Anisotropic elasticity

Summary

This document treats anisotropic elasticity, used for modelings of continuous mediums 3D and 2D (C_PLAN, D_PLAN, AXIS), or layers of the composite hulls.

The springy medium can be anisotropic according to the 3 directions (orthotropic elasticity is spoken), or in isotropic in two directions (one speaks about transverse isotropic elasticity).

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1 Introduction

The objective of this document is to give the form of the matrices of flexibility and Hooke for elastic materials orthotropic, isotropic transverse and isotropic in the cases 3D, 2D-constraints, 2D - plane deformations and axisymetry.

We speak about “matrices” of Hooke because, by preoccupation with a simplification, we did not adopt the notation of a tensor of order 4.

In any rigour, for linear elastic materials, the constraints are linear functions of the deformations.

One writes: $\sigma_{ij}=H_{ijkl}\varepsilon_{kl}$

The symmetrical nature of $\{\sigma\}$ and $[\varepsilon]$ and adoption for these tensors of order 2 of a vectorial form allows to write:

$$\{\sigma\}=[H]\{\varepsilon\}$$

where $\{\sigma\}$ and $[\varepsilon]$ are the vectorial representation of the tensors of order 2 $\{\sigma\}$ and $[\varepsilon]$ and where $[H]$ is a matrix 6×6 .

2 Topology of the matrices of Hooke

2.1 Orthotropism

One can show the symmetry of the matrix of Hooke $[H]$.

We thus have twenty and one independent components in the case 3D.

$$[H]=\begin{vmatrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} & H_{16} \\ & H_{22} & H_{23} & H_{24} & H_{25} & H_{26} \\ & & H_{33} & H_{34} & H_{35} & H_{36} \\ SYM & & & H_{44} & H_{45} & H_{46} \\ & & & & H_{55} & H_{56} \\ & & & & & H_{66} \end{vmatrix}$$

An orthotropic material has two orthogonal plans of elastic symmetry.

This wants to say that if one calls $[H']$ the matrix $[H]$ after symmetry (S)
$[H']=[H]$.

The relations obtained between the coefficients make it possible to write that $[H]$ is defined by nine independent components.

In the axes of orthotropism:

$$[H]=\begin{vmatrix} H_{11} & H_{12} & H_{13} & 0 & 0 & 0 \\ & H_{22} & H_{23} & 0 & 0 & 0 \\ & & H_{33} & 0 & 0 & 0 \\ SYM & & & H_{44} & 0 & 0 \\ & & & & H_{55} & 0 \\ & & & & & H_{66} \end{vmatrix}$$

9 coefficients thus should be provided.

2.2 Transverse isotropy

The transverse isotropy is a restriction of the orthotropism in where one has the isotropy in one of the two orthogonal plans of elastic symmetry.

The matrix $[H]$ will have the same form as for the orthotropism but with additional relations between the components.
Five components are enough to determine $[H]$.

2.3 Isotropy

The material is isotropic if $[H]$ remain invariant in any change of reference mark.

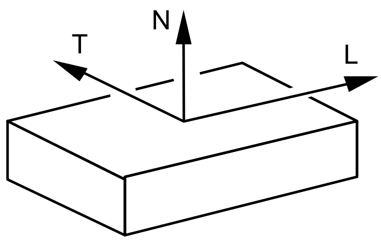
Two coefficients are enough to determine $[H]$.

3 Matrix of Hooke and flexibility

3.1 Notations

Instead of using indices 1.2 and 3 to locate the axes, one will use the corresponding indices L, T and N :

1. L for longitudinal
2. T for transverse
3. N for normal



The coefficients which intervene are the following:

Keyword	Notation	significance
E_L	E_L	Longitudinal Young modulus
E_T	E_T	Transverse Young modulus
E_N	E_N	Normal Young modulus
G_LT	G_{LT}	Modulus of rigidity in the plan (L,T)
G_TN	G_{TN}	Modulus of rigidity in the plan (T,N)
G_LN	G_{LN}	Modulus of rigidity in the plan (L,N)
NU_LT	ν_{LT}	Poisson's ratio in the plan (L,T)
NU_TN	ν_{TN}	Poisson's ratio in the plan (T,N)
NU_LN	ν_{LN}	Poisson's ratio in the plan (L,N)

Notice very important:

ν_{LT} is different from ν_{TL} :

If one applies a traction according to L

$\epsilon_{LL}=\frac{\overline{\sigma_{LL}}}{E_L}$ *(law of Hooke following a direction).*

This traction is accompanied, proportionally, of a contraction according to $T, -\nu_{LT}.\frac{\sigma_{LL}}{E_L}$

and of a contraction according to $N, -\nu_{LN}.\frac{\sigma_{LL}}{E_L}$.

