Diagonalisation of the thermal matrix of mass

Summary:

To improve the regularity of the solution in the problems of transitory thermics, one of the solutions consists with “lumper” (i.e.: to condense on the diagonal) the thermal matrix of mass (matrix of capacity).

This possibility is accessible by modelings PLAN_DIAG, AXIS_DIAG and 3D_DIAG for the phenomenon THERMICS. It is activated at the time of the call to the orders of thermal calculation THER_LINEAIRE and THER_NON_LINE.

When these modelings are used, only the linear finite elements (2D and 3D) have a matrix of lumpée mass. Indeed, the direct diagonalisation does not give satisfactory results for the quadratic finite elements. Consequently, for the quadratic finite elements 2D, one carries out a cutting in linear elements, which are lumpés. On the other hand, for the quadratic finite elements 3D, one does not make a diagonalisation of the matrix of mass.

The theoretical results are illustrated by the thermomechanical calculation of a cylinder subjected to a thermal shock.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Contents

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

1 Introduction

One is interested in transitory thermal calculations where intervene of abrupt variations of the loadings - for example, thermal shocks. In certain cases, it is noted that the temperature oscillates spatially and temporally. Moreover, if one observes a profile of temperature at a given moment of the transient, the temperature can in certain nodes exceed the terminals min. and max. imposed by the initial conditions and the boundary conditions. This physically unacceptable result what is called violates the “principle of the maximum”.

The diagonalisation of the matrix of mass can solve these problems of going beyond the maximum. This is detailed in the note [bib1]. One is satisfied here to recall the principal results of them.

One points out the principle of the maximum in the continuous case, then one expresses sufficient conditions which make it possible to check it for the discrete equations. It is shown in particular that the diagonalisation of the thermal matrix of mass is one of these sufficient conditions and one presents various methods for diagonaliser $\mathbf{M}$.

Another sufficient condition depends on the thermal matrix of rigidity (conduction). One more particularly studies from this point of view the finite elements of thermics used in *Code_Aster*.

It results from it that in the case of the linear elements, all the sufficient conditions to check the principle of the maximum are gathered. In particular, the diagonalisation mass makes it possible indeed to obtain a regular solution. On the other hand, for the quadratic elements, one cannot prevent the oscillations.

One thus describes the solution suggested in *Code_Aster* : the modelings developed in 2D (AXIS_DIAG, PLAN_DIAG) function with linear elements (if the mesh is of order 2, one cuts out it in linear elements for thermal calculation). In 3D, only the linear elements are treated.

A digital study of a thermal shock on a cylinder makes it possible to illustrate these results.

2 Principle of the maximum

2.1 Statement of the principle for the continuous case

One gives here one of the statements possible of the principle of the maximum for the operator of heat (in the absence of terms of source, and in isotropic homogeneous linear thermics) [bib2].

That is to say Ω open limited $\mathbb{R}^n$ of border Γ , of which adherence is noted $\bar{\Omega}$.

That is to say $u(x,t)$ such as:

$$\frac{\partial u}{\partial t}-\Delta u\leq 0 \text{ sur } \Omega\times]0,T[,(T>0)$$

of class C^2 compared to x and u of class C^1 compared to t on $\Omega'\times]0,T[$

Then $\underset{\bar{\Omega}\times[0,T]}{Max}\, u=\underset{P}{Max}\, u$, where $P=(\bar{\Omega}\times\{0\})\cup(\Gamma\times[0,T])$ is the border of the cylinder $\Omega\times]0,T[$.

This result thus ensures that the maximum of u is inevitably reached either at the time of the initial conditions or on an edge of the field during the transient.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

2.2 Respect of the principle of the maximum at the discrete level

The equation of heat (thermal conduction) is considered:

$$\operatorname{div}(\lambda.\nabla T)+s(x,t)=\rho C_p\frac{\partial T}{\partial t} \quad \textbf{+ limiting Conditions + initial condition } T(t_0,x)=T^0(x)$$

with T temperature
s heat per unit of volume (internal sources)
t variable of time
x variable of space
λ thermal coefficient of conductivity
ρC_p voluminal heat with constant pressure

Limiting types of conditions (into linear):

- Imposed temperature: condition limits of Dirichlet

$$T(x,t)=T_{imp}(x,t)\,\text{sur}\,\Gamma_{imp}$$

- Imposed normal flow: condition of Neumann defining flow entering the field

$$-\mathbf{q}(x,t).\mathbf{n}=f(x,t)\,\text{sur}\,\Gamma_{flux}$$

- Exchange: condition limits of Fourier modelling the convectifs exchanges on the edges of the field

$$-\mathbf{q}(x,t).\mathbf{n}=h(x,t)\left(T_{ext}(x,t)-T(x,t)\right)\text{sur}\,\Gamma_{échange}$$

The variational formulation of the problem is the following one: [bib3]

$$\int_\Omega \rho C_p\frac{\partial T}{\partial t}.v\,d\Omega+\int_\Omega \lambda\nabla T.\nabla v\,d\Omega+\int_{\Gamma_{échange}} hT.v\,d\Gamma=\int_\Omega s.v\,d\Omega+\int_{\Gamma_{flux}} f.v\,d\Gamma+\int_{\Gamma_{échange}} hT_{ext}.v\,d\Gamma$$

$\forall v$ checking $v=T_{imp}(x,t)\,\text{sur}\,\Gamma_{imp}$

After discretization in space of this equation, one obtains the system:

$$\mathbf{M}\left\{\frac{\partial \mathbf{T}}{\partial t}(t)\right\}+\mathbf{K}\mathbf{T}(t)=\mathbf{F}(t)\,.$$

with $\mathbf{T}(t)$: vector of the nodal temperatures

$\mathbf{M}$: thermal matrix of mass $\quad \mathbf{M}=\sum_e\int_{\Omega_e}\rho C\,\mathbf{N}\mathbf{N}^T dV$

