

## Method of taking into account of the interaction Floor-Material

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### Summary:

In the seismic studies, one is often brought to carry out spectra of floor for buildings where the equipment is modelled by massS added to the floor. These additional masses impact the spectra of floor. The method suggested here makes it possible to modify the spectra of floor by taking of account the interaction of the masses and the floor.

## Contents

|       |   |   |
|-------|---|---|
| 1     | Introduction of method IPM.....                         | 3 |
| 1.1   | Method.....   | 3 |
| 1.2   | Takings into account of the materials in modelings..... | 3 |
| 2     | Development of the method.....                          | 5 |
| 2.1   | Principles of the method.....                           | 5 |
| 2.2   | Calculation of the transfer transfer functions.....     | 7 |
| 2.2.1 | Transfer transfer function for the model B.....         | 7 |
| 2.2.2 | Transfer transfer function in model A.....              | 8 |
| 2.2.3 | Total transfer transfer function.....                   | 8 |
| 2.2.4 | Initial conditions.....                                 | 8 |
| 3     | Description of the versions of the document.....        | 9 |
| 4     | Bibliography.....                                       | 9 |

## 1 Introduction of method IPM

The problems of the taking into account of the dynamic coupling between the principal and secondary structures in the development of the vertical spectra of floor are an important point dynamic study of the structure.

On certain tapes of floor equipment is sometimes installed whose total mass reported to that of the band of floor can reach some for hundred even a few tenS of for hundred. However, the interaction between this equipment and the band of floor will tend in general to attenuate amplifications observed, especially when there is coincidence between the frequency of the floor and the Eigen frequencies of the equipment which is installed there.

The investigations carried out for this reason showed that the ratios of mass equipment/structure were likely in certain cases to generate a coupling dynamic, qualified Interaction Floor-Material, and having for principal effect, when the Eigen frequencies of the equipment are close to those of the support, to modify the dynamic behavior of the floor compared to the "rigid" case.

This document falls under a will to produce realistic spectra of floor taking of account the effects of dynamic coupling like prescribes it the guides of the ASN ([1]).

### 1.1 Method

The interaction material-floor rests on the principle that a heavy equipment laid out on a floor modifies the behavior of the floor. Simplifying assumptions are retained for the modeling of the floor and the equipment. Both will be represented by oscillators 1D in the vertical direction.

The approach described in this note leads to the construction of spectra of floor corrected by the IPM. These spectra of floor are obtained starting from the accélérogrammes rough vertical (i.e. not taking into account the IPM) resulting from a preliminary transitory seismic analysis.

### 1.2 Takings into account of the materials in modelings

Before any modeling of a building, a reflection must take a lead in the taking into account of the materials in the assessment of the masses.

Three rules can be released on the taking into account of the materials (or secondary structures) in modeling [1] :

- The small equipment (a few hundred kilogrammes) are not taken into account. They are indeed covered by the contractual overload on the floors;
- If the mass of the material is low (see Figure 1) compared to that of its support, its dynamic behavior has little influence on the dynamic behavior of the carrying structure. One is satisfied to represent this material by masses added in the model of building;
- For elements of important mass having a sufficiently flexible support, the question of the dynamic coupling between the structure and the material arises then which is attached there.

To summarize, when a material is supposed to be uncoupled from the floor, its mass is simply taken into account in the model of floor like an added distributed or located mass. On the other hand, when a material is supposed to be coupled, it is necessary to take into account the material in the total model in order to characterize the phenomena of coupling. A modal analysis of the equipment posed on the panel of floor must be realized, it is necessary to release the principal modes from them. The materials are then represented in the total model like simple oscillators.

However, it happens sometimes that certain equipment was taken into account as a mass added without attaching stiffness to it because their relatively low mass, taken individually, did not justify it or when the dynamic coupling of equipment had been considered to be nonsignificant by the modélisateurs.

