Straight-line method are equivalent for the wave propagation in 1D

Summary

This document describes the straight-line method are equivalent and its implementation in *Code_Aster* in the order DEFI_SOL_EQUI [U4.81.31].

The straight-line method are equivalent makes it possible to calculate the answer of a unidimensional column of ground (1D – a component) or multidimensional (1D – three components), laminated horizontally with the vertical propagation of waves of shearing (SV). She gives an account of way approximate of behavior of the grounds under cyclic loading. The method of resolution of the linear equivalent is an iterative procedure, where one evaluates with each iteration for each soil horizon, of the linear viscoelastic characteristics equivalent starting from the curves of degradation of the modulus of rigidity G and of the hysteretic increase in damping measured in experiments.

The simplicity of the implementation, the quality of the results and the speed of calculations make of it a method very much used in engineering. The straight-line method are equivalent monodimensional is thus established in many computer codes (SHAKE, FLUSH, CYBERQUAKE, DEEPSOIL, EERA,…).

The application of the straight-line method are equivalent valid rest as long as the voluminal deformation of the ground can be neglected. In the contrary case, it is necessary to turn to non-linear models integrating the coupling average shearing-pressure (or deformations déviatoriques-dilatancy) such as the cyclic elastoplastic model of Hujeux [R7.01.23] (cyclic Law of behavior of Hujeux for the grounds).

Traditionally the linear model are equivalent is allowed until deformations of distortion in free field about 10^{-3} (Guide ASN/2/01). This remains an order of magnitude of validity, which it is necessary to adapt to the situation and the phenomena that one wishes to model.

Contents

1 Introduction

Idriss and Seed (1967) [1] were the first to propose a linear approach are equivalent for the analysis of response of site to a seismic request. Later, Schnabel and al. (1972) [2] established this method in the frequential field with program SHAKE. The straight-line method are equivalent since is established in several computer codes (SHAKE, FLUSH, CYBERQUAKE, DEEPSOIL, EERA,…) and in **Code_hasster** under the operator `DEFI_SOL_EQUI`, [U4.84.31].

This method is adapted for the evaluation of the answer of profiles of ground made up of horizontal layers subjected to the vertical propagation of waves seismic. A modeling 1D – A component considers only the propagation waves of shearing according to a horizontal direction, whereas a modeling 1D – Three components considers the vertical propagation of three plane waves (SV, HS and P).

Indeed the ground can be in practice regarded as viscoelastic linear for distortions checking $\gamma=2\varepsilon_{xz}\leq 1{,}0\times 10^{-5}$. For higher distortions, LE behavior of the ground is described by experimental curves giving the variation of the modulus of rigidity G and of the critical percentage of damping D according to the distortion (noted gamma $\gamma=2\varepsilon_{xz}$) (see figure 1-1). it is admitted that this approach is acceptable until distortions checking: $\gamma=2\varepsilon_{xz}\leq 1{,}0\times 10^{-3}$. beyond, it is advisable to employ a nonlinear model of behavior coupling dilatancy and distortions.

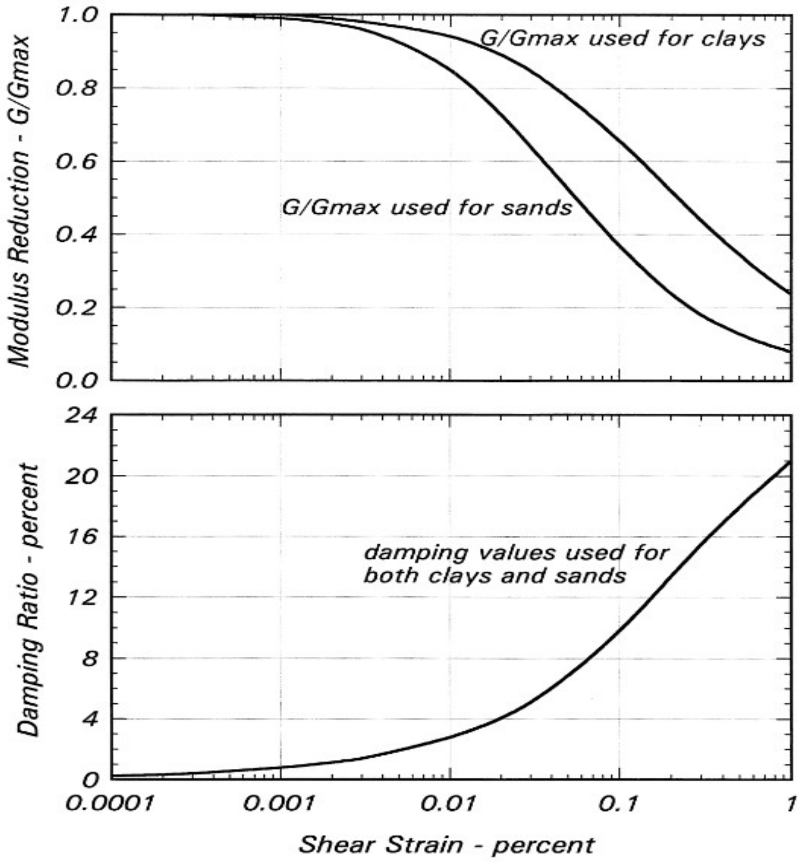


Figure 1-1 Example of curve G/Gmax-Gamma and D-Gamma [9].