$\mathbf{K}$: thermal matrix of rigidity $\quad \mathbf{K}=\sum_e\left(\int_{\Omega_e}\lambda\nabla\mathbf{N}.\nabla\mathbf{N}^T dV+\int_{\Gamma_{échange_e}} h\mathbf{N}.\mathbf{N}^T d\Gamma\right)$

$\mathbf{F}$: vector of the second member $\quad \mathbf{F}=\sum_e\left(\int_{\Omega_e} s\mathbf{N}\,d\Omega+\int_{\Gamma_{flux_e}} f\,\mathbf{N}\,d\Gamma+\int_{\Gamma_{échange_e}} hT_{ext}\mathbf{N}\,d\Gamma\right)$

$\mathbf{N}$: (functions of form)

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

For the discretization in time, one applies one θ - method $(\theta\in[0,1])$, which leads to:

$$(\mathbf{M}+\theta\Delta t\,\mathbf{K})\,\mathbf{T}^{n+1}=(\mathbf{M}+(\theta-1)\Delta t\,\mathbf{K})\,\mathbf{T}^{\mathbf{n}}+\theta\langle\mathbf{F}^{n+1}\rangle+(1-\theta)\langle\mathbf{F}^{n}\rangle$$

where $\mathbf{T}^{n}$, $\mathbf{T}^{n+1}$ are the vectors of the nodal temperatures at the moments t_n, t_{n+1} .

2.3 Conditions sufficient for the respect of the principle of the maximum at the discrete level

One of the characteristics of nonrespect of the principle of the maximum is the appearance of oscillations (temporal or space): if one observes the variation in the temperature in a node in the course of time, one notes that the solution oscillates and exceeds the values minimal and maximum determined by the initial conditions and the limiting conditions. Or, at a given moment, one observes space oscillations.

One thus seeks sufficient conditions on Δt , $\mathbf{K}$ and $\mathbf{M}$ so that the solution does not oscillate in the course of time ([bib1], [bib4], [bib5]). Indeed, one cannot obtain requirements and sufficient. One thus seeks conditions of nonoscillation of the solution in the course of time. If those are checked, it will be checked that the space oscillations also disappeared, and then the respect of the principle of the maximum is assured.

Assumptions:

To be able to express these sufficient conditions of nonoscillation, two assumptions should be added:

- *one places oneself at the elementary level. The respect of the properties at the elementary level is enough so that the conditions of nonoscillation are checked for the assembled matrices.*
- *it is considered that* ***the matrix of rigidity*** $\mathbf{K}$ ***is formed only of the voluminal term***

$$\mathbf{K}_{\mathbf{V}}=\int_{\Omega_e}\lambda\nabla\mathbf{N}.\nabla\mathbf{N}^{T}dV$$

This assumption is not valid for all the limiting conditions (see paragraph 3.1.3).

The sufficient conditions of nonoscillation amount expressing conditions on the step of time and the diagonal and extra-diagonal terms of $\mathbf{M}$ and $\mathbf{K}$ so that certain properties of these matrices are checked (based on the monotony of the matrices) [bib1]:

$$M_{ij}+\theta.\Delta tK_{ij}\leq 0 \qquad i\neq j \qquad \text{éq 2.3-1}$$

$$M_{ij}+(\theta-1)\Delta tK_{ij}\geq 0 \qquad i\neq j \qquad \text{éq 2.3-2}$$

$$M_{ii}+(\theta-1)\Delta tK_{ii}\geq 0 \qquad \forall i \qquad \text{éq 2.3-3}$$

In the case general, the extra-diagonal terms can be of unspecified sign. A fast study makes it possible to determine the conditions on Δ T according to their signs so that the preceding equations are checked:

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

	$K_{ij} \geq 0$	$K_{ij} \leq 0$
$M_{ij} \geq 0$	$M_{ij} + \theta . \Delta t K_{ij} \leq 0 \quad i \neq j$ [éq 2.3-1] unconditionally distorts except $M_{ij} = K_{ij} = 0$	$\max_{i \neq j} \left(\frac{M_{ij}}{-\theta K_{ij}} \right) \leq \Delta t \leq \min_{i} \left(\frac{M_{ii}}{(1-\theta) K_{ii}} \right)$
$M_{ij} \leq 0$	$M_{ij} + (\theta - 1) \Delta t K_{ij} \geq 0 \quad i \neq j$ [éq 2.3-2] unconditionally distorts except $M_{ij} = K_{ij} = 0$	$\max_{i \neq j} \left(\frac{M_{ij}}{(1-\theta) K_{ij}} \right) \leq \Delta t \leq \min_{i} \left(\frac{M_{ii}}{(1-\theta) K_{ii}} \right)$
	Whatever is Δt and forms it $\mathbf{M}$, there is **risk** oscillations.	Interval to be respected on Δt. The diagonalisation of $\mathbf{M}$ allows to remove the lower limit.

The sufficient conditions to avoid the oscillations are then:

$$K_{ij} \leq 0 \quad i \neq j$$
$$\Delta t_{\min} \leq \Delta t \leq \Delta t_{\max}$$

with:

- $\Delta t_{\max} = \min_{i} \left(\frac{M_{ii}}{(1-\theta) K_{ii}} \right)$ and
- $\Delta t_{\min} = \max_{i \neq j} \left(\frac{M_{ij}}{(1-\theta) K_{ij}}, \frac{M_{ij}}{-\theta K_{ij}} \right)$,

Consequently, it is necessary initially that the elementary matrices check $K_{ij} \leq 0$ (it is the case of **linear finite elements** studied further).

With regard to the interval on the step of time:

If the oscillations are due to a step of too large time $(\Delta t > \Delta t_{\max})$, one can advise:

- **that is to say to choose a diagram of integration in time of the implicit type** $(\theta = 1)$, to eliminate the upper limit from the interval.
- **that is to say to decrease** Δt. (In practice, it is difficult to know an order of magnitude of $\Delta t_{\max}$).