The first index indicates the axis where the effect of the loading is exerted and the second index indicates the direction of the loading.

Then one exerts a traction according to T, then a traction according to N; one obtains:

$$\left.\begin{aligned}\varepsilon_{LL}&=\frac{\sigma_{LL}}{E_L}-\nu_{TL}\frac{\sigma_{TT}}{E_T}-\nu_{NL}\frac{\sigma_{NN}}{E_N}\\ \varepsilon_{TT}&=-\nu_{LT}\frac{\sigma_{LL}}{E_L}+\frac{\sigma_{TT}}{E_T}-\nu_{NT}\frac{\sigma_{NN}}{E_N}\\ \varepsilon_{NN}&=-\nu_{LN}\frac{\sigma_{LL}}{E_L}-\nu_{TN}\frac{\sigma_{TT}}{E_T}+\frac{\sigma_{NN}}{E_N}\end{aligned}\right\}(S)$$

The matrix of flexibility $[H]^{-1}$ being symmetrical; one from of deduced:

$$\frac{\nu_{LT}}{E_L}=\frac{\nu_{TL}}{E_T}$$

$$\frac{\nu_{LN}}{E_L}=\frac{\nu_{NL}}{E_N}$$

$$\frac{\nu_{TN}}{E_T}=\frac{\nu_{NT}}{E_N}$$

3.2 Case 3D

3.2.1 0rthotropie

3.2.1.1 Matrix of flexibility

$$\begin{bmatrix}\varepsilon_{LL}\\ \varepsilon_{TT}\\ \varepsilon_{NN}\\ 2\varepsilon_{LT}\\ 2\varepsilon_{LN}\\ 2\varepsilon_{TN}\end{bmatrix}=\begin{bmatrix}\frac{1}{E_L} & \frac{-\nu_{TL}}{E_T} & \frac{-\nu_{NL}}{E_N} & 0 & 0 & 0\\ \frac{-\nu_{LT}}{E_L} & \frac{1}{E_T} & \frac{-\nu_{NT}}{E_N} & 0 & 0 & 0\\ \frac{-\nu_{LN}}{E_L} & \frac{-\nu_{TN}}{E_T} & \frac{1}{E_N} & 0 & 0 & 0\\ & & & \frac{1}{G_{LT}} & 0 & 0\\ & SYM & & & \frac{1}{G_{LN}} & 0\\ & & & & & \frac{1}{G_{TN}}\end{bmatrix}\begin{bmatrix}\sigma_{LL}\\ \sigma_{TT}\\ \sigma_{NN}\\ \sigma_{LT}\\ \sigma_{LN}\\ \sigma_{TN}\end{bmatrix}$$

$[H]^{-1}$ – Orthotropism

3.2.1.2 Matrix of Hooke

$$\begin{bmatrix}\sigma_{LL}\\ \sigma_{LL}\\ \sigma_{NN}\\ \sigma_{LT}\\ \sigma_{LN}\\ \sigma_{TN}\end{bmatrix}=\frac{1}{\Delta}\begin{bmatrix}\frac{(1-\nu_{TN}\nu_{NT})}{E_T E_N} & \frac{(\nu_{TL}+\nu_{NL}\nu_{TN})}{E_T.E_N} & \frac{(\nu_{NL}+\nu_{TL}\nu_{NT})}{E_T.E_N} & 0 & 0 & 0\\ \frac{(\nu_{LT}+\nu_{LN}\nu_{NT})}{E_L E_N} & \frac{(1-\nu_{NL}\nu_{LN})}{E_L.E_N} & \frac{(\nu_{NT}+\nu_{NL}.\nu_{LT})}{E_L.E_N} & 0 & 0 & 0\\ \frac{(\nu_{LN}+\nu_{LT}.\nu_{TN})}{E_L.E_T} & \frac{(\nu_{TN}+\nu_{TL}.\nu_{LN})}{E_L.E_T} & \frac{(1-\nu_{LT}.\nu_{TL})}{E_L.E_T} & 0 & 0 & 0\\ & & & GLT*\Delta & 0 & 0\\ & SYM & & & GLN*\Delta & 0\\ & & & & & GTN*\Delta\end{bmatrix}\begin{bmatrix}\varepsilon_{LL}\\ \varepsilon_{TT}\\ \varepsilon_{NN}\\ 2\varepsilon_{LT}\\ 2\varepsilon_{LN}\\ 2\varepsilon_{TN}\end{bmatrix}$$