For recall, the modulus of rigidity G is related to the Young modulus E and with the celerity of the waves of shearing V_s by the following formula:

$$G=\frac{E}{2(1+\nu)}=\rho . V_s^2 \quad (1)$$

Withvec ρ density, and ν Poisson's ratio.

2 Viscoelastic relation stress-strain in a dimension

The linear model are equivalent is based on the viscoelastic rheological model of Kelvin-Voigt. This last, represented in Figure 2-1, is made up in parallel of a linear spring and a linear shock absorber representing an internal resistance to the deformation.

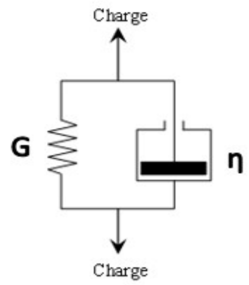


Figure 2-1 : Representation of the rheological model of Kelvin-Voigt.

The relation constitutive of this rheological model is the following one:

$$\sigma_{xz}=G\gamma+H\frac{\partial\gamma}{\partial t}=G\gamma+H\dot{\gamma} \tag{2}$$

with σ_{xz} shear stress, γ the distortion (equalizes with $2\varepsilon_{xz}$), G modulus of rigidity and H the module of viscosit é. The point represents the temporal derivative.

In the case of a column of ground in one dimension, while noting $u(z,t)$ horizontal displacement with a depth z and at one moment t given, the distortion and its temporal drift are defined starting from the field of displacement $u(z,t)$:

$$\gamma=\frac{\partial u(z,t)}{\partial z} \tag{3}$$

And its temporal derivative:

$$\dot{\gamma}=\frac{\partial\gamma(z,t)}{\partial t}=\frac{\partial^2 u(z,t)}{\partial z\partial t} \tag{4}$$

In the case of a harmonic loading $u(z,t)=U(z)e^{i\omega t}$, the distortion is written :

$$\gamma(z,t)=\frac{\partial u(z,t)}{\partial z}=\frac{dU(z)}{dz}e^{i\omega t}=\Gamma(z)e^{i\omega t} \tag{5}$$

And its temporal derivative:

$$\dot{\gamma}(z,t)=i\omega\gamma(z,t) \tag{6}$$

with $U(z)$ and $\Gamma(z)$ respectively the amplitude of displacement and the distortion.

The relation stress-strain constitutive of the model of Kelvin-Voigt is then defined:

$$\sigma_{xz}(z,t)=G\gamma+H\dot{\gamma}=G(1+i\eta\omega)\gamma(z,t)=G^*\gamma(z,t) \tag{7}$$

with G^* complex modulus of rigidity $G^*=(G+i\eta\omega)$; the factor of loss is equal here to $\eta\omega$.
During a symmetrical cyclic loading, the answer of the ground present hysteresis loops or loops in the stress-strain plan (see Figure 2-2).

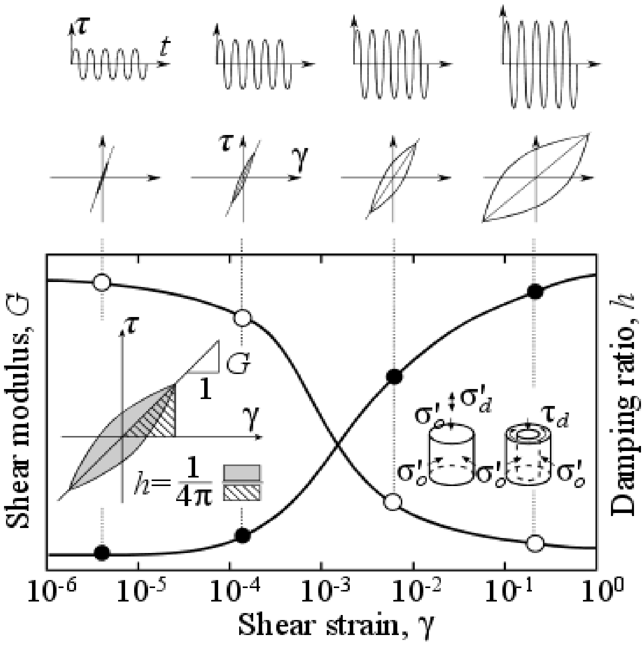


Figure 2-2. Response of a ground to a symmetrical cyclic loading, σ_{xz} shear stress, γ the distortion, D damping [4].

These loops represent the quantity of deformation energy dissipated by the ground during the loading. This energy can be quantified by defining the reduced damping coefficient D (see Figure 2-2) ground by the following relation:

$$D=\frac{W_d}{4\pi W_s} \tag{8}$$

where W_d represent the energy dissipated in a complete cycle of loading (equal to the interior surface formed by the loop stress-strain, in gray on Figure 2-1), and W_s the elastic energy stored by the ground during the loading for the level of distortion γ_c (lower triangle, in hatched on Figure 2-2).