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Rather often, the problem of the oscillations arises for steps of small times $\left(\Delta t<\Delta t_{\min}\right)$; indeed, to take into account the variations of the solution (for example, at the time of a thermal shock), one is brought to choose a fine discretization in time. In this case, to avoid the oscillations, one can suggest:

- **that is to say to increase the step value of time**. In practice, this is not always possible because Dt can be imposed by the nature of the problem (fast variation of the loading). Moreover, it is difficult to have an order of magnitude of $\Delta t_{\min}$.
- **that is to say to decrease the size of the meshs** and thus to increase the number of elements. Indeed, the value of $\Delta t_{\min}$ depends directly on the space discretization:

  The forms of the elementary matrices are indeed:

$$M_{ij}=\int_{\Omega_e}\rho CN_i N_j dV$$
$$K_{ij}=\int_{\Omega_e}\lambda\nabla N_i\nabla N_j dV$$

  For the elements 2D, the terms of $\mathbf{M}$ are thus of the form $\rho C\times\text{Surface}$ whereas those of $\mathbf{K}$ are only function of λ . This solution remains the best if one is not limited by the cost calculation, because the thermal solution and especially mechanics will be all the more precise.

- **Maybe of diagonaliser the matrix** $\mathbf{M}$, which removes the lower limit of the interval.
  It is the solution suggested here.

In the continuation of the study, one is interested only in the problem of the oscillations which appear for the steps of too small times: $\Delta t<\Delta t_{\min}$. One more precisely presents the method of diagonalisation of the matrix $\mathbf{M}$ chosen, and various types of elements to which it applies.

3 Method of diagonalisation selected and types of elements

3.1 Limiting elements and conditions such as the extra-diagonal terms of $\mathbf{K}$ are negative

It was seen that the diagonalisation of $\mathbf{M}$ is effective only when the extra-diagonal terms of the matrix of rigidity $\mathbf{K}$ are negative. In the contrary case, one of the sufficient conditions of non-oscillation is unconditionally false, whatever the form of $\mathbf{M}$.

For each finite element used in thermics in *Code_Aster,* one checks if the elementary matrix of rigidity of the element has negative extra-diagonal terms, while resting mainly on [bib11], who gives the analytical expressions of the elementary matrices for the classical finite elements. One summarizes here the observations made in [bib11].

3.1.1 Linear elements

3.1.1.1 Elements TRIA3, TETRA4, PENTA6

The elementary matrix $\mathbf{K}$ is function of the cotangents of the angles. If one of the angles is blunt ($\geq 90°$), certain extra-diagonal terms of $\mathbf{K}$ are positive. If all the angles are acute, the property is checked.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

One has the same type of result in 3D for the tetrahedron with 4 nodes and the pentahedron with 6 nodes.

3.1.1.2 Elements QUAD4 and HEXA8

Certain extra-diagonal terms of $\mathbf{K}$ can be positive if the element is lengthened too much in a direction. If not, the property is checked.
One has the same type of result in 3D for element HEXA8.

3.1.1.3 Element 3D pyramid with 5 nodes

For this element, the functions of form are not any more of the polynomials but rational fractions in x, y, z. For this kind of element, one does not have the expression, even approximate of $\mathbf{K}$.

3.1.2 Quadratic elements

3.1.2.1 Element TRIA6

In $\mathbf{K}$, certain extra-diagonal terms are necessarily positive.

3.1.2.2 Element QUAD9

In the same way, on the analytical expression of the terms of $\mathbf{K}$, it is noted that some of the diagonal terms extra - are necessarily positive.

3.1.2.3 Element QUAD8

For this element, there is not the expression supplements $\mathbf{K}$ for the real element. But for the element of reference, one notes that certain extra-diagonal terms are positive.

3.1.3 Limiting conditions

The matrix $\mathbf{K}$ is not always reduced at the end $\mathbf{K_V}=\int_{\Omega_e} \lambda \nabla \mathbf{N}.\nabla \mathbf{N}^T dV$. According to the limiting conditions used, a surface term can be added (case of the exchange or non-linear flow). This term can be positive.

For a condition of exchange for example, the term $\mathbf{K_{échange}}=\int_{\Gamma_{échange_e}} h\mathbf{N}.\mathbf{N}^T d\Gamma$ is added to the matrix $\mathbf{K}$. It is always positive and thus does not check properties 2.3 stated above ($K_{ij}\geq 0$). The risk of oscillation is thus always present even by diagonalisant the matrix masses. In this case, to refine the grid where the limiting condition is applied, makes it possible to reduce the positive contribution of the limiting condition and often to remove the oscillations.

3.1.4 Conclusion on the elements: properties of the matrices $\mathbf{K}$

For the linear elements, if the real element is not too irregular, extra-diagonal terms of $\mathbf{K}$ are quite negative. For the quadratic elements (in 2D) and certain limiting conditions, certain extra-diagonal terms of $\mathbf{K}$ are positive. Even while diagonalisant $\mathbf{M}$, one cannot ensure that the solution will not oscillate.

In *Code_Aster*, to eliminate the problems from oscillation and going beyond the maximum, one diagonalise only matrices of mass for the thermal calculations carried out on linear elements. For the quadratic elements, one saw that one could not diagonaliser directly the matrix of mass. One thus cuts out these elements in linear elements which themselves are lumpés. This is applied to the quadratic elements 2D in *Code_Aster*, but not with the quadratic elements 3D, not for reasons of method but because automatic cutting is difficult to implement in 3D.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

3.2 Method of diagonalisation: Integration with the nodes of the elements

If the elementary matrix of mass is calculated by digital integration, its terms are written in the form [bib8]

$$M_{ij}=\int_{\Omega_c} \rho CN_i N_j \,\mathrm{dV} \simeq \sum_{q=1}^{N} W_q (\rho CN_i N_j)_q$$

where $N_i \rho CN_j$ is evaluated with $\mathrm{q}^{\text{ième}}$ not integration
and W_q is the weight of integration associated with this point.

Classically, the points of integration are the points of Gauss; the position of N points and their weight are defined so that the diagram integrates the polynomials of degree exactly $2\mathrm{N}+1$. If one chooses **points of integration to the nodes of the element**, one obtains: $M_{ij}=0$ for $i\neq j$. This method of integration is also called method of Newton-Dimensions.

