	10.5	9.93	10

$N°$ **NODE**	$COOR\_$	$COOR\_$	$COOR\_Z$	**Reference**	**Aster** $SIYY(MPa)$	**Tolerance** $(\%)$

				$\sigma_r(MPa)$		
2286	0	0.14	0.5	10.3	10.25	10
2582	0	13.1	0.5	14.9	14.8	10

3 Summary of the results

Results are coherent with the analytical solution.

4 References

1. Abousleiman, Y. Ekbote, S. "Solution for the inclined Borehole in has Porothermoelastic Transversly Isotropic Medium". Newspaper of Applied Mechanics. Vol. 72 (2005)