# Coupling fluid-structure for the structures tubular and coaxial hulls

**Summary:**

This document describes the various models of coupling fluid-structure available starting from the operator CALC_FLUI_STRU. These models make it possible to simulate the forces of coupling fluid-rubber band in the following configurations:

- tube bundles under transverse flow (primarily for the tubes of Steam Generator),
- passage stem of order/plate of housing (exclusively for the control rods),
- coaxial cylindrical hulls under annular flow (space ferments/envelope of heart,…),
- tube bundles under axial flow (fuel assemblies,…).

For each configuration, the model of forces fluid-rubber bands is initially presented. The resolution of the modal problem is then described. The methods of resolution employed integrate specificities of the various models of forces fluid-rubber bands.

# 1 General presentation

## 1.1 Recalls

The dynamic fluid forces being exerted on a structure moving can be classified in two categories:

- forces **independent** movement of the structure, at least in the range of small displacements; they are mainly random forces generated by the turbulence or the diphasic nature of the flow,
- fluid forces **dependent** movement of the structure, known as "**forces fluid-rubber bands**", persons in charge of the coupling fluid-structure.

In this document, one is interested in the four models of forces **fluid-rubber bands** integrated in the operator CALC_FLUI_STRU. The data-processing aspects related to the integration of these models were the object of notes of specifications [feeding-bottle. 1], [feeding-bottle. 2].

## 1.2 Modeling

The dependence of the forces fluid-rubber bands with respect to the movement of the structure is translated, for the low amplitudes, by **a matrix of transfer enters the force fluid-rubber band and it vector displacement**. The projection of the equation of the movement of the system coupled fluid-structure on the modal basis of the structure alone is written, in **field of Laplace** :

$$\{[M_{ii}]s^2+[C_{ii}]s+[K_{ii}]-[B_{ij}(U,s)]\}(q)=(Q_t)$$ **éq. 1.2- 1**

where $[M_{ii}],[C_{ii}]et[K_{ii}]$ the diagonal matrices of mass, damping and stiffness structural in air indicate respectively;
$(q)$ indicate the vector of the displacements generalized in air;
$(Q_t)$ is the vector of the generalized random excitations (independent forces);
and $[B_{ij}(U,s)]$ represent the matrix of transfer of the forces fluid-rubber bands, projected on the basis of modal structure alone. This matrix depends in particular on $U$, speed characteristic of the flow, as well as frequency of the movement via the variable of Laplace $s$.

A priori, $[B_{ij}(U,s)]$ is an unspecified matrix whose diagonal terms, if they are not worthless, introduce a coupling between modes. In addition, terms of $[B_{ij}(U,s)]$ evolve in a nonlinear way with the complex frequency $s$.

**With each model of force fluid-rubber band is associated a specific matrix of transfer. In all the cases, the formulation of the modal problem under flow can be characterized by the relation [éq. 1.2-1].**

For the various types of configurations being able to be simulated using the operator CALC_FLUI_STRU, the representations of the matrices of transfer of the forces fluid-rubber bands are clarified in the continuation of this document.

# 2 Excitation fluid-rubber band acting on the tube bundles under transverse flow (primarily for the tubes of Steam Generator)

Two methods of simulation of the excitation fluid-rubber band are available in *Code_Aster*.

The first goes up at the end of the Seventies. It is very widespread in the scientific community, within which it is known under the denomination of “method of Connors”. The results provided by this method depend mainly on the value which one allots to the one of his principal parameters of entry: the “constant of Connors”. Conservative values thus have being determined for this constant on the basis of test many carried out in the world. The method of Connors is well adapted to the dimensioning of the tube banks against the vibratory risk at the stage of the design. It is described hereafter in the paragraph § 2.5.

The second integrates more physics that the method of Connors. However, the complete modeling of the phenomena being too complex compared to current knowledge, this second method remains based on a set of experimental correlations, known as correlations fluid-rubber bands. The integration of this second model of excitation fluid-rubber band in *Code_Aster* was approached in the note of specifications [bib.1]. The note of principle of software FLUSTRU [feeding-bottle. 3] constitutes the theoretical documentation of reference. She is recalled in her broad outlines in the paragraphs § 2.1 to 2.4 hereafter.

## 2.1 Description of the studied configuration

One considers a tube bundle excited by a transverse external flow. The transverse external flows tend to destabilize the mechanical system when the rate of the flow increases. An industrial case is that of the vibrations of the tubes of Steam Generator. On this component, the transverse flows are observed as starter tube bundle (monophasic flow liquidates), and the curved part of the tubes (diphasic flow) [Figure 2.1-a].

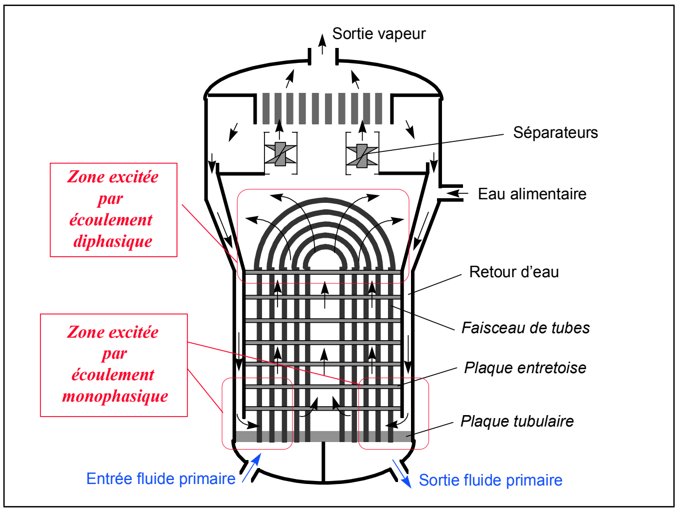


**Figure 2.1-a: Diagram of steam generator**

From the point of view of the coupling fluid-rubber band, the study of the dynamic behavior of the various tubes of a beam subjected to a transverse flow is brought back under investigation equivalent tube; the definition of the equivalent tube depends on the environment of the tube to treat.

When the tube considered has vibratory characteristics appreciably different from those of its neighbors, this tube can be comparable to only one tube, vibrating in the middle of a rigid tube bundle.

In the contrary case, the problem is more complex because one must consider a mechanical system with coupling between tubes of the beam and thus comprising a large number of degrees of freedom.

To treat this kind of configuration, a model was developed at Department TTA, "the total model" [feeding-bottle. 7]; this model allows the definition of a system equivalent to a degree of freedom, which represents the complete coupled system.

The approach adopted to lead calculations can be summarized in the following way [Figure 2.1-b]:

- Taking into account the telegraphic nature of the structures studied, the calculation of the fluid coupling - elastic in the tube bundle is carried out by describing the tube by its curvilinear X-coordinate.
- In calculation, the fluid environment of the tube is characterized, at the same time by the physical properties of the fluid circulating inside the tube (fluid primary education), and by those of the fluid circulating outside the tube (fluid exiting secondary). These physical properties, such as the density, can vary along the tube, according to the curvilinear X-coordinate.
- The rate of flow taken into account for the calculation of coupling fluid-rubber band is the component, normal with the tube in the plan of the tube, the speed of the secondary fluid. This speed can vary along the tube.
- In order to be able to take into account the various possible types of excitation, several zones of excitation can be defined along the structure. In the case of the steam generator, for example, one may find it beneficial to distinguish, on the one hand the zones where the excitation is exerted by a fluid in a monophasic state, which are in foot of tube, and on the other hand, the zone where the excitation is diphasic atstrong rate of vacuum, localised in the curved part of the tube.
- The calculation of coupling is carried out starting from the mechanical characteristics of the structure in "fluid at rest". The forces fluid-rubber bands of coupling are estimated starting from adimensional correlations which are obtained on analytical experiments in similarity. On each zone of excitation, one can thus apply the adequate correlations; the zones of excitation must be disjoined.

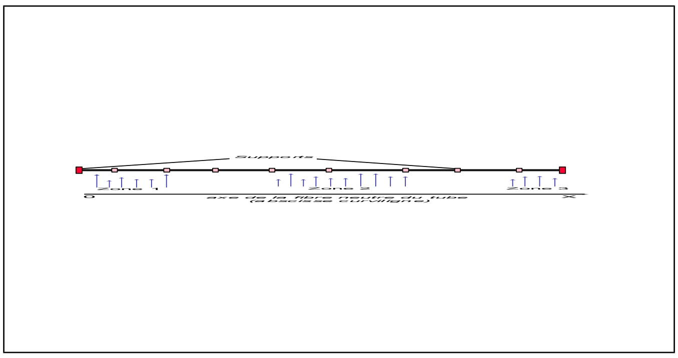


**Figure 2.1-b: representation of the configuration studied**

**For this configuration of coupling fluid-rubber band, the following notations will be used:**

$L$ Overall length of the tube

| | |
|---|---|
| $L_k$ | Length of the zone $k$ |
| | Diameter external of the tube |
| $d_i$ | Internal diameter of the tube |
| $\varphi_i$ | Modal deformation of the mode $i$ |
| $\rho_i(x)$ | Density of the external fluid to the curvilinear X-coordinate $x$ |
| $\rho_i(x)$ | Density of the fluid interns with the curvilinear X-coordinate $x$ |
| $\rho_t$ | Density of the tube (structure alone) |
| $\rho_{eq}(x)$ | Density equivalent to the curvilinear X-coordinate $x$ |
| $U$ | Speed of the external fluid specified by the user in the operator DEFI_FLUI_STRU |
| $V(x)$ | Speed of the external fluid to the curvilinear X-coordinate $x$ |
| $V_k(x)$ | Speed of the external fluid to the curvilinear X-coordinate $x$ (zone of excitation $k$ ) defined by the product of $U$ and of a profile speed specified by the user in the operator DEFI_FLUI_STRU |
| $U_k$ | Mean velocity of the external fluid calculated from $V_k(x)$ for the zone of excitation $k$ |
| $\bar{U}$ | Average speeds $U_k$ on all the zones of excitation |

## 2.2 Stages of calculation

- The first stage of calculation consists in calculating the structural features in “fluid at rest”. One proceeds by considering an equivalent mass of the tube; this equivalent mass gathers, on the one hand the mass of the tube alone, and on the other hand the masses added by the fluids internal and external.