Notice 1: Axisymmetric problems:

If the points of integration are with the nodes, one will have, for any type of element, of the worthless masses on the axis of symmetry.

Indeed, $M_{ij}=\int \rho C\, N_i N_j\, 2\pi\, r\, \mathrm{dr}\, dz$

$$M_{ij}\simeq \delta_{ij}\, 2\pi\, \rho C W_i \,\mathrm{Jac}\, r(x_i)$$

If the point of integration i is a node of the axis, $r(x_i)=0$ and the corresponding mass is worthless.

*For the axisymmetric elements, **method of integration to the nodes is thus not adapted close to the axis** . In this case, it is necessary to integrate into the points of Gauss the elements which touch the axis of revolution, by using usual modeling (`AXIS`).*

Notice 2: other possible methods of diagonalisation:

Other methods are studied in [bib1], to test diagonaliser in particular the quadratic elements. In practice, it are not retained at the present time in Code_Aster.

- *Setting at the level of the diagonal terms ([bib9], [bib10]): Hinton suggests the setting at the level of the diagonal terms of the matrix $\mathbf{M}$ consistent, so that the total mass of the element is preserved. One notes that the lumpées masses are always positive, even for the elements quadrangles to 8 and 9 nodes.*
- *Summation by line ([bib10]): One summons the values of M_{ij} by line and one concentrates the result on the diagonal. Unfortunately, this process can lead to negative masses, in particular for the quadrangle with 8 nodes.*

Notice 3:

For the quadratic elements, one notes in [bib1] that, even while diagonalisant with the method of setting at the level of the diagonal terms, one obtains oscillations. One cannot thus use these elements within the framework of the diagonalisation (i.e. for a relatively coarse grid with respect to the speed of the thermal transient).

One can of course use the quadratic elements in thermics, on condition that adapting the smoothness of the grid to the stiffness of the thermal shock.

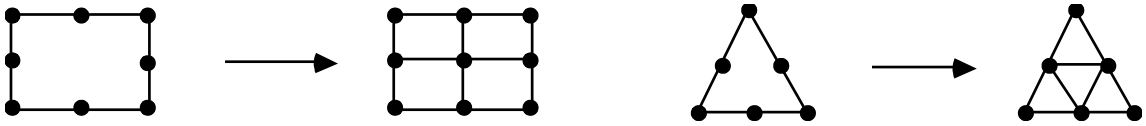
Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

4 Implementation in *Code_Aster*

In order to eliminate the oscillations from the temperature in space and time, modelings AXIS_DIAG, PLAN_DIAG and 3D_DIAG carry out the diagonalisation of the matrices of mass during thermal calculation linear (THER_LINEAIRE) and non-linear (THER_NON_LINE). To guarantee the effectiveness of it, it was seen that it should be carried out on linear elements.

If the grid is linear, one carries out simply a diagonalisation of the matrices of mass by integration to the nodes.

In the case of a quadratic grid, in 2D, one carries out a thermal calculation ISO-P2: calculation on a QUAD9 is brought back to a calculation on 4 QUAD4; in the same way, one passes from a TRIA6 to 4 TRIA3.



This makes it possible not to lose the smoothness of the discretization of the grid, as well for the solution of the thermal problem as for that of the mechanical problem. Indeed, one shows in [bib1] that this solution is preferable with that which consists in carrying out thermal calculation on linear meshs which are based on the nodes tops of the quadratic meshs (what is normal since the discretization is finer).

Modelings available are thus:

4.1 Modelings 2D

Modeling	**PLAN_DIAG**	**AXIS_DIAG**
Mesh	Element	Element
TRIA3	THPLTL3	THAXTL3
QUAD4	THPLQL4	THAXQL4
SEG2	THPLSL2	THAXSL2
TRIA6	THPLTL6	THAXTL6
QUAD9	THPLQL9	THAXQL9
SEG3	THPLSL3	THAXSL3

Comments on elementary calculations 2D:

For the linear elements: the terms of mass (matrix to the first member and vector with the second member) are lumpés by integration with the nodes. The new elements have options of elementary calculations identical to the classical elements. The only modified elementary options are thus MASS_THER and CHAR_THER_EVOL.

For the quadratic elements: calculation is ISO-P2. Calculation on an element QUAD9 (resp. TRIA6) is brought back has a calculation on 4 linear elements QUAD4 (resp. 4 TRIA3) whose terms of mass are lumpés by the preceding method. The matrices and vectors of each of the 4 linear elements are assembled on the level of the elementary routine of calculation. By homogeneity, on the elements of edges, one calculates the elementary terms on 2 SEG2, then one assembles.

Elements THPLTL6, THAXTL6,THPLQL9, THAXQL9 the functions of form of the linear elements have in which they are cut out.

Caution:

There is no element associated with mesh QUAD8. Consequently, if the grid is composed of quadratic meshs, it is initially necessary to change the QUAD8 into QUAD9 has the assistance of the order CRÉA-MAILLAGE :

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

```
CREA_MAILLAGE (MODI_MAILLE: (OPTION: 'QUAD8_9')).
```

Into axisymmetric: so elements of the grid touch the axis, one should not integrate into the nodes which are on the axis. It is thus necessary to isolate this layer from elements and to affect modeling AXIS to him.

4.2 Modeling 3D

Modeling	**3D_DIAG**

Mesh	**Element**
HEXA8	THER_HEXA8_D
PENTA6	THER_PENTA6_D
TETRA4	THER_TETRA4_D
QUAD4	THER_FACE4_D
TRIA3	THER_FACE3_D

Comments on elementary calculations 3D:

For the linear elements: as in 2D, the terms of mass (matrix to the 1st member and vector with the 2nd member) are lumpés by integration with the nodes (3rd family of points of Gauss).

For the quadratic elements, it would be necessary to cut out those in linear elements. This cutting is delicate to implement, because it results in creating a new element (PENTA18) for nodes in the middle of each quadrilateral face (and it would also be necessary to create a new element PYRAM14).

