

## WTNV145 - Application of a pressure distributed on the lips of a junction of crack XFEM for the hydraulic case

---

### Summary:

It is a question of a first test of validation in order to make sure of the good performance of the finite element method extended in the model of coupling HM in fractured saturated medium. In this test we seek to check if the pressure of pore is taken uniformly worthless in all the field of study (*i.e* in order to place itself in a mechanical case "classical" without coupling), which there exists well a discontinuity of the field of displacements on both sides of the interface. The results are compared with an analytical solution and the results got with a mechanical test XFEM (similar to the tests ssnv203, modelings F and H).

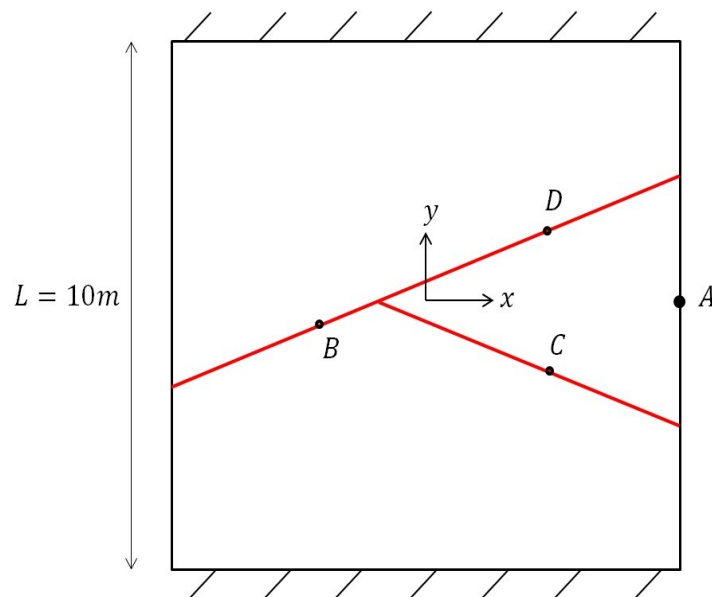
## 1 Problem of reference

### 1.1 Geometry of the problem 2D (modeling A)

It is about a square block on side  $L=10\text{ m}$ . This block presents two discontinuity discontinuities of the type interfaces (interface nonwith a grid which is introduced into the model via the level-sets thanks to the operator `DEFI_FISS_XFEM`). First is located by the level-set normal of equation  $lsn_1=Y-0.5X-0.2$  and entirely the block in the horizontal direction crosses. The second interface is located by the level-set normal of equation  $lsn_2=Y+0.5X+0.2$ . It connects on the lower lip of the first interface. The second interface thus exists only in the part of the block such as  $lsn_1 < 0$ . The junction point between the two interfaces checks  $lsn_1=lsn_2=0$  and has as coordinates  $(-0.4, 0)$ . The field is thus cut out in 3 blocks, a lower block, a higher block and an intermediate block located between the two interfaces. Points  $A(5,0)$ ,  $B(-3,-1.3)$ ,  $C(3,-1.7)$  and  $D(3,1.7)$  will be used for the application of the boundary conditions and the evaluation of the sizes tested.

One represents on the Figure 1.1-a geometry of the block.

Figure 1.1-a: Geometry of the problem 2D



### 1.2 Geometry of the 3D problem (modeling B)

It is about a block height  $LZ=10\text{ m}$ , length  $LX=10\text{ m}$  and of width  $LY=2\text{ m}$ . This block presents two discontinuity discontinuities of the type interfaces (interface nonwith a grid which is introduced into the model via the level-sets thanks to the operator `DEFI_FISS_XFEM`). First is located by the level-set normal of equation  $lsn_1=Z-0.5X-0.2$  and entirely the block in the horizontal direction crosses. The second interface is located by the level-set normal of equation  $lsn_2=Z+0.5X+0.2$ . It connects on the lower lip of the first interface. The second interface thus exists only in the part of the block such as  $lsn_1 < 0$ . The curve of junction between the two

interfaces checks  $l_{sn_1} = l_{sn_2} = 0$  and has as an equation  $\begin{cases} X = -0.4 \\ Z = 0 \end{cases}$ . The field is thus cut out in 3 blocks, a lower block, a higher block and an intermediate block located between the two interfaces. Points  $A_1(5, -1, 0)$ ,  $A_2(5, 1, 0)$ ,  $B(-3, -1, -1.3)$ ,  $C(3, -1, -1.7)$  and  $D(3, -1, 1.7)$  will be used for the application of the boundary conditions and the evaluation of the sizes tested.