  An equivalent density is thus defined along the tube according to the curvilinear X-coordinate $x$ by the expression:

  $$\rho_{eq}(x)=\frac{1}{(d_e^2-d_i^2)}[\rho_i(x).d_i^2+\rho_t.(d_e^2-d_i^2)+\rho_e(x).d_{eq}^2] \quad \textbf{éq. 2.2- 1}$$

  with

  $$d_{eq}^2=\frac{2.C_m.d_e^2}{\pi} \quad \textbf{éq. 2.2- 2}$$

  In the equation [**éq. 2.2-1**], the term $\rho_e(x).d_{eq}^2$ represent the mass added by the external fluid. This term depends, via the parameter $C_m$, arrangement of the tube bundle (not square or triangular), and containment of the beam (not reduced). For calculations of coupling fluid-rubber band of the tube bundles subjected to a transverse flow, one usually uses, to estimate the coefficient $C_m$, given analytical expressions starting from experimental results. The whole of the data necessary to the estimate of the coefficient $C_m$ is collected by the operator DEFI_FLUI_STRU.

- Knowing the equivalent density of the tube, the elementary matrices of water mass and stiffness at rest are then calculated by means of the profile of equivalent density, by the operator CALC_MATR_ELEM ; the options are used MASS_FLUI_STRU and RIGI_FLUI_STRU. The operator CALC_MODES allows, after assembly of the elementary matrices, to directly calculate the modes out of water at rest of the studied structure.

- The forces fluid-rubber bands of coupling are calculated by the operator CALC_FLUI_STRU starting from the adimensional correlations established on analytical models in similarity. These forces of coupling, $[B_{ij}(U,s)]$ , dependent on the movement of the structure are then taken into account in the general equation of the movement [**éq. 1.2-1**] to calculate the characteristics of the system coupled flow-structure for a given speed of flow.

## 2.3 Form of the matrix of transfer of the forces fluid-rubber bands

In the case of the tube bundles excited by a transverse flow, the forces fluid-rubber bands of coupling are forces distributed along the structure. They are characterized by linear adimensional coefficients of added damping and stiffness, respectively named $C_d$ and $C_k$ . The expression of the coefficients of the matrix of transfer of the forces fluid-rubber bands projected on the basis of modal structure in "fluid at rest" is then the following one:

$$B_{ij}(U,s)=[(\int_L \frac{1}{2}\rho_e(x)V(x)d_e C_d(x,s_r)\varphi_i^2(x)dx)s+\int_L \frac{1}{2}\rho_e(x)V^2(x)C_k(x,s_r)\varphi_i^2(x)dx].\delta_{ij}$$

**éq. 2.3- 1**

Dependence of the coefficients $C_d$ and $C_k$ with respect to the movement the structure and rate of the flow of the fluid is translated by their evolution according to the reduced frequency complexes, $s_r$ defined by:

$$s_r=\frac{s.D}{U}$$

**éq. 2.3-2**

The expression [**éq. 2.3-1**] watch which one retains a diagonal matrix of transfer. That implies:

- the various clean modes of the structure are rather distant from/to each other so that one can suppose that there is not coupling between modes.
- the modal deformations of the structure in "fluid at rest" are not disturbed by the setting in flow of the fluid.

These two assumptions could be checked in experiments on the tube bundles subjected to a transverse flow.

In practice, taking into account the various zones of excitation taken into account along the structure, the diagonal coefficients of the matrix of efforts fluid-rubber bands projected on modal basis are written:

$$B_{ii}(U,s)=\sum_k [(\int_{L_k} \frac{1}{2}\rho_e(x)V_k(x)d_e C_{dk}(\frac{sd_e\bar{U}}{UU_k})\varphi_i^2(x)dx)s+\int_{L_k} \frac{1}{2}\rho_e(x)V_k^2(x)C_{kk}(\frac{sd_e\bar{U}}{UU_k})\varphi_i^2(x)dx]$$

**éq. 2.3- 3**

where $C_{dk}$ and $C_{kk}$ indicate respectively the adimensional coefficients of coupling, damping and stiffness, retained for the zone of excitation $k$ . Fluid speed $\frac{UU_k}{\bar{U}}$ intervening in the reduced frequency complexes in argument of the coefficients of coupling corresponds to the mean velocity on the zone of excitation $k$ , after renormalisation of the profile $V_k(x)$ , so that its average on all the zones of excitation is worth $U$ .

**It is in addition very important to note that each modal deformation taken into account in equations 2.3-1, 2.3-3, etc is actually only via its component in translation according to the direction of the bearing pressure. This is with the fact that the stiffness and added damping coefficients which appear in these equations were given (in experiments) only for the direction of the bearing pressure. This remark applies to all methods of calculating of**

**instabilities fluid-rubber bands of tubes of Steam Generator presented in this document, including with the method of Connors introduced to the paragraphs § 2.5.1 and § 2.5.2. He results from it in particular that the generalized matrices of mass, damping and stiffness which appear in the equations associated with calculations with instability fluid-rubber band of the tubes of Steam Generator (as for example equation 2.4-1) are not matrices generalized with the usual direction of the term, i.e. being pressed on the three components in translation and the three components in rotation, but generalized matrices which one can describe as “directed according to a privileged direction” insofar as they all are calculated on the basis of component only in translation of the modal deformations according to the direction of the bearing pressure. This remark applies only to the application “vibrations of the tubes of Steam Generator” and, inside this application, with the calculation of instabilities fluid-rubber bands.**

## 2.4 Resolution of the modal problem under flow

In the configuration “Tube bundle subjected to a transverse flow”, the problem is solved on the modal basis characterizing the structure in “fluid at rest”.

Generally, the characteristics of the system coupled flow-structure are obtained by seeking the solutions of the equation:

$$\{[M_{ii}]s^2+[C_{ii}]s+[K_{ii}]-[B_{ij}(U,s)]\}(q)=0$$ **éq. 2.4- 1**

where $[M_{ii}],[C_{ii}]et[K_{ii}]$ indicate respectively the diagonal matrices of mass, damping and stiffness structural features in “fluid at rest”;

$(q)$ indicate the vector of the displacements generalized in “fluid at rest”.

As the matrix of efforts fluid-rubber bands retained is diagonal, and that the modal deformations are supposed not to be modified under flow, the problem of coupling fluid-rubber band is reduced to the resolution of $N$ scalar problems, $N$ indicating the number of modes taken into account in the modal base.

For each mode $i$ and each rate of flow $U$ , the problem to be solved is written:

$$M_{ii}s^2+[C_{ii}-\sum_k(\int_{L_k}\frac{1}{2}\rho_e(x)V_k(x)d_eC_{dk}(\frac{sd_e\bar{U}}{UU_k})\varphi_i^2(x)dx)]s+K_{ii}-\sum_k(\int_{L_k}\frac{1}{2}\rho_e(x)V_k^2(x)C_{kk}(\frac{sd_e\bar{U}}{UU_k})\varphi_i^2(x)dx)=0$$

$$\mathbf{M}_{ii}s^2+\left[\mathbf{C}_{ii}-\left(\sum_k\left(\int_{L_k}\frac{1}{2}\rho_e(x)V_k(x)d_eCd_k\left(\frac{sd_e\overline{U}}{UU_k}\right)\varphi_i^2(x)dx\right)\right)\right]s$$

$$+\left[\mathbf{K}_{ii}-\left(\sum_k\left(\int_{L_k}\frac{1}{2}\rho_e(x)V_k^2(x)Ck_k\left(\frac{sd_e\overline{U}}{UU_k}\right)\varphi_i^2(x)dx\right)\right)\right]=0$$

**éq. 2.4- 2**

It will be noted that the equation [**éq. 2.4-2**] is non-linear in $s$ ; its solutions are obtained using an iterative method of Broyden type.

For each mode $i$ , a solution is obtained $s_i$ equation [**éq. 2.4-2**]. One deduces then from $s_i$ , for this mode, the pulsation $\omega_i$ and damping $\xi_i$ system coupled flow-structure, by using the relation:

$$s_i=-\xi_i\omega_i+J\omega_i\sqrt{1-\xi_i^2} \quad \text{with} \quad J^2=-1$$ **éq. 2.4-3**

The coupled system dynamically becomes unstable when one of the damping coefficients $\xi_i$ becomes negative or cancels themselves.

## 2.5 Method of Connors

### 2.5.1 Case of a single zone of fluid excitation

In 1978, H.J. Connors proposes to determine the critical velocity $V_{cn}$ associated with the mode of order $n$ of a tube of Steam generator (Steam Generator) according to the relation [10]:

$$\frac{V_{cn}}{f_n D_e} = K \sqrt{\frac{\bar{m}\,\delta_n}{\bar{\rho}_s D_e^2}}$$

In this relation:

$V_{cn}$ indicate the critical velocity inter-tubes of instability for mode N, $f_n$ indicate the Eigen frequency of order N of the tube [1], $D_e$ indicate the diameter external of the tube, $K$ indicate the constant of Connors, $\overline{m}$ indicate the linear density of reference of the tube including the effects of added mass, $\delta_n$ indicate the decrement logarithmic curve of mode N in fluid at rest, i.e. including the damping of the structure and that brought by the external fluid at rest, and $\overline{\rho}_s$ indicate the density of reference of the secondary fluid.