One diagonalise thus currently only the linear elements 3D.

With regard to the pyramids with 5 nodes, integration with the nodes was tested but does not function well. Cf [§ 3.1.1.3] (it is not known if all the extra-diagonal terms are negative). Modeling '3D_DIAG'thus does not exist for the pyramids with 5 nodes. In any case these elements are minority in a grid 3D: it are generated only by the voluminal free maillor of GIBI, which creates some with the need of them to supplement the hexahedral grid.

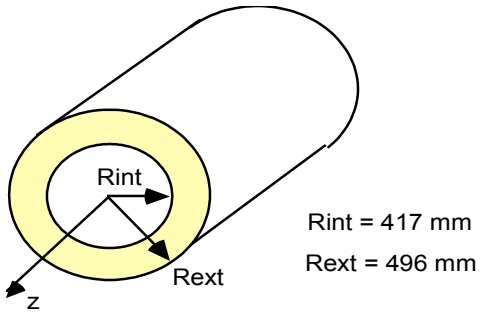
Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

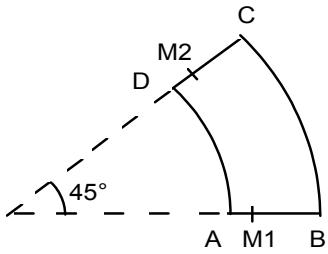
5 Thermal calculation of a cylinder subjected to a cold shock

One illustrates on a digital example what was shown previously; namely that the diagonalisation is effective to check the principle of the maximum.
One takes as a starting point the industrial example of the cooling of a moulded elbow: one applies a cold thermal shock ($289\,°C$ with $20\,°C$) on a fissured elbow. During the transient of cooling, the temperature calculated in certain nodes reached $310\,°$ without diagonalisation of the matrices of mass. For the example treated here, one restricts with a hollow roll of the same dimension than the elbow to which one applies a cold thermal shock.

5.1 Data

A presumedly infinite hollow roll is studied. As there is no dependence compared to z (infinite cylinder), one limits the study to a calculation plan. By reason of symmetry, one nets only one portion of the structure.

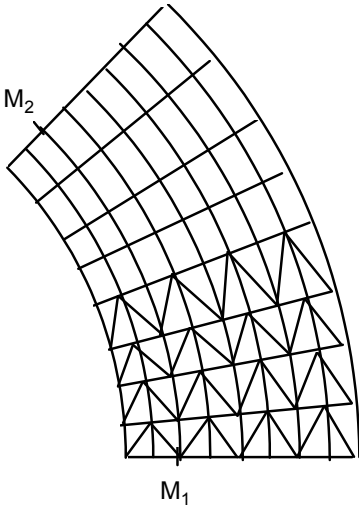




Coordinates of the points:

	$x\ (mm)$	$y\ (mm)$	$z\ (mm)$
$M1$	436.75	0.	0.
$M2$	$436.75\cos 45°$	$436.75\sin 45°$	0.

Calculations are carried out on a linear grid (meshs TRIA3-QUAD4):



Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Characteristics of the grid:

Many nodes: 90
Number and type of meshs: 64 TRIA3,32 QUAD4

Characteristics of material:

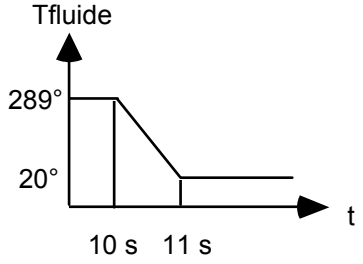
$\lambda = 19{,}97 \text{ W/m °C}$
$\rho Cp = 4.89488\,10^{6} \text{ J/m}^3 \text{ °C}$

Limiting conditions and loading:

To ensure invariance by rotation, one forces conditions of null heat flow on the faces AB and CD. The external wall is supposed perfectly insulated. On the internal skin AD, the heat transfer between the cylinder and the fluid are modelled by a coefficient of high convectif exchange:

$$h = 40\,000 \text{ W/m² °C}.$$

The cold thermal shock applied to the moulded elbows is represented by a linear variation of the temperature of the fluid circulating in the pipe: $289° \rightarrow 20°$ in 12s. In order to accentuate the problem of going beyond the maximum and thus to better highlight the influence of the diagonalisation, a more brutal shock is adopted: $289° \rightarrow 20°$ in 1s.



The following discretization in time is adopted:

of $t = 0\,s$ with $t = 10\,s$ 1 pas de time
,
of $t = 10\,s$ with $t = 11\,s$ 2 pas de time
,
of $t = 11\,s$ with $t = 25\,s$ 7 pas de time
,
of $t = 25\,s$ with $t = 60\,s$ 10 pas de time
,

Numerically, the value retained for the parameter of the discretization in time is $\theta = 0{,}57$.

5.2 Results

The following figures show the profiles of temperature in the thickness of the cylinder at the moment $t = 15\text{s}$ (moment when the goings beyond the maximum are largest) without diagonalisation of the matrices of mass.

One gives also the temporal evolution T(t) with the nodes M_1 and M_2 located at a quarter thickness of the internal skin.

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

Without diagonalisation, it is noted that the temperature oscillates in time and in space exceed the maximum value of $289\,°$ at the beginning of the transient.

