Finite elements of joint mechanics and finite elements of hydraulic joint coupled

Summary:

This documentation relates to the description of the finite elements of linear and quadratic mechanical joint. Modelings in pure mechanics (xxx_JOINT) the two types of grids support, while hydraulic modelings coupled (xxx_JOINT_HYME) are implemented only for grids quadratic. One carries out the elimination of fictitious degrees of freedom of pressure for quadratic modelings in pure mechanics. ItS modelings make it possible to simulate the evolution of a crack along a predetermined way. The second type of element also takes into account the interaction of mechanics with a flow of fluid inside the crack.

One presents successively the following points:

1) geometry of the elements
2) local reference mark with the joint and matrix of passage of the total reference mark to the local reference mark
3) jump of displacement in the joint
4) gradient of pressure of fluid
5) nodal vector of the efforts intérieurs as well as the elementary tangent matrix

1 Geometry of the elements

1.1 Geometry of the linear joint 2D

The element of joint in 2D is a quadrangle with four nodes (QUAD4) with two small sides and two large sides [12] and [34] (see figure 1) which represent the two segments Γ^+ and Γ^- of an interface (or lips of a crack) between two pennies two-dimensional fields. To distinguish the sides Γ^+ and Γ^-, the local classification of the nodes must be done obligatorily as on figure Ci below:

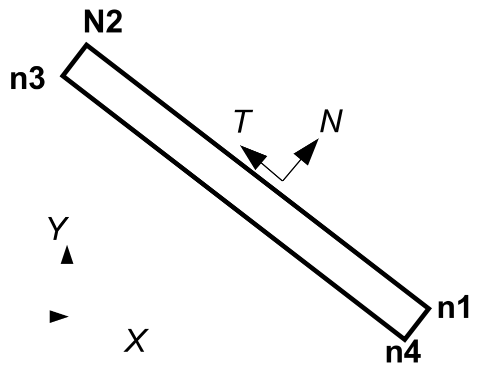

Figure 1 : Element of joint 2D with good local classification.

By convention the face Γ^- is given by nodes 3 and 4 and the face Γ^+ by nodes 1 and 2. The normal n face is directed Γ^- towards the face Γ^+.

1.2 Geometry quadratic joint 2D

These elements are quadrangle with 8 nodes (QUAD8), nodes 1,2,3,4,5,7 carry mechanical degrees of freedom of displacement. Nodes 6 and 8 carry degrees of freedom of pressure so representing the flow of fluid for hydraulic coupled modeling. They are eliminated for pure mechanical modelings.

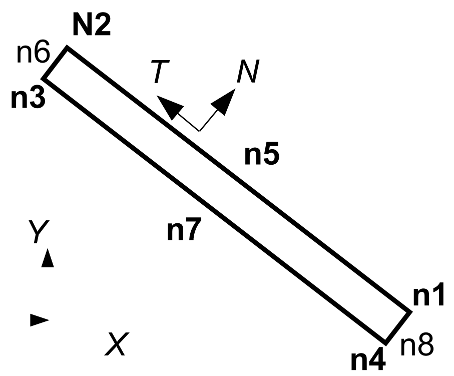

Figure 2 : Element of joint HYME 2D with good local classification.

By convention the face Γ^- is given by nodes 3,4,7 and the face Γ^+ by nodes 1,2,5. The normal n face is directed Γ^- towards the face Γ^+.

1.3 Geometry of the linear joint 3D

The elements of joint in 3D make it possible to represent a surface S between two pennies voluminal fields Ω^+ and Ω^-. They are compatible with the grid of under fields. If volume is with a grid with HEXA8, the joints to be used are also HEXA8 (hexahedrons with eight nodes). If volume is with a grid with PENTA6 or of TETRA4, the joints to be used are PENTA6 (pentahedrons with six nodes).
To distinguish the upper surfaces S^+ (related to Ω^+) and lower S^- (related to Ω^-), it is necessary to impose a local classification of the nodes quite specific (see figure 3).