One represents on the Figure 1.2-a geometry of the block.

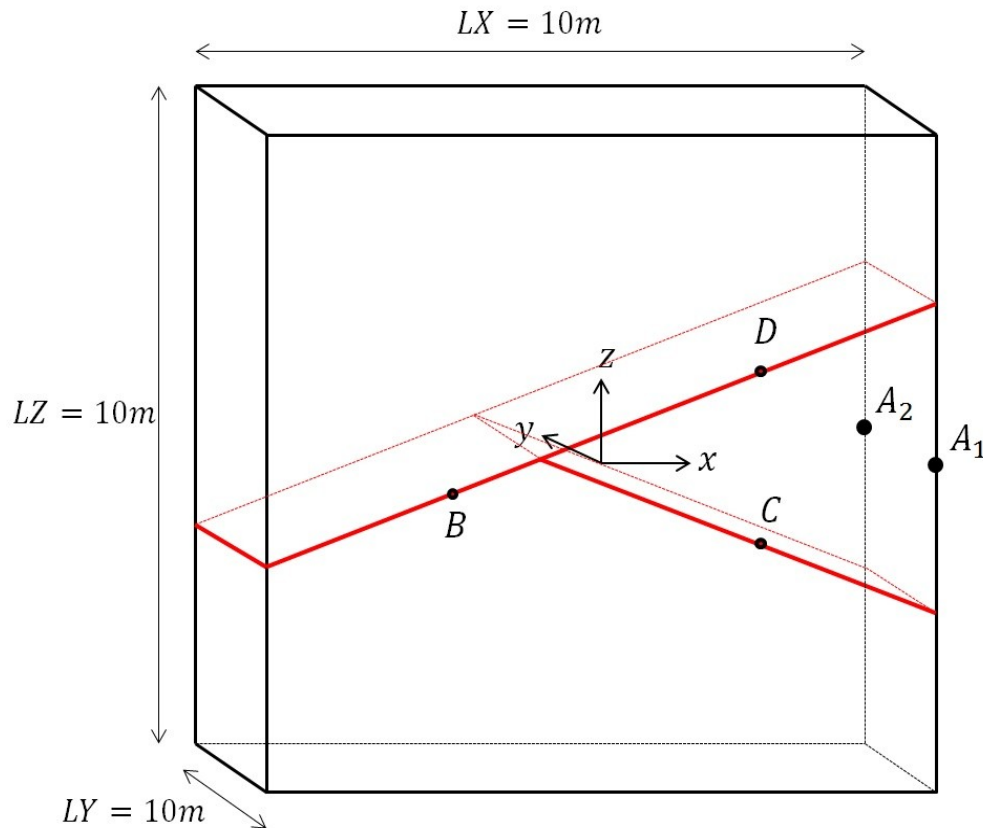


Figure 1.2-a: Geometry of the problem 3D

## 1.3 Properties materials

Parameters given in the Table 1.3-1, correspond to the parameters used for modeling in the hydro-mechanical coupled case. The mixing rate used is 'LIQU\_SATU'.

Liquid (water)	Viscosity $\mu_w$ (en Pa.s)	$10^{-3}$
	Module of compressibility $\frac{1}{K_w}$ (en Pa <sup>-1</sup> )	$5.10^{-10}$
	Density of the liquid $\rho_w$ (en kg/m <sup>3</sup> )	1

Elastic parameters	Young modulus $E(en MPa)$	5800
	Poisson's ratio $\nu$	0
	Thermal dilation coefficient $\alpha(en K^{-1})$	0
Parameters of coupling	Coefficient of Biot $b$	1
	Initial homogenized density $r_0(en kg/m^3)$	2,5
	Intrinsic permeability $K^{int}(en m^2/s)$	1,01937 <sup>-19</sup>

**Table 1.3-1 : Properties of material**

In addition the forces related to gravity (in the conservation equation of the momentum) are neglected. The pressure of pore of reference is taken worthless  $p_1^{ref}=0 MPa$  and the porosity of material is  $\varphi = 0,15$ .

