

## WTNV141 – Validation of an evolutionary loading for a saturated hydraulic problem

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### Summary:

The purpose of this test is to validate the keyword `EVOL_CHAR` in THM in 2D and 3D. One has an analytical solution.

## 1 Problem of reference

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### 1.1 Geometry

In 2D, a square on side is considered 1m . In 3D, a cube on side is considered 1m .

### 1.2 Properties of material

Intrinsic permeability:  $\kappa = 0.05 \text{ m}^2 \cdot \text{Pa}^{-1} \cdot \text{s}^{-1}$

Coefficient of Biot:  $b = 1$ .

Young modulus:  $E = 2.5 \text{ Pa}$

Poisson's ratio:  $\nu = 0.25$

There are thus the following coefficients of Lamé:  $\lambda = \mu = 1.0 \text{ Pa}$

### 1.3 Boundary conditions and loadings

The boundary conditions are the conditions of Dirichlet corresponding to the analytical solution one has.

In 2D, one defines  $A = 2 \times \pi^2 \times \kappa$  and the voluminal mechanical loading is defined:

$$f(t, x, y) = \begin{bmatrix} \cos(\pi x) \sin(\pi y) \\ \sin(\pi x) \cos(\pi y) \end{bmatrix} \pi e^{-At} [-(\lambda + 2\mu) + b]$$

In 3D, one defines  $A = 3 \times \pi^2 \times \kappa$  and the voluminal mechanical loading is defined:

$$f(t, x, y, z) = \begin{bmatrix} \cos(\pi x) \sin(\pi y) \sin(\pi z) \\ \sin(\pi x) \cos(\pi y) \sin(\pi z) \\ \sin(\pi x) \sin(\pi y) \cos(\pi z) \end{bmatrix} \pi e^{-At} [-(\lambda + 2\mu) + b]$$

### 1.4 Initial conditions

The initial conditions correspond to the analytical solution one has.

## 2 Reference solution

### 2.1 Method of calculating

One points out the system of equations which one solves:

$$\begin{cases} -\nabla \cdot \sigma(u) + b \nabla p = f \\ \partial_t (\nabla \cdot u) - \kappa \Delta p = 0 \end{cases}$$

where  $\sigma(u) = \lambda \nabla \cdot u I_d + 2\mu \varepsilon(u)$  and  $I_d$  indicate the matrix identity in dimension  $d$ .

In 2D, there is the following analytical solution:

$$p(t, x, y) = e^{-At} \sin(\pi x) \sin(\pi y)$$
$$u(t, x, y) = - \begin{bmatrix} \cos(\pi x) \sin(\pi y) \\ \sin(\pi x) \cos(\pi y) \end{bmatrix} \frac{e^{-At}}{2\pi}$$

In 3D, there is the following analytical solution:

$$p(t, x, y) = e^{-At} \sin(\pi x) \sin(\pi y) \sin(\pi z)$$
$$u(t, x, y) = - \begin{bmatrix} \cos(\pi x) \sin(\pi y) \sin(\pi z) \\ \sin(\pi x) \cos(\pi y) \sin(\pi z) \\ \sin(\pi x) \sin(\pi y) \cos(\pi z) \end{bmatrix} \frac{e^{-At}}{3\pi}$$

### 2.2 Sizes and results of reference

One compares the digital solution with the analytical solution in 3 points distinct from the grid (in displacements and pressure).

### 2.3 Uncertainties on the solution

Analytical solution

## 3 Modeling A

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### 3.1 Characteristics of modeling

Modeling used is D\_PLAN\_HMS. The duration of simulation is fixed at  $T_1 = 0.1 s$  with 10 pas de time.

### 3.2 Characteristics of the grid

2048 meshes TRIA6.

### 3.3 Sizes tested and results

One tests the values of the digital solution at the final moment  $T_1 = 0.1 s$  in 3 nodes.

Name node	$X ( m )$	$Y ( m )$	Analytical solution		Relative error
N25	0.75	0.75	PRE1	4.53E-1	0.7%
			DX	7.21E-2	0.2%
			DY	7.21E-2	0.2%
N40	0,875	0,125	PRE1	1.33E-1	0.75%
			DX	5.10E-2	0.2%
			DY	-5.10E-2	0.2%
N35	0,375	0,625	PRE1	7.73E-1	0.8%
			DX	-5.10E-2	0.2%
			DY	5.10E-2	0.2%

Table 3.3-1

### 3.4 Remarks

The digital results are in very good agreement with the analytical results.

## 4 Modeling B

### 4.1 Characteristics of modeling

Modeling used is 3D\_HMS . Durea of simulation is fixed at  $T_1 = 0.01 s$  with 4 pas de time.

### 4.2 Characteristics of the grid

1000 meshes HEXA20 .

### 4.3 Sizes tested and results

One tests the values of the digital solution at the final moment  $T_1 = 0.1 s$  in 3 distinct nodes.

Name node	$X ( m )$	$Y ( m )$	$Z ( m )$	Analytical solution		Relative error	
N1948	0.8	0.2	0.2	PRE1	2.00E-1	1.2%	
					DX	2.92E-2	0.2%
					DY	-2.92E-2	0.2%
					DZ	-2.92E-2	0.2%
N1900	0.2	0.8	0.2	PRE1	2.00E-1	1.2%	
					DX	-2.92E-2	0.2%
					DY	2.92E-2	0.2%
					DZ	-2.92E-2	0.2%
N2380	0.2	0.2	0.8	PRE1	2.00E-1	1.2%	
					DX	-2.92E-2	0.2%
					DY	-2.92E-2	0.2%
					DZ	2.92E-2	0.2%

Table 4.3-1

### 4.4 Remarks

The digital results are in very good agreement with the analytical results.

## 5 Summary of the results

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The digital results are in perfect agreement with the analytical solutions.