WTNV143 - Application of a pressure distributed on the lips of a crack XFEM in a model hydro-mechanical

Summary:

The goal of this test is to make sure of the good performance of the wide finite element method (XFEM) in a hydro-mechanical model, in order to represent one fractured saturated medium.

In this test, it is supposed that the pressure of pore is taken uniformly worthless in all the field of study: the hydro-mechanical model corresponds then with a model "classical" mechanics without coupling. One checks that there exists well a discontinuity of the field of displacement on both sides of the interface. The results are compared with an analytical solution and the results got with a mechanical test XFEM (similar to the tests `ssnv203`, modelings F and H).

1 Problem of reference

1.1 Geometry of the problem

It is about a column height $LZ=5m$, length $LX=1m$ and of width $LY=1m$. This column presents in $Z=\frac{LZ}{2}$ a discontinuity of the type interfaces. The column is thus entirely crossed by discontinuity.

One represents on the Figure 1.1-a geometry of the column.

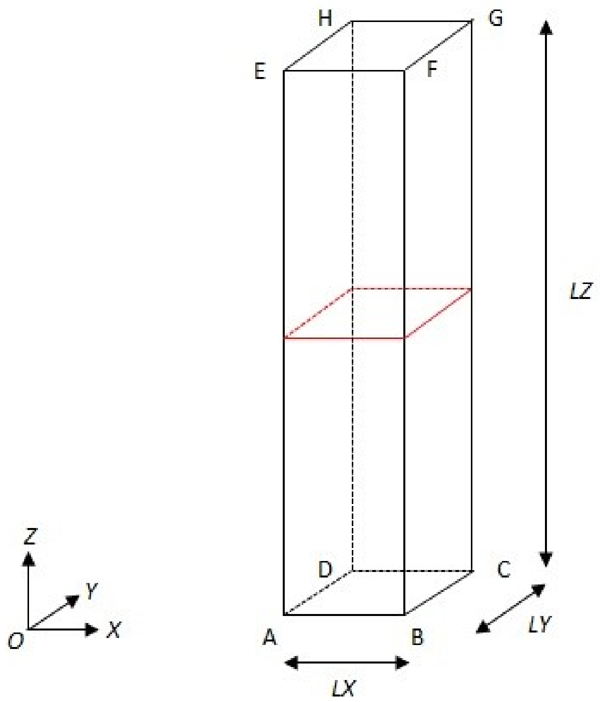


Figure 1.1-a: Geometry of the problem

1.2 Properties materials

Parameters given in the Table 1.2-1 , correspond to the parameters used for modeling in the hydraulic coupled case. The mixing rate used is `'LIQU_SATU'` . The parameters specific to this mixing rate are given but do not have any influence on the solution (because we chose to measure a pressure of pore uniformly worthless in all the field). Only the elastic parameters affect the solution of the pseudo-coupled problem.

Liquid (water)	Viscosity $\mu_w(en\ Pa.s)$	10^{-3}
	Module of compressibility $\frac{1}{K_w}(en\ Pa^{-1})$	5.10^{-10}
	Density of the liquid $\rho_w(en\ kg/m^3)$	1
Elastic parameters	Young modulus $E(en\ MPa)$	5800
	Poisson's ratio ν	0
	Thermal dilation coefficient $\alpha(en\ K^{-1})$	0
Parameters of coupling	Coefficient of Biot b	1
	Initial homogenized density $r_0(en\ kg/m^3)$	2,5
	Intrinsic permeability $K^{int}(en\ m^2/s)$	$1{,}01937^{-19}$

Table 1.2-1 : Properties of material

In addition the forces related to gravity (in the conservation equation of the momentum) are neglected. The pressure of pore of reference is taken worthless $p_1^{ref}=0\ MPa$ and the porosity of material is $\varphi = 0{,}15$.

One takes $\nu=0$ in order to have a unidimensional problem according to the direction y .

1.3 Boundary conditions

The boundary conditions which one can apply to the field are of two types:
- conditions of the Dirichlet type,
- conditions of the Neuman type.

The conditions of Dirichlet are:
- on [ABCD] and [EFGH] displacements are blocked in all the directions ($u_x=0$, $u_y=0$ and $u_z=0$),
- in all the field the pressure of pore is worthless $p_1=0$, as well as the degree of freedom enriched associated with this pressure by pore $Hp_1=0$.

The conditions of Neuman are:
- on [ABCD] and [EFGH] mass water flows are worthless $\boldsymbol{M}.\boldsymbol{n}=0$,
- on each lip of the interface one imposes a pressure distributed uniform $p=10\ MPa$ via `AFFE_CHAR_MECA` and of the keyword `CRACK` of `PRES_REP`.

Gravity:
- to the whole of the field, one applies the voluminal loading of gravity of intensity $g=9{,}81 m.s^{-2}$ via `AFFE_CHAR_MECA` with the keyword `CHAR_MECA_PESA_R`.

2 Reference solution

2.1 Method of calculating

It is about an analytical solution. Taking into account the boundary conditions, displacements can be obtained starting from the analytical resolution of the conservation equation of the momentum.

