WTNP122 - Modeling of a bar saturated with compressible gas slightly non-linear (monophasic flow) subjected to a shock with pressure

Summary:

This case test aims to validate:

- the diagrams finished volumes developed for the modeling of the diphasic flows.
- the hydraulic modeling saturated with finite elements D_PLAN_HS

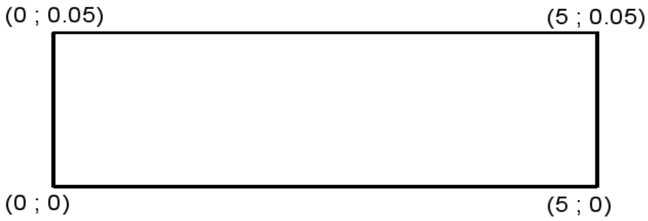
The diphasic problem here will be degenerated in a monophasic problem gas which one knows the analytical solution. It is the monodimensional modeling of a bar saturated with gas subjected to a shock with pressure.

1 Problem of reference

The objective of this case test is to compare the solution obtained with the various diagrams with an analytical solution.

1.1 Geometry

A bar is considered $1\mathrm{D}$ of $5\mathrm{m}$ of length. Concretely the field with a grid will make $[0\mathrm{m},5\,m]\times[0\mathrm{m}\,;0{,}05\,m]$ (in the case of modeling in triangle, it is important not to have too "flattened" triangles, the choice height of the field is thus not pain-killer).



The duration of simulation is of $100\mathrm{s}$ and the number of steps of time is of 100.

1.2 Properties of materials

One gives here only the properties whose solution depends, knowing that the command file contains other data of material which do not play any part in the solution of with the dealt problem.

Gas	Molar mass $(kg\,.mol^{-1})$	0,0001
	Viscosity $(kg\,.m^{-1}.s^{-1})$	1
	Relative permeability (m^2)	1
Dissolved gas	Coefficient of Henry $(Pa\,.mol^{-1}.m^3)$	10000000000
Liquid	Relative permeability (m^2)	1
Homogenized parameters	Permeability $K_{int}(m^2)$	10^{-7}
	Porosity	1
	Fick gas $(m^2.s^{-1})$	0
	Liquid Fick $(m^2.s^{-1})$	0

Table 1.2-1 : Properties of materials

1.3 Boundary conditions and loadings

The limiting conditions are the following ones:

- conditions of Neumann on the right of field:

$$\frac{\partial(\delta P_g)}{\partial x}(t, x=5, y)=0\,Pa$$

- condition of Dirichlet on the left part of the field:

$$P_g(t, x=0, y)=0\,Pa$$

1.4 Initial conditions

The variation of initial gas pressure compared to the pressure of reference is of $\delta P_g(t=0,x,y)=10^4\,Pa$.

One also has $P_g^{ref}(t=0,x,y)=10^4\,Pa$ what amounts studying a problem slightly nonlinear (to be linear one would have to choose $P_g^{ref}(t=0,x,y)=10^{10}\,Pa$) (because one a: $P_g(t=0,x,y)=P_g^{ref}+\delta P_g(t=0,x,y)$).

2 Analytical solution

2.1 Method of calculating

The non stationary and monodimensional monophasic problem can be written in a general form of the type:

$$N\frac{\partial P}{\partial t}-K_{int.}\Delta P=0$$
$$P(t=0)=P_0$$
$$P(t,x=0)=0$$
$$\frac{\partial P}{\partial x}(t,x,L)=0$$

This problem admits an analytical solution obtained by development in Fourier series.

$$P=\sum_{k=0}^{K}\frac{4\mathrm{P}_0}{(2\mathrm{k}+1)\pi}\exp\left(\frac{-K_{int}}{N}\omega_k^2 t\right)\sin(\omega_k x)\ \text{ with }\ \omega_k=\left(k+\frac{1}{2}\right)\frac{\pi}{L}$$

The number of terms K of this series is in the following way given:

That is to say n_x the number of points x_i where the solution is evaluated at one moment t.
One poses:

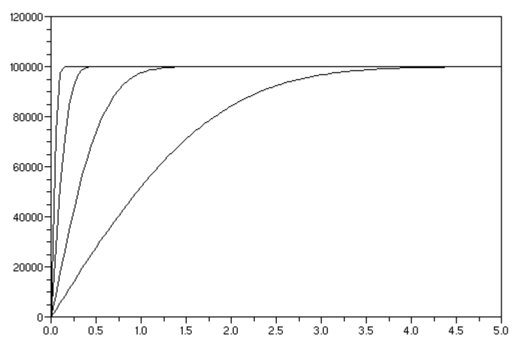
$$a_k^i=\frac{4}{(2\mathrm{k}+1)\pi}\exp\left(\frac{-K_{int}}{N}\omega_k^2 t\right)\sin(\omega_k x_i)$$

So that the solution can be written: $P(x_i)=\sum_{k=0}^{K}P_0.a_k^i$

One chooses K such as: $\frac{1}{n_x}\sqrt{\sum_{i=1}^{nx}(a_k^i)^2}<\epsilon$

In practice we took $\varepsilon=10^{-10}$.

The paces of the analytical solution at times 1,10,100,1000 are shown on the figure below:



The following table gives the number of terms according to time:

Time	Many series terms
1	194
10	64
100	22
1000	8

Table 2.1-1 : Representation amongst term according to time

2.2 Simplifying assumptions

It is considered that the medium is completely saturated with gas and one imposes in aster a pressure of worthless liquid on all the nodes. One imposes an initial gas pressure P_g^{ref} and one gives boundary conditions which corresponds to a variation of this pressure of reference, That is to say then δP_g this variation of pressure. The conservation equation of the gas mass will be written:

$$\frac{\partial(\varphi\,\delta P_g)}{\partial t}+div\left(\frac{K_{int}k_g}{\mu_g}(P_g^{ref}+\delta P_g)\,div(P_g^{ref}+\delta P_g)\right)=0$$

While supposing δP_g small in front P_g^{ref}, this equation becomes:

$$\frac{\partial(\varphi\,\delta P_g)}{\partial t}+\frac{K_{int}k_g}{\mu_g}P_g^{ref}\Delta(\delta P_g)=0$$

It is thus δP_g that one will identify with the solution of the model analytical equation.

In order to find the coefficients of the model problem, one will take:

$$\varphi=1$$
$$k_g=\mu_g=1$$

and it will be made so that

$$K_{int}P_{ref}=10^{-3}\ .$$

3 Modeling A

3.1 Characteristics of modeling

Modeling D_PLAN_HH2SUDA. This modeling corresponds with modeling Volume Finished diecentered on the edges for mobilities (the fickiens terms are centered). The hydraulic mixing rate is LIQU_AD_GAZ_VAPE.

3.2 Characteristics of the grid

One uses a grid made up of 100 elements QUAD9.

