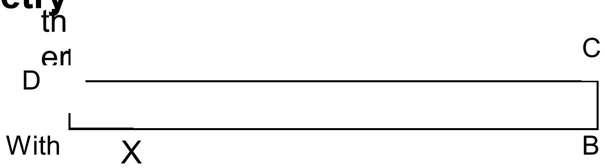
WTNA102 - Diffusion of dissolved air (axi)

Summary:

One considers here a problem at temperature and saturation constants. By boundary conditions suitable one imposes a water pressure and a steam pressure constants. A gas pressure is imposed on an edge of the field (worthless flows on other side). Only the air pressures dryness and of dissolved air connected by the law of Henry evolve. This problem is brought back in an equation for the air pressure dryness of type “equation of heat”. The reference solution will be then a thermal calculation DE Code_Aster.

1 Problem of reference

1.1 Geometry



Coordinates of the points (m) :

A	0	0	C	1	0,5
B	1	0	D	0	0,5

1.2 Properties of material

One gives here only the properties whose solution depends, knowing that the command file contains other data material (moduli of elasticity, thermal conductivity…) who finally do not play any part in the solution of with the dealt problem.

Liquid water	Density ($kg.m^{-3}$)	10^3
	Specific heat with constant pressure ($J.K^{-1}$)	0
	Dynamic viscosity of liquid water ($Pa.s$)	0.001
	thermal dilation coefficient of the liquid (K^{-1})	0
	Permeability relating to water	$kr_w(S)=0.5$
Vapor	Specific heat ($J.K^{-1}$)	0
	Molar mass ($kg.mol^{-1}$)	0.01
Gas	Specific heat ($J.K^{-1}$)	0
	Molar mass ($kg.mol^{-1}$)	0.01
	Permeability relating to gas	$kr_{gz}(S)=0.5$
	Viscosity of gas ($kg.m^{-1}.s^{-1}$)	0.001
Dissolved air	Specific heat ($J.K^{-1}$)	0
	Constant of Henry ($\mathrm{Pa.m^3.mol^{-1}}$)	50000
Initial state	Porosity	1
	Temperature (K)	300
	Gas pressure (Pa)	$1.01\,10^5$
	Steam pressure (Pa)	1000
	Capillary pressure (Pa)	10^6
	Initial saturation in liquid	0.4
Constants	Constant of perfect gases	8.32

Coefficients homogenized	Homogenized density ($kg.m^{-3}$)	2200
	Isotherm of sorption	$S(p_c)=0.4$
	Coefficient of Biot	0
	Fick Vapor ($m^{2.}s^{-1}$)	0
	Fick dissolved air ($m^{2.}s^{-1}$)	$FA=6^{-10}$
	Intrinsic permeability (m^2)	1^{-19}

1.3 Boundary conditions and loadings

On the whole of the field, one wants:

$$p_w=cte=p_w^0$$
$$\frac{1}{K_w}=0\Rightarrow\rho_w=cte=\rho_w^0$$
$$p_{vp}=cte=p_{vp}^0$$
$$F_{vp}=0$$
$$S(p_c)=cte=S_0$$
$$T=cte=T_0$$
$$\phi=1$$
$$M_{as}^{ol}=M_{ad}^{ol}=M_{vp}^{ol}$$

On all the edges: Hydraulic flows and worthless thermics.

One now will linearize p_{vp} according to p_w.

Writing of p_{vp} linear function of p_w :

Section 4.2.3 of the reference document `Code_Aster` [R7.01.11] the relation gives us: $\frac{dp_{vp}}{p_{vp}}=\frac{M_{vp}^{ol}}{RT}\frac{dp_w}{\rho_w}$. If this expression is linearized one obtains: $p_{vp}=\frac{p_{vp}^0}{RT}\frac{M_{vp}^{ol}}{\rho_w^0}p_w+\left(p_{vp}^0-\frac{p_{vp}^0}{RT}\frac{M_{vp}^{ol}}{\rho_w^0}p_w^0\right)$ that one can write in the form:

$$p_{vp}=A\,p_w+B \qquad \text{éq 1.3-1}$$

with $A=\frac{p_{vp}^0}{RT}\frac{M_{vp}^{ol}}{\rho_w^0}$ and $B=p_{vp}^0-\frac{p_{vp}^0}{RT}\frac{M_{vp}^{ol}}{\rho_w^0}p_w^0$