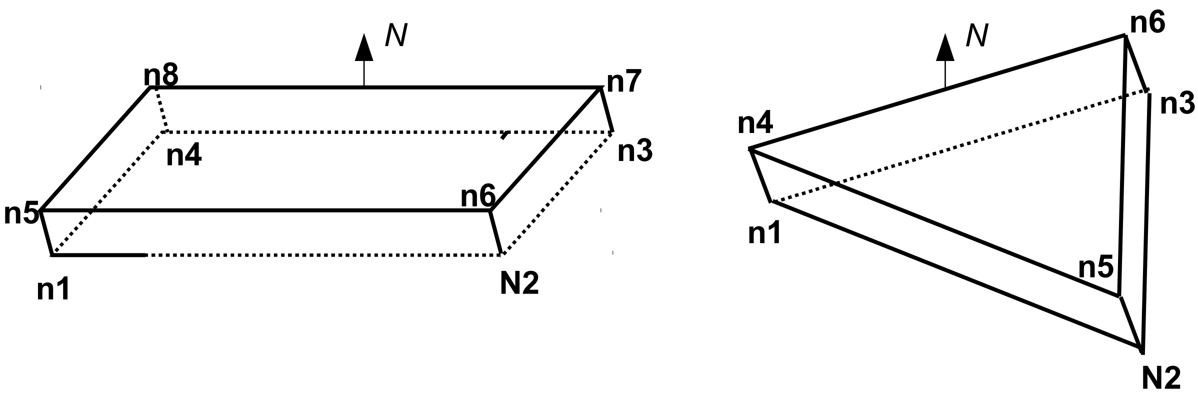

Figure 3 : Diagram of the elements of joint 3D HEXA8 and PENTA6 with good local classification.

By convention the face Γ^- is given by nodes 1,2,3,4 forHEXA8 (or 1,2,3 for PENTA6) and the face Γ^+ by nodes 5.6, 7, 8 forHEXA8 (or 4,5,6 for PENTA6). The normal n face is directed Γ^- towards the face Γ^+.

1.4 Geometry quadratic joint 3D

These elements of joint usable for modelings are coupled HYME or for modelings in pure mechanics by the elimination of DDL of pressure. In 3D they make it possible to represent a surface S between two pennies voluminal fields *in pure mechanics* Ω^+ and Ω^-. They are compatible with the grid of under fields. If volume is with a grid with HEXA20, the joints to be used are also HEXA20 (hexahedrons with twenty nodes). If volume is with a grid with PENTA15 or of TETRA4, the joints to be used are PENTA15 (pentahedrons with 15 nodes).

To distinguish the upper surfaces S^+ (related to Ω^+) and lower S^- (related to Ω^-) it is necessary to impose a local classification of the nodes quite specific (see figure 4).

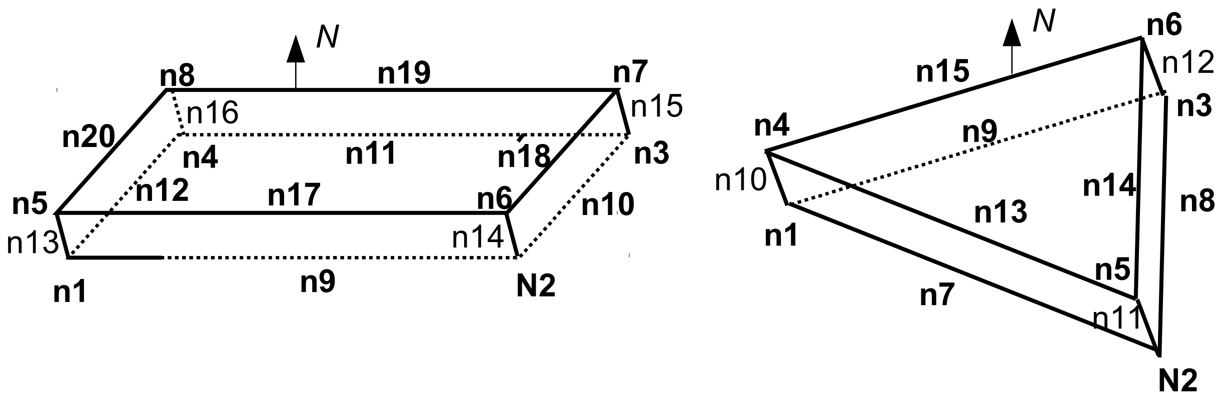

Figure 4 : Diagram of the coupled elements of joint HYME in 3D `HEXA20` and `PENTA15` with good local classification.