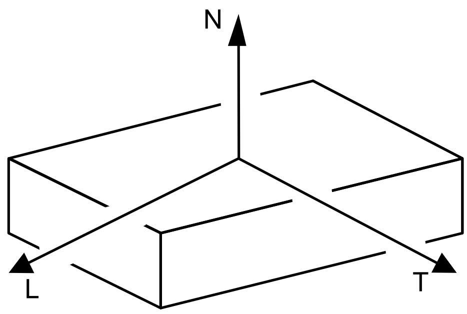
$[H]$ – Orthotropism with:

$$\frac{\nu_{TL}}{E_T}=\frac{\nu_{LT}}{E_L};\frac{\nu_{NL}}{E_N}=\frac{\nu_{LN}}{E_L};\frac{\nu_{NT}}{E_N}=\frac{\nu_{TN}}{E_T}$$

$$\frac{1}{\Delta}=\frac{E_L E_T E_N}{\begin{pmatrix} 1-\nu_{TN}\nu_{NT} \\ -\nu_{NL}\nu_{LN} \\ -\nu_{LT}\nu_{TL} \\ -2\nu_{TN}\nu_{NL}\nu_{LT}\end{pmatrix}}$$

3.2.2 Transverse isotropy

The isotropy is here defined in the plan (L,T), and the direction of orthotropism is thus N. One can draw the attention of the reader to the fact that this convention differs from a usual convention which indicates by “longitudinal direction” the direction of orthotropism of isotropic transverse materials.



3.2.2.1 Matrix of flexibility

The matrix $[H]^{-1}$ can be deduced directly from the matrix $[H]^{-1}$ - Orthotropism by using the properties of the transverse isotropy.

In the plan (L,T) :

$$E_L=E_T$$
$$\nu_{TL}=\nu_{LT}$$
$$G_{LT}=\frac{E_L}{2(1+\nu_{LT})}$$

In the plans (L,N) and (T,N) :

$$\nu_{NT}=\nu_{NL}$$
$$\nu_{LN}=\nu_{TN}$$
$$G_{TN}=G_{LN}$$
$$\frac{\nu_{NT}}{E_N}=\frac{\nu_{LN}}{E_L}$$

$$\begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \\ 2\varepsilon_{LN} \\ 2\varepsilon_{TN} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_L} & \frac{-\nu_{LT}}{E_L} & \frac{-\nu_{NL}}{E_N} & 0 & 0 & 0 \\ \frac{-\nu_{TL}}{E_L} & \frac{1}{E_L} & \frac{-\nu_{NT}}{E_N} & 0 & 0 & 0 \\ \frac{-\nu_{LN}}{E_L} & \frac{-\nu_{TN}}{E_L} & \frac{1}{E_N} & 0 & 0 & 0 \\ & & & \frac{2(1+\nu_{LT})}{E_L} & 0 & 0 \\ & SYM & & & \frac{1}{G_{LN}} & 0 \\ & & & & & \frac{1}{G_{TN}} \end{bmatrix} \begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \\ \sigma_{LN} \\ \sigma_{TN} \end{bmatrix}$$

$[H]^{-1}$ - Transverse isotropy

3.2.2.2 Matrix of Hooke

The matrix $[H]$ have same symmetries as $[H]^{-1}$.

$$\begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \\ \sigma_{LN} \\ \sigma_{TN} \end{bmatrix} = \frac{1}{\Delta'} \begin{bmatrix} \frac{1-\nu_{NL}.\nu_{LN}}{E_L.E_N} & \frac{\nu_{LT}+\nu_{NL}\nu_{LN}}{E_L.E_N} & \frac{\nu_{NL}+\nu_{LT}\nu_{NL}}{E_L.E_N} & 0 & 0 & 0 \\ \frac{\nu_{TL}+\nu_{NL}\nu_{LN}}{E_L.E_N} & \frac{1-\nu_{NL}.\nu_{LN}}{E_L.E_N} & \frac{\nu_{LN}+\nu_{LT}\nu_{LN}}{E_L.E_N} & 0 & 0 & 0 \\ \frac{\nu_{LN}+\nu_{LT}.\nu_{LN}}{E_L^2} & \frac{\nu_{TN}+\nu_{LT}.\nu_{TN}}{E_L^2} & \frac{1-\nu_{LT}^2}{E_L^2} & 0 & 0 & 0 \\ & & & \frac{E_L.\Delta'}{2(1+\nu_{LT})} & & \\ & & & & G_{LN}.\Delta' & \\ & & & & & G_{LN}.\Delta' \end{bmatrix} \begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \\ 2\varepsilon_{LN} \\ 2\varepsilon_{TN} \end{bmatrix}$$