On the edge AB : $p_{vp}=A\,p_w+B$
$p_{gz}=115000$ and $p_c=10^6$

2 Reference solution

2.1 Method of calculating

2.1.1 Calculation of the conservation of the mass of air

The conservation of the gas mass is written:

$$\frac{dm_{air}}{dt}+div\left(\boldsymbol{M}_{as}+\boldsymbol{M}_{ad}\right) \qquad \text{éq 2.1.1-1}$$

It is written that the total water mass and the total mass of air are preserved (because there is no gas water flow nor at the edge) and one obtains:

$$m_{air}=m_{as}+m_{ad}=S_0(\rho_{ad}-\rho_{ad}^0)+(1-S_0)(\rho_{as}-\rho_{as}^0)$$

thus

$$d\,(m_{as}+m_{ad})=S_0\,d\,\rho_{ad}+(1-S_0)\,d\,\rho_{as} \qquad \text{éq 2.1.1-2}$$

$d\,\rho_{as}=\frac{M_{as}^{ol}}{RT}dp_{as}$ and $d\,\rho_{ad}=\frac{M_{ad}^{ol}}{K_H}dp_{as}$

$$\frac{dm_{air}}{dt}=div\left(\frac{M_{ad}^{ol}}{K_H}S_0+(1-S_0)\frac{M_{as}^{ol}}{RT}dp_{as}\right)$$

Calculation speeds:

$$\frac{\boldsymbol{M}_{as}}{\rho_{as}}=\lambda_{gz}(-\nabla p_{as}) \qquad \text{éq 2.1.1-3}$$

since $F_{\boldsymbol{vp}}=0$ and $\nabla p_{\boldsymbol{vp}}=0$

and

$$\boldsymbol{M}_{ad}=\rho_{ad}\lambda_{lq}(-\nabla p_{lq})-F_{ad}\nabla C_{ad} \quad \text{with} \quad C_{ad}=\rho_{ad}$$

Like $\nabla p_{lq}=\nabla p_w+\nabla p_{ad}=\nabla p_{ad}=\frac{RT}{K_H}\nabla p_{as}$

$$\boldsymbol{M}_{ad}=\rho_{ad}\lambda_{lq}\frac{RT}{K_H}(-\nabla p_{as})-\frac{M_{ad}^{ol}}{K_H}F_{ad}\nabla p_{as}$$

[éq 2.1.1-1] can then be simplified in the following form:

$$C\frac{dp_{as}}{dt}=L\,div(\nabla p_{as})$$

with

$$C=\frac{M_{ad}^{ol}}{K_H}S_0+(1-S_0)\frac{M_{as}^{ol}}{RT}$$

and

$$L=\rho_{as}^0\lambda_{gz}+\frac{RT}{K_H}\rho_{ad}^0\lambda_{lq}+\frac{M_{as}^{ol}}{K_H}F_{ad}$$

Equation of the heat whose one knows the result.

2.2 Results of reference

With the preceding digital values, one finds:

$$p_{as}=10^5 \Rightarrow p_{ad}^0=\frac{RT}{K_H}p_{as}^0=4992$$

$\rho_{as}^0=\frac{M_{as}^{ol}}{RT}p_{as}^0=0.4$ and $\rho_{ad}^0=\frac{M_{ad}^{ol}}{RT}p_{ad}^0=0.02$

$$\rho_{vp}^0=\rho_{vp}=4.10^{-3}$$

The constants of the equation of heat are then:

$$C=2.48\,10^{-6}$$
$$L=1.4\,10^{-16}$$

2.3 Uncertainties

Uncertainties are rather large because the analytical solution is a solution approached because of linearization of the equations.