For`HEXA20`, the nodes medium 13,14,15,16 carry degrees of freedom of pressure for the flow of fluid, and all the others carry mechanical degrees of freedom of displacement.
For `PENTA15`, the nodes medium 10,11,12 carry degrees of freedom of pressure for the flow of fluid, and all the others carry mechanical degrees of freedom of displacement.

By convention the face Γ^- is given by nodes 1,2,3,4,9,10,11,12 for`HEXA20` (or 1,2,3,7,8,9 for `PENTA15`) and the face Γ^+ by nodes 5.6, 7, 8,17,18,19,20 for`HEXA20` (or 4,5,6,13,14,15 for `PENTA15`). The normal n face is directed Γ^- towards the face Γ^+ .

1.5 Automatic construction of meshs of joint

The order `MODI_MAILLAGE` used with the keyword `ORIE_FISSURE` allows to impose the good local classification of the meshs of joint in 2D or 3D (see Doc. [U4.23.04]).

In addition techniques are available automatically to create elements of joint in a grid which is deprived by it (see [U2.05.07]).

2 Local reference mark and matrix of passage

It is necessary to build a local reference mark with the element to define the jump of displacement δ (input datum of the laws of behavior: to see [R7.02.11] and [R7.01.25]). In addition, the matrix passage is defined $\boldsymbol{R}$ total reference mark with the local reference mark. This part is valid for modelings of joint in pure mechanics and for coupled modelings hydraulic.

2.1 Case 2D

That is to say $(\boldsymbol{X},\boldsymbol{Y})$ the total reference mark. The direction given by the large sides [12] and [34] of the element of joint 2D makes it possible to define a local reference mark $(\boldsymbol{n},\boldsymbol{t})$ with the element of joint (see figure 1):

$$\boldsymbol{t}=\frac{\vec{12}}{\|\vec{12}\|}\ ,\ \boldsymbol{n}=\boldsymbol{t}\wedge(\boldsymbol{X}\wedge\boldsymbol{Y})$$

The matrix of passage of the total reference mark to the local reference mark is expressed:

$$\boldsymbol{R}=\begin{bmatrix} n_x & n_y \\ t_x & t_y \end{bmatrix}$$

2.2 Case 3D

One notes $(\boldsymbol{X},\boldsymbol{Y},\boldsymbol{Z})$ the total reference mark. For the construction of the local reference mark to the element of joint, one uses the base covariante surface element corresponding. If one notes $\boldsymbol{s}(\xi^1,\xi^2)$ the parameterized position of a point of the surface element:

$$\boldsymbol{s}(\xi^1,\xi^2)=\sum_{n=1}^{Nb} N_n(\xi^1,\xi^2)\,\boldsymbol{s^n}$$

where N_n and s^n the function of form and the geometrical position of the node indicate respectively n , and Nb the number of nodes of the surface element. One defines the local base covariante $(\boldsymbol{a_1},\boldsymbol{a_2})$ in the following way:

$$\boldsymbol{a_1}=\frac{\partial\boldsymbol{s}}{\partial\xi_1}=\sum_{n=1}^{Nb}\frac{\partial N_n}{\partial\xi^1}\boldsymbol{s^n}\quad \boldsymbol{a_2}=\frac{\partial\boldsymbol{s}}{\partial\xi_2}=\sum_{n=1}^{Nb}\frac{\partial N_n}{\partial\xi^2}\boldsymbol{s^n}$$

These two vectors are by way of vectors tangent with the element in a given point. The local direct orthonormal base $(\boldsymbol{n},\boldsymbol{t},\boldsymbol{\tau})$ is then built in the following way:

$$\boldsymbol{t}=\frac{\boldsymbol{a_1}}{\|\boldsymbol{a_1}\|}\quad \boldsymbol{n}=\frac{\boldsymbol{t}\wedge\boldsymbol{a_2}}{\|\boldsymbol{a_2}\|}\quad \boldsymbol{\tau}=\boldsymbol{n}\wedge\boldsymbol{t}$$

The matrix of passage of the total reference mark to the local reference mark is given by:

$$\boldsymbol{R}=\begin{bmatrix} n_x & n_y & n_z \\ t_x & t_y & t_z \\ \tau_x & \tau_y & \tau_z \end{bmatrix}$$

3 Jump of displacement

The joints have authority to represent two faces in glance, they utilize only the functions of interpolation and the points of integration of the elements *surface* (in 3D) or *linear* (in 2D) corresponding:

In 2D: for the joint `QUAD4` (or the joint `HYME QUAD8`), the linear element is it `SEG2`
In 3D: for the joint `PENTA6` (or the joint `HYME PENTA15`) the surface element is it `TRIA3`
for the joint `HEXA8` (or the joint `HYME HEXA20`) the surface element is it `QUAD4`.