$[H]$ – Transverse isotropy

$$\frac{1}{\Delta'} = \frac{E_L^2.E_N}{\begin{bmatrix} 1-2\nu_{NL}.\nu_{LN}-\nu_{LT}^2 \\ -2\nu_{NL}\nu_{LN}\nu_{LT} \end{bmatrix}}$$

3.2.3 Cubic elasticity

Cubic elasticity corresponds to a matrix of elasticity of the form :

$$\begin{bmatrix} y_{1111} & y_{1122} & y_{1122} & & & \\ y_{1122} & y_{1111} & y_{1122} & & & \\ y_{1122} & y_{1122} & y_{1111} & & & \\ & & & y_{1212} & & \\ & & & & y_{1212} & \\ & & & & & y_{1212} \end{bmatrix}$$

Being given cubic symmetry, it remains to determine 3 coefficients:

$$E_L = E_N = E_T = E, G_{LT} = G_{LN} = G_{TN} = G, \nu_{LN} = \nu_{LT} = \nu_{LN} = \nu$$

To reproduce cubic elasticity with `ELAS_ORTH`, it is enough to calculate the coefficients of the orthotropism such that the matrix of elasticity obtained is form above:

$$y_{1111} = \frac{E(1-\nu^2)}{(1-3\nu^2-2\nu^3)}$$
$$y_{1122} = \frac{E\nu(1+\nu)}{(1-3\nu^2-2\nu^3)}$$
$$y_{1212} = G_{LT} = G_{LN} = G_{TN}$$

therefore, as long as $(1-3\nu^2-2\nu^3) \neq 0$ (i.e. ν different from 0.5).

$\frac{y_{1122}}{y_{1111}} = \frac{\nu}{1-\nu}$ what provides $\nu = \frac{1}{1+\frac{y_{1111}}{y_{1122}}}$ then $E = y_{1111}\frac{(1-3\nu^2-2\nu^3)}{(1-\nu^2)}$

3.2.4 Isotropy

3.2.4.1 Matrix of flexibility according to E and ν

$$\begin{bmatrix} \varepsilon_{LL} \\ \varepsilon_{TT} \\ \varepsilon_{NN} \\ 2\varepsilon_{LT} \\ 2\varepsilon_{LN} \\ 2\varepsilon_{TN} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ & \frac{1}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ & & \frac{1}{E} & 0 & 0 & 0 \\ & & & \frac{1}{G}=\frac{2(1+\nu)}{E} & 0 & 0 \\ & SYM & & & \frac{1}{G}=\frac{2(1+\nu)}{E} & 0 \\ & & & & & \frac{1}{G}=\frac{2(1+\nu)}{E} \end{bmatrix} \begin{bmatrix} \sigma_{LL} \\ \sigma_{TT} \\ \sigma_{NN} \\ \sigma_{LT} \\ \sigma_{LN} \\ \sigma_{TN} \end{bmatrix}$$

$[H]^{-1}$ – Complete isotropy

3.2.4.2 Matrix of Hooke according to E and ν

$$\begin{bmatrix}\sigma_{LL}\\ \sigma_{TT}\\ \sigma_{NN}\\ \sigma_{LT}\\ \sigma_{LN}\\ \sigma_{TN}\end{bmatrix}=\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & \nu & 0 & 0 & 0\\ & 1-\nu & \nu & 0 & 0 & 0\\ & & 1-\nu & 0 & 0 & 0\\ & SYM & & \frac{1-2\nu}{2} & 0 & 0\\ & & & & \frac{1-2\nu}{2} & 0\\ & & & & & \frac{1-2\nu}{2}\end{bmatrix}\begin{bmatrix}\varepsilon_{LL}\\ \varepsilon_{TT}\\ \varepsilon_{NN}\\ 2\varepsilon_{LT}\\ 2\varepsilon_{LN}\\ 2\varepsilon_{TN}\end{bmatrix}$$

$[H]$ – Complete isotropy

3.2.4.3 Matrix of flexibility according to the coefficients of Lamé λ and μ

The law of Hooke takes the following shape with the coefficients of Lamé λ and μ .