The purpose of this method is to correct a posteriori this lack with modeling upstream by a taking into account of the dynamic coupling between the primary structures and the equipment. The equipment concerned can be

made up of a sum of small equipment of which total mass and modal characteristics check the exposed criterion Figure 1 (for example : a line of electrical equipment boxes) or of single equipment checking this same criterion in mass and frequencies

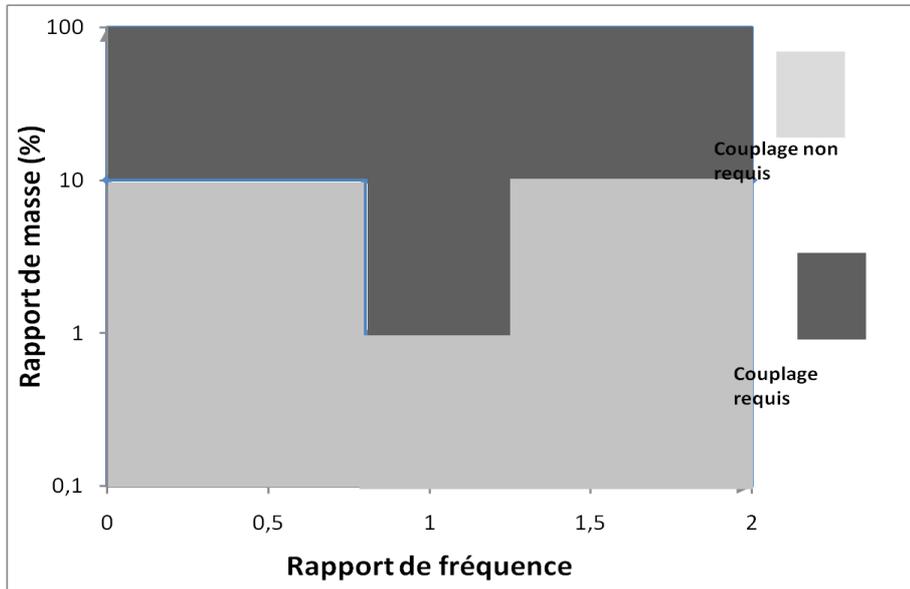


Figure 1 : Criteria of coupling/decoupling [2]

## 2 Development of the method

### 2.1 Principles of the method

The method suggested rests on the simple assumption that the answer of the floor without Interaction Material Floor - (IPM) can be comparable to the answer of an oscillator 1D of frequency  $f_1$ , of mass  $m_1 + m_2$  and of damping  $\xi_1$  (see Figure 2), where we defined  $m_1$  as being mass of the floor and  $m_2$  mass of the equipment.

It is the mode of inflection of the panels of floor which is taken into account by modeling unidimensional floor. We consider moreover that the modal mass mobilized by the mode of inflection is the total mass of the floor, which is a conservative assumption. The answer of the floor without IPM amounts considering that the equipment of mass  $m_2$  is rigidly related to the floor.

The equipment as for him is characterized by a set of oscillators 1D of frequencies and variable masses. We suppose indeed that the heavy equipment laid out on the floor has several Eigen frequencies in the vertical direction. The modal characteristics of the equipment (participative frequencies and masses) are to be determined upstream calculation by the user. These data can be found in the notes of qualification or the notes of dimensioning. A calculation by finite elements can also provide this kind of information. Each mechanical characteristic of the oscillators representing the equipment will carry the index  $i$  in the continuation of this document.

From the mass  $m_2$  equipment and mass  $m_1$  floor, one defines the ratio of mass  $\lambda = m_2/m_1$ .

The method used to obtain the corrected spectra of floor rests on the following principle:

- The accélérogramme resulting from calculations of response of the building and the floors characterizes the movement of the mass  $m_1 + m_2$  in model A (see Figure 2);
- The fascinating accélérogramme corrected of account the IPM is calculated by considering that the equipment is characterized by a series of simple oscillators;
- From the corrected accélérogramme, spectra of corrected floorS are calculated for various values of the critical percentage of damping.