It is pointed out that the decrement logarithmic curve $\delta_n$ is defined as:

$$\delta_n = \frac{2\pi\,\xi_n}{\sqrt{1 - \xi_n^2}}$$

Where $\xi_n$ indicate the reduced modal damping of mode N. $\xi_n$ being about the percent, it is legitimate to pose $\sqrt{1 - \xi_n^2} = 1$, and thus the approximation:

$$\frac{V_{cn}}{f_n D_e} = K \sqrt{\frac{2\pi\overline{m}\,\xi_n}{\overline{\rho}_s D_e^2}}$$

The constant of instability *K* is in experiments given starting from test results of instability. In the studies of vibratory dimensioning of the tube bundles of Steam Generator, the values usually adopted for this constant are:

- *K = 4* in the event of diphasic transverse flow on the level of the chignon,
- *K = 2.9* in the event of monophasic transverse flow with the top of the tubular plate.

While regarding as linear density of reference of the tube $\overline{m}$ its average linear density, one can determine $\overline{m}$ in the form:

$$\overline{m} = \frac{\pi}{4} \frac{\left(D_e^2 - D_i^2\right)}{L_{tube}} \int_{tube} \rho_{eq}(s)\,ds$$

1 In any rigour, one should consider the value of $f_n$ under flow. However, the method of Connors does not envisage the calculus of the ascribable variation of frequency to the flow. At first approximation, one thus considers for $f_n$ the value of the frequency in fluid at rest.

Where $D_i$ indicate the internal diameter of the tube, $L_{tube}$ indicate its length, S indicates the curvilinear X-coordinate along the tube and $\rho_{eq}(s)$ indicate the density equivalent of the tube to the X-coordinate S:

$$\rho_{eq}(s) = \rho_t + \frac{D_i^2}{D_e^2 - D_i^2}\rho_p(s) + \frac{2C}{\pi}\frac{D_e^2}{D_e^2 - D_i^2}\rho_s(s)$$

Where $\rho_t$ indicate the density of the tube presumedly independent of the curvilinear X-coordinate, $\rho_p(s)$ and $\rho_s(s)$ respectively indicate the density of the primary education fluid and secondary fluid with the curvilinear X-coordinate $s$, and $C$ is defined by:

$C = \frac{\pi}{2}\frac{(\Delta / D_e)^2 + 1}{(\Delta / D_e)^2 - 1}$, where $\Delta$ indicate an equivalent diameter given by the relations:

1) $\Delta / D_e = \left(1.07 + 0.56\frac{P}{D_e}\right)\frac{P}{D_e}$ for a square step (*C* is worth approximately *2.0* for the French Steam Generators)
2) $\Delta / D_e = \left(0.96 + 0.50\frac{P}{D_e}\right)\frac{P}{D_e}$ for a triangular step (*C* is worth approximately *2.2* for the French Steam Generators)

In the same way, while regarding as density of reference of the secondary fluid $\overline{\rho}_s$ its average density, one can determine $\overline{\rho}_s$ in the form:

$$\overline{\rho}_s = \frac{1}{L_{tube}} \int_{tube} \rho_s(s) ds$$

Mode N is unstable if the critical velocity $V_{cn}$ is lower at the effective speed $V_{en}$ associated with mode N, thus defined:

$$V_{en} = \sqrt{\frac{\int_{tube} \frac{\rho_s(s)}{\overline{\rho}_s} V^2(s)\phi_n^2(s) ds}{\int_{tube} \frac{m(s)}{\overline{m}} \phi_n^2(s) ds}}$$

Where $\phi_n(s)$ indicate the modal deformation of the mode $n$, $V(s)$ indicate the rate of the flow under operation ( $m/s$ ), $m(s)$ indicate the linear density of the tube including the effects of added mass ( $kg/m$ ) supposed to vary along the tube, obtained like:

$$m(s) = \frac{\pi}{4}(D_e^2 - D_i^2)\rho_{eq}(s)$$

One defines the report of instability for mode N within the meaning of Connors as being the report:

$$R_{Cn} = \frac{V_{en}}{V_{cn}}$$

### 2.5.2 Case of several zones of fluid excitation

The approach of application of the method of Connors deserves to be specified if the tube is subjected to a multiform excitation on behalf of the fluid, in particular, if the latter is monophasic in bottom of beam and diphasic in the chignon. It is pointed out that such a situation is taken into account in software GEVIBUS [11].

To extrapolate the method of Connors to this case general, one proceeds by generalizing the establishment of the approach suggested by Connors [10].

That is to say $W_n$ the energy added by the flow during a cycle to a vibrating tube in its mode N:

$$W_n = \sum_{i=1}^{Nex} C_i \int_{Lex_i} \rho_s(s)\, V^2(s)\, \phi_n^2(s)\, ds$$

Where, compared to the talk of Connors, the dependence of the constant $C_i$ at the zone of excitation $i$ is added, $Nex$ indicate the full number of zones of excitation, and $Lex_i$ indicate the length of $i$ - ème zone of excitation.

That is to say $E_n$ the energy dissipated during a cycle by the vibrating tube in its mode N:

$$E_n = C_2 f_n^2 \delta_n \int_{tube} m(s)\, \phi_n^2(s)\, ds$$

While equalizing $W_n$ and $E_n$, i.e. while placing themselves at instability, by introducing like Connors the variables of reference $\overline{\rho}_s$ and $\overline{m}$ (although they do not appear essential), while posing $\frac{C_i}{C_2} = \frac{1}{K_i^2}$, where $K_i$ indicate the constant of Connors associated with $i$ - ème zone of excitation, and while seeking to reveal effective speed $V_{en}$ such as Connors defines it, one obtains all done calculations the expression:

$$\frac{\int_{tube} \frac{\rho_s(s)}{\overline{\rho}_s} V^2(s)\, \phi_n^2(s)\, ds}{\int_{tube} \frac{m(s)}{\overline{m}} \phi_n^2(s)\, ds} \sum_{i=1}^{Nex} \frac{1}{K_i^2} \frac{\int_{Lex_i} \rho_s(s)\, V^2(s)\, \phi_n^2(s)\, ds}{\int_{tube} \rho_s(s)\, V^2(s)\, \phi_n^2(s)\, ds} \left[\frac{1}{f_n D_e}\right]^2 = \delta_n \frac{\overline{m}}{\overline{\rho}_s D_e^2}$$

From where:

$$\frac{\int_{tube} \frac{\rho_s(s)}{\overline{\rho}_s} V^2(s)\, \phi_n^2(s)\, ds}{\int_{tube} \frac{m(s)}{\overline{m}} \phi_n^2(s)\, ds} = (V_{en})^2 = \frac{\int_{tube} \rho_s(s)\, V^2(s)\, \phi_n^2(s)\, ds}{\sum_{i=1}^{Nex} \frac{1}{K_i^2} \int_{Lex_i} \rho_s(s)\, V^2(s)\, \phi_n^2(s)\, ds} (f_n D_e)^2 \delta_n \frac{\overline{m}}{\overline{\rho}_s D_e^2}$$

One from of deduced in the case of a multiform excitation the form of the critical velocity $\mathrm{V_{cn}}$ associated with the mode of order $n$ :

$$\frac{\mathrm{V_{cn}}}{\mathrm{f_n D_e}} = \sqrt{\frac{\int_{\mathrm{tube}} \rho_s(s) \mathrm{V}^2(s) \phi_n^2(s) ds}{\sum_{i=1}^{\mathrm{Nex}} \frac{1}{\mathrm{K_i}^2} \int_{\mathrm{Lex_i}} \rho_s(s) \mathrm{V}^2(s) \phi_n^2(s) ds}} \sqrt{\frac{\overline{\mathrm{m}}\,\delta_n}{\overline{\rho}_s \mathrm{D_e^2}}}$$

It is checked that, when the excitation is of comparable nature on the whole of the excited zones $\mathrm{K_{i(i=1,Nex)}} = \mathrm{K}$, the relation above finds the form suggested by Connors:

$$\frac{\mathrm{V_{cn}}}{\mathrm{f_n D_e}} = \mathrm{K} \sqrt{\frac{\overline{\mathrm{m}}\,\delta_n}{\overline{\rho}_s \mathrm{D_e^2}}}$$

### 2.5.3 Alternative of the method

In the method of Connors introduced in the paragraphs above (paragraphs § 2.5.1 and § 2.5.2), the calculation of the report of instability takes into account only one component of the modes: direction defined in `DEFI_FLUI_STRU`. An alternative of this method consists in taking into account the three components in translation of the modes. Effective speed and the critical velocity are written then:

$$V_{en} = \sqrt{\frac{\int_{tube} \left(\frac{\rho_s(s)}{\bar{\rho}_s} V^2(s) \phi_n^2(s) ds\right)}{\frac{M_n}{\bar{m}}}}$$

$$\frac{\mathrm{V_{cn}}}{\mathrm{f_n D_e}} = \mathrm{K} \sqrt{\frac{\overline{\mathrm{m}}\,\delta_n}{\overline{\rho}_s \mathrm{D_e^2}}}$$

Where $M_n$ indicate the generalized mass (not directed according to a direction privileged and fascinating thus in account at the same time three components in translation and three components in rotation) of the mode $n$ , and $\varphi_n$ indicate the modal deformation of the mode $n$ . By $\phi_n^2$ one hears here the sum of the squares of the three components in translation of $\varphi_n$ . The three components in rotation are not taken into account in the calculation of $\varphi_n^2$ .