3 Modeling A

3.1 Characteristics of modeling A

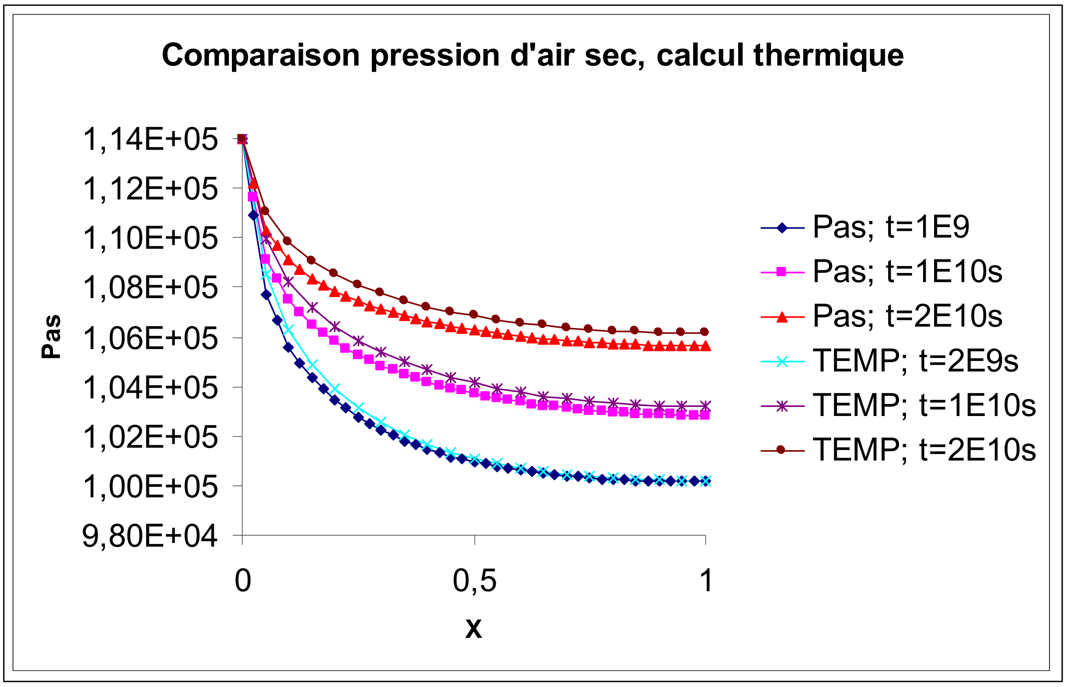
Modeling AXIS_HH2D. 20 elements QUAD8.

Discretization in time: 100 pas de time of $5E7\,s$ each one.

3.2 Sizes tested and results

X (m)	Time (s)	$PRE2$ Aster	$PRE2$ thermal calculation	Tolerance (%)
0.2	$3\,10^9$	$8.1\,10^3$	$7.9\,10^3$	10.0
0.2	$5\,10^9$	$9.7\,10^3$	$9.5\,10^3$	10.0

Values obtained by Code_Aster are tested in nonregression with a tolerance of $0.01\,\%$.



4 Modeling B

4.1 Characteristics of modeling B

Modeling AXIS_HH2S. 20 elements QUAD8.

Discretization in time: 100 pas de time of $5\,10^7 s$ each one.

4.2 Sizes tested and results

X (m)	Time (s)	$PRE2$ Aster	$PRE2$ thermal calculation	Tolerance ($\%$)
0.2	$3\,10^9$	$8.1\,10^3$	$7.9\,10^3$	10.0
0.2	$5\,10^9$	$9.7\,10^3$	$9.5\,10^3$	10.0

Values obtainedbe by Code_Aster are youstées in nonregression with a tolerance of $0.01\,\%$.

5 Modeling C

5.1 Characteristics of modeling C

Modeling 3D_HH2S. 200 elements HEXA20. This test consists of a bar and cannot thus have the same analytical solution as previously. It is obtained same manner by a thermal calculation.

Discretization in time: 100 pas de time of $5E7 s$ each one.