One takes  $\nu=0$  in order to have a unidimensional problem.

## 1.4 Boundary conditions and loadings

### Case 2D

The boundary conditions which one can apply to the field are of two types:

- conditions of the Dirichlet type,
- conditions of the Neuman type.

The conditions of Dirichlet are:

- following displacements  $x$  are blocked in all the field. The problem is thus one-way according to  $y$ ,
- displacements according to  $y$  are blocked on the lower face and the higher face of the block,
- in order to block displacements of rigid body of the block located between the two interfaces, the point is embedded  $A$ .

Conditions of Neuman are:

- a mechanical pressure distributed constant is applied  $P=10 MPa$  on each lip of the interfaces,
- the pressure of pore is fixed at  $p_1=0.2 MPa$  in the lower block,
- the pressure of pore is fixed at  $p_2=0.4 MPa$  in the intermediate block located between the two interfaces,
- the pressure of pore is fixed at  $p_3=0.6 MPa$  in the higher block.

### Case 3D

The boundary conditions which one can apply to the field are of two types:

- conditions of the Dirichlet type,
- conditions of the Neuman type.

The conditions of Dirichlet are:

- following displacements  $x$  and displacements according to  $y$  are blocked in all the field. The problem is thus one-way according to  $z$ ,
- displacements according to  $z$  are blocked on the lower face and the higher face of the block,
- in order to block displacements of rigid body of the block located between the two interfaces, the points are embedded  $A_1$  and  $A_2$ .

The conditions of Neuman are:

- a mechanical pressure distributed constant is applied  $P=10\text{ MPa}$  on each lip of the interfaces,
- the pressure of pore is fixed at  $p_1=0.2\text{ MPa}$  in the lower block,
- the pressure of pore is fixed at  $p_2=0.4\text{ MPa}$  in the intermediate block located between the two interfaces,
- the pressure of pore is fixed at  $p_3=0.6\text{ MPa}$  in the higher block.

## 2 Reference solution

### 2.1 Method of calculating

Taking into account the boundary conditions, displacements can be obtained starting from the analytical resolution of the conservation equation of the momentum.

By neglecting gravity, the equation is written (in total constraints):

$$\text{Div}(\boldsymbol{\sigma}) = \mathbf{0}$$

In the case of a coupled modeling, the tensor of the total constraints is written:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' - bp\mathbf{1}$$

$\boldsymbol{\sigma}'$  is the tensor of the constraints in the skeleton,  $b$  the coefficient of Biot and  $p$  and pressure of pore in the solid mass. The Fish module  $\nu$  being null, and being in the elastic case, one has  $\boldsymbol{\sigma}' = E \boldsymbol{\epsilon}$ .

$\nu$  being null, the boundary conditions and the loading make the problem unidimensional according to  $y$ . Only  $\epsilon_{yy}$  is nonnull and thus in each block:

$$\boldsymbol{\sigma} = E \epsilon_{yy} \mathbf{e}_y \otimes \mathbf{e}_y - bp_i \mathbf{1}$$

with  $p_i$  pressure of pore imposed in the current block.

Thus in each block, the tensor of the total constraints is written in the form:

$$\boldsymbol{\sigma} = \sigma_{xx} \mathbf{e}_x \otimes \mathbf{e}_x + \sigma_{yy} \mathbf{e}_y \otimes \mathbf{e}_y$$

However in each block, the boundary conditions on the level as of lips of the interfaces which delimit the blocks are written  $\boldsymbol{\sigma} \cdot \mathbf{n} = -P \mathbf{n}$ , therefore:

$$\begin{aligned} (\sigma_{xx} \mathbf{e}_x \otimes \mathbf{e}_x + \sigma_{yy} \mathbf{e}_y \otimes \mathbf{e}_y) \cdot \mathbf{n} &= -P \mathbf{n} \\ \sigma_{xx} (\mathbf{e}_x \cdot \mathbf{n}) \mathbf{e}_x + \sigma_{yy} (\mathbf{e}_y \cdot \mathbf{n}) \mathbf{e}_y &= -P \mathbf{n} \\ (\sigma_{xx} (\mathbf{e}_x \cdot \mathbf{n}) \mathbf{e}_x + \sigma_{yy} (\mathbf{e}_y \cdot \mathbf{n}) \mathbf{e}_y) \cdot \mathbf{e}_y &= -P (\mathbf{e}_y \cdot \mathbf{n}) \\ \sigma_{yy} (\mathbf{e}_y \cdot \mathbf{n}) &= -P (\mathbf{e}_y \cdot \mathbf{n}) \\ \sigma_{yy} &= -P \quad \text{car } \mathbf{e}_y \cdot \mathbf{n} \neq 0 \end{aligned}$$

Finally in each block

$$\sigma_{yy} = E \epsilon_{yy} - bp_i = -P \quad \text{that is to say} \quad \epsilon_{yy} = \frac{-P + bp_i}{E}$$

with  $p_i$  pressure of pore imposed in the current block.

### 2.2 Sizes and results of reference

In particular one is interested in displacements according to the direction  $y$  in each block:

- in the lower block, according to the boundary conditions,  $u_y(y) = \frac{-P+bp_1}{E} * (\frac{L}{2} + y)$
- in the lower block, displacements are symmetrical compared to the axis  $(Ox)$ . Indeed, this block is symmetrical compared to the axis  $(Ox)$  and the boundary conditions and the loadings which are applied to him also follow this symmetry (mechanical pressure distributed on the lips of crack and embedding of the point  $A$ ). Vertical displacements are thus written:  $u_y(y) = \frac{-P+bp_2}{E} * y$
- in the higher block, according to the boundary conditions,  $u_y(y) = \frac{-P+bp_3}{E} * (\frac{-L}{2} + y)$

## 2.3 Uncertainty on the solution

No, it is about an analytical solution.

## 3 Modeling A

### 3.1 Characteristics of modeling

It is about a modeling `D_PLAN_HM` using quadratic elements HM-XFEM.

### 3.2 Characteristics of the grid

The block on which one carries out modeling is divided into 25 `QUAD8`.

### 3.3 Sizes tested and results

One tests the value of displacement vertical for the nodes *B* , *C* and *D* on both sides of the interface. The tolerance is fixed at  $10^{-6}$  . These values are summarized in the table below:

Sizes tested	Type of reference	Value of reference	Tolerance
DY (node B in lower part)	'ANALYTICAL'	-6.251724137931E-3	1, E-06
DY (node B with the top)	'ANALYTICAL'	1.0210344827586E-2	1, E-06
DY (node C in lower part)	'ANALYTICAL'	-5.575862068966E-3	1, E-06
DY (node C with the top)	'ANALYTICAL'	2.813793103448E-3	1, E-06
DY (node D in lower part)	'ANALYTICAL'	-2.8137931034482E-3	1, E-06
DY (node D with the top)	'ANALYTICAL'	5.34827586206896E-3	1, E-06