These two energies are calculated in the case of a harmonic loading $\gamma(t)=\gamma_c e^{i\omega t}$ with the model of Kelvin-Voigt:

$$W_d=\oint_{\sigma_{xzc}} \sigma_{xz} d\gamma=\oint_t^{t+\frac{2\pi}{\omega}} \Re[\sigma_{xz}]\Re[\frac{d\gamma}{dt}]dt=\pi\eta\omega\gamma_c^2 \tag{9}$$

And:

$$W_s=\frac{1}{2}\tau_c\gamma_c=\frac{1}{2}G\gamma_c^2 \tag{10}$$

For the viscoelastic model of Kelvin-Voigt, the energy dissipated during a cycle of loading thus depends on the frequency of request. However the experiment shows that for the grounds, the energy dissipated during shearing is almost independent the speed of deformation. Damping results primarily from irreversible plastic deformations at the level of the grain. Damping in the ground is of rather hysteretic nature that viscous [5]. The model of Kelvin-Voigt is thus modified so that the dissipative properties are equivalent to actual material.

3 Relation stress-strain with three dimensions

One is interested in this part to define a scalar equivalent quantity on the basis of a stress and strain stateS tensorial. One leaves the tensorial decomposition of the tensor of deformation $\boldsymbol{\varepsilon}$ partly voluminal and deviatoric:

$$\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}_d+\frac{1}{3}tr(\boldsymbol{\varepsilon})\boldsymbol{I} \quad (11)$$

where $\boldsymbol{\varepsilon}_d$ is the tensor deviatoric of the deformations. Linear modeling are equivalent being interested in behaviour in shearing of the column of ground, the standard of the tensor of deviatoric deformation ε_d is the most suitable candidate because one can connect it to the shearing strain γ for a monodimensional loading:

$$\gamma=\sqrt{3}\varepsilon_d \quad (12)$$

This expression allows, starting from calculation ε_d obtained by the approach 1D – 3 components, to go back to the value of γ to use to make evolve the elastic module.

In the case of the wave propagation plane P, SV and HS perpendicular to the plans of stratigraphy (i.e vertical incidence for a horizontal stratigraphy), the calculation of the deviatoric deformation is summarized with the following expression:

$$\varepsilon_d=\sqrt{\frac{2}{3}(\boldsymbol{\varepsilon}_d:\boldsymbol{\varepsilon}_d)}$$
$$\varepsilon_d=\frac{2}{3}\sqrt{\varepsilon_{yy}^2+3\varepsilon_{xy}^2+3\varepsilon_{yz}^2} \quad (13)$$

The selected equivalent constraint is the deviatoric constraint q , calculated in postprocessing via by the following expression:

$$q=\sqrt{\frac{3}{2}\boldsymbol{S}:\boldsymbol{S}} \text{ and } q=3G\varepsilon_d \quad (14)$$

where $\boldsymbol{S}$ is the tensor deviatoric, defined by the following expression:

$$\boldsymbol{S}=\boldsymbol{\sigma}-\frac{1}{3}tr(\boldsymbol{\sigma})\boldsymbol{I} \quad (15)$$

4 Formulations of the linear behavior are equivalent

4.1 Formulation of Schnabel

Schnabel and al. (1972) [2] connect viscosity to the damping of material D, by $H\omega=2GD$. Damping criticizes material D is the value measuredE in laboratory and deferredE on the curves D - gamma.
The complex modulus of rigidity is written then:

$$G^*=(G+i\eta\omega)=(G+2i.GD)=G(1+2i.D) \quad (16)$$

Stiffness G is taken equal to the secant module G_s curves ($\sigma_{xz}-\gamma$) (see Figure 2-2). The value of the complex modulus of rigidity is equalE with:

$$|G^*|=G\sqrt{1+4D^2} \quad (17)$$

The module of the complex modulus of rigidity $|G^*|$ and D increaseNT thus jointly. Figure 4.1-1 represent the value of $\frac{|G^*|}{G}$ according to D. For example, for a reduced damping coefficient D from 20%, the module of the complex modulus of rigidity is higher of 8% than the value of G.

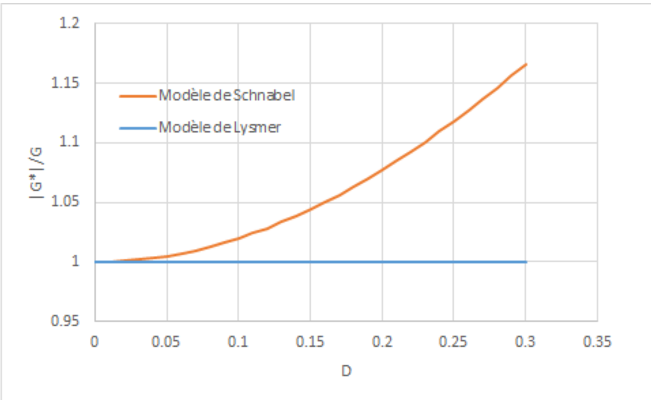


Figure 4.1-1. Evolution of $\frac{|G^*|}{G}$ according to D for the model of Schnabel, compared to that for the model of Lysmer.

To solve this problem, Lysmer formulated a second linear model are equivalent.