By neglecting gravity, the equation is written (in total constraints):

$$\mathbf{Div}(\boldsymbol{\sigma})+r_0\boldsymbol{g}=\mathbf{0}$$

In the case of a coupled modeling, the tensor of the total constraints is written:

$$\boldsymbol{\sigma}=\boldsymbol{\sigma}'-p_1\mathbf{1}$$

$\boldsymbol{\sigma}'$ is the tensor of the constraints in the skeleton and p_1 and pressure of pore in the solid mass. The Fish module ν being null, and being in the elastic case, one has $\boldsymbol{\sigma}'=E\boldsymbol{\epsilon}$.

However $\forall y\ p_1=0$ thus finally $E*\mathbf{Div}(\boldsymbol{\epsilon})+r_0\boldsymbol{g}=\mathbf{0}$

In the case 2D, ν being null, the boundary conditions and the loading make the problem unidimensional according to y. Only ϵ_{yy} is nonnull and:

$$E*\frac{\partial\epsilon_{yy}}{\partial y}-r_0 g=0 \quad \text{that is to say} \quad \frac{\partial^2 u_y}{\partial y^2}=\frac{r_0}{E}g \quad \text{and thus finally} \quad u_y(y)=\frac{r_0 g}{2E}y^2+Ay+B$$

with A and B constants of integration to be determined. One separately solves the problem in each of the two under-blocks. In order to determine the constants of integration, one uses the boundary condition on displacements $u_y=0$ with the lower and higher ends of the column and the condition of Neumann $\sigma_{yy}=-p$ on the level of the interface.
Finally, one obtains displacement according to the direction y on both sides of the interface:

- displacements with the top of the interface in $y=\frac{LY^+}{2}$ are written $u_y\left(\frac{LY^+}{2}\right)=\frac{p}{E}\left(\frac{LY}{2}\right)-\frac{r_0 g}{2E}\left(\frac{LY}{2}\right)^2$
- displacements below the interface in $y=\frac{LY^-}{2}$ are written $u_y\left(\frac{LY^-}{2}\right)=-\frac{p}{E}\left(\frac{LY}{2}\right)-\frac{r_0 g}{2E}\left(\frac{LY}{2}\right)^2$

In the case 3D, the reference solution is rigorously the same one:

- displacements with the top of the interface in $z=\frac{LZ^+}{2}$ are written $u_z\left(\frac{LZ^+}{2}\right)=\frac{p}{E}\left(\frac{LZ}{2}\right)-\frac{r_0 g}{2E}\left(\frac{LZ}{2}\right)^2$

- displacements below the interface in $z=\frac{LZ^-}{2}$ are written $u_z\left(\frac{LZ^-}{2}\right)=-\frac{p}{E}\left(\frac{LZ}{2}\right)-\frac{r_0 g}{2E}\left(\frac{LZ}{2}\right)^2$
- displacements according to x and y are worthless.

2.2 Sizes and results of reference

One tests the value of displacements on the lips lower and higher of the interface:

	Vertical displacements	Horizontal displacements
Lower lip	-4.323558729E-03	0
Upper lip	4.297130927-03	0

2.3 Uncertainties on the solution

The no solution is analytical.

2.4 Bibliographical references

1 Reference material R7.02.12 (eXtended Finite Method Element).

3 Modeling A

3.1 Characteristics of modeling

It is about a modeling D_PLAN_HM using quadratic elements HM-XFEM. The bar on which one carries out modeling is divided into 5 QUAD8. The interface is nonwith a grid and cuts the central element. Thus there are 3 classical elements HM-XFEM and 2 elements HM. As indicated on the Figure 3.1-a, 3 elements XFEM undergo under cutting under triangles (to carry out the integration of Gauss-Legendre on both sides of lips of the interface, but these triangular subelements are not elements of the grid).

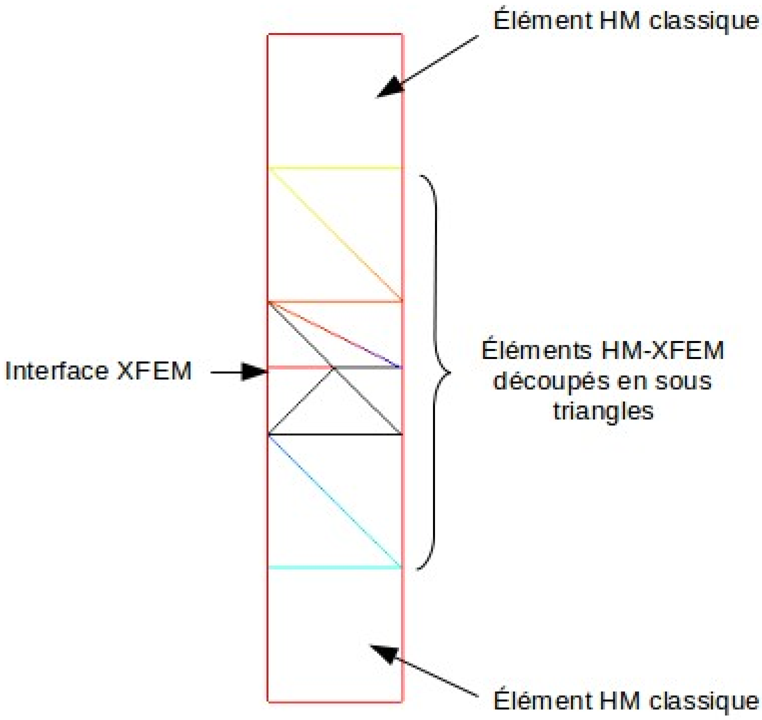


Figure 3.1-a: Characteristics of modeling

3.2 Characteristics of the grid

The grid consists of 5 meshs quadratic quadrangles (QUAD8).