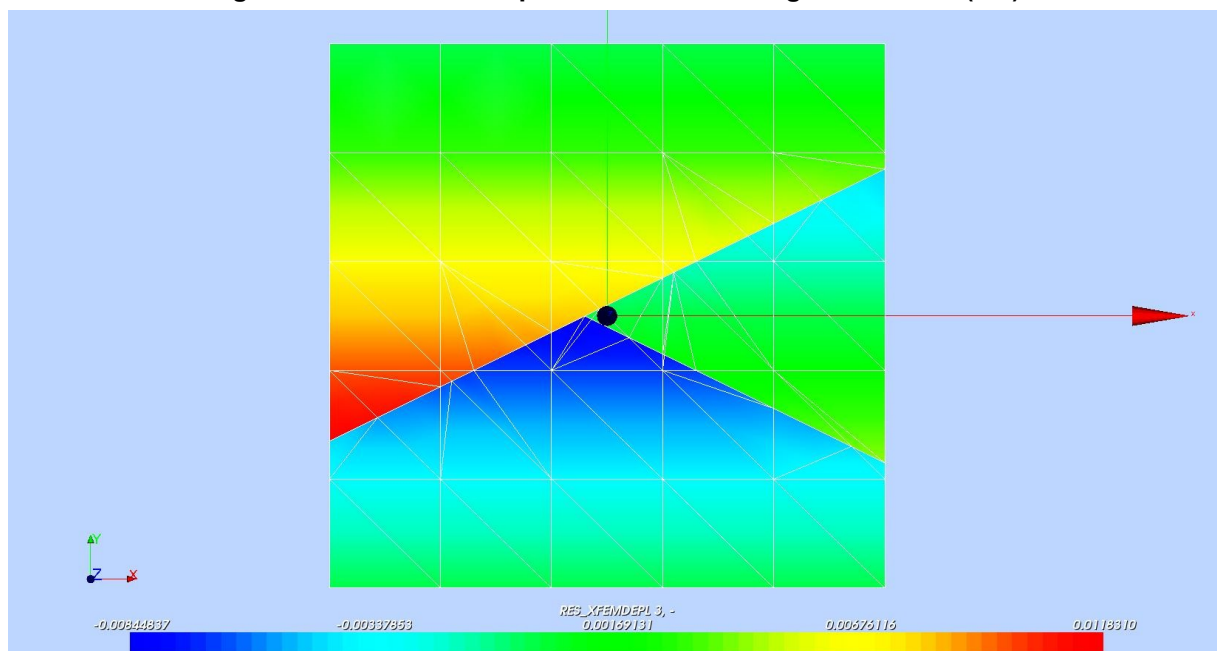
### 3.4 Remarks

There is also post-treaty the field of displacements according to the direction *y* (Figure 3.4-a) thanks to SALOMÉ.

One can then note (starting from the Figure 3.4-a) a frank discontinuity of the field of displacements related to the presence of the interfaces crossing the solid mass. That suggests the good taking into account of enrichment in the approximation of the field of displacements, including on the level of the junction of the interfaces.

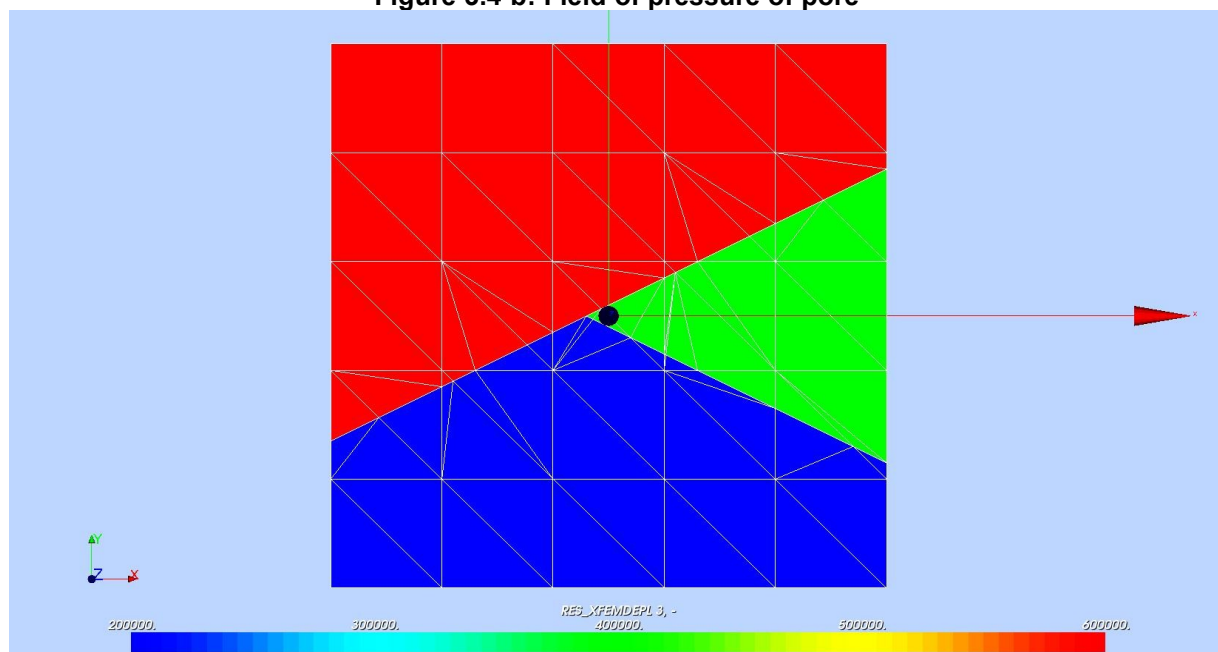


Figure 3.4-a: Field of displacements according to direction (OY)



Finally there is post-treated the field of pressure of pore  $p$  (Figure 3.4-b) thanks to SALOMÉ. There still, one observes a clear discontinuity at the level as of interfaces separating the blocks. The pressure of pore is quite constant in each block.