$$\sigma_{ij}=\lambda\,\varepsilon_{kk}\,\delta_{ij}+2\,\mu\,\varepsilon_{ij}$$

By using the system of equations (S), one obtains:

$$\begin{bmatrix}\sigma_{LL}\\ \sigma_{TT}\\ \sigma_{NN}\\ \sigma_{LT}\end{bmatrix}=\frac{1}{1-\nu_{LT}.\nu_{TL}}\begin{bmatrix}E_L & \nu_{TL}.E_T & 0 & 0\\ \nu_{LT}.E_L & E_T & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & G_{LT}\end{bmatrix}\begin{bmatrix}\varepsilon_{LL}\\ \varepsilon_{TT}\\ \varepsilon_{NN}\\ 2\varepsilon_{LT}\end{bmatrix}$$

$[H]$ – Orthotropism planes in plane constraints

3.2.4.4 Matrix of Hooke according to the coefficients of Lamé λ and μ

$$\begin{bmatrix}\sigma_{LL}\\ \sigma_{TT}\\ \sigma_{NN}\\ \sigma_{LN}\\ \sigma_{LT}\\ \sigma_{TN}\end{bmatrix}=\begin{bmatrix}\lambda+2\mu & \lambda & \lambda & 0 & 0 & 0\\ & \lambda+2\mu & \lambda & 0 & 0 & 0\\ & & \lambda+2\mu & 0 & 0 & 0\\ & SYM & & \mu & 0 & 0\\ & & & & \mu & 0\\ & & & & & \mu\end{bmatrix}\begin{bmatrix}\varepsilon_{LL}\\ \varepsilon_{TT}\\ \varepsilon_{NN}\\ 2\varepsilon_{LN}\\ 2\varepsilon_{LT}\\ 2\varepsilon_{TN}\end{bmatrix}$$

$[H]$ – Isotropy supplements with the coefficients of Lamé

3.3 Orthotropic in plane deformations and axisymmetric case 2 D

3.3.1 Matrix of flexibility

$$\begin{bmatrix}\varepsilon_{LL}\\ \varepsilon_{TT}\\ 0\\ 2\varepsilon_{LT}\end{bmatrix}=\begin{bmatrix}\frac{1}{E_L} & -\frac{\nu_{TL}}{E_T} & -\frac{\nu_{NL}}{E_N} & 0\\ -\frac{\nu_{LT}}{E_L} & \frac{1}{E_T} & -\frac{\nu_{NL}}{E_N} & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G_{LT}}\end{bmatrix}\begin{bmatrix}\sigma_{LL}\\ \sigma_{TT}\\ \sigma_{NN}\\ \sigma_{LT}\end{bmatrix}$$

$[H]^{-1}$ – Orthotropism planes in plane deformations and axisymetry

3.3.2 Matrix of Hooke

$$\begin{bmatrix}\sigma_{LL}\\ \sigma_{TT}\\ \sigma_{NN}\\ \sigma_{LT}\end{bmatrix}=\frac{1}{\Delta}\begin{bmatrix}\frac{(1-\nu_{TN}\nu_{NT})}{E_T E_N} & \frac{(\nu_{TL}+\nu_{NL}\nu_{TN})}{E_T.E_N} & \frac{(\nu_{NL}+\nu_{TL}\nu_{NT})}{E_T.E_N} & 0\\ \frac{(\nu_{LT}+\nu_{LN}\nu_{NT})}{E_L E_N} & \frac{(1-\nu_{NL}\nu_{LN})}{E_L.E_N} & \frac{(\nu_{NT}+\nu_{NL}.\nu_{LT})}{E_L.E_N} & 0\\ \frac{(\nu_{LN}+\nu_{LT}.\nu_{TN})}{E_L.E_T} & \frac{(\nu_{TN}+\nu_{TL}.\nu_{LN})}{E_L.E_T} & \frac{(1-\nu_{LT}.\nu_{TL})}{E_L.E_T} & 0\\ 0 & 0 & 0 & GLT*\Delta\end{bmatrix}\begin{bmatrix}\varepsilon_{LL}\\ \varepsilon_{TT}\\ 0\\ \varepsilon_{LT}\end{bmatrix}$$

$[H]$ – Orthotropism planes in plane deformations and axisymetry

$$\frac{1}{\Delta}=\frac{E_L E_T E_N}{\begin{pmatrix}1-\nu_{TN}\nu_{NT}\\ -\nu_{NL}\nu_{LN}\\ -\nu_{LT}\nu_{TL}\\ -2\nu_{TN}\nu_{NL}\nu_{LT}\end{pmatrix}}$$