Out of pboneant :

$$r(s) = \frac{\rho_s(s)}{\bar{\rho}_s} \quad \text{and} \quad u(s) = \frac{V(s)}{V_{moy}}$$

where $V_{moy}$ is the mean velocity., LE report of instability is written:

$$R_n = \frac{V_{en}}{V_{cn}} = \frac{V_{moy}}{f_n D_e K \left[\frac{2\pi \xi_n M_n}{\bar{\rho}_s D_e^2 \int_{tube} (r(s) u^2(s) \phi_n^2(s) ds)}\right]^{1/2}}$$

The calculation of the report according to this alternative is systematically carried out by Code_Aster when one asks for the implementation of the method of Connors.

The report of instability calculated according to this alternative is provided beside the report of instability calculated according to the method specified in the paragraphs § 2.5.1 and § 2.5.2. Most of the time, the two results are identical. If there exists a variation, the reason of this variation must be required in the contribution of the components in rotation of the mode considered, for example in the contribution of rotations of the right parts of the tubes around their axis. One will then adopt the result more penalizing of both.

# 3 Excitation fluid-rubber band acting on the stem of order on the level of the plate of housing (exclusively for the control rods)

The forces fluid-rubber bands acting on this kind of configuration were identified on model GRAPPE2 of department TTA. The theoretical aspects of the identification of these sources are developed in reference [feeding-bottle. 4]. The integration of model GRAPPE2 in *Code_Aster* is approached in the note of specifications [feeding-bottle. 2].

## 3.1 Description of the studied configuration

Model GRAPPE2 represents the stem of order, the upper part of the guide of bunch, and the thermal cuff of an engine of the type 900 or 1300 MWe [Figure 3.1-a].

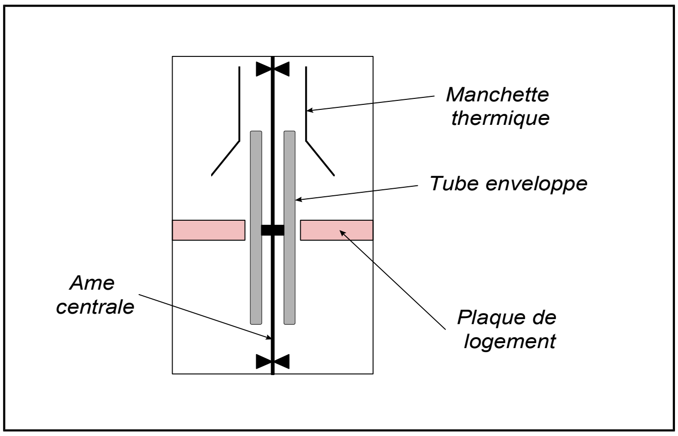


**Figure 3.1-a: General diagram of the model BUNCH 2**

This model primarily consists of a hollow cylindrical tube low thickness, fixed on a full cylindrical central heart. The hollow tube is entirely immersed in water with room temperature. A plate, representing the plate of housing, makes it possible to reproduce annular containment. The flow through the plate can be ascending or descendant. The stem of order can be centered or offset (50% of the average game) on the level of the plate of housing.

Four experimental configurations are thus possible, according to the direction of the flow and the centering or not of the Co stembegs for. The coefficients of forces fluid-rubber bands were identified for each one of these configurations and are available in *Code_Aster*.

Model GRAPPE2 was dimensioned in geometrical, hydraulic similarity and of frequency reduced compared to the configuration engine. The only data of the diameter of the stem of order thus makes it possible, in particular, to deduce the unit from the other geometrical magnitudes.

## 3.2 Stages of calculation

- The first stage of calculation consists in calculating the modal base of the water structure at rest, the locally induced effects of mass added to the level of the containment of the plate of housing being neglected. This stage is carried out by the operator `CALC_MODES`.

With this intention, a homogeneous equivalent density is assigned to the whole of the structure, in order to take into account the apparent mass added by the fluid, except for that induced by the effects of containment on the level of annular space. This equivalent density is defined by:

$$\rho_{eq} = \alpha \frac{\pi R^2}{S} \rho_f + \rho_{tube}$$ **éq. 3.2- 1**

where:

$\alpha$ indicate an adimensional coefficient of containment depend on the studied configuration; $\alpha=1$ is the value used for calculations of control rods. It corresponds to a vibrating roller in an unlimited fluid field.

$R$ indicate the ray external of the tube,

$S$ indicate the surface of the cross-section of the tube,

$\rho_{tube}$ indicate the density of material constituting the vibrating tube.

- The second stage is the taking into account of the coupling with the fluid flow. It is carried out using the operator `CALC_FLUI_STRU`.

## 3.3 Representation of the excitation fluid-rubber band

That is to say $x$ direction of neutral fibre of the tube. The excitation fluid-rubber band identified on model GRAPPE2 is represented by a resulting force and a moment, applied in the same point of X-coordinate $x_o$, corresponding to the central zone of the passage of the stem of order through the plate of housing. The excitation is thus defined, in the physical base, by the relation:

$$\hat{\mathbf{f}}_c(x,s)=\mathbf{F}_c(s)\delta(x-x_0)-\mathbf{M}_c(s)\delta'(x-x_0)$$ **éq. 3.3- 1**

where $\delta'$ indicate the derivative compared to $x$ distribution of Dirac $\delta$.

The resulting force, $\mathbf{F}_c$, acts thus under the effect of transverse displacements of the stem of order; and resulting moment, $\mathbf{M}_c$, acts under the effect of the rotation of the latter.

One notes $\mathbf{X}_T(s)$ the vector of transverse displacements and $\Theta(s)$ the vector of associated rotations, defined by:

$$\mathbf{X}_T(s)=\begin{Bmatrix} 0 \\ u_y(x_0,s) \\ u_z(x_0,s) \end{Bmatrix}$$ **éq. 3.3- 2**

$$\Theta(s)=\begin{Bmatrix} 0 \\ \dfrac{\partial u_y}{\partial x}(x_0,s) \\ \dfrac{\partial u_z}{\partial x}(x_0,s) \end{Bmatrix}$$ **éq. 3.3- 3**

The following relations are used to calculate the forces and the moments resulting fluid-rubber bands starting from the added masses $Cm_1, Cm_2$, added depreciation $Cd_1(V_r), Cd_2(V_r)$ and of the added stiffnesses $Ck_1(V_r), Ck_2(V_r)$, adimensional coefficients identified on model GRAPPE2:

$$\mathbf{F}c(s)=\left[-\frac{1}{2}\rho_f D^2 L_p Cm_1 s^2+\frac{1}{2}\rho_f DUL_p Cd_1(V_r)s+\frac{1}{2}\rho_f U^2 L_p Ck_1(V_r)\right]\mathbf{X}_T(s)$$ **éq. 3.3- 4**

$$\mathbf{M}c(s) = \left[ -\frac{1}{2}\rho_f D^2 L_p{}^3 Cm_2 s^2 + \frac{1}{2}\rho_f DUL_p{}^3 Cd_2(V_r)s + \frac{1}{2}\rho_f U^2 L_p{}^3 Ck_2(V_r) \right]\Theta(s)$$ **éq. 3.3- 5**

In order to simplify the writing of the equations, one notes thereafter:

$$\mathbf{F}c(s) = H_1(s)\mathbf{X}_T(s) \text{ et } \mathbf{M}c(s) = H_2(s)\Theta(s)$$

Speed reduced adimensional $V_r$ is defined here using the relation $V_r = \frac{U}{sD}$ , where $s$ indicate the variable of Laplace.

Expressions [**éq. 3.3-4**] and [**éq. 3.3-5**] utilize the thickness $L_p$ plate of housing. This thickness results from the value of the diameter of the stem of order, $D$, because of geometrical similarity with the configuration engine. The effort fluid-rubber band $\hat{\mathbf{f}}_c(x,s)$ is thus completely characterized by the data of the following sizes:

| | |
|---|---|
| $\rho_f$ | Density of the fluid, |
| $U$ | Rate of the average flow in annular space between stem of ordering and plate of housing, |
| $D$ | Diameter of the stem of order, |
| $Cm_1$ | Coefficient of added mass associated with the translatory movement, |
| $Cd_1(V_r)$ | Added damping coefficient associated with the translatory movement, |
| $Ck_1(V_r)$ | Coefficient of added stiffness associated with the translatory movement, |
| $Cm_2$ | Coefficient of added mass associated with the rotation movement, |
| $Cd_2(V_r)$ | Added damping coefficient associated with the rotation movement, |
| $Ck_2(V_r)$ | Coefficient of added stiffness associated with the rotation movement. |

Adimensional coefficients of added mass, $Cm_1$ and $Cm_2$ , allow the taking into account of the inertial effects induced by local containment of the stem of order the level of the plate of housing. These effects are estimated as follows.