Figure 3.4-b: Field of pressure of pore



## 4 Modeling B

### 4.1 Characteristics of the modélisation

It is about a modeling 3D<sub>HM</sub> using quadratic elements HM-XFEM.

### 4.2 Characteristics of the grid

The block on which one carries out modeling is divided into 25 HEXA20.

### 4.3 Sizes tested and Results

One tests the value of displacement vertical for the nodes *B* , *C* and *D* on both sides of the interface. The tolerance is fixed at  $10^{-6}$  . These values are summarized in the table below:

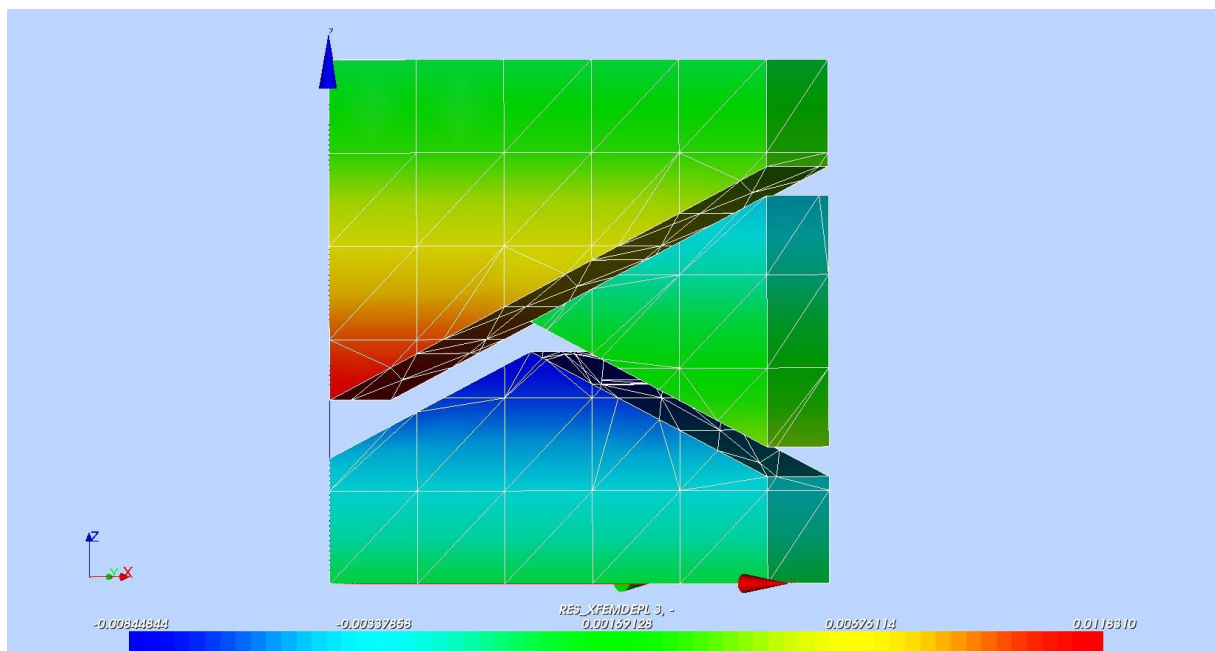
Sizes tested	Type of reference	Value of reference	Tolerance
DZ (node B in lower part)	'ANALYTICAL'	-6.251724137931E-3	1, E-06
DZ (node B with the top)	'ANALYTICAL'	1.0210344827586E-2	1, E-06
DZ (node C in lower part)	'ANALYTICAL'	-5.575862068966E-3	1, E-06
DZ (node C with the top)	'ANALYTICAL'	2.813793103448E-3	1, E-06
DZ (node D in lower part)	'ANALYTICAL'	-2.8137931034482E-3	1, E-06
DZ (node D with the top)	'ANALYTICAL'	5.34827586206896E-3	1, E-06

### 4.4 Remarks

There is also post-treaty the field of displacements according to the direction *z* (Figure 4.4-a) thanks to SALOMÉ.