For the calculation of the corrected accélérogramme, we employ a frequential method founded on the use of transfer transfer functions. This method thus supposes the calculation of the transform of Fourier of the starting accélérogramme (or accélérogramme gross).

A calculation of opposite transform of Fourier makes it possible to find the accélérogramme corrected after having multiplied the FFT (Fast Fourier Transform) of the starting accélérogramme by the transfer transfer function between the movement of the mass  $m_1$  in the model A and the model B.

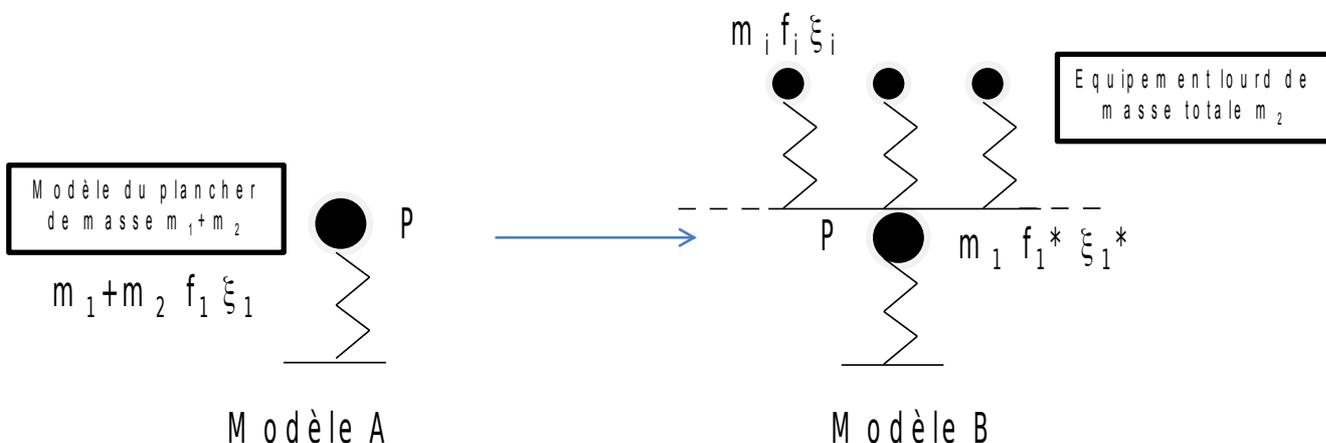


Figure 2 : Descriptions of the models

We define the following notations:

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- $\omega_1 = \sqrt{\frac{k_1}{m_1 + m_2}}$
- $f_1 = \frac{1}{2\pi} \sqrt{\frac{k_1}{m_1 + m_2}}$
- $C_1$  indicate the damping coefficient
- $\xi_1 = \frac{C_1}{2(m_1 + m_2)\omega_1}$
- $f_1^* = f_1 \sqrt{1 + \lambda}$
- $\xi_1^* = \xi_1 \sqrt{1 + \lambda}$
- $\lambda = \frac{m_2}{m_1}$
- $f_i \in 0,40 \text{ Hz}$
- $m_i = \alpha_i m_2$
- $\sum_i \alpha_i = 1$
- $\lambda_i = \alpha_i \lambda$

In these notations, we indicated by  $\alpha_i$  unit effective masses associated with each mode of heavy equipment and by  $\lambda_i$  the report of mass associated with each one of these modes. We will note  $N$  the full number of oscillators constituting the equipment laid out on the floor.

In the model A, the oscillator has a mass  $m_2 + m_1$  because we consider that the accélérogramme, calculated gross during a preliminary transitory analysis, takes into account the added masses. That thus supposes that the model finite elements of the floor integrates the added masses of heavy equipment.