3.3 Sizes tested and results

The results (resolution with STAT_NON_LINE) are synthesized in the table below for the direction y . To test all the nodes of the bar at the same time, it is calculated MIN and it MAX.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DY (in lower part) MIN	'ANALYTICAL'	-4.323558729E-03	0.0001
DY (in lower part) MAX	'ANALYTICAL'	-4.323558729E-03	0.0001
DY (with the top) MIN	'ANALYTICAL'	4.297130927-03	0.0001
DY (with the top) MAX	'ANALYTICAL'	4.297130927-03	0.0001

The results (resolution with STAT_NON_LINE) are synthesized in the table below for the direction x . To test all the nodes of the bar at the same time, it is calculated MIN and it MAX.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DX (in lower part) MIN	'ANALYTICAL'	0	0.0001
DX (in lower part) MAX	'ANALYTICAL'	0	0.0001
DX (with the top) MIN	'ANALYTICAL'	0	0.0001
DX (with the top) MAX	'ANALYTICAL'	0	0.0001

One can then note (starting from the Figure 3.3-a) a frank discontinuity of the field of displacements related to the presence of the interface crossing the solid mass. That suggests the good taking into account of enrichment in the approximation of the field of displacements by the Heaviside function.

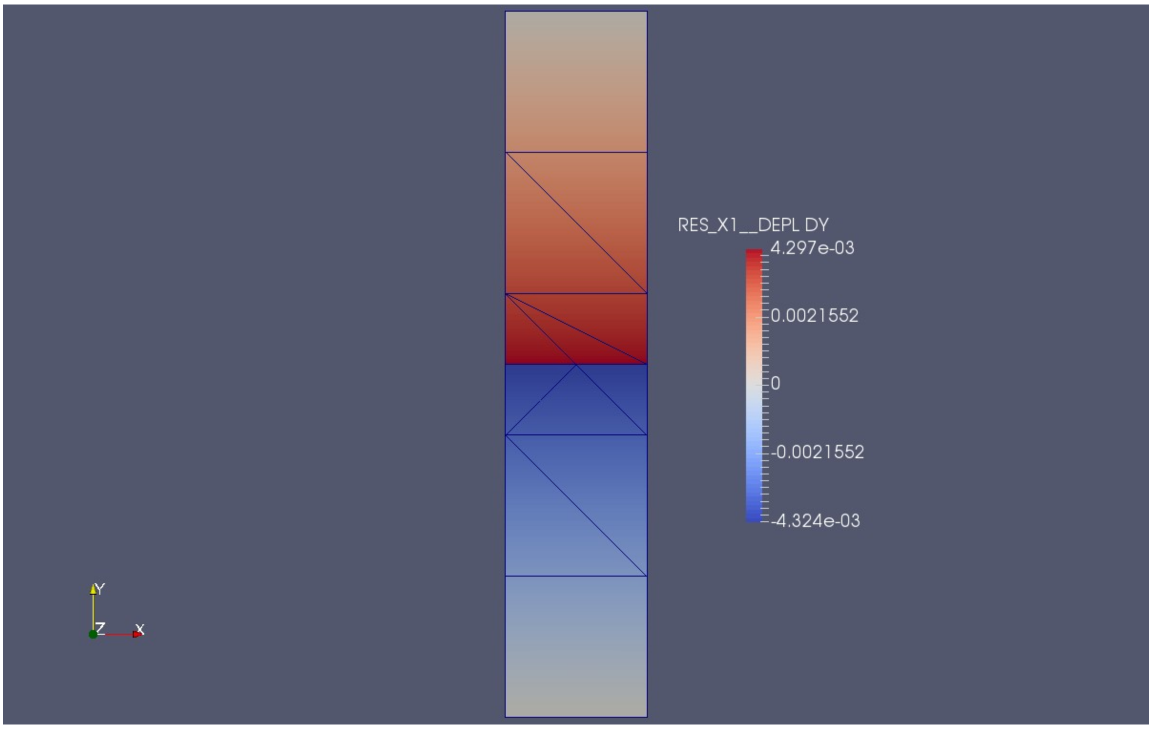


Figure 3.3-a: Field of displacements according to direction (OY)

One carries out then the second modeling which uses the same parameters as the preceding one, but this time Ci in carrying out the calculation XFEM (which is then a calculation of pure mechanics) with the operator MECA_STATIQUE instead of the operator STAT_NON_LINE in case HM-XFEM. The got results are strictly identical to the precedents.

4 Modeling B

4.1 Characteristics of modeling

This modeling is strictly identical to the preceding one. In order to test that model HM-XFEM is functional with TRIA6, we choose to subdivide the 5 QUAD8 preceding modeling, each one in 2 TRIA6. What makes 10 quadratic triangular elements on the whole. As previously the triangular elements contained in the extreme quadrangles are not nouveau riches. *A contrario* those contained in the 3 central quadrangles are enriched by the Heaviside function.

4.2 Characteristics of the grid

The grid consists of 10 quadratic triangular meshs (TRIA6).