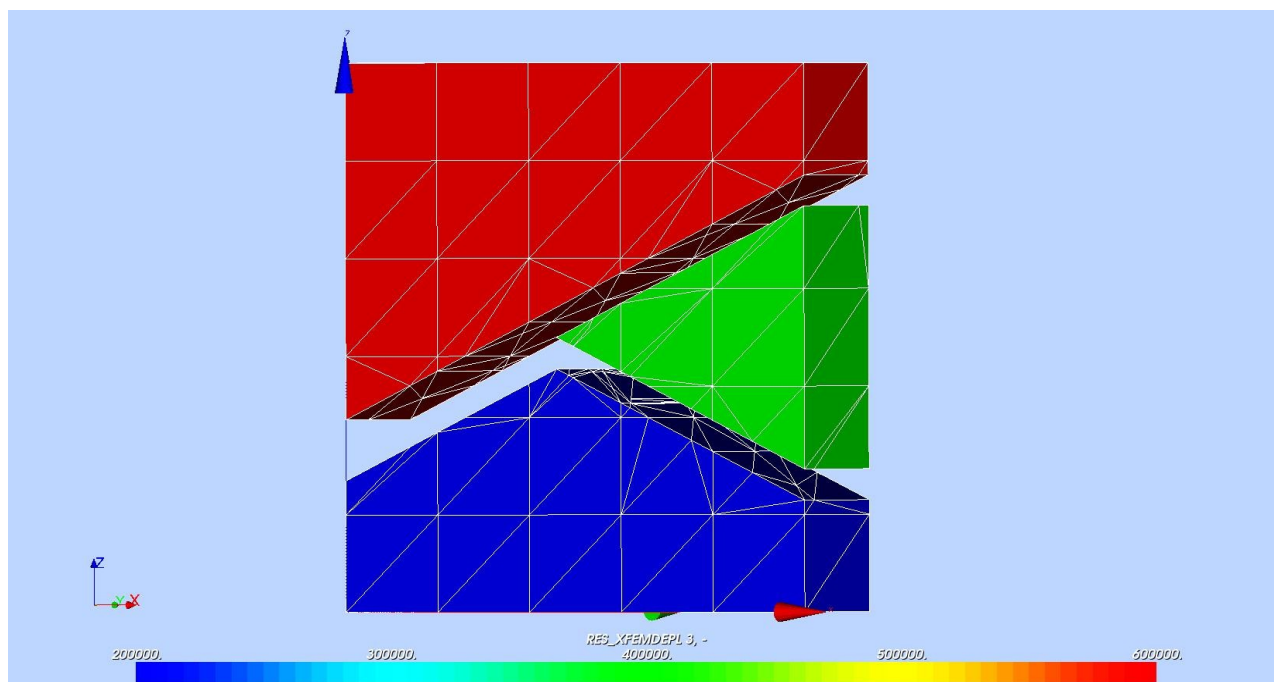
One can then note (starting from the Figure 4.4-a) a frank discontinuity of the field of displacements related to the presence of the interfaces crossing the solid mass. That suggests the good taking into account of enrichment in the approximation of the field of displacements, including on the level of the junction of the interfaces.

Figure 4.4-a: Field of displacements according to direction (OZ) and deformed



Finally there is post-treated the field of pressure of pore  $p$  (Figure 4.4-b) thanks to SALOMÉ. There still, one observes a clear discontinuity at the level of the interfaces separating the blocks. The pressure of pore is quite constant in each block.

Figure 4.4-b: Field of pressure of pore and deformation



## 5 Conclusion

---

For modeling `D_PLAN_HM` and modeling `3D_HM`, the results agree with the analytical solution. For elements `HM-XFEM` multi-Heaviside, the following features from now on are validated:

- `MODI_MODELE_XFEM`
- `CHAR_MECA_PRES`
- `POST_CHAM_XFEM`
- the calculation of the matrices and elementary vectors if the degrees of freedom Heaviside mechanics and hydraulics were introduced into the developments of the model of coupling `HM`.