4.2 Formulation of Lysmer

The formulation of Lysmer consists in rewriting the modulus of complex rigidity G^* to make sure that the report $\frac{|G^*|}{G}$ that is to say always equal to one. The complex modulus of rigidity is written then:

$$G^* = G\left(\left(1-2D^2\right)+2iD\sqrt{1-D^2}\right) \quad (18)$$

The energy dissipated in a cycle is then:

$$W_d = 2\pi G\sqrt{1-D^2}\gamma_c^2 \quad (19)$$

4.3 Comparison of the energies dissipated by the two formulations

Figure 4.3-1 watch that the module complexes shearing $|G^*|$ is overestimated with the formulation of Schnabel, and rigorously equal to G for the formulation of Lysmer. The energy dissipated during a cycle of loading is also compared between the two formulations according to D.

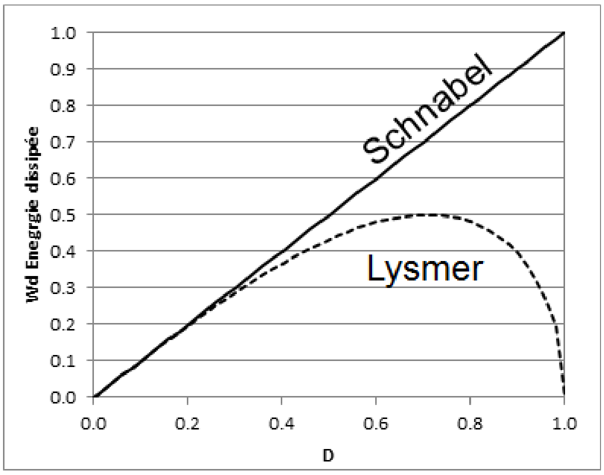


Figure 4.3-1 : Comparison of the energy dissipated during a cycle of loading for two formulations of the linear equivalent according to damping D [5].

The energy dissipated between the two formulations is the same one until approximately $D=0,2$, then the formulation of Lysmer dissipates less energy. The field of validity of the linear model are equivalent makes that the field of interest of these curves is on the interval $[0,\ 0.3]$. It is noticed whereas even if $D_{Schnabel} < D_{Lysmer}$, the model of Schnabel can dissipate more energy that the model of Lysmer.

5 Taking into account simplified of water pressure via the model of Byrne

The modeling simplified of the influence of the water pressure on the answer in shearing of a column of ground with `DEFI_SOL_EQUI` rest on the following assumptions:

- the plastic voluminal deformations developed during a seismic loading are primarily due to the shearing strain there undergone by the ground,
- incompressibility of pore water, the variations of water pressure in a sand saturated during a loading with cyclic shearing can be estimated E S starting from the voluminal deformations obtained at the time of a test of cyclic shearing carried out in conditions drained with the same constraint of containment .

With these assumptions, the evaluation of the increase in water pressure during an earthquake (loading in cyclic shearing) requires to evaluate the plastic voluminal deformations generated E S by this earthquake in saturated conditions. These deformations are function of the cumulated plastic voluminal deformation and D E it amplitude of the al. and shearing strain applied (Martin, 1975; Byrn E 1991). Martin and al. (1975) consider that slip between grains generating one increment of voluminal deformation plastic $\Delta\varepsilon_v^p$ during a drained shear test or not drained is the same one. Very forced hydrostatic transferred by the slip from the grains to pore water generates one increase in the pressure of water, according to the following relation:

$$\Delta u = E_r \Delta\varepsilon_v^p \quad (20)$$

where $\Delta\varepsilon_v^p$ represent the increment of voluminal deformation plastic accumulated for one period of the history of shearing strain, Δu is the increment of water pressure and E_r is the module of discharge/refill of the skeleton for the level of effective constraint current (Martin and al. 1975).

- Evaluation of $\Delta\varepsilon_v^p$

The evaluation of $\Delta\varepsilon_v^p$ is carried out using the model of Byrne (1991):

$$\frac{\Delta\varepsilon_v^p}{\gamma} = C_1 \exp\left(-C_2 \frac{\varepsilon_v^p}{\gamma}\right) \quad (21)$$

Where ε_v^p is the voluminal office plurality of deformation (nap of $\Delta\varepsilon_v^p$) since the beginning of calculation and C_1 and C_2 are the parameters of the model, which depend on the relative density D_r or of the value of test SPT $(N_1)_{60}$. Byrne (1991) proposes an empirical relation in order to identify the parameters C_1 and C_2 starting from the value $(N_1)_{60}$:

$$C_1 = \frac{8{,}7}{(N_1)_{60}^{1{,}25}}$$

$$C_2 = \frac{0{,}4}{C_1} \quad (22)$$

These expressions make it possible to obtain the increment of voluminal deformation plastic for a harmonic loading. For a seismic loading (i.e not uniform), the evaluation of the plastic deformations is made by semi-cycle successive in an incremental way, via one method of counting of cycles per passage to zero shearing strain. This method of counting on the deformation restricts the use of this approach to the case 1D – 1 components only.

- Evaluation of E_r

The evaluation of E_r is made using the expression suggested by Wu (1995), determined starting from the experimental tests:

$$E_r = M \sigma'_v = M(\sigma'_{v0} - u) \quad (23)$$

where the value of M depends on the relative density D_r or of the value of test SPT $(N_1)_{60}$. Wu (2001) proposes the following correlation for the calculation of M :

$$M=10(N_1)_{60}+160 \quad (24)$$

Cette expression is integrated in the order DEFI_SOL_EQUI during the evaluation of the module E_r .