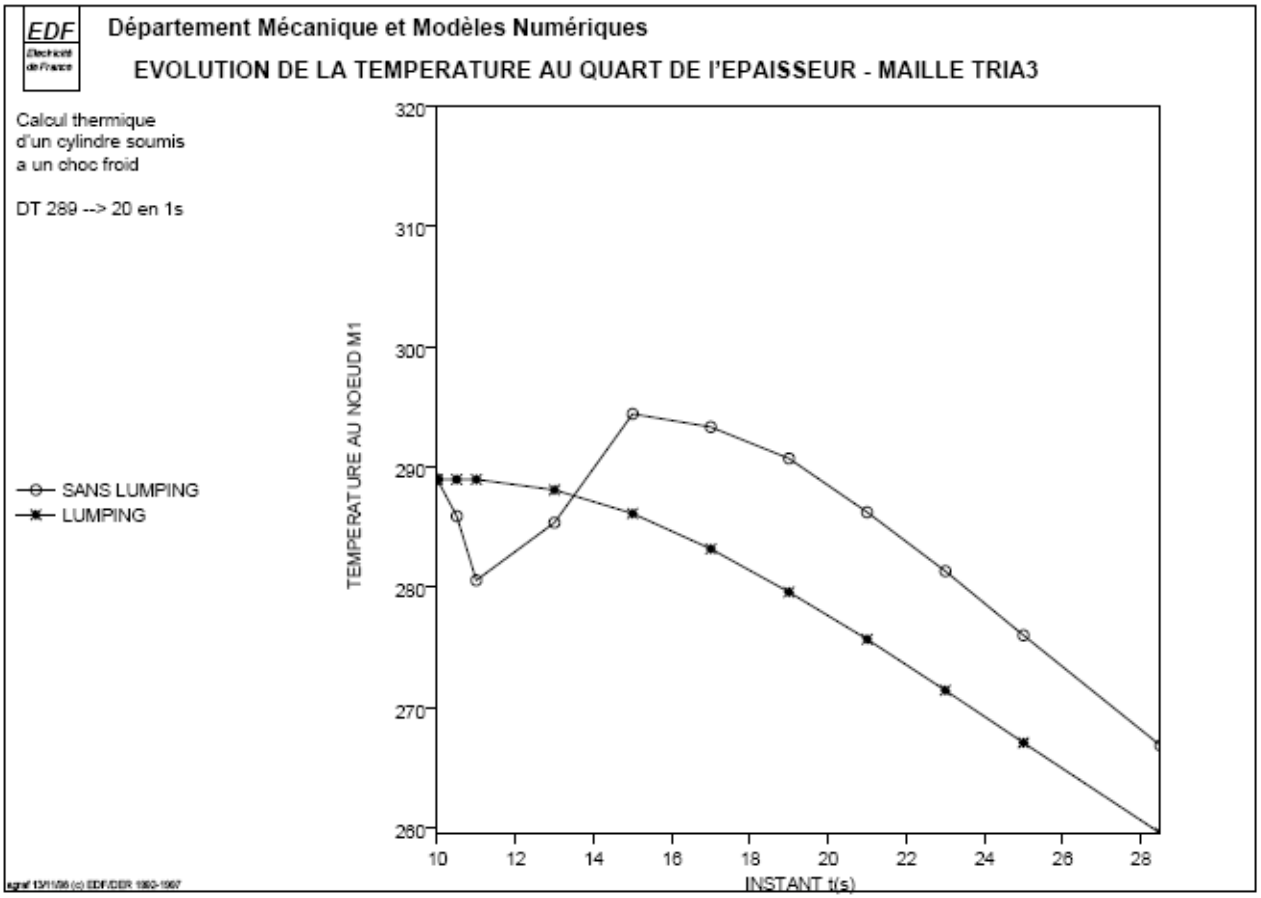
With diagonalisation on the linear elements, one observes a regular solution without going beyond the maximum.

A similar study was conducted on linear elements 3D (tetrahedron with 4 nodes, pentahedron with 6 nodes, hexahedron with 8 nodes). The results lead to the same conclusions: with diagonalisation, the oscillations of the temperature disappear for calculation on the linear elements 3D.

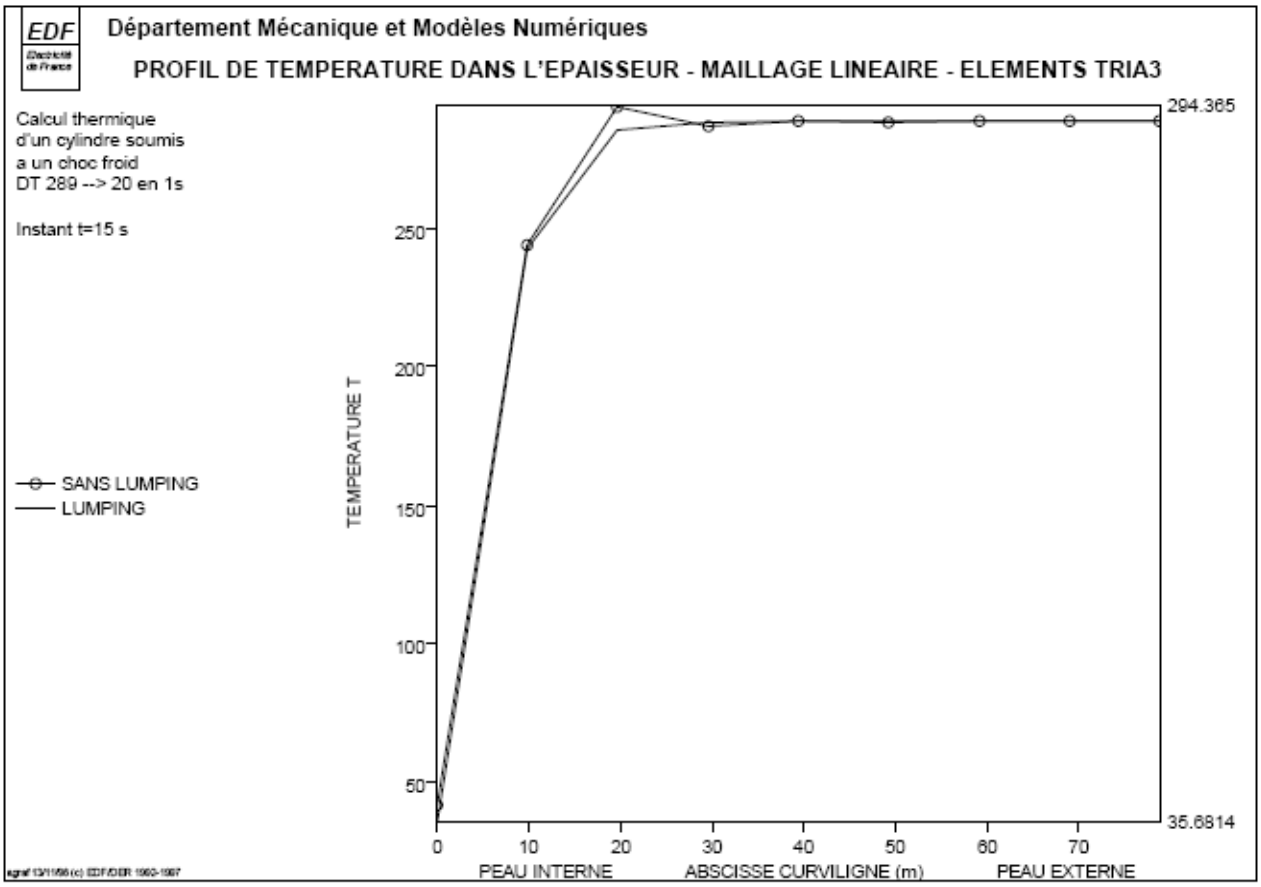
Notice complementary concerning thermomechanical calculation:

Another study was carried out in [bib1] estimating the consequences of the thermal diagonalisation on the mechanical results. It is noted that calculation ISO-P2 (quadratic elements divided into linear elements, whose matrices of mass are lumpées) provides satisfactory results. One eliminates the space oscillations from the temperature. But in the studied case, with a relatively coarse grid, the mechanical solution remains not very precise. Although the thermal solution is correct, to improve the solution in constraints, the grid should in any case be refined.

For the meshs TRIA3, the diagonalisation leads to a regular solution without going beyond the maximum:

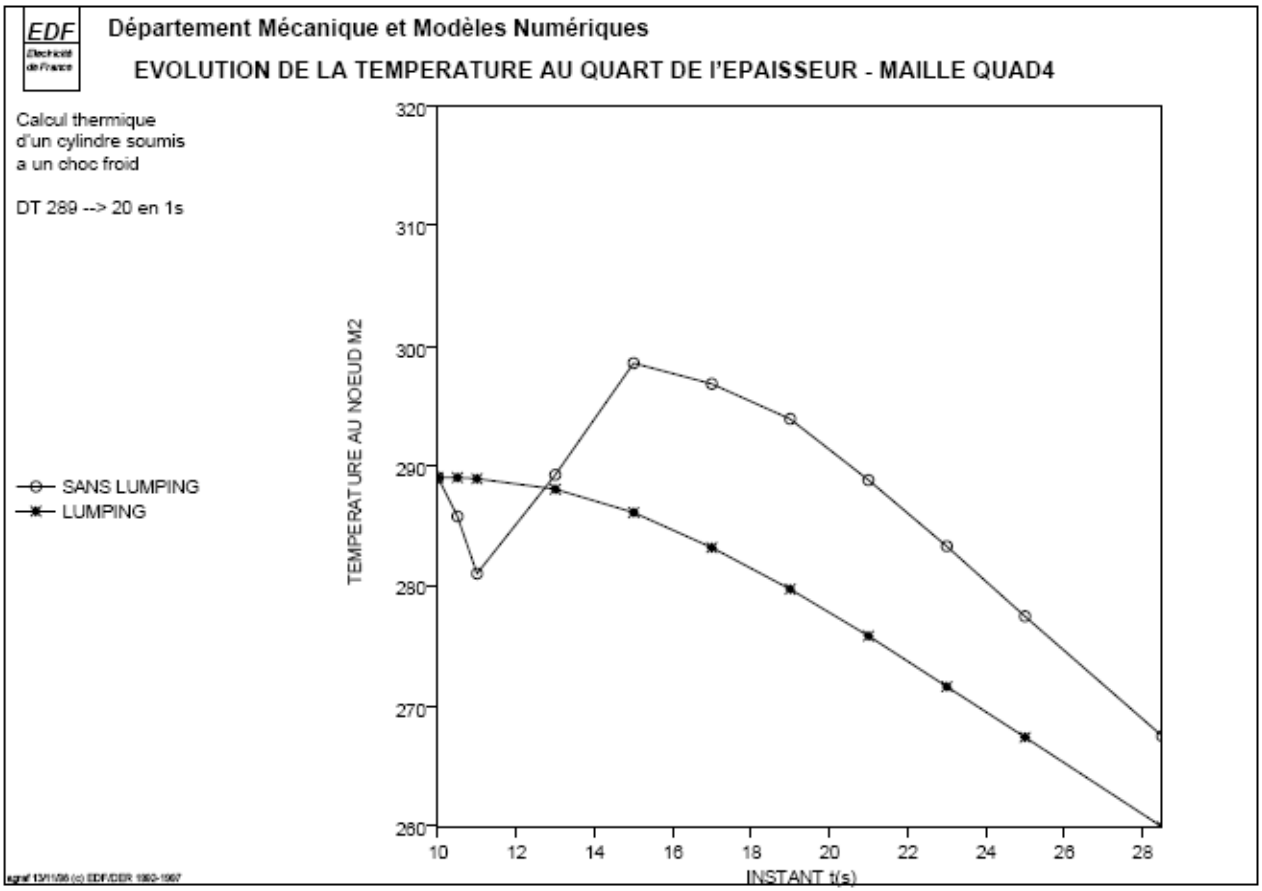


Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

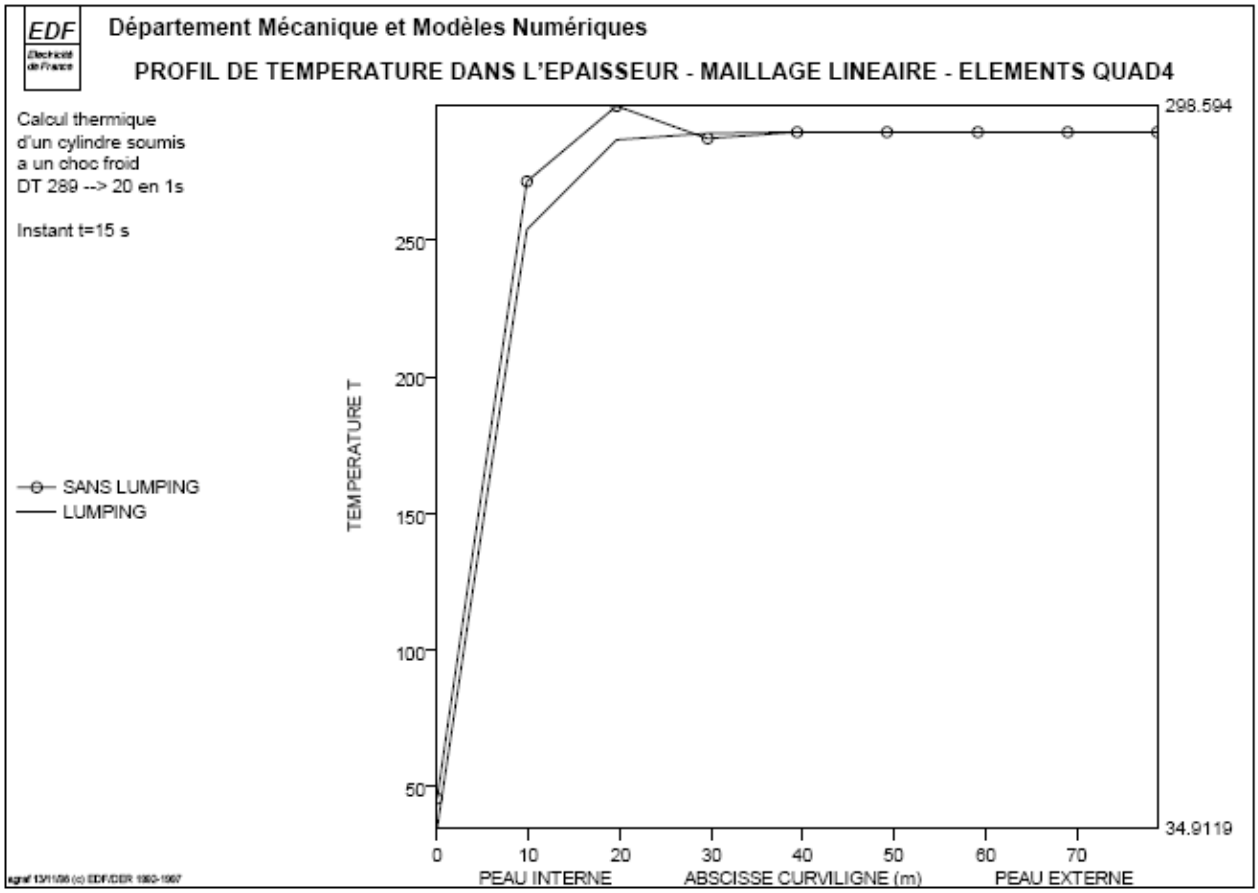


Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

For the meshs QUAD4, the diagonalisation conduit with a regular solution without going beyond the maximum:



Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)



Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

6 Conclusion

Modelings AXIS_DIAG, PLAN_DIAG and 3D_DIAG are proposed in order to solve the problems of going beyond the maximum with oscillation of the solution in space and times which appear during certain transitory thermal calculations with abrupt variation of the loading.

At the discrete level, the analysis leads to a sufficient condition of not-oscillation on the step of the discretization in time which must belong with an interval:

$$\Delta t_{\min} \leq \Delta t \leq \Delta t_{\max}$$

where values of $\Delta t_{\min}$ and $\Delta t_{\max}$ depend on the coefficients of matrices of mass and rigidity thermics as well as parameter θ discretization in time.

In practice, if the oscillations come from a step of too large time $\left(\Delta t > \Delta t_{\max}\right)$, one suggests the choice of an implicit scheme in time $\left(\theta = 1\right)$. If the steps of time are too small, the diagonalisation of the matrix of mass can make it possible to remove the oscillations.