Calculations necessary to the modification of the spectra of floor proceed in the following way:

- Identification of the parameters (frequency  $f_1$  and damping  $\xi_1$  support; depreciation  $\xi_i$  heavy equipment, report of mass  $\lambda$ , unit effective masses  $\alpha_i$  and frequencies  $\lambda_i$ );
- Reading of the accélérogramme at the point of the studied floor  $\ddot{x}(t)$ ;
- Transform of Fourier of the accélérogramme,  $\hat{\ddot{x}}(\omega)$ ;
- Calculation of the transfer transfer function enters the movement of the mass  $m_1$  in model A and the model B (see Figure 2),  $H(\omega)$ ;
- Product  $\hat{\ddot{x}}(\omega)H(\omega)$ ;
- Opposite transform of Fourier,  $\ddot{y}(t)$ ;
- Calculation of the spectrum associated with  $\ddot{y}(t)$  with desired damping  $\xi$ .

We obtain after calculations, a series of spectra with the desired values of damping. These spectra constitute the spectra of floor corrected to include in the collections of spectra. The calculation perhaps carried out for several points of the floor.

## 2.2 Calculation of the transfer transfer functions

In this section, we propose to develop the equations leading to the correction of the spectra of floor by the IPM. The transfer transfer function between the points P of models A and B, to see Figure 2 and Figure 3, is equal to the transfer transfer function between the base and the point P in the model B divided by the transfer transfer function between the base and the point P in model A.

### 2.2.1 Transfer transfer function for the model B

Below equations of the movement for an excitation  $\ddot{x}_b$  at the base of model b:

$$m_i \ddot{x}_i + C_i(\dot{x}_i - \dot{x}_1) + k_i(x_i - x_1) = -m_i \ddot{x}_b \text{ for } i \in [2; N] \quad (1)$$

And:

$$m_1 \ddot{x}_1 + C_1 \dot{x}_1 + \sum_{i=2, N} C_i(\dot{x}_1 - \dot{x}_i) + k_1 x_1 + \sum_{i=2, N} k_i(x_1 - x_i) = -m_1 \ddot{x}_b \quad (2)$$

For a harmonic request of pulsation  $p$  of amplitude  $\hat{x}_b$ , the equations are written:

$$-m_i p^2 \hat{x}_i + ip C_i(\hat{x}_i - \hat{x}_1) + k_i(\hat{x}_i - \hat{x}_1) = m_i p^2 \hat{x}_b \text{ for } i \in [2; N] \quad (3)$$

And:

$$-m_1 p^2 \hat{x}_1 + ip C_1 \hat{x}_1 + \sum_{i=2, N} ip C_i(\hat{x}_1 - \hat{x}_i) + k_1 x_1 + \sum_{i=2, N} k_i(\hat{x}_1 - \hat{x}_i) = m_1 p^2 \hat{x}_b \quad (4)$$

That one can rewrite:

$$(-p^2 + 2ip\omega_i \xi_i + \omega_i^2) \hat{x}_i - 2(2ip\omega_i + \omega_i^2) x_1 = p^2 \hat{x}_b \text{ for } i \in [2; N] \quad (5)$$

And:

$$(-p^2 + 2ip\omega_1 \xi_1 + \omega_1^2) \hat{x}_1 + \sum_{i=2, N} (\lambda_i \omega_i \xi_i + \lambda_i \omega_i^2) \hat{x}_1 - \lambda (2ip\omega_i \xi_i + \omega_i^2) \hat{x}_i = p^2 \hat{x}_b \quad (6)$$

From the equation 5, we obtain:

$$\frac{\hat{x}_i}{\hat{x}_b} = \frac{(2ip\omega_i + \omega_i^2) \frac{\hat{x}_1}{\hat{x}_b} + p^2}{-p^2 + 2ip\omega_i \xi_i + \omega_i^2} \text{ for } i \in [2; N] \quad (7)$$