That is to say $H$ the thickness of the annular flow on the level of containment, deduced from $D$ by geometrical similarity compared to the configuration engine; $\alpha$ indicate the adimensional coefficient of containment introduced by the relation [**éq. 3.2-1**]. One obtains [feeding-bottle then. 4]:

$$\frac{1}{2}\rho_f D^2 L_p Cm_1 = \left[ \rho_f \frac{\pi D^3}{8H} - \alpha\rho_f \frac{\pi D^2}{4} \right] L_p = \rho_f \frac{\pi D^2}{4}\left[ \frac{D}{2H} - \alpha \right] L_p$$

$$\frac{1}{2}\rho_f D^2 L_p{}^3 Cm_2 \theta = \rho_f \frac{\pi D^2}{4}\left[ \frac{D}{2H} - \alpha \right] \int_{L_p} \theta (x - x_o)^2 dx = \rho_f \frac{\pi D^2}{4}\left[ \frac{D}{2H} - \alpha \right] \theta \frac{L_p{}^3}{3}$$

One from of deduced the values from $Cm_1$ and $Cm_2$ by:

$$Cm_1 = \frac{\pi}{2}\left[ \frac{D}{2H} - \alpha \right]$$ **éq. 3.3- 6**

$$Cm_2 = \frac{Cm_1}{3} = \frac{\pi}{6}\left[ \frac{D}{2H} - \alpha \right]$$ **éq. 3.3- 7**

Coefficients $Cd_1, Ck_1, Cd_2$ and $Ck_2$ are directly deduced from measurement and are expressed in the form of adimensional correlations.

## 3.4 Projection on modal base and expression of the terms of the matrix of transfer of effort fluid-rubber band

**Decomposition of the movement on modal basis**

One notes $\Phi_\mathbf{j}(x)$ modal deformation of $j$ ème mode of the structure. The decomposition of the vector of displacements in the modal base is expressed in the form:

$$\mathbf{u}(x,s)=\sum_{j=1}^{N}\Phi_j(x)q_j(s)=\sum_{j=1}^{N}\begin{Bmatrix}DX_j(x)\\DY_j(x)\\DZ_j(x)\end{Bmatrix}q_j(s) \quad \text{éq. 3.4- 1}$$

Where $DX_j$, $DY_j$ and $DZ_j$ correspond to the three components of translation characterizing the modal deformations calculated using *Code_Aster*.

**Calculation of the generalized excitation associated with mode I**

The generalized excitation $\mathbf{Q}_i(s)$ associated with the mode $i$ is defined by the relation:

$$\mathbf{Q}_i(s)=\int_0^L\hat{\mathbf{f}}_c(x,s).\Phi_i(x)dx \quad \text{éq. 3.4- 2}$$

where $L$ indicate the length of the structure on which one wants to impose excitations GRAPPE2.

Transfer transfer functions $H_1(s)$ and $H_2(s)$ being defined starting from the relations [**éq. 3.3-4**] and [**éq. 3.3-5**], one from of deduced, taking into account the expressions [**éq. 3.3-1**], [**éq. 3.3-4**] and [**éq. 3.3-5**]:

$$\begin{aligned}\mathbf{Q}_i(s) &=\sum_{j=1}^{N}\int_0^L H_1(s)\begin{Bmatrix}0\\DY_j(x_o)\\DZ_j(x_o)\end{Bmatrix}q_j(s)\delta(x-x_o).\begin{Bmatrix}0\\DY_i(x)\\DZ_i(x)\end{Bmatrix}dx\\ &-\sum_{j=1}^{N}\int_0^L H_2(s)\begin{Bmatrix}0\\DY'_j(x_o)\\DZ'_j(x_o)\end{Bmatrix}q_j(s)\delta'(x-x_o).\begin{Bmatrix}0\\DY_i(x)\\DZ_i(x)\end{Bmatrix}dx\end{aligned} \quad \text{éq. 3.4- 3}$$

From where, after integration:

$$\begin{aligned}\mathbf{Q}_i(s) &=\sum_{j=1}^{N}\Big\{H_1(s)\big[DY_i(x_o).DY_j(x_o)+DZ_i(x_o).DZ_j(x_o)\big]\\ &+H_2(s)\big[DY'_i(x_o).DY'_j(x_o)+DZ'_i(x_o).DZ'_i(x_o)\big]\Big\}\quad q_j(s)\\ &=\sum_{j=1}^{N}B_{ij}(s)q_j(s)\end{aligned} \quad \text{éq. 3.4- 4}$$

**Note:**

$DY'_i(x_o)=DRZ_i(x_o)$ and $DZ'_i(x_o)=-DRY_i(x_o)$

## 3.5 Resolution of the modal problem under flow

The modal problem is solved by supposing, at first approximation, that the diagonal terms of the matrix of transfer of the efforts fluid-rubber bands $\lfloor \mathbf{B}(s) \rfloor$ are dominating compared to the extra-diagonal terms.

The matrix $\lfloor \mathbf{B}(s) \rfloor$ being thus reduced to its diagonal, the modal deformations are not disturbed by the taking into account of the coupling fluid-rubber band; the only modified parameters are the Eigen frequencies and modal reduced depreciation.

The modal problem under flow breaks up then into $N$ independent scalar problems, solved by a method of the Broyden type:

$$\left(\mathbf{M}_{ii} + \mathbf{M}^{aj}{}_{ii}\right)s^2 + \left(\mathbf{C}_{ii} + \mathbf{C}^{aj}{}_{ii}(s)\right)s + \left(\mathbf{K}_{ii} + \mathbf{K}^{aj}{}_{ii}(s)\right) = 0 \quad \text{éq. 3.5- 1}$$

where
- $\mathbf{M}^{aj}{}_{ii}$ indicate the generalized mass added by the fluid,
- $\mathbf{C}^{aj}{}_{ii}(s)$ indicate the generalized damping added by the fluid,
- $\mathbf{K}^{aj}{}_{ii}(s)$ indicate the generalized stiffness added by the fluid.

$\mathbf{M}^{aj}{}_{ii}$, $\mathbf{C}^{aj}{}_{ii}(s)$ and $\mathbf{K}^{aj}{}_{ii}(s)$ are calculated using the relations:

$$\mathbf{M}^{aj}{}_{ii} = +\frac{1}{2}\rho_f D^2 L_p \left[ Cm_1 \left\{ DY_1^2(x_o) + DZ_i^2(x_o) \right\} + L_p{}^2 Cm_2 \left\{ DY'^2_i(x_o) + DZ'^2_i(x_o) \right\} \right] \quad \text{éq. 3.5- 2}$$

$$\mathbf{C}^{aj}{}_{ii}(s) = -\frac{1}{2}\rho_f D U L_p \left[ Cd_1(V_r) \left\{ DY_1^2(x_o) + DZ_i^2(x_o) \right\} + L_p{}^2 Cd_2(V_r) \left\{ DY'^2_i(x_o) + DZ'^2_i(x_o) \right\} \right] \quad \text{éq. 3.5- 3}$$

$$\mathbf{K}^{aj}{}_{ii}(s) = -\frac{1}{2}\rho_f U^2 L_p \left[ Ck_1(V_r) \left\{ DY_1^2(x_o) + DZ_i^2(x_o) \right\} + L_p{}^2 Ck_2(V_r) \left\{ DY'^2_i(x_o) + DZ'^2_i(x_o) \right\} \right] \quad \text{éq. 3.5-4}$$

$\mathbf{C}^{aj}{}_{ii}$ and $\mathbf{K}^{aj}{}_{ii}$ depend implicitly on $s$ via the fallback speed $V_r = \dfrac{U}{sD}$.

The three sizes necessary to dimension these terms are thus only $\rho_f, D$ and $U, L_p$ being deduced from $D$ thanks to the geometrical property of similarity.

As that was indicated previously, adimensional coefficients $Cd_1(V_r)$, $Ck_1(V_r)$, $Cd_2(V_r)$ and $Ck_2(V_r)$ are resulting from the empirical correlations identified in experiments on model GRAPPE2.

# 4 Excitation fluid-rubber band acting on two coaxial cylindrical hulls under annular flow (example: space ferments/envelope of heart)

The integration of this model of excitation fluid-rubber band in *Code_Aster* was approached in the note of specifications [feeding-bottle. 2]. The note of principle of model MOCCA_COQUE [feeding-bottle. 5] constitutes the theoretical documentation of reference.

## 4.1 Description of the studied configuration

The studied hardware configuration is made up of two coaxial cylindrical hulls, separated by an annular space in which runs out a viscous incompressible monophasic fluid [Figure 4.1 - has]. The flow is done in the direction of the axis of revolution of the cylinders; to fix the notations, one supposes in the continuation of the document that it is the axis $\mathbf{x}$.

One notes:

| | |
|---|---|
| $L$ | the common length of the two cylindrical hulls, |
| $R_1(\theta,x,t)$ | the interior ray of annular space, |
| $R_2(\theta,x,t)$ | the ray external of annular space, |
| $R(\theta,x,t)$ | the average radius $\left(R(\theta,x,t)=\dfrac{R_1(\theta,x,t)+R_2(\theta,x,t)}{2}\right)$, |
| $H(\theta,x,t)$ | annular game $(H(\theta,x,t)=R_2(\theta,x,t)-R_1(\theta,x,t))$, |
| $\mathbf{e}_r,\mathbf{e}_\theta,\mathbf{e}_x$ | vectors of the base of cylindrical coordinates. |

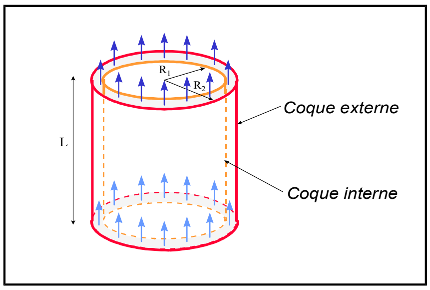


**Figure 4.1-a: general diagram coaxial hulls**

## 4.2 Stages of calculation

- The first stage of calculation consists in determining the modal base in air of the structure. This operation is carried out by the operator `CALC_MODES`. This calculation is necessary because the decomposition of the matrix of transfer of the forces fluid-rubber bands $[\mathbf{B}(s)]$ is expressed in this base.
- The second stage relates to the taking into account of the forces fluid-rubber bands. It intervenes in the operator `CALC_FLUI_STRU` . This stage breaks up into eight sub-tasks:

### 4.2.1 Preprocessings

1°/ Determination of the characteristic geometrical magnitudes, starting from the topology of the grid: common length of the two hulls, average radius, average annular game.
2°/ Characterization of the modal deformations in air: determination of the orders of hull, the principal plans, the numbers of wave and the coefficients of deformations of beam associated with each mode of the structure, both for the hull interns the external hull.