One calls N_n the function of form of the node n surface element[1]. U^{+n} and U^{-n} nodal displacements of the segments indicate respectively Γ^+ and Γ^- in 2D or faces S^+ and S^- in 3D.
In the local reference mark, the jump of displacement $\boldsymbol{\delta}$ is discretized starting from the functions of form N_n. At the point of gauss g, it is expressed like the difference of displacements of the faces (or segments) + and -:

$$\boldsymbol{\delta}_g=\sum_{n=1}^{Nb}\boldsymbol{R}(\boldsymbol{U^{+n}}-\boldsymbol{U^{-n}})N_n^g$$

1 Thereafter, one uses the generic term: “surface” for the 2D as for the 3D.

where Nb is the number of nodes of the surface element and where $\boldsymbol{R}$ matrix of passage in 2D, in 3D, which makes it possible to express nodal displacements in the local reference mark. One can synthesize the expression the preceding one in a matrix $\boldsymbol{M}_g^U$ who acts on the vector of nodal displacements of the element: $\boldsymbol{U}$, to build the jump of displacement in the local reference mark:

$$\boldsymbol{\delta}_g = \boldsymbol{M}_g^U \boldsymbol{U}$$

The matrix $\boldsymbol{M}_g^U$ is of dimension $ndim \times Nddl_U$, with $Nddl_U$ many degrees of freedom *mechanics* :

$Nddl_U = 8$ for the joint `2D`,

$Nddl_U = 24$ for the joint `3D HEXA`

$Nddl_U = 18$ for the joint `3D PENTA`

$Nddl_U = 12$ for the joint `HYME 2D`

$Nddl_U = 48$ for the joint `HYME 3D HEXA`

$Nddl_U = 36$ for the joint `HYME 3D PENTA`

4 Gradient of the pressure of fluid

Elements of joint `HYME` have, besides the mechanical degrees of freedom $\boldsymbol{U}$, nodal degrees of freedom of pressure of fluid (one by node) noted $\boldsymbol{P}$.
For `QUAD8`, the nodes following nodes 6,7,8 carry these degrees of freedom of pressure. The quadratic element of reference used for the approximation of the pressures is it `SEG3`.
For`HEXA20`, nodes 13,14,15,16,17,18,19,20 carry these degrees of freedom of pressure. The surface element of reference used for the approximation of the pressures is it `QUAD8`.
For `PENTA15`, the nodes according to 10,11,12,13,14,15 carry these degrees of freedom of pressure. The surface element of reference used for the approximation of the pressures is it `TRIA6`.

The law of flow of the fluid (cubic law, to see [R7.01.25]) utilized the gradient of pressure in the direction of the flow estimated with the point of gauss *G* in a classical way:

$$\nabla p_g = \sum_{n=1}^{Nb} \boldsymbol{P_n} \nabla N_n^g$$

where Nb is the number of nodes of pressure and N_n^g the value of the function of form of the node n at the point of gauss g. To simplify the writing one notes:

$$\nabla p_g = \boldsymbol{M}_g^P \boldsymbol{P}$$

The matrix $\boldsymbol{M}_g^U$ is of dimension $(ndim-1) \times Nddl_P$: with $Nddl_P$ many degrees of freedom *fluid* :

