

WTNA113 – Modeling of incompressible water injection in a saturated medium

Summary:

One studies a cylindrical bar saturated with top of which a flow is injected. The associated law of behavior of the fluid is of type LIQU_SATU. Modeling is hydraulic (HM). The fluid is incompressible and the medium is infinitely rigid. It is thus about a stationary problem comprising an analytical solution. This test aims to validate the good taking into account of a boundary condition of type hydraulic flow in the case of an axisymmetric modeling.

1 Problem of reference

1.1 Presentation

One studies in this case test the hydraulic behavior of a saturated porous environment consisted only one fluid: water in its liquid phase. The associated law of behavior of the fluid is of type LIQU_SATU. Modeling is hydraulic (HM). LE fluid is incompressible and the medium is infinitely rigid. It is thus about a stationary problem comprising an analytical solution. This test aims to validate the good taking into account of a boundary condition of type hydraulic flow in the case of an axisymmetric modeling.

1.2 Geometry

A bar length is represented $L=1m$. Its width l does not intervene in the analytical solution because the problem is purely 1D. One takes here $l=0,2m$. The N1 point corresponds to the point of coordinate $(0,0)$.

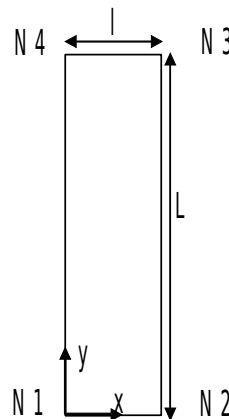


Illustration 1:
vertical bar

1.3 Properties of material

Liquid water	Density $\rho_l (kg.m^{-3})$	10^3
	Dynamic viscosity of liquid water $\mu_l (Pa.s)$	0.001
	Compressibility $K_w (Pa^{-1})$	0
Solid	Drained Young modulus $E (Pa)$	$7.5 \cdot 10^{15}$
	Poisson's ratio ν	0.3
State of reference	Porosity ϕ	0.2
	Temperature $T^{ref} (K)$	293
	Liquid pressure $P^{ref} (Pa)$	0
	Steam pressure $P_v^{ref} (Pa)$	0.1
Constants	Constant of perfect gases R	8.32
Homogenized coefficients	Homogenized density $r_0 (kg.m^{-3})$	2200
	Coefficient of Biot B	0.6
	Intrinsic permeability $K_{int} (m^2)$	$K_{int} = 10^{-12}$

Table 1.3-1: Data materials

The gravity of water is neglected.

1.4 Boundary conditions and loadings

On all the edges

blocked displacements $u_x = u_y = u_z = 0$

Higher edge:

hydrous flow: $q = 1 kg.s^{-1}.m^{-2}$

Lower edge:

Pressure of liquid: $P_{lq} = P_0 = 1 atm$

Side edges:

null flow

1.5 Initial conditions

The initial pressure is of $1 atm$. All the other fields are worthless.

2 Reference solution

The reference solution is unidimensional because it depends only on the vertical coordinate (loading 1D). The system saturated with water is brought back to solve the problem of conservation of the mass:

$$\frac{\partial(\phi \rho_l)}{\partial t} - \operatorname{div}\left(K_{\text{int}} \frac{\rho_l}{\mu_l} \nabla P_l\right) = q$$

where Q is the flow imposed on the edge.

- The liquid is incompressible: $\rho_l = \text{cst}$
 - The matrix is incompressible (infinite rigidity): porosity thus remains constant $\phi = \text{cst}$.
- One thus obtains in this case a steady flow which is summarized with the writing of flow:

$$-\operatorname{div}\left(K_{\text{int}} \frac{\rho_l}{\mu_l} \nabla P_l\right) = q$$

If one notes P_L pressure in $Y=L$, one thus obtains after discretization:

$$K_{\text{int}} \frac{\rho_l}{\mu_l} \frac{P_L - P_0}{L} = q$$

What gives with the data indicated previously: $P_L = 1.1 \cdot 10^6 \text{ Pa}$

3 Modeling A

3.1 Characteristics of modeling A

- Modeling in plane deformations `AXIS_HMS`.
- Hydraulic behavior `LIQU_SATU`.
- 10*10 elements `Q8`.
- Discretization in 1 pas de times of 100 S.

3.2 Result of modeling A

One has the results of the nodes N3 and N4 which each one correspond to the pressure P_L

Table of results at the various moments:

N° NODE	Sequence number	$PREI(Pa)$	Tolerance (%)
N3	1	1.110^6	1
N4	1	1.110^6	1

Table 1: Results

The results are in perfect agreement with the analytical solution.

4 Modeling B

It acts of the same modeling but with a modeling of the type `AXIS_HMD`. The results are logically exactly the same ones as above.

5 Modeling C

It acts of the same modeling but with a modeling of the type `AXIS_THMS`. The temperature is blocked everywhere. It is just a question of making sure that this modeling is well treated. The results are logically exactly the same ones as above.

6 Summary of the results

The results are in perfect agreement with the analytical solution.