### 4.2.2 Resolution of the modal water problem at rest

3°/ Calculation of the matrix of mass added by the fluid $\left[\mathbf{M}_{aj}\right]$ in the modal base of the structure in air
4°/ Calculation of the modal characteristics of the water structure at rest while solving: $\left[\left(\left[\mathbf{M}_i\right]+\left[\mathbf{M}_{aj}\right]\right)s^2+\left[\mathbf{K}_i\right]\right](\mathbf{q})=0$

One obtains the new structural features out of water at rest $\mathbf{M}_i{}^e, \mathbf{K}_i{}^e, f_i{}^e$ (generalized mass and stiffness, Eigen frequency of the mode $i$ ) as well as the modal deformations $\psi_i{}^e$, expressed in the base in air.
5°/ Calculation of the water deformations at rest in the physical base, by basic change: $\left[\varphi_i{}^e\right]=\left[\varphi_i{}^a\right].\left[\psi_i{}^e\right]$

### 4.2.3 Resolution of the modal problem under flow

For each rate of flow:

6°/ Calculation of $\left[B(s)\right]$ in the modal base in air.
This calculation is carried out by solving the non stationary fluid problem according to the method specified in the paragraph § 4.3.1.
7°/ Calculation of the forces fluid-rubber bands induced by the effects of added damping and stiffness, in the modal water base at rest.

$$\left[\mathbf{B}^e(s)\right]={}^t\left[\psi_i{}^e\right]\left[\left[\mathbf{B}(s)\right]-\left[\mathbf{M}_{aj}\right]s^2\right]\left[\psi_i{}^e\right]$$

8°/ Resolution of the modal problem by neglecting the extra-diagonal terms of this last matrix, **by the method of Broyden** (buckles on the sub-tasks 6° and 7°).

$$\mathbf{M}_i{}^e s^2+\mathbf{C}_i{}^e s+\mathbf{K}_i{}^e-\mathbf{B}_{ii}{}^e(s)=0$$

Modal characteristics of the structure: $\mathbf{M}_i{}^{ec}, f_i{}^{ec}, \xi_i{}^{ec}$ (generalized mass, Eigen frequency and damping of the mode $i$ , under flow) are given. The modal deformations are supposed to be identical to those out of water at rest.

End of loop on the rates of flow

**Note:**

- *The calculation of the terms of the matrix of transfer of the forces fluid-rubber bands requires the resolution of the non stationary fluid problem (sub-task 6°). This resolution is it - even possible only if one beforehand determined certain geometrical magnitudes characteristic of the configuration, as well as the coefficients of the analytical forms of the modal deformations of the structures (preprocessings 1° and 2°).*
- *If the user chooses to carry out the first stage (calculation of the modal base by the operator `CALC_MODES`) by taking directly into account the effects of added mass, those should not be any more taken into account by the operator `CALC_FLUI_STRU` . For that, the keyword `MASS_AJOU` order `DEFI_FLUI_STRU` must be well informed by `'NOT'` . Under - tasks 3° with 7° become then:*

*3°/ Calculation of the effects of mass added by the fluid, in the modal base of the water structure, in order to be able to cut off these effects of the effort total fluid-rubber band, since the terms of added mass are already taken into account.*
*4°/ Removed sub-task.*
*5°/ Removed sub-task.*

*For each rate of flow*

*6°/ Calculation of the matrix $\left[\mathbf{B}(s)\right]$ in the modal water base.*
*7°/ Calculation of the forces fluid-rubber bands induced by the effects of damping and stiffness added in the modal water base:*

$$\left[\mathbf{B}^e(s)\right]=\left[\mathbf{B}(s)\right]-\left[\mathbf{M}_{aj}\right]s^2$$

*The sub-tasks 1°, 2° and 8° are not modified.*

*For each rate of flow*

*6°/ Calculation of the matrix $\left[\mathbf{B}(s)\right]$ in the modal water base.*
*7°/ Calculation of the forces fluid-rubber bands induced by the effects of damping and stiffness added in the modal water base:*

$$\left[\mathbf{B}^e(s)\right]=\left[\mathbf{B}(s)\right]-\left[\mathbf{M}_{aj}\right]s^2$$

*The sub-tasks 1°, 2° and 8° are not modified.*

## 4.3 Resolution of the non stationary fluid problem

### 4.3.1 Simplifying assumptions

Some assumptions on the nature of the flow make it possible to simplify the non stationary Navier-Stokes equations, at the base of the problem fluid-structure.

**H1** It is supposed that the flow is the superposition of an average flow stationary, obtained when the structures are fixed, and from a non stationary flow induced by the movement of the walls.

**H2** It is supposed that the vibrations of structure are of low amplitude with respect to the thickness of the average annular flow.

**H3** One supposes that the disturbances speed induced by the vibratory movements are, on average on a ray, primarily directed in the directions $\theta$ and $x$ : one supposes thus that the vibratory movement induced a helicoid movement of fluid around the structures rather than a radial movement compared to these last. These disturbances speed define order 1.

**H4** One supposes finally that the speed and pressure field is uniform, with order 1, in the radial direction.

These simplifying assumptions make it possible to solve the fluid problem analytically. The matrix of transfer of the forces fluid-rubber bands $\lfloor \mathbf{B}(s) \rfloor$ is deduced from the non stationary flow resulting from this resolution.

### 4.3.2 Analysis in disturbances

With the help of the assumptions stated previously, the analysis in disturbances of the fluid problem led to search the non stationary flow in the form:

$$U_r = 0 + 0 + \text{ordre 2}$$ **éq. 4.3.2-1**

$$U_\theta = 0 + \tilde{u}_\theta(\theta, x, t) + \text{ordre 2}$$ **éq. 4.3.2-2**

$$U_x = \overline{U}(x) + \tilde{u}_x(\theta, x, t) + \text{ordre 2}$$ **éq. 4.3.2-3**

$$P = \overline{P}(x) + \tilde{p}(\theta, x, t) + \text{ordre 2}$$ **éq. 4.3.2-4**

with:

$$R_1 = \overline{R}_1 + \tilde{r}_1(\theta, x, t)$$ **éq. 4.3.2-5**

$$R_2 = \overline{R}_2 + \tilde{r}_2(\theta, x, t)$$ **éq. 4.3.2-6**

The variables are defined $\tilde{h}$ and $\tilde{R}$ like: $\tilde{h} = \tilde{r}_2 - \tilde{r}_1$ and $\tilde{R} = \frac{\tilde{r}_2 + \tilde{r}_1}{2}$ .

By limiting the development of the Navier-Stokes equations to the first order, one obtains two systems of equations characterizing the stationary part and the disturbed part of the flow, the second system being a linear system.

The resolution of the stationary fluid problem leads thus to:

$$\overline{U}(x) = \overline{U} \text{ constant and } \frac{\partial \overline{P}}{\partial x} = -\frac{1}{\overline{H}} \rho \overline{C}_f \overline{U}^2$$ **éq. 4.3.2-7**

In the equation [**éq. 4.3.2-7**], $\rho$ indicate the density of the fluid and $\overline{C}_f$ the stationary part of the coefficient of friction to the wall. The incompressible fluid being supposed, its density is not not broken up partly stationary and fluctuating part. $C_f$ is deduced from the law of Nikuradzé characterizing the flows in control:

$$C_f = C_{fo}(R_e, \varepsilon) R_e^{m(R_e,\varepsilon)} \text{ with } R_e = \frac{2\overline{H}U_x}{\nu}$$ **éq. 4.3.2-8**

where m indicates the value of an exhibitor, $\nu$ indicate the kinematic viscosity of the fluid and $\varepsilon$ the surface roughness.