4.3 Sizes tested and results

The results (resolution with STAT_NON_LINE) are synthesized in the table below according to the direction y . The tolerance is fixed at 10^{-6} . To test all the nodes of the bar at the same time, it is calculated MIN and it MAX.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DY (in lower part) MIN	'ANALYTICAL'	-4.323558729E-03	0.0001
DY (in lower part) MAX	'ANALYTICAL'	-4.323558729E-03	0.0001
DY (with the top) MIN	'ANALYTICAL'	4. 297130927 -03	0.0001
DY (with the top) MAX	'ANALYTICAL'	4. 297130927 -03	0.0001

The results (resolution with STAT_NON_LINE) are synthesized in the table below according to the direction x . To test all the nodes of the bar at the same time, it is calculated MIN and it MAX.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DX (in lower part) MIN	'ANALYTICAL'	0	0.0001
DX (in lower part) MAX	'ANALYTICAL'	0	0.0001
DX (with the top) MIN	'ANALYTICAL'	0	0.0001
DX (with the top) MAX	'ANALYTICAL'	0	0.0001

One can then note, as in the case of modeling A, a frank discontinuity of the field of displacement related to the presence of the interface crossing the solid mass. That suggests the good taking into account of the enrichment of the approximation of the field of displacements by the Heaviside function.

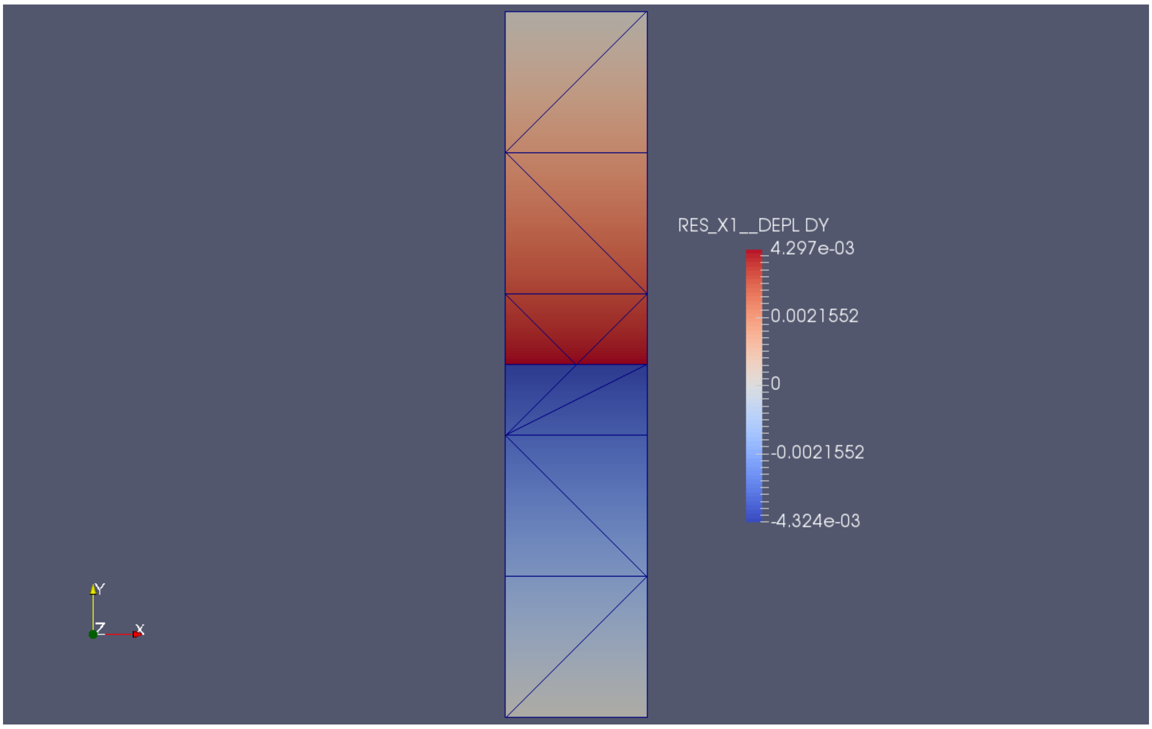


Figure 4.3-a : Field of displacements according to direction (OY)

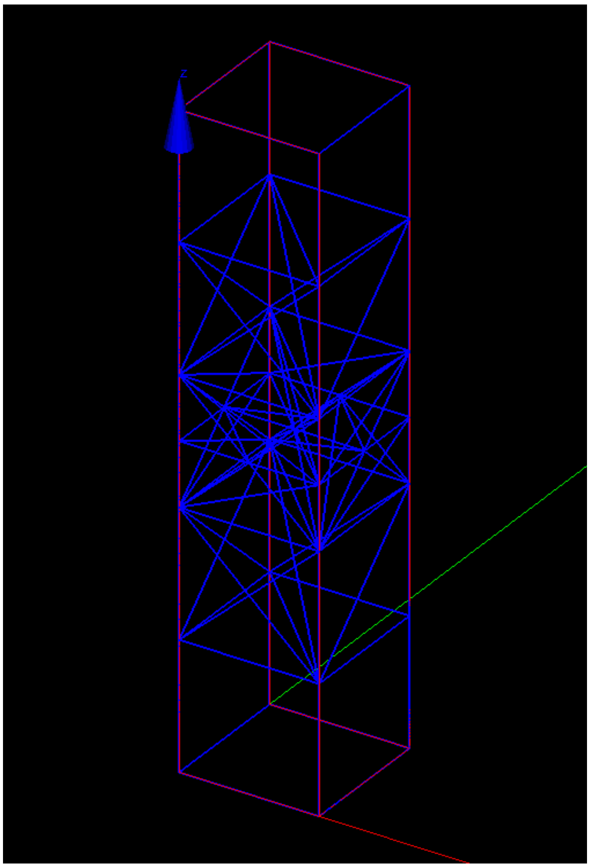
One carries out then the second modeling which uses the same parameters as the preceding one, but this time Ci by leading the calculation XFEM (which is then a pure calculation of mechanics) with the operator `MECA_STATIQUE` instead of the operator `STAT_NON_LINE` in case HM-XFEM. The got results are strictly identical to the precedents.