3.3 Sizes tested and results

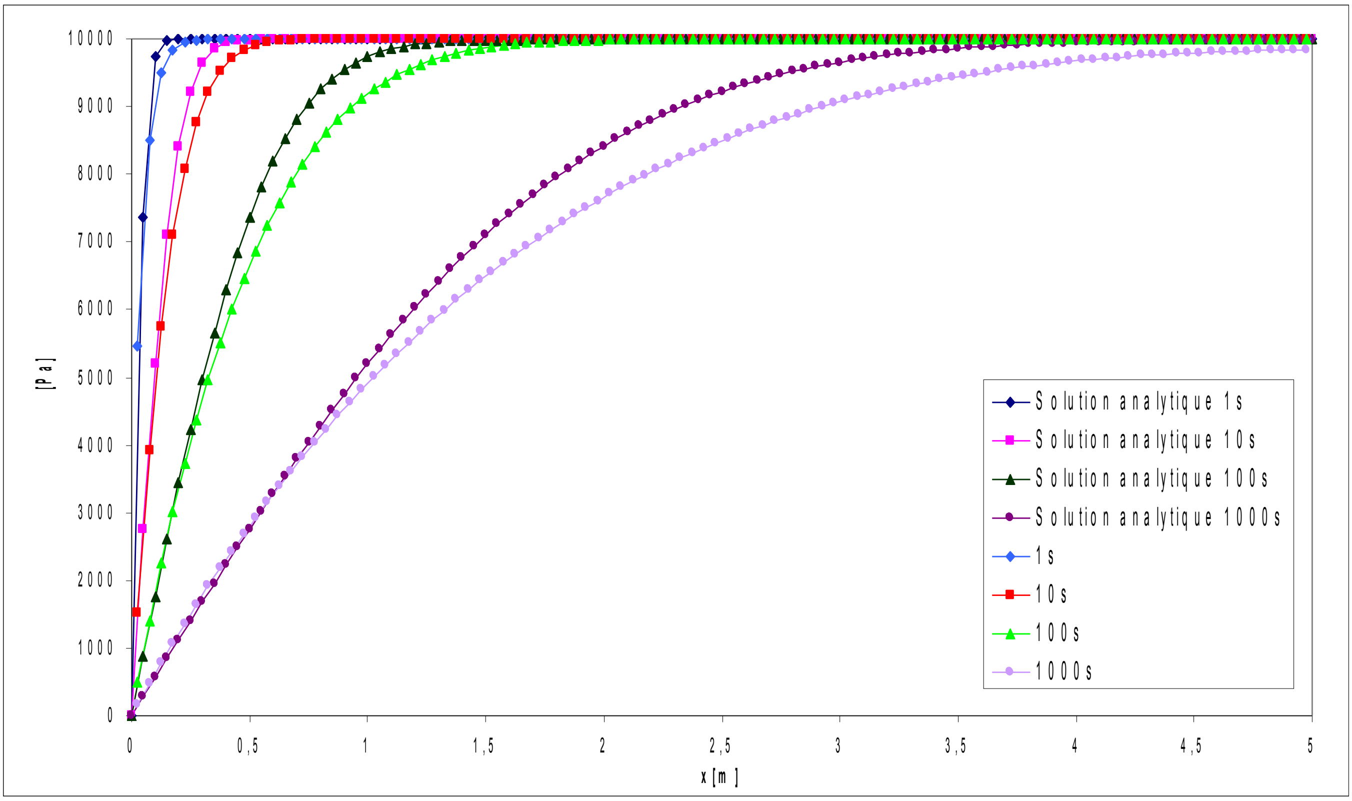


Illustration 1: Profiles of gas pressure

One for the first time carries out tests on 4 nodes with 1 moment by comparing the results with the analytical solution and second once by carrying out a test of nonregression.

Size	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	$(0{,}075 ; 0)$ *N304*	100	analytical	1331.0	9.0%
PRE2	$(0{,}075 ; 0)$ *N304*	100	not regression	1444.1	1.0%
PRE2	$(0{,}075 ; 1)$ *N293*	100	analytical	1331.0	9.0%
PRE2	$(0{,}075 ; 1)$ *N293*	100	not regression	1444.1	1.0%
PRE2	$(0{,}05 ; 0{,}05)$ *N469*	100	analytical	889.3176	12.0%
PRE2	$(0{,}05 ; 0{,}05)$ *N469*	100	not regression	990.9	1.0%
PRE2	$(0{,}075 ; 0{,}5)$ *NQ95*	100	analytical	1331.0	9.0%
PRE2	$(0{,}075 ; 0{,}5)$ *NQ95*	100	not regression	1444.1	1.0%

Table 3.3-1 : Values tested

4 Modeling B

4.1 Characteristics of modeling B

Modeling D_PLAN_HH2S. This modeling corresponds with modeling classical Finite elements. The hydraulic mixing rate is LIQU_AD_GAZ_VAPE.

4.2 Characteristics of the grid

The grid consists of 100 elements QUAD8.

4.3 Sizes tested and results

One for the first time carries out tests on 2 nodes with 1 moment by comparing the results with the analytical solution and second once by carrying out a test of nonregression.