- Evaluation of the interstitial pressure ratio r_u

By combining the equations (20) and (23) one can estimate the report directly r_u by the following equation:

$$r_u=\frac{\Delta u}{\sigma'_{v0}}=1-\exp(-M\varepsilon_v^p) \quad (25)$$

The value of r_u is estimated for each semi-cycle in an incremental way, starting from the calculation of ε_v^p obtained with the assistance the expression (21) .

It is supposed that the impact of the water pressure in the modulus of rigidity can be estimated using the following expression:

$$G_{max}=G_{0max}(1-r_{ueff})^{0,5} \quad (26)$$

where G_{0max} is the modulus of maximum rigidity to the confining pressure in-situ and r_{ueff} is estimated like a factor χ_{ru} maximum value of r_u , r_{umax} , in a way similar to the weighting coefficient of the straight-line method are equivalent classical § 7 .

$$r_{ueff}=\chi_{ru}r_{umax} \quad (27)$$

Therefore, the method presented below and applied by Kteich and al. (2018) to the estimate of liquefaction and compressings sismo-armatures to the town of Urayasu to Japan during the earthquake of 2011 consists in enriching behaviour in shearing describes by the curves G/G_{max} according to the shearing strain, by taking account of the influence of the water pressure for the calculation of the modulus of rigidity degraded §6.3. Nevertheless, it does not intervene for obtaining the hysterical value of damping starting from the curves D according to the effective deformation.

6 Establishment of the linear equivalent in Code_ hasster

The order DEFI_SOL_EQUI [U4.84.31] allows to calculate the answer of a unidimensional column of ground a seismic request with the linear model are equivalent (formulation of Schnabel or Lysmer) in convolution or déconvolution.

The method of resolution of the linear model are equivalent is an iterative procedure, where are evaluated with each iteration for each layer of the linear characteristics equivalent starting from the curves of degradation of the modulus of rigidity G and of the hysteretic increase in damping.

The order provides in exit the temporal signals, the Spectra of Answer of Oscillator (SRO), the temporal strains and stresses with various depths as well as the degraded properties of ground in agreement with the levels of calculated shearing strains.

The user must give as starter order:

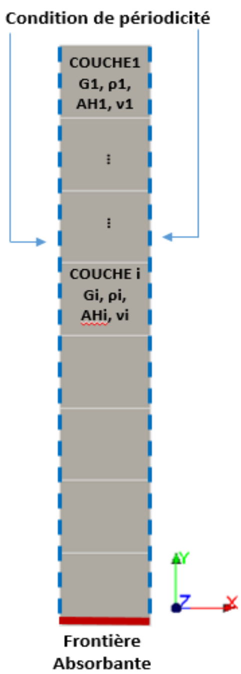
- A table giving the report G/G_{max} according to Gamma,
- A table giving the reduced damping of material D according to Gamma,
- A table describing characteristic initial materials of the various layers (Young modulus E, density RHO, Poisson's ratio NAKED, damping hysteretic AMOR_HYST, possibly the value of $(N_1)_{60}$ N1 for obtaining the parameters of the model of Byrne). Damping hysteretic is equal to twice critical damping $AH=2D$ (see justification with the §4.2.).
- The grid 2D/3D column respecting a size of mesh $L_{max}<\frac{V_s}{8\,f_{coupure}}$,
- A temporal signal (one component or three components) imposed in condition of free field or levelling rock.

6.1 Limiting conditions of the model

The subjacent model is a column of ground 2D with the boundary conditions following:

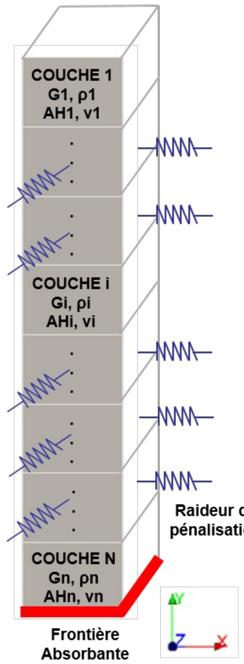
- *Flat rims and left of the column* : Condition of periodicity. That means that displacements of the nodes of the faces left and right-hand side in opposite are made equal. Infinite horizontal layers are thus modelled.
- *Low of the column* : hasffectation of an element of absorbing border (cf [R4.02.05] Elements of absorbing border). The substratum is thus modelled like an infinite medium. According to the coefficient given to the element, part of the waves is thought of the interface, while the other part is dissipated (by radiative damping).

The absorbing border adopts modeling D_PLAN_ABSO . The other elements adopt modeling D_PLAN .



The subjacent model is a column of ground 3D with the boundary conditions following:

- *Edges side* : hasffectation of one stiffness of penalty of $1{,}0\times 10^{15}$ in order to simulate conditions of periodicity and to impose the same dynamic behavior on the two horizontal directions. Infinite horizontal layers are thus modelled.
- *Low of the column* : hasffectation of an element of absorbing border (cf [R4.02.05] Elements of absorbing border). The substratum is thus modelled like an infinite medium. According to the coefficient given to the element, part of the waves is thought of the interface, while the other part is dissipated (by radiative damping).