5 Modeling C

5.1 Characteristics of modeling

It is about a modeling 3D_HM using quadratic elements HM-XFEM. The column on which one carries out modeling is divided into 5 HEXA20. The interface is nonwith a grid and cuts the central element. Thus there are 3 classical elements HM-XFEM and 2 elements HM (two hexahedrons which form the ends of the column). As indicated on the Figure 5.1-a, 3 elements XFEM undergo under cutting under tetrahedrons (to carry out the integration of Gauss-Legendre on both sides of lips of the interface, but these subelements tetrahedrons are not elements of the grid).

Figure 5.1-a: Characteristics of modeling



5.2 Characteristics of the grid

The grid consists of 5 meshs hexahedral quadratic (HEXA20).

5.3 Sizes tested and results

The results (resolution with STAT_NON_LINE) are synthesized in the tables below. To test all the nodes of the column at the same time, it is calculated MIN and it MAX.

Sizes tested	Type of reference	Value of reference	Tolerance (%)

DZ (in lower part) MIN	'ANALYTICAL'	-4.323558729E-03	0.0001
DZ (in lower part) MAX	'ANALYTICAL'	-4.323558729E-03	0.0001
DZ (with the top) MIN	'ANALYTICAL'	4. 297130927 -03	0.0001
DZ (with the top) MAX	'ANALYTICAL'	4. 297130927 -03	0.0001

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DX (in lower part) MIN	'ANALYTICAL'	0	0.0001
DX (in lower part) MAX	'ANALYTICAL'	0	0.0001
DX (with the top) MIN	'ANALYTICAL'	0	0.0001
DX (with the top) MAX	'ANALYTICAL'	0	0.0001

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DY (in lower part) MIN	'ANALYTICAL'	0	0.0001
DY (in lower part) MAX	'ANALYTICAL'	0	0.0001
DY (with the top) MIN	'ANALYTICAL'	0	0.0001
DY (with the top) MAX	'ANALYTICAL'	0	0.0001

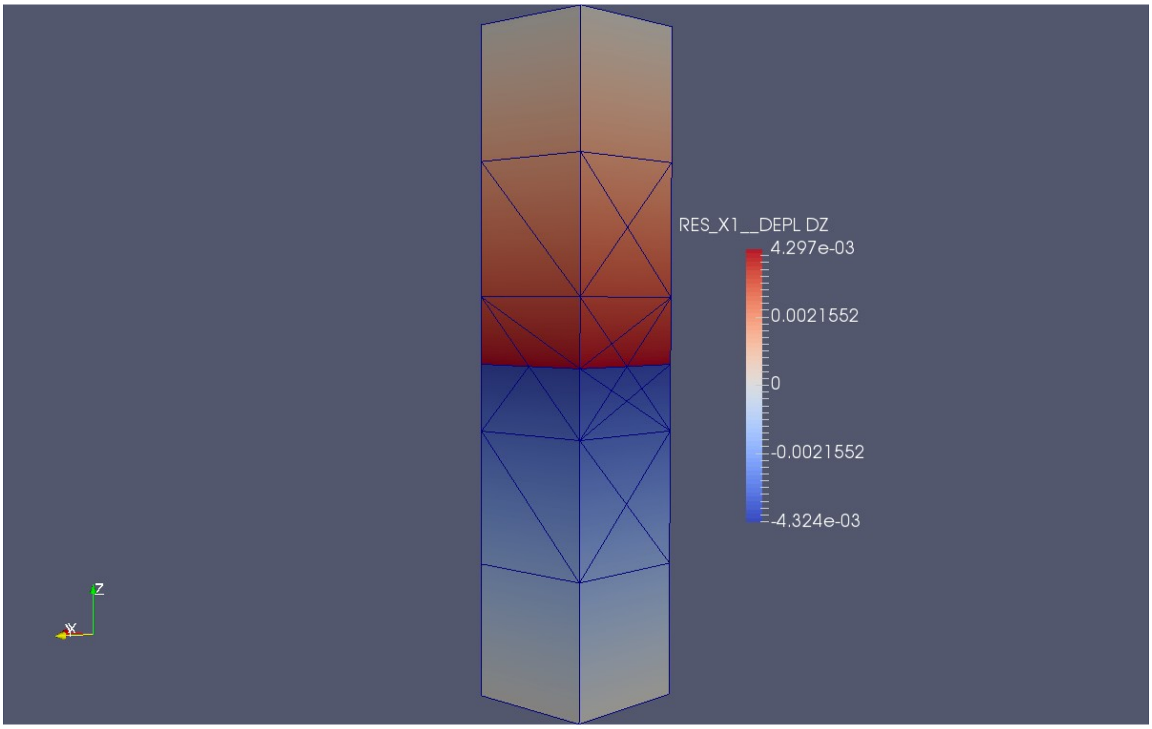


Figure 5.3-a: Field of displacements according to direction (OZ)

One carries out then the second modeling which uses the same parameters as the preceding one, but this time Ci by leading the calculation XFEM (which is then a pure calculation of mechanics) with the operator MECA_STATIQUE instead of the operator STAT_NON_LINE in case HM-XFEM. The got results are strictly identical to the precedents.