It results from this:

$$\overline{C}_f = \overline{C}_{fo}(\overline{R}_e, \varepsilon) \overline{R}_e^{m(\overline{R}_e,\varepsilon)}$$

$$\tilde{C}_f = C_f(\tilde{R}_e) - \overline{C}_f(\tilde{R}_e) \quad \text{with } \overline{R}_e = \frac{2\overline{H}\overline{U}}{\nu} \text{ and } \tilde{R}_e = \frac{2\overline{H}\tilde{u}_x}{\nu}$$

$$= (m+2)\,\overline{C}_f \frac{\tilde{u}_x}{\overline{U}} + \text{ordre 2}$$

The linear differential connection of a nature 1 characterizing the non stationary part of the flow induced by the movements of walls is written in the field of Laplace:

$$\begin{cases} \dfrac{\partial \tilde{u}_x}{\partial x} + \dfrac{1}{\overline{R}}\dfrac{\partial \tilde{u}_\theta}{\partial \theta} = -\dfrac{\overline{U}}{\overline{H}}\left[\dfrac{\partial \tilde{h}}{\partial x} + \dfrac{s}{\overline{U}}\tilde{h}\right] - \dfrac{\overline{U}}{\overline{R}}\left[\dfrac{\partial \tilde{R}}{\partial x} + \dfrac{s}{\overline{U}}\tilde{R}\right] \\ \overline{U}\dfrac{\partial \tilde{u}_\theta}{\partial x} + \left[s + \overline{C}_f \dfrac{\overline{U}}{\overline{H}}\right]\tilde{u}_\theta + \dfrac{1}{\rho \overline{R}}\dfrac{\partial \tilde{p}}{\partial \theta} = 0 \\ \overline{U}\dfrac{\partial \tilde{u}_x}{\partial x} + \left[s + \overline{C}_f (m+2)\dfrac{\overline{U}}{\overline{H}}\right]\tilde{u}_x + \dfrac{1}{\rho}\dfrac{\partial \tilde{p}}{\partial x} = \overline{C}_f \left[\dfrac{\overline{U}}{\overline{H}}\right]^2 \tilde{h} \end{cases}$$ **éq. 4.3.2-9**

Three boundary conditions of input-output make it possible to solve this system. The first of these conditions is obtained by supposing that the flow is sufficiently regular upstream of annular space, so that the tangential component the speed of entry can be neglected:

$u_\theta = 0$ in $x = 0$ **éq. 4.3.2-10**

The two others are obtained by applying the conservation equation of the kinetic energy, in its quasi-stationary form, between the infinite upstream and the entry of annular space, then between the exit of annular space and the infinite downstream. One obtains then respectively, with the order of the disturbances:

$$\begin{cases} \displaystyle\int_{\overline{R}_1}^{\overline{R}_2} \left[\tilde{p} + \rho\overline{U}\,\tilde{u}_x\left(1 + \overline{C}_{d_e}\right) + \frac{1}{2}\rho\,\tilde{C}_{d_e}\overline{U}^2\right]\overline{U}\,r dr = 0 \quad \text{en } x = 0 \\ \displaystyle\int_{\overline{R}_1}^{\overline{R}_2} \left[\tilde{p} + \rho\overline{U}\,\tilde{u}_x\left(1 - \overline{C}_{d_s}\right) - \frac{1}{2}\rho\,\tilde{C}_{d_s}\overline{U}^2\right]\overline{U}\,r dr = 0 \quad \text{en } x = L \end{cases}$$ **éq. 4.3.2-11**

In these expressions, $\overline{C}_{d_e}$ and $\overline{C}_{d_s}$ the stationary parts of the singular loss ratios represent of load of entry and exit. They take into account the dissipation of induced energy, when the walls are fixed, by possible abrupt evolutions of the geometry at the entry or the exit of annular space. In most case, these coefficients can be estimated simply using data of the literature (Idel' cik for example). When the geometrical configuration of entry or exit is very particular, these coefficients can also be given using a two-dimensional code of mechanics of the fluids adapted under investigation problems to fixed walls, of type N3S.

$\tilde{C}_{d_e}$ and $\tilde{C}_{d_s}$ are the non stationary parts of the singular loss ratios of load. These coefficients take into account the disturbances of the lines of separation induced by the movements of structure. They can be modelled thanks to a quasi-stationary approach of comparable nature that introduced for the estimate of the coefficient of friction of wall.

The system [**éq. 4.3.2-9**] is solved analytically, using the limiting conditions [**éq. 4.3.2-10**] and [**éq. 4.3.2-11**], by clarifying the functions $\tilde{h}$ et $\tilde{R}$ characterizing the second member.

Disturbances $\tilde{r}_1(\theta,x,s)$ and $\tilde{r}_2(\theta,x,s)$ defining the movement of the walls, the disturbed parts of the annular game and average radius are then defined, in the field of Laplace, by:

$$\tilde{h}(\theta,x,s)=\tilde{r}_2(\theta,x,s)-\tilde{r}_1(\theta,x,s)$$ **éq. 4.3.2-12**

$$\tilde{R}(\theta,x,s)=\frac{\tilde{r}_1(\theta,x,s)+\tilde{r}_2(\theta,x,s)}{2}$$ **éq. 4.3.2-13**

### 4.3.3 Decomposition on modal basis

That is to say $N$ the number of oscillatory modes of the structure in the studied waveband. The decomposition on the basis of modal movement of the walls is expressed in the following way:

$$\tilde{r}_1(\theta,x,s)=\sum_{i=1}^{N}\mathbf{cos}\left[k_{1i}(\theta-\theta_{1i})\right].r_{1i}^{*}(x).\alpha_i(s)$$ **éq. 4.3.3-1**

$$\tilde{r}_2(\theta,x,s)=\sum_{i=1}^{N}\mathbf{cos}\left[k_{2i}(\theta-\theta_{2i})\right].r_{2i}^{*}(x).\alpha_i(s)$$ **éq. 4.3.3-2**

where $k_{1i}$ and $k_{2i}$ the orders of hull represent of $^{i\,ème}$ mode for the respective movements of the hulls internal and external,
$\theta_{1i}$ and $\theta_{2i}$ allow to characterize the principal plans of these modes,
$r_{1i}^{*}(x)$ and $r_{2i}^{*}(x)$ are deduced from the deformations of beam of the structures internal and external associated with the mode considered,
and $\alpha_i(s)$ represent generalized displacement.

**Note:**

*Functions $r_{1i}^{*}(x)$ and $r_{2i}^{*}(x)$ are represented, within the framework of the analytical resolution, in the form of linear combinations of sine, cosine, hyperbolic sine and hyperbolic cosine:*

$$r_{1i}^{*}(x)=A_{1i}\,\mathbf{cos}\left(\frac{\delta_{1i}}{L}x\right)+B_{1i}\,\mathbf{sin}\left(\frac{\delta_{1i}}{L}x\right)+C_{1i}ch\left(\frac{\delta_{1i}}{L}x\right)+D_{1i}sh\left(\frac{\delta_{1i}}{L}x\right)$$ **éq. 4.3.3-3**

$$r_{2i}^{*}(x)=A_{2i}\,\mathbf{cos}\left(\frac{\delta_{2i}}{L}x\right)+B_{2i}\,\mathbf{sin}\left(\frac{\delta_{2i}}{L}x\right)+C_{2i}ch\left(\frac{\delta_{2i}}{L}x\right)+D_{2i}sh\left(\frac{\delta_{2i}}{L}x\right)$$ **éq. 4.3.3-4**

*with $\delta_{1i}$ and $\delta_{2i}$ numbers of wave of $^{i\,ème}$ mode for the movements of the hulls internal and external respectively.*

Solutions of the fluid problem $\tilde{p}$, $\tilde{u}_x$ et $\tilde{u}_\theta$ are required in the form of decompositions on modal basis deduced from those of $\tilde{r}_1$ et $\tilde{r}_2$ clarified by the relations [**éq. 4.3.3-1**] and [éq 4.3.3-2]. One obtains thus, in the field of Laplace:

$$\tilde{p}(\theta,x,s)=\sum_{i=1}^{N}\left[\frac{p_{1i}^*(x,s)}{k_{1i}^2}\mathbf{cos}\left[k_{1i}(\theta-\theta_{1i})\right]+\frac{p_{2i}^*(x,s)}{k_{2i}^2}\mathbf{cos}\left[k_{2i}(\theta-\theta_{2i})\right]\right]\alpha_i(s) \qquad \textbf{éq. 4.3.3-5}$$

$$\tilde{u}_x(\theta,x,s)=\sum_{i=1}^{N}\left(u_{1i}^*(x,s)\mathbf{cos}\left[k_{1i}(\theta-\theta_{1i})\right]+u_{2i}^*(x,s)\mathbf{cos}\left[k_{2i}(\theta-\theta_{2i})\right]\right)\alpha_i(s) \qquad \textbf{éq. 4.3.3-6}$$

$$\tilde{u}_\theta(\theta,x,s)=\sum_{i=1}^{N}\left[\frac{v_{1i}^*(x,s)}{k_{1i}}\mathbf{sin}\left[k_{1i}(\theta-\theta_{1i})\right]+\frac{v_{2i}^*(x,s)}{k_{2i}}\mathbf{sin}\left[k_{2i}(\theta-\theta_{2i})\right]\right]\alpha_i(s) \qquad \textbf{éq. 4.3.3-7}$$

### 4.3.4 Expression of the terms of the fluid matrix of transfer of the elastic forces -

The surface effort fluid-rubber band, $\mathbf{F}$ , is the resultant of the field of pressure and the viscous and turbulent constraints exerted by the flow on the walls of the structure moving.

$$\mathbf{F}=-P\,\mathbf{n}+\tau_\theta\mathbf{t}_\theta+\tau_x\mathbf{t}_x \qquad \textbf{éq. 4.3.4-1}$$

The effort generalized fluid-rubber band associated with $i$ ème oscillatory mode of the structure, $\mathbf{Q}_i(s)$ , is written as follows:

$$\mathbf{Q}_i(s)=\int_{S_i}\mathbf{F}.\mathbf{X}_i\,ds_i \qquad \textbf{éq. 4.3.4-2}$$