Size	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance ($\%$)
PRE2	$(0{,}05;0)$ *N104*	100	analytical	889.3176	12.0%
PRE2	$(0{,}05;0)$ *N104*	100	not regression	992.8591566774	1.0%
PRE2	$(0{,}05;1)$ *N103*	100	analytical	889.3176	12.0%
PRE2	$(0{,}05;1)$ *N103*	100	not regression	992.85915667759	1.0%

Table 4.3-1 : Values tested

5 Modeling C

5.1 Characteristics of modeling

Modeling D_PLAN_HH2SUDA. This modeling corresponds with modeling Finished Volumes diecentered on the edges for mobilities (the fickiens terms are centered). The hydraulic mixing rate is LIQU_AD_GAZ_VAPE.

5.2 Characteristics of the grid

The grid consists of 200 elements TRIA7.

5.3 Sizes tested and results

One carries out tests on 2 nodes with 1 moment by comparing the results with the analytical solution and on 3 nodes with 1 moment by carrying out a test of nonregression.

Size	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	$(0{,}075\,;0)$ $N360$	100	analytical	1331.0	11.0%
PRE2	$(0{,}075\,;0)$ $N360$	100	not regression	1454.4	1.0%
PRE2	$(0{,}075\,;0{,}025)$ $N505$	100	analytical	1331.0	10.0%
PRE2	$(0{,}075\,;0{,}025)$ $N505$	100	not regression	1450.4	1.0%
PRE2	$(0{,}016\,;0{,}0158)$ $NT70$	100	not regression	353,978	1.0%

Table 5.3-1: Values tested

6 Modeling D

6.1 Characteristics of modeling

Modeling D_PLAN_HH2S. This modeling corresponds with modeling Finite elements. The hydraulic mixing rate is LIQU_AD_GAZ_VAPE.

6.2 Characteristics of the grid

The grid consists of 200 elements TRIA6.

6.3 Sizes tested and results

One for the first time carries out tests on 2 nodes with 1 moment by comparing the results with the analytical solution and second once by carrying out a test of nonregression.

Size	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance ($\%$)
PRE2	$(0{,}05\,;0)$ $N103$	100	analytical	889.3176	12.0%
PRE2	$(0{,}05\,;0)$ $N103$	100	not regression	992,699	1.0%
PRE2	$(0{,}05\,;0{,}05)$ $N203$	100	analytical	889.3176	12.0%
PRE2	$(0{,}05\,;0{,}05)$ $N203$	100	not regression	993.06	1.0%

Table 6.3-1 : Values tested

7 Modeling E

7.1 Characteristics of modeling

Modeling D_PLAN_HS. This modeling corresponds with modeling Finite elements. The hydraulic mixing rate is GAS.

7.2 Characteristics of the grid

The grid consists of 200 elements TRIA6.

7.3 Sizes tested and results

One carries out tests on 2 nodes at the moment $t=100\,ans$ for the first time by comparing the results with the analytical solution and second once by carrying out a test of nonregression.

Size	**Points** (x,y)	**Time (S)**	**Standard reference**	**Reference**	**Tolerance (** $\%$ **)**
PRE2	$(0{,}05\,;0{,}05)$ *N103*	100	analytical	889.3176	12.0%
PRE2	$(0{,}05\,;0{,}05)$ *N103*	100	not regression	992,699	1.0%
PRE2	$(0{,}05\,;0)$ *N203*	100	analytical	889.3176	12.0%
PRE2	$(0{,}05\,;0)$ *N203*	100	not regression	993.06	1.0%

Table 7.3-1 : Values tested

8 Summary of the results

This case test makes it possible to test the diagrams volumes finished in various configurations on a modeling of gas flow:

- the diagram finished volumes decentred edge;
- on various types of meshs (triangles and rectangles).

These same cases are also carried out with the classical diagrams finite elements. All the results are very close to the analytical solution.

From a performance point of view, one will recommend the use of the eccentric diagram edge.