The absorbing border adopts modeling `3D _ ABSO` . The other elements adopt modeling `3D` .

6.2 Formulations of the linear model are equivalent

The order models damping by a damping hysteretic (operator `DEFI_MATERIAU` keyword `AMOR_HYST`). With to leave the Young modulus E and of damping hysteretic AH informed in `DEFI_MATERIAU`, **Code_hasster** built the complex module E^* according to the equation:

$$E^*=E(1+i.AH) \tag{28}$$

The construction of the complex module thus corresponds to the formulation of Schnabel described with the § 4.1 if $AH=2\mathrm{D}$. This is why it is necessary well to specify $2D_{min}$ in initial value in the table of ground.

However if the user chose the formulation of Lysmer, the order calculates starting from the data materials E and AH , a module E_1 and a damping AH_1 equivalent such as the formulation of Lysmer is found:

$$E_1(1+i\,AH_1)=E\left(\left(1-\frac{AH^2}{2}\right)+ii\,AH\sqrt{1-\frac{AH^2}{4}}\right) \tag{29}$$

By equalizing the real and imaginary parts of two preceding equations, the following relations are obtained:

$$E_1=E\left(1-\frac{AH^2}{2}\right) \text{ and } AH_1=AH\sqrt{1-\frac{AH^2}{4}} \tag{30}$$

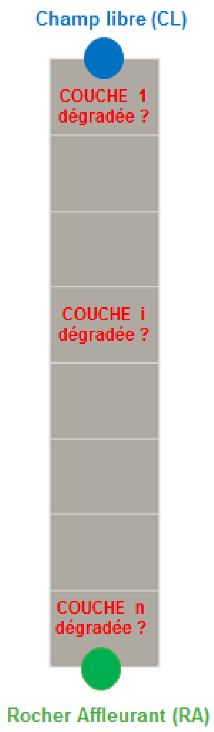
The straight-line method are equivalent does not present in this form of dependence of damping to the frequency.

6.3 Method of resolution

The procedure of resolution of the problem of the order is the following iterative procedure:

1. Assignment of the moduli of initial rigidity and damping ($G=G_{max}$, $D=D_{min}$) in each point of discretization of the column of ground.
2. Calculation of the complex modules of shearing according to the formulation of the linear model are equivalent selected (Schnabel or Lysmer) from G and D informed.
3. Calculation of the harmonic answer of the column of ground: excitation by a unit white vibration at the base of the column (noted edge `RA`): $A_{RA}^{harm}=1$ on the selected waveband `DYNA_VIBRA` , `TYPE_CALCUL = 'HARM'` .
4. Harmonic accelerations with each depth are recovered and the transfer transfer functions between the levelling rock `RA` , and the free field `CL` , and various layers i are calculated.

   While noting ${A^{harm}}_i$, the harmonic acceleration of the layer i , it comes:



$$Fonction\,de\,transfert_{CL/RA}=\frac{1+{A^{harm}}_{CL}}{{A^{harm}}_{RA}}=\frac{1+{A^{harm}}_{CL}}{1}$$

$$Fonction\,de\,transfert_{i/RA}=\frac{1+A^{harm}{}_{i}}{1}$$

Calculation being moving relative (`CHARGEMENT=' MONO_APPUI'`), it is advisable to add the movement of training to harmonic accelerations of each layer. Here, the movement of training is 1, which explains the expression of the numerator of the transfer transfer function $1+A^{harm}{}_{CL}$.

If calculation is moving absolute (`CHARGEMENT= 'ONDE_PLANE'`), the transfer transfer functions are reduced to the harmonic answer of each layer.

1. Calculation of the transform of Fourier (FFT) of the entry signal (passage into frequential).
2. Convolution/déconvolution of the entry signal in the column (Multiplication of the transfer transfer functions by the FFT of the entry signal). If the imposed signal is in levelling rock `RA`, it is convoluted by using the transfer transfer functions defined above. If the imposed signal is in free field CL, one calculates the signal levelling rock corresponding with the transfer transfer function RA/CL for `CHARGEMENT=' MONO_APPUI'` :
$$Fonction\,de\,transfert_{RA/CL}=\frac{1}{1+A^{harm}{}_{CL}}$$
3. Calculation of the deformations $\gamma(f)$ in each layer.
In the case of a modeling 1D – 3 components, the value ofS deformations $\varepsilon_{xy}(f)$, $\varepsilon_{yz}(f)$ and $\varepsilon_{yy}(f)$ are recovered in order to calculate the equivalent deformation.
4. Calculation of the transforms of opposite Fourier $\gamma(t)$ (return into temporal).
In the case of a modeling 1D – 3 components, the value of $\varepsilon_d(t)$ is obtainedE fromS deformations $\varepsilon_{xy}(t)$, $\varepsilon_{yz}(t)$ and $\varepsilon_{yy}(t)$ by the following expression:
$$\varepsilon_d=\frac{2}{3}\sqrt{\varepsilon_{yy}^2+3\varepsilon_{xy}^2+3\varepsilon_{yz}^2}$$
The value of $\gamma(t)$ is then calculated from $\varepsilon_d(t)$.
5. Calculation in each layer of the definite shearing strain effective like:
$$\gamma_{effective}=\chi\cdot\gamma_{max}$$
with γ_{max} maximum shearing strain calculated in the layer and χ weighting coefficient of the effective deformation (`COEF_GAMMA`, by default is equal to $0{,}65$, cf § 7).
6. Evaluation for each layer of the modulus of rigidity and damping corresponding to $\gamma_{effective}$ on the curves G/G_{max} - Gamma and D - Gamma corresponding.