$Nddl_P = 3$ for the joint `HYME 2D`

$Nddl_P = 8$ for the joint `HYME 3D HEXA`

$Nddl_P = 6$ for the joint `HYME 3D PENTA`

5 Interior efforts and tangent matrix

5.1 Mechanical case pure

The formulation of the mechanical problem (see [R7.02.11] and [R7.01.25]) utilized the work of the efforts along the discontinuity, which is not other than the energy of surface related to the cracking of the structure:

$$W_s(\boldsymbol{\delta})=\sum_g \omega_g \psi(\boldsymbol{\delta}_g)$$

with ψ density of energy of surface and ω_g weight of the point of gauss g. That makes it possible to define the vector of the interior efforts:

$$\boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}=\frac{\partial W_s(\boldsymbol{\delta})}{\partial \boldsymbol{U}}=\sum_g \omega_g \frac{\partial \psi}{\partial \boldsymbol{\delta}_g}\frac{\partial \boldsymbol{\delta}_g}{\partial \boldsymbol{U}}$$

In the preceding expression, the first term is given by the cohesive law of behavior (see [R7.02.11]). That corresponds to the vector forced $\vec{\boldsymbol{\sigma}}_g$ (or forces cohesive) at the point of gauss g :

$$\frac{\partial \psi}{\partial \boldsymbol{\delta}_g}=\vec{\boldsymbol{\sigma}}_g$$

The second term is resulting from the definition of the jump displacement in the part 3 :

$$\frac{\partial \boldsymbol{\delta}_g}{\partial \boldsymbol{U}}=\boldsymbol{M}_g^{\boldsymbol{U}}$$

The nodal vector of the interior forces is thus expressed in the following way:

$$\boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}=\sum_g \omega_g \boldsymbol{M}_g^{\boldsymbol{U}t}\vec{\boldsymbol{\sigma}}_g$$

Within the framework of an algorithm of Newton, to solve the nonlinear problem of balance, it is useful to have the elementary tangent matrix, i.e. the derivative of the interior forces compared to nodal displacements. In the case of the element of joint, it is expressed simply:

$$\boldsymbol{K}^{\boldsymbol{UU}}=\frac{\partial \boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}}{\partial \boldsymbol{U}}=\sum_g \omega_g \boldsymbol{M}_g^{\boldsymbol{U}t}\frac{\partial \vec{\boldsymbol{\sigma}}_g}{\partial \boldsymbol{\delta}_g}\boldsymbol{M}_g^{\boldsymbol{U}}$$

The latter is pressed on the tangent operator: $\frac{\partial \vec{\boldsymbol{\sigma}}_g}{\partial \boldsymbol{\delta}_g}$ specific to the cohesive law of behavior adopted (see [R7.02.11]).

5.2 Hydraulic coupled case

Joints HYME, besides the nodal efforts related to mechanics $\boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}$ on which one defers[2] pressure of fluid at the point of gauss on the normal component $\vec{\boldsymbol{p}}_g=(p_g,0,0)$ (expressed in the local reference mark with the crack):

$$\boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}=\sum_g \omega_g \boldsymbol{M}_g^{\boldsymbol{U}t}(\vec{\boldsymbol{\sigma}}_g-\vec{\boldsymbol{p}}_g)$$

have nodal efforts for the flow of fluid on the nodes which carry ddl of pressure.

2 To take into account coupling HM

The formulation of the hydraulic problem (see [R7.01.25]) utilized the work of the efforts of the fluid along the way of the flow (inside the crack):

$$W_F(\nabla p)=\sum_g \omega_g H(\nabla p)$$

with H density of energy of surface and ω_g weight of the point of gauss g. That makes it possible to define the vector of the interior efforts:

$$\boldsymbol{F}_{\text{int}}^{\boldsymbol{P}}=\frac{\partial W_F(\nabla p)}{\partial \boldsymbol{P}}=\sum_g \omega_g \frac{\partial H}{\partial \nabla p_g}\frac{\partial \nabla p_g}{\partial \boldsymbol{P}}$$

In the preceding expression, the first term is given by the cubic law of behavior of the fluid (see [R7.01.25]). That corresponds to hydraulic flow $\vec{\boldsymbol{w}}_{\boldsymbol{g}}$ at the point of gauss g :

$$\frac{\partial H}{\partial \nabla p_g}=\vec{\boldsymbol{w}}_{\boldsymbol{g}}$$

According to the definition of the gradient of pressure in 4. The second term is given by:

$$\frac{\partial \nabla p_g}{\partial \boldsymbol{P}}=\boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{P}}$$

The nodal vector of the interior forces is thus expressed in the following way:

$$\boldsymbol{F}_{\text{int}}^{\boldsymbol{P}}=\sum_g \omega_g \boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{Pt}} \vec{\boldsymbol{w}}_{\boldsymbol{g}}$$