6 Modeling D

6.1 Characteristics of modeling

This modeling is strictly identical to the preceding one. In order to test that model HM-XFEM is functional with PENTA15, we choose to subdivide the 5 HEXA20 preceding modeling, each one in 2 PENTA15. What makes 10 quadratic pentahedral elements on the whole. As previously the pentahedral elements contained in the extreme hexahedrons are not nouveau riches. *A contrario* those contained in the 3 central hexahedrons are enriched by the Heaviside function.

6.2 Characteristics of the grid

The grid consists of 10 quadratic pentaedric meshs (PENTA15).

6.3 Sizes tested and results

The results (resolution with STAT_NON_LINE) are synthesized in the tables below. To test all the nodes of the column at the same time, it is calculated MIN and it MAX.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DZ (in lower part) MIN	'ANALYTICAL'	-4.323558729E-03	0.0001
DZ (in lower part) MAX	'ANALYTICAL'	-4.323558729E-03	0.0001
DZ (with the top) MIN	'ANALYTICAL'	4. 297130927 -03	0.0001
DZ (with the top) MAX	'ANALYTICAL'	4. 297130927 -03	0.0001

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DX (in lower part) MIN	'ANALYTICAL'	0	0.0001
DX (in lower part) MAX	'ANALYTICAL'	0	0.0001
DX (with the top) MIN	'ANALYTICAL'	0	0.0001
DX (with the top) MAX	'ANALYTICAL'	0	0.0001

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DY (in lower part) MIN	'ANALYTICAL'	0	0.0001
DY (in lower part) MAX	'ANALYTICAL'	0	0.0001
DY (with the top) MIN	'ANALYTICAL'	0	0.0001
DY (with the top) MAX	'ANALYTICAL'	0	0.0001

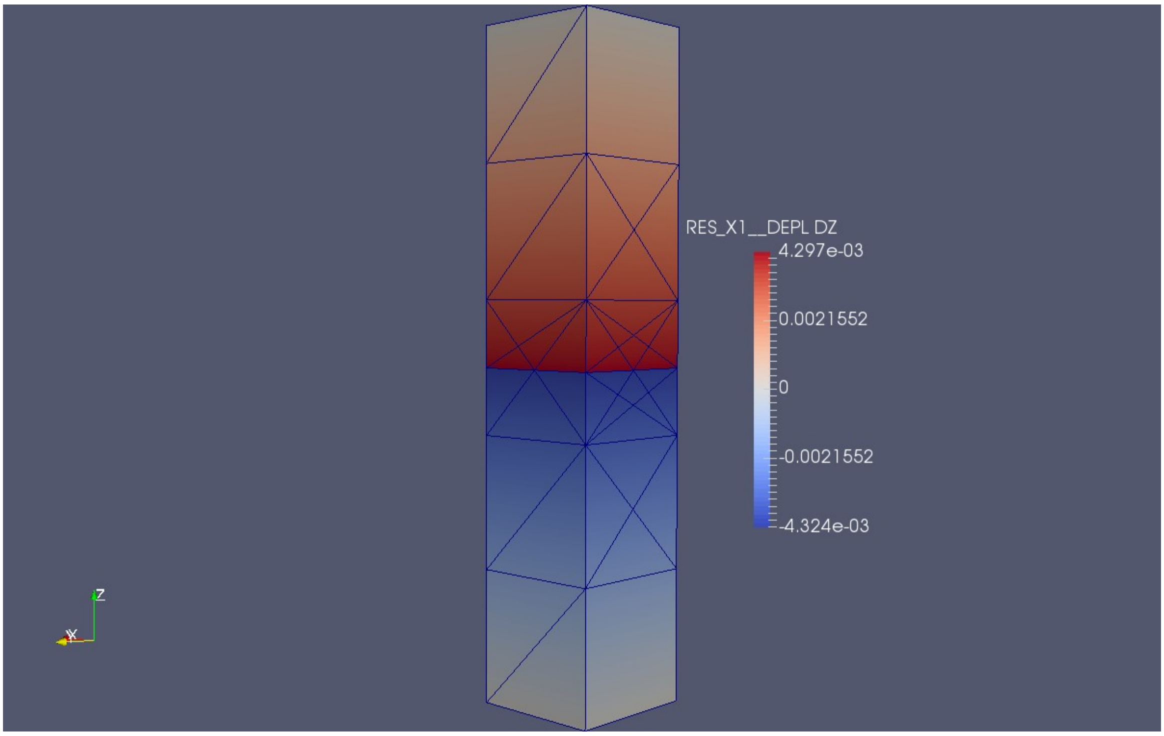


Figure 6.3-a : Field of displacements according to direction (OZ)

One carries out then the second modeling which uses the same parameters as the preceding one, but this time Ci by leading the calculation XFEM (which is then a pure calculation of mechanics) with the operator MECA_STATIQUE instead of the operator STAT_NON_LINE in case HM-XFEM. The got results are strictly identical to the precedents.

