WTNP115 – Désaturation of a porous environment without air on unit cell

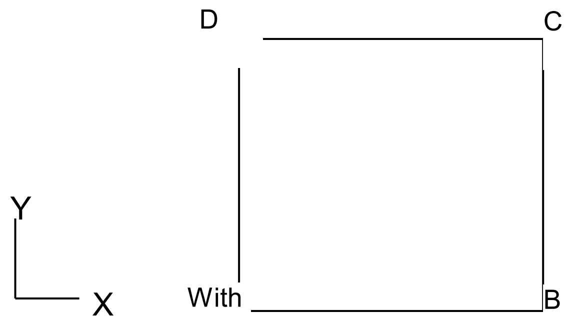
Summary:

One heats a porous environment whose pores are filled with a mixture of water and steam. Initial saturation in liquid is of 50%, the loading is a uniform heat flux on the edges of the field. The modeling made by only one element corresponds to the modeling of a homogeneous problem in space.

The reference solution is an approximate analytical solution.

1 Problem of reference

1.1 Geometry



Coordinates of the points (m) :

A	0	0	C	100	100
B	100	-0	D	0	100

1.2 Properties of material

One gives here only the properties whose solution depends, knowing that the command file contains other data of material (moduli of elasticity, thermal conductivity…) who finally do not play any part in the solution of with the dealt problem.

Liquid water	Density ($kg.m^{-3}$)	10^3
	Heat with constant pressure ($J.K^{-1}$)	4180
	thermal dilation coefficient of the liquid (K^{-1})	0.
Vapor	Heat-storage capacity ($J.K^{-1}$)	1900
	Initial enthalpy (latent heat of vaporization)	2,5E6.
	Molar mass ($kg.mol^{-1}$)	0.018
Skeleton	Heat-storage capacity with constant constraint ($J.K^{-1}$)	1050
Initial state	Porosity	0.3
	Temperature (K)	300
	Pressure of liquid (Pa)	1E5
	Steam pressure (Pa)	3700
	Initial saturation in liquid	0.5
Constants	Constant of perfect gases	8.315
Homogenized coefficients	Homogenized density ($kg.m^{-3}$)	2200
	Isotherm of sorption	$S(P_c)=0.5-10^{-12}(P_c-P_{vp}^0-P_c^0)$
		With $P_{vp}^0=3700$

$$P_c^0 = -10^5$$

1.3 Boundary conditions and loadings

On all the edges:

Heat flux $\mathbf{q}_{ext}.\mathbf{n}=10^6$

Hydraulic flow no one

2 Reference solution

2.1 Method of calculating

2.1.1 Calculation of the steam pressure starting from the temperature

We suppose the linear curve of saturation. She is thus written:

$$S=S_0+S'\Delta P_c \quad \text{éq 2.1.1-1}$$

[R7.01.11 éq 3.2.1-2] give then:

$$\begin{aligned}\Delta m_{lq}&=\rho_{lq}\phi S'\Delta P_c-\rho_{lq}^0\phi^0 S_{lq}^0\\ \Delta m_{vp}&=\left(\rho_{vp}-\rho_{vp}^0\right)\phi^0(1-S_0)-S'\rho_{vp}^0\phi^0\Delta P_c\end{aligned} \quad \text{éq 2.1.1-2}$$

It is written that the total water mass is preserved (because there is no water flow at the edge) and one obtains:

$$\begin{aligned}&\Delta m_{lq}+\Delta m_{vp}=0 \qquad \Rightarrow\\ &\left(\rho_{lq}-\rho_{vp}\right)S'\Delta P_c+\left(\rho_{vp}-\rho_{vp}^0\right)(1-S_0)=0\end{aligned} \quad \text{éq 2.1.1-3}$$

[R7.01.11 éq 4.4-1] gives in addition

$$\begin{aligned}\ln\left(\frac{p_{vp}}{p_{vp}^0}\right)&=\frac{M_{vp}^{ol}}{RT}\frac{1}{\rho_{lq}}\Delta P_{lq}+\\ &\frac{M_{vp}^{ol}}{R}\left(h_{vp}^0-h_{lq}^0\right)\left(\frac{1}{T^0}-\frac{1}{T}\right)+\frac{M_{vp}^{ol}}{R}\left(C_{vp}^p-C_{lq}^p\right)\left(\ln\left(\frac{T}{T^0}\right)+\frac{T^0}{T}-1\right)\end{aligned} \quad \text{éq 2.1.1-4}$$

The coupling of the equations [éq 2.1.1-3] and [éq 2.1.1-4], for which it is necessary to add the equation of perfect gases for the vapor, is a strongly nonlinear system which we will solve in small disturbance, which makes it possible to linearize it.
All done calculations, one obtains:

$$\left\{\begin{aligned}&\Delta P_{vp}\left(\left(\rho_{lq}-\rho_{vp}^0\right)S'+\frac{(1-S_0)M_{vp}^{ol}}{RT^0}\right)-\left(\rho_{lq}-\rho_{vp}^0\right)S'\Delta P_{lq}=(1-S_0)p_{vp}^0\frac{M_{vp}^{ol}}{R}\frac{\Delta T}{T^{0^2}}\\ &\frac{\Delta P_{vp}}{p_{vp}^0}-\frac{M_{vp}^{ol}}{\rho_{lq}RT^0}\Delta P_{lq}=\frac{M_{vp}^{ol}}{R}\left(h_{vp}^0-h_{lq}^0\right)\frac{\Delta T}{T^{0^2}}\end{aligned}\right. \quad \text{éq 2.1.1-5}$$