The following notations are defined:

$$\begin{aligned} \Delta_i &= -p^2 + 2ip\omega_i \xi_i + \omega_i^2 \text{ for } i \in [2; N] \\ \bar{x}_j &= \frac{\hat{x}_j}{\hat{x}_b} \text{ for } j \in [2; N] \\ \bar{x}_i &= \frac{(\Delta_i + p^2)}{\Delta_i} x_1 + \frac{p^2}{\Delta_i} \text{ for } i \in [1; N] \end{aligned}$$

In using these notations in the equation 6, one obtains:

$$\Delta_1 \bar{x}_1 + \sum_{i=2, N} [\lambda_i (\Delta_i + p^2)] \bar{x}_1 - \lambda_i [\Delta_i + p^2] \bar{x}_i = p^2 \quad (8)$$

EN defining absolute displacement by  $X_i = \hat{x}_1 + \hat{x}_b$ , the transfer transfer function to item 2 is worth:

$$T_B(p) = \frac{X_1}{\hat{x}_b} = 1 + \bar{x}_1 = 1 + p^2 \left( \frac{1 + \sum_{i=2,N} \lambda_i (\Delta_i + p^2)}{\Delta_1 + \sum_{i=2,N} \lambda_i (\Delta_i + p^2) - \lambda_i \frac{(\Delta_i + p^2)^2}{\Delta_i}} \right) \quad (9)$$

## 2.2.2 Transfer transfer function in model A

With the same notations relating to only the simple oscillator representing the support in model a:

$$(m_1 + m_2) \ddot{x}_1 + C_1 \dot{x}_1 + k_1 x_1 = -(m_1 + m_2) \ddot{x}_b \quad (10)$$

And:

$$-(m_1 + m_2) p^2 \hat{x}_1 + ipC_1 \hat{x}_1 + k_1 \hat{x}_1 = (m_1 + m_2) p^2 \hat{x}_b \quad (11)$$

For a harmonic request of pulsation  $p$  of amplitude  $\hat{x}_b$ , the equations are written:

$$(-p^2 + 2i\omega \xi_{1p} + \omega_1) \hat{x}_1 = p^2 \hat{x}_b \quad (12)$$

EN defining absolute displacement by  $X_1 = \hat{x}_1 + \hat{x}_b$ , the transfer transfer function to item 2 is worth:

$$T_A(p) = \frac{X_1}{\hat{x}_b} = 1 + \bar{x}_1 = 1 + \frac{p^2}{\Delta_1} \quad (13)$$

## 2.2.3 Total transfer transfer function

The total transfer transfer function  $H(\omega)$  is then calculated by carrying out the simple report enters  $T_A$  and  $T_B$ :

$$H(\omega) = \frac{T_B(\omega)}{T_A(\omega)} \quad (14)$$

It is checked well that when  $\lambda = 0$  or  $f_i \rightarrow \infty$ , for  $i \in [2; N]$ , then  $H(\omega) = 1$ ; it thus affects there, in these two cases, no the rough spectrum of entry.

## 2.2.4 Initial conditions

The transformation of the acceleration of the node P between model A and B supposes that the initial conditions are identical between these two models.

It is checked that these conditions are worthless by comparing the initial value compared to the maximum value of the signal:

$$\frac{|\ddot{x}_A(0)|}{\max(|\ddot{x}_A(t)|)} < tol \quad (15)$$

If this condition is not satisfied and that the option of correction is selected, one modifies the initial value of the signal of model A to 0.

## 3 Description of the versions of the document

| Version<br>Aster | Author (S)<br>Organization (S) | Description of the modifications |
|------------------|--------------------------------|----------------------------------|
| 12.7             | N.GREFFET<br>EDF/R & D /AMA    | Initial version of the document. |

## 4 Bibliography

[1] **ASN (2006)**. *Guide/2/01 - Taking into account of the seismic risk to the design of civil engineer works of basic nuclear facilities except for long-term storages of the radioactive waste*, ASN, Internal report.

[2] **Betbeder-Matibet, J., (2003)**. *Paraseismic prevention - Volume 3*. Lavoisier.