   In the case of the simplified modeling of the water pressure via the model of Byrne, the following formula is used for the evaluation of the modulus of rigidity:
$$G/G_{max}(\gamma_{effective}).(1-\chi_{ru}r_{umax})^{0,5}$$

where r_{umax} represent the maximum of history of evolution of r_u and χ_{ru} is a coefficient of weighting which makes it possible to modulate the impact of r_{umax} on the effective deformation (COEF_KSI, by default is equal to $0{,}667$). One thus multiplies the value of G/G_{max} evaluated with $\gamma_{effective}$ by the expression taking account of the rise of water pressure.

Damping is évalu é with the expression of $\gamma_{effective}$ without taking into account of the water pressure .

7. If $\left|\frac{G_{k+1}-G_k}{G_k}\right|>\theta$ (with $\theta=5\%$ by default (RESI_RELA)), return at stage 2 with the new modules G_{k+1} for each layer.
8. If convergence is reached, postprocessing is carried out. The Macro-order calculates them accelerations, constraints, deformations, into temporal (FFT reverses), and SRO for each layer. The degraded properties of the column are also given.

7 Choice of the weighting coefficient of the effective deformation

The weighting coefficient of the effective deformation χ is a parameter-key connecting the conditions of loading during the earthquake to the laboratory tests which are carried out to measure the curves G/G_{max} and D - gamma (Kramer, 1996 [7]). Its value is taken lower than 1. The introduction of this parameter into the theory of the linear model are equivalent is practical because it makes it possible to adjust the results calculated with a recorded answer. However when one wishes to predict the answer of a column of ground, it is necessary to make the choice of a value of χ.

In the literature, the practices and the recommendations on the value of this coefficient are rather different:

- In the instruction manual of SHAKE (2012 and previous versions) [8], Schnabel and al. (2012) recommend to take it equal to a value of 0.65. This value remained today like the value reference used in practice of engineering without more consideration. However this recommendation remains empirical.
- In the instruction manual of DEEPSOIL (2012) [9], Y. Hashash recommends to take this parameter with a value of $0{,}65$.
- In the instruction manual of SHAKE91 (1992) [10], Idriss and Sun recommends to vary this coefficient according to the magnitude M_w earthquake of the entry signal, according to the formula:

$$\chi=\frac{(M_w-1)}{10} \quad (31)$$

- According to Idriss and Sun [10], values of χ are thus understood in the interval $[0.4,\ 0.75]$, which corresponds to earthquakes magnitude M_w of 5 with $8{,}5$.
- In the instruction manual of EERA (2000) [5], this coefficient is noted R_y and depends on the magnitude on the earthquake according to the formulation on Idriss and Sun [10] above.
- Dickenson (1994) [11] watch that the answer calculated with $\chi=0{,}65$ can be slightly improved by using a value of $0{,}35$ with $0{,}55$ for earthquakes magnitude $M_s=6-7$, and enters $0{,}55$ and $0{,}7$ for earthquakes magnitude $M_s=7-8$, M_s being magnitude of the waves of surface.
- Yoshida et al. (2002) [12] estimate that the value of χ must vary according to the seismic level of the recorded signal and that it actually varies enters $0{,}2$ and 1. It specifies that there is not good method for the evaluation of χ. Yoshida and al. show that the use of the weighting coefficient for the evaluation of the modulus of rigidity and damping is at the origin of two defects in the theory of the linear equivalent:

  1. The over-estimate of the maximum shear stress which implies an over-estimate of maximum acceleration to the peak.

  The mechanism is described on Figure 7-1. Feature full O-A-B represents stress-strain curve of the ground calculated starting from the relation G - Gamma given by the curve of test. Let us take the example where a maximum deformation γ_{max} is calculated. The estimate of the modulus of rigidity (secant module - straight line OA) is made compared to the effective deformation γ_{eff}. However this implies that with this modulus of secant rigidity (OA) to the deformation γ_{max}, the maximum shear stress becomes τ_1 and either τ_2. There is thus an over-estimate of the maximum constraint of shearing which implies an over-estimate of acceleration to the peak. This over-estimate remains moderate with weak deformation but can be important for strong earthquakes. Yoshida and al. (2012) watch that with strong deformation (on the plastic stage), over-estimate is in $\frac{1}{\chi}$, that is to say $50\ \%$

for $\chi=0{,}65$. To mitigate this difficulty, the first idea is to take χ equal to 1 but this solution implies another problem explained low.