7 Modeling E

7.1 Characteristics of modeling

This modeling is strictly identical to the preceding ones. In order to test that model HM-XFEM is functional with TETRA10, we choose to subdivide the 5 HEXA20 modeling C, each one in 6 TETRA10. What makes 30 elements quadratic tetrahedrons on the whole. As previously the elements tetrahedrons contained in the extreme hexahedrons are not nouveau riches. *A contrario* those contained in the 3 central hexahedrons are enriched by the Heaviside function.

7.2 Characteristics of the grid

The grid consists of 30 quadratic tetrahedral meshs (TETRA10).

7.3 Sizes tested and results

The results (resolution with STAT_NON_LINE) are synthesized in the tables below. To test all the nodes of the column at the same time, it is calculated MIN and it MAX.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DZ (in lower part) MIN	'ANALYTICAL'	-4.323558729E-03	0.0001
DZ (in lower part) MAX	'ANALYTICAL'	-4.323558729E-03	0.0001
DZ (with the top) MIN	'ANALYTICAL'	4. 297130927 -03	0.0001
DZ (with the top) MAX	'ANALYTICAL'	4. 297130927 -03	0.0001

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DX (in lower part) MIN	'ANALYTICAL'	0	0.0001
DX (in lower part) MAX	'ANALYTICAL'	0	0.0001
DX (with the top) MIN	'ANALYTICAL'	0	0.0001
DX (with the top) MAX	'ANALYTICAL'	0	0.0001

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DY (in lower part) MIN	'ANALYTICAL'	0	0.0001
DY (in lower part) MAX	'ANALYTICAL'	0	0.0001
DY (with the top) MIN	'ANALYTICAL'	0	0.0001
DY (with the top) MAX	'ANALYTICAL'	0	0.0001

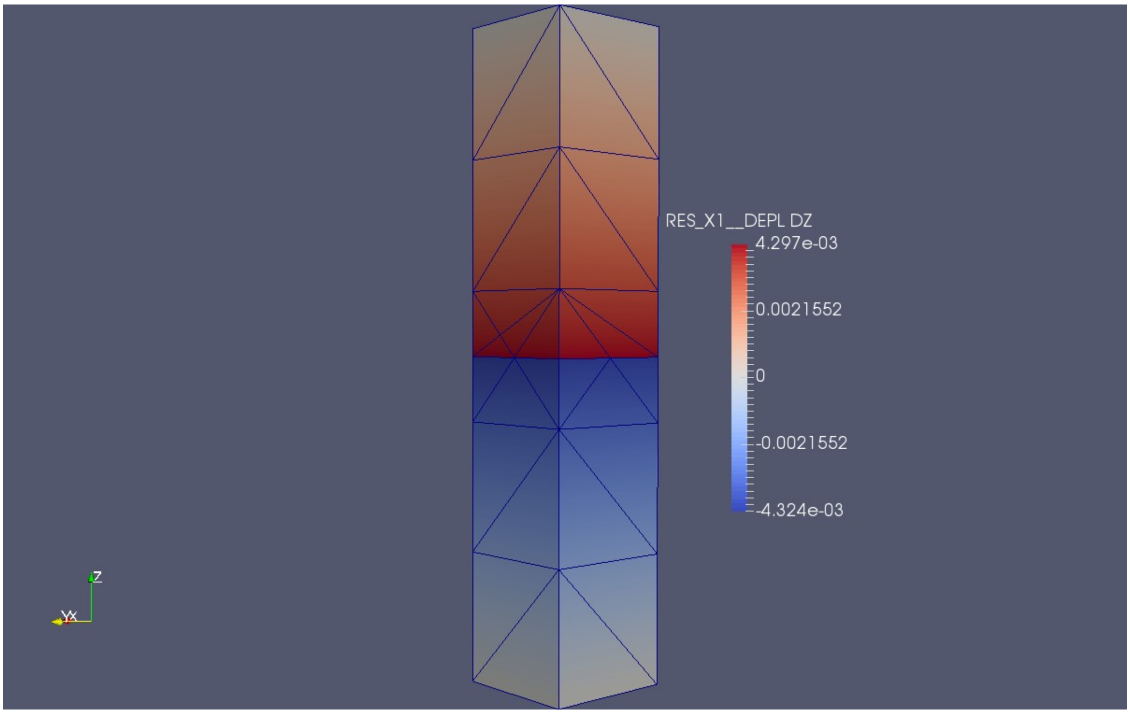


Figure 7.3-a : Field of displacements according to direction (OZ)

One carries out then the second modeling which uses the same parameters as the preceding one, but this time Ci by leading the calculation XFEM (which is then a pure calculation of mechanics) with the operator `MECA_STATIQUE` instead of the operator `STAT_NON_LINE` in case HM-XFEM. The got results are strictly identical to the precedents.