5.2 Sizes tested and results

X (m)	Time (s)	$PRE2$ Aster	$PRE2$ thermal calculation	Relative error
0.2	$3\,10^9$	14682	14617	0.45%
0.2	$5\,10^9$	14953	14935	0.12%

6 Modeling D

6.1 Characteristics of modeling D

Modeling `3D_HH2D`. 200 elements `HEXA20`.

Discretization in time: 100 pas de time of $5E7\,s$ each one.

6.2 Sizes tested and results

X (m)	Time (s)	$PRE2$ Aster	$PRE2$ thermal calculation	Relative error
0.2	$3\,10^9$	14687	14617	0.48%
0.2	$5\,10^9$	14954	14935	0.13%

7 Modeling E

7.1 Characteristics of modeling E

Axisymmetric modeling THHM2D with blocked temperatures and displacements. 20 elements QUAD8.

Discretization in time: 100 pas de time of $5E7\,s$ each one.

7.2 Sizes tested and results

X (m)	Time (s)	$PRE2$ Aster	$PRE2$ thermal calculation	Tolerance
0.2	$3\,10^9$	7944	7900	1%
0.2	$5\,10^9$	9557	9500	1%

Values obtained by Code_Aster are tested in nonregression with a tolerance of 0.01%.

8 Modeling F

8.1 Characteristics of modeling F

Axisymmetric modeling THHM2S with blocked temperatures and displacements. 20 elements QUAD8.

Discretization in time: 100 pas de time of $5E7\,s$ each one.

8.2 Sizes tested and results

X (m)	Time (s)	$PRE2$ Aster	$PRE2$ thermal calculation	Tolerance
0.2	$3\,10^9$	7954	7900	1%
0.2	$5\,10^9$	9566	9500	1%

Values obtained by Code_Aster are tested in nonregression with a tolerance of 0.01%.

9 Modeling G

9.1 Characteristics of modeling G

Axisymmetric modeling THH2D with blocked temperatures. 20 elements QUAD8.

Discretization in time: 100 pas de time of $5E7 s$ each one.

9.2 Sizes tested and results

X (m)	Time (s)	$PRE2$ Aster	$PRE2$ thermal calculation	Tolerance
0.2	$3\,10^9$	7945	7900	10%
0.2	$5\,10^9$	9557	9500	10%

Values obtained by Code_Aster are tested in nonregression with a tolerance of 0.01%.

10 Modeling H

10.1 Characteristics of modeling H

Axisymmetric modeling THH2S with blocked temperatures. 20 elements QUAD8.

Discretization in time: 100 pas de time of $5E7\,s$ each one.

10.2 Sizes tested and results

X (m)	Time (s)	$PRE2$ Aster	$PRE2$ thermal calculation	Tolerance
0.2	$3\,10^9$	7944	7900	$10\,\%$
0.2	$5\,10^9$	9560	9500	$10\,\%$

Values obtained by Code_Aster are tested in nonregression with a tolerance of $0.01\,\%$. Modeling H

11 Modeling I

11.1 Characteristics of modeling I

Axisymmetric modeling HH2MD with blocked displacements. 20 elements QUAD8.

Discretization in time: 100 pas de time of $5E7\,s$ each one.

11.2 Sizes tested and results

X (m)	Time (s)	$PRE2$ Aster	$PRE2$ thermal calculation	Tolerance
0.2	$3\,10^9$	7944	7900	1%
0.2	$5\,10^9$	9557	9500	1%

Values obtained by Code_Aster are tested in nonregression with a tolerance of 0.01%.

12 Modeling J

12.1 Characteristics of modeling J

Axisymmetric modeling HH2MD with blocked displacements. 20 elements QUAD8.

Discretization in time: 100 pas de time of $5E7\,s$ each one.

12.2 Sizes tested and results

X (m)	Time (s)	$PRE2$ Aster	$PRE2$ thermal calculation	Tolerance
0.2	$3\,10^{9}$	7948	7900	1%
0.2	$5\,10^{9}$	9560	9500	1%

Values obtained by Code_Aster are tested in nonregression with a tolerance of 0.01%.

13 Summary of the results

The resultS of Code_Aster soNT in very good agreement with the analytical solution.