2.1.2 Calculation of the temperature

[R7.01.11 éq 3.2.4.3 - 1] gives:

$$\Delta Q' = -3\alpha_{gz}^{m} T \Delta p_{vp} + C_{\varepsilon}^{0} \Delta T \qquad \text{éq 2.1.2-1}$$

(since the other dilation coefficients are worthless).
[éq 3.2.4.3 - 2] gives:

$$\alpha_{gz}^{m} = \frac{\phi\left(1 - S_{lq}\right)}{3T} \qquad \text{éq 2.1.2-2}$$

One thus obtains:

$$\Delta Q' = -\phi\left(1 - S_{lq}\right)\Delta p_{vp} + C_{\varepsilon}^{0} \Delta T \qquad \text{éq 2.1.2-3}$$

In this problem, $\Delta Q'$ is anything else only the heat brought per unit of volume.

While calling Vol the total volume of the part and $Surf$ its side surface and Δt the time of application of flows:

$$\Delta Q' = \Delta t \frac{Surf}{Vol} \mathbf{q}_{ext} . \mathbf{n} \qquad \text{éq 2.1.2-4}$$

2.1.3 System to be solved

$$\begin{bmatrix} \left(\rho_{lq} - \rho_{vp}^{0}\right) S' + \dfrac{\left(1 - S_0\right) M_{vp}^{ol}}{R T^{0}} & -\left(\rho_{lq} - \rho_{vp}^{0}\right) S' & -\left(1 - S_0\right) p_{vp}^{0} \dfrac{M_{vp}^{ol}}{R T^{0\,2}} \\ \dfrac{1}{p_{vp}^{0}} & -\dfrac{M_{vp}^{ol}}{\rho_{lq} R T^{0}} & -\dfrac{M_{vp}^{ol}}{R} \dfrac{\left(h_{vp}^{0} - h_{lq}^{0}\right)}{T^{0\,2}} \\ 0 & -\phi\left(1 - S_{lq}\right) & C_{\varepsilon}^{0} \end{bmatrix} \begin{bmatrix} \Delta P_{vp} \\ \Delta P_{lq} \\ \Delta T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \Delta t \dfrac{Surf}{Vol} \mathbf{q}_{ext} . \mathbf{n} \end{bmatrix}$$

éq 2.1.3-1

2.2 Results of reference

One gives the value of the temperature, the pressure of liquid and the steam pressure, solution of the system [éq 2.1.3-1] with the data summarized in the paragraphs [§1.2] and pointed out Ci below. For the calculation of the heat-storage capacities, one uses the following relations:

$$(1-\phi^0)\rho_s = r_0 - \rho_{lq}^0 S_l^0 \phi^0 - (1-S_l^0)\phi^0 \rho_{vp}^0$$
$$C_\sigma^0 = (1-\phi)\rho_s C_\sigma^s + \rho_{lq} S_l \phi C_{lq}^p + (1-S_l)\phi \rho_{vp} C_{vp}^p$$

$C_\varepsilon^0 = C_\sigma^0$, this last relation being true because the dilation coefficient of the grains is null.

S_0	S'	T^0	p_{vp}^0	h_{vp}^0	ρ_{vp}^0 (calculated)	ρ_{lq}
5,00E-01	-1,00E-12	3,00E+02	3,70E+03	2,50E+06	2,67E-02	1,00E+03

r_0	ϕ^0	ρ_s (calculated)	C_σ^s	C_{lq}^p L	C_{vp}^p	C_ε^0 (calculated)
2,20E+03	3,00E-01	2,93E+03	1,05E+03	4,18E+03	1,90E+03	2,78E+06

$\mathbf{q}_{ext}.\mathbf{n}$	Δt	$Surf$	Vol
1,00E+06	1000	400	1,00E+04

After resolution, the following results are got:

ΔP_{vp}	3.E+03
P_{lq}	-1E+07
ΔT	14

2.3 Uncertainties

Uncertainties are rather large because the analytical solution is a solution approached because of linearization of the equations.

3 Modeling A

3.1 Characteristics of modeling A

Modeling in plane deformations. An element $Q8$
Discretization in time: only one step of time: $10^3 s$.

3.2 Values tested

Node	Type of value	Moment (s)	Reference (analytical)	*Aster*	Difference (%)
$NO1$	DEPL/TEMP	10^3	14	14.4	2.7%
$NO1$	DEPL/PRE1	10^3	-1.10^7	$-1.3\ 10^7$	30%
$NO1$	VARI_ELNO/V4		$3.\ 10^3$	$3.9\ 10^3$	30%

One thus finds results relatively close to the analytical results. Uncertainty remaining rather broad because of linearization of the equations.