For the linear elements, one shows that the diagonalisation makes it possible indeed to avoid the oscillations of the solution. For the quadratic elements, a diagonalisation direct is not enough to avoid the oscillations. For this kind of element, one cuts out them in linear elements, and one carries out a diagonalisation of the linear elements resulting by integration with the nodes (this is carried out only in 2D for the moment).

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)

7 Bibliography

1) M.A. REDON, J.M. PROIX: Diagonalisation of the thermal matrix of mass. Note EDF/DER/HI-75/97/008/0

2) H. BREZIS: Functional analysis, Theory and applications. Masson (1983).

3) Handbook of reference. Document [R5.02.01]: Linear algorithm of thermics.

4) H. OUYANG, D. XIAO: Criteria for eliminating oscillation in analysis of heat-conduction equation by finite-element method. Communications in numerical methods in engineering. Vol.10, 453-460 (1994).

5) B. NITROSSO: Study of the principle of the discrete maximum - September 1996. Communication interns EDF.

6) O.C. ZIENKIEWICZ: Finite element method (3ème edition). Mc Graw-Hill (1977).

7) I. FRIED, D.S. MELKUS: Finite element farmhouse matrix diagonalisation by numerical integration with disastrous No convergence loss. Int. Newspaper of solids and structures. Vol.11, 461-466 (1975).

8) G. COHEN, A. ELMKIES: Triangular finite elements P2 with condensation of mass for the equation of the waves. Research report INRIA n°2418 - September 1994.

9) E. HINTON, A. ROCK, O.C. ZIENKIEWICZ: With note one farmhouse diagonalisation in related processes in the finite element method. Int. J. Earthquake Engineering and Structural Dynamics, Vol.4, 245-249 (1976).

10) T. HUGHES: The finite element method. Linear static and dynamic finite element analysis. Prentice Hall (1987).

11) J.P. GREGOIRE: Collection of elementary matrices of mass and the Laplacian for the principal elements of simple form. Note EDF HI-76/96/012 - June 1996.

8 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
5	J.M.PROIX-R&D/AMA	Initial text

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2021 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)