8 Modeling F

8.1 Characteristics of modeling

This modeling is strictly identical to the preceding ones. In order to test that model HM-XFEM is functional with PYRAM13, we choose to subdivide the 5 HEXA20 modeling C, each one in 6 PYRAM13. What makes 30 quadratic pyramidal elements on the whole. As previously the pyramidal elements contained in the extreme hexahedrons are not nouveau riches. *A contrario* those contained in the 3 central hexahedrons are enriched by the Heaviside function.

8.2 Characteristics of the grid

The grid consists of 30 quadratic pyramidal meshs (PYRAM13).

8.3 Sizes tested and results

Results got with *Code_Aster* (resolution with STAT_NON_LINE) are synthesized in the tables below. To test all the nodes of the column at the same time, it is calculated MIN and it MAX.

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DZ (in lower part) MIN	'ANALYTICAL'	-4.323558729E-03	0.1
DZ (in lower part) MAX	'ANALYTICAL'	-4.323558729E-03	0.1
DZ (with the top) MIN	'ANALYTICAL'	4.297130927-03	0.1
DZ (with the top) MAX	'ANALYTICAL'	4.297130927-03	0.1

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DX (in lower part) MIN	'ANALYTICAL'	0	0.0001
DX (in lower part) MAX	'ANALYTICAL'	0	0.0001
DX (with the top) MIN	'ANALYTICAL'	0	0.0001
DX (with the top) MAX	'ANALYTICAL'	0	0.0001

Sizes tested	Type of reference	Value of reference	Tolerance (%)
DY (in lower part) MIN	'ANALYTICAL'	0	0.0001
DY (in lower part) MAX	'ANALYTICAL'	0	0.0001
DY (with the top) MIN	'ANALYTICAL'	0	0.0001
DY (with the top) MAX	'ANALYTICAL'	0	0.0001

Note:

The tolerance for the test on DZ is raised, one passes from 10^{-6} *with* 10^{-3} *. Indeed, the functions of form of the pyramids are not polynomials but rational fractions. That explains the least good precision obtained for this linear problem compared to preceding modelings.*

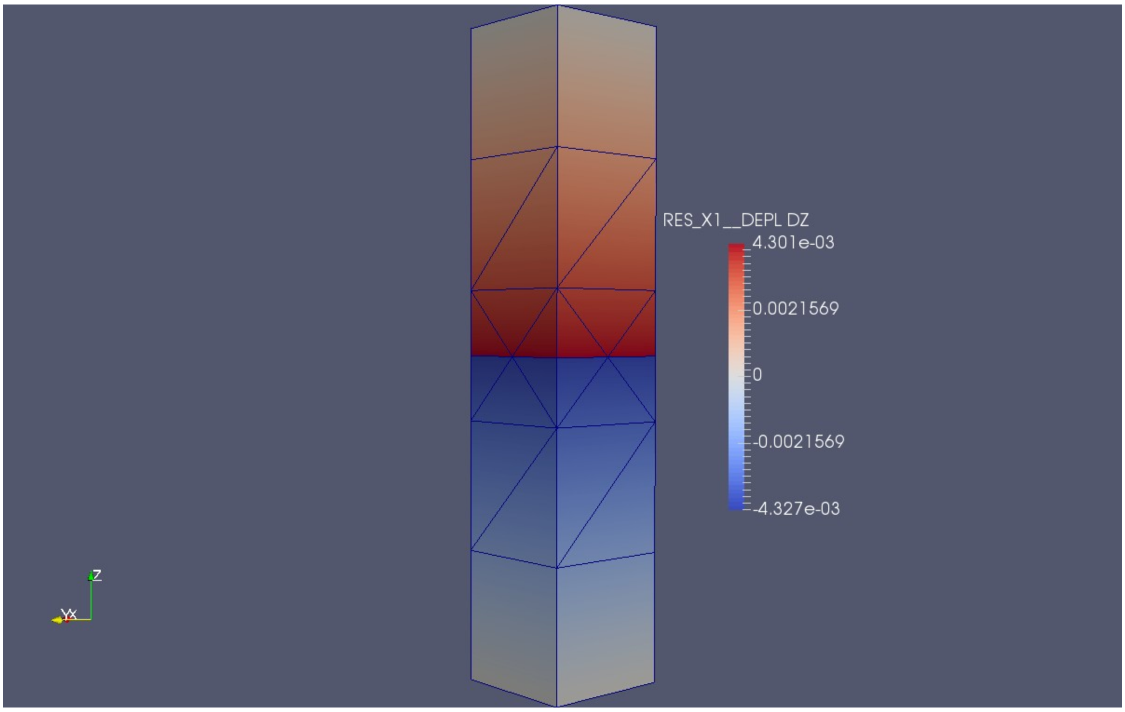


Figure 8.3-a : Field of displacements according to direction (OZ)

One carries out then the second modeling which uses the same parameters as the preceding one, but this time Ci by leading the calculation XFEM (which is then a pure calculation of mechanics) with the operator `MECA_STATIQUE` instead of the operator `STAT_NON_LINE` in case HM-XFEM. The got results are strictly identical to the precedents.

9 Summary of the results

For each finite element HM-XFEM tested, the results agree with the analytical solution. For the elements HM-XFEM, the features following from now on are validated:

- MODI_MODELE_XFEM ,
- application of a pressure distributed,
- POST_CHAM_XFEM,
- the calculation of the matrices and elementary vectors if the degrees of freedom Heaviside mechanics were introduced into the developments of the model of coupling HM.