Within the framework of an algorithm of Newton, to solve the nonlinear problem of balance, it is useful to have the tangent matrix, i.e. the derivative of the interior forces compared to the degrees of freedom. The derivative of the interior effort on the nodal degrees of freedom of displacement compared to displacement gives the term identical to that of the matrix obtained in pure mechanics in 5.1 :

$$\boldsymbol{K}^{\boldsymbol{UU}}=\frac{\partial \boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}}{\partial \boldsymbol{U}}=\sum_g \omega_g \boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{Ut}}\frac{\partial \vec{\boldsymbol{\sigma}}_{\boldsymbol{g}}}{\partial \boldsymbol{\delta}_{\boldsymbol{g}}}\boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{U}}$$

In the case of hydraulic coupling, them $\boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}$ depend explicitly on the pressure (see expression above) from where:

$$\boldsymbol{K}^{\boldsymbol{UP}}=\frac{\partial \boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}}{\partial \boldsymbol{P}}=\sum_g \omega_g \boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{Ut}}\boldsymbol{X}_{\boldsymbol{g}}$$

with $\boldsymbol{X}_{\boldsymbol{g}}=\frac{\partial \vec{\boldsymbol{p}}_{\boldsymbol{g}}}{\partial \boldsymbol{P}}=(-N_g^i,0,0)$ with $i=1$ à Nb, Nb many nodes of pressure (or many degrees of freedom of pressure per element) and N_n^g the value of the function of form of the node n at the point of gauss g.

The derivative of the interior efforts on the nodal degrees of freedom of pressure compared to displacement gives:

$$\boldsymbol{K}^{\boldsymbol{PU}}=\frac{\partial \boldsymbol{F}_{\text{int}}^{\boldsymbol{P}}}{\partial \boldsymbol{U}}=\sum_g \omega_g \boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{Pt}}\frac{\partial \vec{\boldsymbol{w}}_{\boldsymbol{g}}}{\partial \boldsymbol{\delta}_{\boldsymbol{g}}}\boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{U}}$$

Lastly, the derivative of the interior efforts on the degrees of freedom of pressure compared to the nodal pressures gives:

$$\boldsymbol{K}^{PP}=\frac{\partial \boldsymbol{F}_{\mathrm{int}}^{P}}{\partial \boldsymbol{P}}=\sum_{g}\omega_g \boldsymbol{M}_g^{Pt}\frac{\partial \vec{\boldsymbol{w}}_g}{\partial \nabla p_g}\boldsymbol{M}_g^{P}$$

The elementary tangent matrix (nonsymmetrical) is expressed in the following way:

$$\boldsymbol{K}=\begin{bmatrix}\boldsymbol{K}^{UU} & \boldsymbol{K}^{UP}\\ \boldsymbol{K}^{PU} & \boldsymbol{K}^{PP}\end{bmatrix}$$

The latter is pressed on the components of the tangent operator: $\frac{\partial \vec{\boldsymbol{\sigma}}_g}{\partial \boldsymbol{\delta}_g}$, $\frac{\partial \vec{\boldsymbol{w}}_g}{\partial \boldsymbol{\delta}_g}$ and $\frac{\partial \vec{\boldsymbol{w}}_g}{\partial \nabla p_g}$ specific to the cohesive law of behavior adopted (see [R7.01.25]).

6 Features and validation

See documentations [R7.02.11] and [R7.01.25].

7 Description of the versions of the document

Index document	Version Aster	Author (S) Organization (S)	Description of the modifications
B	7.2	J.Laverne, EDF-R&D/AMA	
C	8.4	J.Laverne, EDF-R&D/AMA	
D	9.1	J.Laverne, EDF-R&D/AMA	card 9807 integration of the elements of joint 3D
E	10.4	J.Laverne, EDF-R&D/AMA	Card 14831 addition of coupled modelings *_HYME
F	11.4	K.Kazymyrenko, J.Laverne EDF-R&D/AMA	Card 18711 activation of quadratic modelings in pure mechanics
F	12.2	K.Kazymyrenko, J.Laverne EDF-R&D/AMA	Card 23070 modeling P2P2 for the hydraulic part