3.4 Orthotropic case 2 D in plane constraints

3.4.1 Matrix of flexibility

$$\begin{bmatrix}\varepsilon_{LL}\\ \varepsilon_{TT}\\ \varepsilon_{NN}\\ 2\varepsilon_{LT}\end{bmatrix}=\begin{bmatrix}\frac{1}{E_L} & -\frac{\nu_{TL}}{E_T} & 0 & 0\\ -\frac{\nu_{LT}}{E_L} & \frac{1}{E_T} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G_{LT}}\end{bmatrix}\begin{bmatrix}\sigma_{LL}\\ \sigma_{TT}\\ \sigma_{NN}\\ \sigma_{LT}\end{bmatrix}$$

$[H]^{-1}$ – Orthotropism planes in plane constraints

3.4.2 Matrix of Hooke

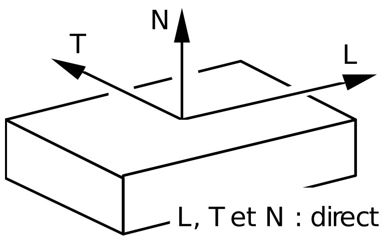
$$\begin{bmatrix}\sigma_{LL}\\ \sigma_{TT}\\ 0\\ \sigma_{LT}\end{bmatrix}=\frac{1}{1-\nu_{LT}.\nu_{TL}}\begin{bmatrix}E_L & \nu_{TL}E_T & 0 & 0\\ \nu_{LT}E_L & E_T & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & G_{LT}\end{bmatrix}\begin{bmatrix}\varepsilon_{LL}\\ \varepsilon_{TT}\\ \varepsilon_{NN}\\ 2\varepsilon_{LT}\end{bmatrix}$$

$[H]$ – Orthotropism in plane constraints

4 Use in *Code_Aster*

In *Aster* , the definition of the constant orthotropic elastic characteristics or functions of the temperature is carried out by the order `DEFI_MATERIAU` , keywords `ELAS_ORTH`, `ELAS_ISTR`, `ELAS_ISTR_FO` or `ELAS_ORTH_FO` for the elements of hull and the solid elements isoparametric or the layers constitutive of a composite (see the order `DEFI_COMPOSITE`).

To define the reference mark of orthotropism (L, T, N) bound to the elements, one can refer to documentations [U4.42.03] `DEFI_COMPOSITE` and [U4.42.01] `AFFE_CARA_ELEM` .



L, T et N : directions d'orthotropie longitudinale, transversale et normale

/ `ELAS_ORTH = _F (`	♦	`E_L = ygl`	Longitudinal Young modulus.
	♦	`E_T = ygt`	Transverse Young modulus.
	◊	`E_N = ygn`	Normal Young modulus.
	♦	`GL_T = glt`	Modulus of rigidity in the plan LT .
	◊	`G_TN = gtn`	Modulus of rigidity in the plan TN .
	◊	`G_LN = gln`	Modulus of rigidity in the plan LN .
	♦	`NU_LT = nult`	Poisson's ratio in the plan LT .
	◊	`NU_TN = nutn`	Poisson's ratio in the plan TN .
	◊	`NU_LN = nuln`	Poisson's ratio in the plan LN .

Notice important:

The talk of this note of reference is based on the convention of the books of J.L.Batoz and D.Gay. Documentation U of `DEFI_MATERIAU` described these choices, and the coefficient `NU_LT` be interpreted in the following way in Aster:

if one exerts a traction according to the axis L causing a deformation according to this axis equalizes with $\varepsilon_L = \frac{\sigma_l}{\mathrm{ygl}}$, there is a deformation according to the axis T equalize with:

$$\varepsilon_t = -\mathrm{nult} * \frac{\sigma_l}{\mathrm{ygl}} .$$

5 Bibliography

1) J.C. MASSON: Matrix of Hooke for orthotropic materials, Report interns Applications in Mechanics, n°79-018, CiSi, 1979.

2) D. GAY: Composite materials, Hermes Edition, 1987

3) J.L. BATOZ, G. DHATT: Modeling of the structures by finite elements, Volume 1, Hermes Edition

6 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
6.4	A. ASSIRE, EDF-R&D/AMA	Initial text
8.4	*A. ASSIRE, X. DESROCHES, J.M. PROIX* EDF-R&D/AMA	Tiny corrections