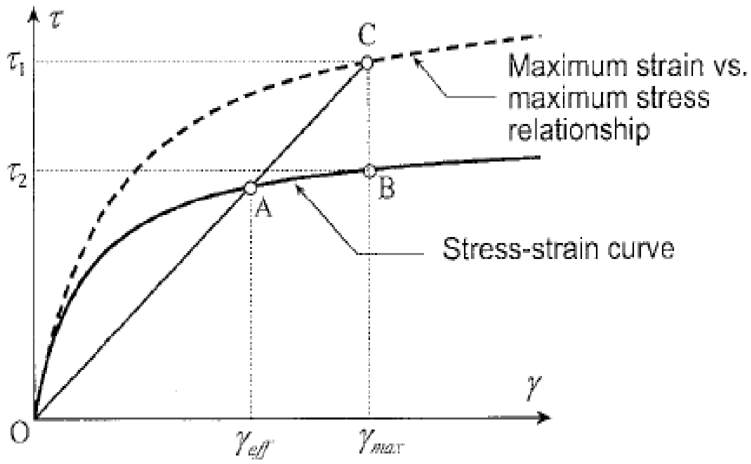


Figure 7-1 : Diagram illustrating the over-estimate of the maximum constraint of shearing with the straight-line method are equivalent [12].

2. The undervaluation of high frequency amplification

Yoshida and al. [12] illustrate the second defect of the straight-line method are equivalent by Figure 7-2 where amplification (ratio of the FFT of accelerations) was calculated with SHAKE for a survey instrumented (borehole) in bay of Tokyo and was compared with the measured ratio. The amplification calculated by SHAKE is lower from $7\,Hz$. Yoshida and al. [12] show that the problem comes owing to the fact that the modulus of rigidity and damping are evaluated starting from the effective deformation, and that for all the frequencies. The modulus of rigidity is then weaker and damping larger than necessary for the field of the high frequencies, where the amplitudes are smaller than the effective deformation. This point shows simplification to use the same weighting coefficient of the effective deformation for the behavior with high and low frequencies.

This is why certain authors recommend to vary χ according to the magnitude. χ must take low values for weak earthquakes and more important values for stronger earthquakes.

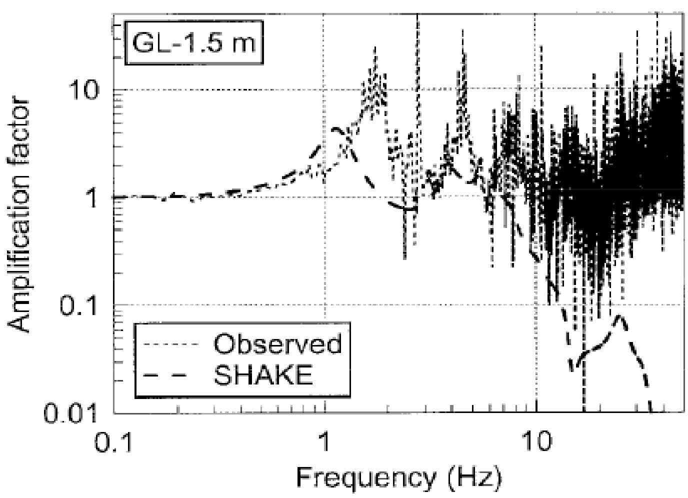


Figure 7-2. Comparison of amplification between SHAKE and measurements of the borehole [12].

Items 1 and 2 are thus contradictory since it would be necessary to increase χ to lower the over-estimate of the maximum constraint of shearing (and thus of acceleration to the peak) and to reduce χ to avoid the undervaluation of high frequency amplification.

The choice of χ must thus be done according to the problem posed by taking account of the two preceding remarks.

Modifications of the linear model are equivalent were proposed to solve these difficulties (Model FDEL, Assimaki and al. [13], Model DYNEQ Yoshida and al. [12]). In these models, the weighting coefficient of the effective deformation, the modulus of rigidity and damping depend on the frequency.

These models remain however little used in practice because of the moderate results.

For model FDEL, Assimaki et al. [13] and Furumoto (2000) [14] show a good improvement of the results compared to the classical model while Yoshida and al. [12] and Kwak and al. [15] put forward the results less good than the classical model with model FDEL.

For model DYNEQ, Yoshida and al. [12] show an unquestionable improvement in the quality of the results got particularly in déconvolution compared to the classical model (described here) and with model FDEL. This improvement is nevertheless to moderate by work of Kwak and al. [15].

8 Choice of the factor loading of the rise of effective water pressure

The factor loading of the rise of effective water pressure χ_{ru} is a parameter-key allowing to consider the effect of the rise of water pressure on the degradation of the modulus of rigidity. The introduction of this parameter is practical because it makes it possible to adjust the influence of the increase in the water pressure in behaviour in shearing of the grounds.

At the time of the thesis of Kteich (2018), Uparametric study on the real case of the town of Urayasu was conducted to gauge this factor compared someNT observations of liquefaction in the city with the predictions for various values of χ_{ru}. The value allowing to obtain all observations of occurrence of liquefaction is equal to 0.6667.

9 Checking and validation

The reader will refer to the documentation of use of the order DEFI_SOL_EQUI [U4.84.31] or with the CAS-tests ZZZZ412 (Validation of the order DEFI_SOL_EQUI), SDLS128 (Seismic request of a column of ground with the linear model are equivalent, Comparison with SHAKE in déconvolution), SDLS141 (seismic Request of a column of ground – Comparison Benchmark PRENOLIN – Comparison with the model of Hujeux in weak deformation), SDLX101E,H (Checking of the chaining *MISS3D - code_aster* in the case of a building of big size), to have cases of applications.

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