Where $S_i$ indicate the surface of the walls of the structure wet by the flow, and the vector $\mathbf{X}_i$ represent it i ème vector deformed modal in this expression. The representation of the speed and pressure field and the representation in the form of a law of wall of the viscous and turbulent constraints exerted on the structure moving make it possible to express the effort generalized fluid-rubber band $\mathbf{Q}_i(s)$ in the following way:

$$\mathbf{Q}_i(s)=\sum_{j=1}^{N}\mathbf{B}_{ij}(s)\alpha_j(s) \qquad \textbf{éq. 4.3.4-3}$$

with $\mathbf{B}_{ij}(s)=\mathbf{B}_{1ij}(s)+\mathbf{B}_{2ij}(s)$

$\mathbf{B}_{1ij}(s)$ and $\mathbf{B}_{2ij}(s)$ the contributions of the hulls indicate respectively interior and external. These contributions are defined by:

$$\begin{aligned}\mathbf{B}_{1ij}(s)= & -\pi\frac{\overline{R}_1}{k_{1i}^2}\mathbf{cos}\left[k_{1i}(\theta_{1i}-\theta_{1j})\right]\delta_{k_{1i}k_{1j}}\int_0^L\left[p_{1i}{}^*(x,s)+\frac{1}{2}\rho\overline{C}_f\overline{U}v_{1i}{}^*(x,s)\right].r_{1j}{}^*(x)dx\\ & -\pi\frac{\overline{R}_1}{k_{2i}^2}\mathbf{cos}\left[k_{2i}(\theta_{2i}-\theta_{1j})\right]\delta_{k_{2i}k_{1j}}\int_0^L\left[p_{2i}{}^*(x,s)+\frac{1}{2}\rho\overline{C}_f\overline{U}v_{2i}{}^*(x,s)\right].r_{1j}{}^*(x)dx\end{aligned}$$

**éq. 4.3.4-4**

and

$$\mathbf{B}_{2ij}(s) = -\pi\frac{\overline{R}_2}{k_{1i}^2}\mathbf{cos}\left[k_{1i}\left(\theta_{1i}-\theta_{2j}\right)\right]\delta_{k_{1i}k_{2j}}\int_0^L\left[p_{1i}^{\ *}(x,s)+\frac{1}{2}\rho\overline{C}_f\overline{U}v_{1i}^{\ *}(x,s)\right].r_{2j}^{\ *}(x)dx -\pi\frac{\overline{R}_2}{k_{2i}^2}\mathbf{cos}\left[k_{2i}\left(\theta_{2i}-\theta_{2j}\right)\right]\delta_{k_{2i}k_{2j}}\int_0^L\left[p_{2i}^{\ *}(x,s)+\frac{1}{2}\rho\overline{C}_f\overline{U}v_{2i}^{\ *}(x,s)\right].r_{2j}^{\ *}(x)dx \quad \text{éq. 4.3.4-5}$$

## 4.4 Resolution of the modal problem under flow

As one explained in the paragraph [§ 4.2], one solves beforehand the modal problem out of water at rest, in order to take into account the inertial coupling between modes. One thus estimates the matrix of mass added by the fluid, while calculating $\left[\mathbf{B}(s)\right]$ for a mean velocity of the flow worthless. The modal characteristics of the system under flow are then obtained by disturbing the water characteristics at rest. One does not take any more account but of damping and the stiffness added: the terms of mass added previously calculated are cut off from the matrix $\left[\mathbf{B}(s)\right]$ . The coupling between modes is then neglected; consequently, the modal deformations remain unchanged compared to those out of water at rest. Only parameters disturbed by the setting in the frequency and reduced modal damping. These parameters are calculated while solving $N$ nonlinear equations mode by mode, implementation of a method of the Broyden type.

# 5 Axial flow (example: fuel assemblies)

The integration of this model of excitation fluid-rubber band in *Code_Aster* was approached in the note of specifications [feeding-bottle. 2]. The note of principle of model MEFISTEAU [feeding-bottle. 6] constitutes the theoretical documentation of reference.

## 5.1 Description of the studied configuration

One considers a beam of $K$ circular cylinders mobile in inflection and subjected to an incompressible flow of viscous fluid, limited by a cylindrical rigid enclosure of circular or rectangular section [Figure 5.1-a].

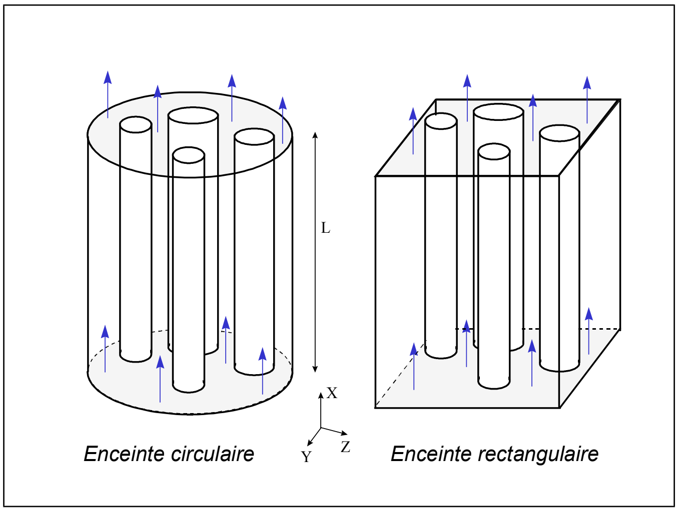


**Figure 5.1-a: Beam under axial flow**

The cylinders all parallel, are directed along the axis of the enclosure. They have a common length, noted $L$. To simplify the notations, it is supposed thereafter that $\mathbf{x}$ is the directing axis. The steady flow axial and is supposed to be uniform in each section. Density of the fluid which can be variable along the axis $\mathbf{x}$ (heat gradients), the rate of the stationary flow also depends on the variable $x$.

## 5.2 Stages of calculation

- The first stage relates to the determination of the modal base in air of the beam. This operation is carried out by the operator `CALC_MODES`. This stage is essential because the forces fluid-rubber bands are projected on this basis.
- The second phase relates to the taking into account of the forces fluid-rubber bands with the operator `CALC_FLUI_STRU`. This stage breaks up into 7 sub-tasks:

### 5.2.1 Preprocessings

1°/ By means of the topology of the grid, deduction of the coordinates of the centers of the cylinders of the beam then checking of the good provision of the cylinders ones compared to the others (it is checked in particular that there is not overlapping between two cylinders) and compared to the rigid enclosure.

2°/ Determination length of excitation of the fluid, commune to all the cylinders, as well as associated discretization along the directing axis.

3°/ Constitution of the tables giving the modal deformations in air of each cylinder of the beam, for each mode taken into account for the coupling fluid-structure. One interpolates for that the deformations at the points of the discretization determined before.

### 5.2.2 Resolution of the modal problem under flow

4°/ Resolution of the disturbed fluid problem. The determination of the potential disturbed speeds requires the inversion of linear systems of high natures calling the implementation of the method of Crout.

For each rate of flow

5°/ Calculation of the matrices of mass, damping and stiffness added giving the matrix of transfer of the forces fluid-rubber bands in the modal base in air:

$$\left[\mathbf{B}_{ij}(s)\right] = -\left[\mathbf{M}_a\right]s^2 - \left[\mathbf{C}_a\right]s - \left[\mathbf{K}_a\right]$$

$\left[\mathbf{M}_a\right]$ full symmetrical; $\left[\mathbf{C}_a\right]\text{et}\left[\mathbf{K}_a\right]$ a priori full and nonsymmetrical.

6°/ Resolution of the modal problem under flow; one solves the complete problem with the vectors and the clean ones

$$\left\{ \left[\mathbf{M}_{ij}\right]s^2 + \left[\mathbf{C}_{ij}\right]s + \left[\mathbf{K}_{ij}\right] - \left[\mathbf{B}_{ij}(s)\right] \right\}.(\mathbf{q}) = (0)$$

One does not neglect the extra-diagonal terms of $\left[\mathbf{B}_{ij}(s)\right]$. After reformulation, the resolution is carried out using algorithm QR: obtaining the masses, frequencies and modal depreciation reduced under flow $\mathbf{M}_i^{ec}, f_i^{ec}, \xi_i^{ec}$, complex modal deformations $\psi_i^{ec}$ expressed in the base in air; of these last, one retains only the real part after minimization of the imaginary part (calculation of a criterion on the imaginary part).

7°/ Restitution of the deformations under flow in the physical base.

$$\left[\phi_i^{ec}\right] = \left[\phi_i\right]\left[\psi_i^{ec}\right]$$

$\left[\phi_i\right]$ is the matrix whose columns are the modal deformations in air, expressed in physical base.

End of loop on the rates of flow

**Note:**

- *The knowledge of the coordinates of the centers of the cylinders (preprocessing 1°) is necessary to the resolution of the disturbed fluid problem (sub-task 4°). This resolution leads to the estimate of the terms of the matrix of transfer of the forces fluid-rubber bands (under - task 5°), which utilize the disturbances of pressure and speed.*
- *The determination a common length of excitation and the creation of an associated discretization (pre - 2° treatment) make it possible to define a field of integration on the structures for projection of the forces fluid-rubber bands on the modal basis. The interpolation of the modal deformations at the same points is thus necessary (preprocessing 3°).*
- *The dynamic behavior of the beam under flow can also be studied using a simplified representation of the beam (with equivalent tubes). The stages of calculation for the taking into account of the coupling fluid-structure are then identical to those described previously, the only differences appearing in the preprocessings. This second approach is described more precisely in the note [feeding-bottle. 2]. In the stage 1° of the pre - treatments, the coordinates of the centers of the cylinders of the beam are then specified directly by the user, who also establishes the correspondence between the cylinders of the beam and the beams of the simplified representation given by the grid. In the stage 3° of the preprocessings, this correspondence makes it possible to assign to the cylinders of the beam, at the points of discretization determined in the stage 2°, the modal deformations of the beams of the simplified representation.*

## 5.3 Resolution of the non stationary fluid problem

### 5.3.1 Simplifying assumption

**H1** The field non stationary fluid speeds is analytically given by supposing that the disturbed flow $\tilde{\varphi}$ is potential in all the fluid field, and that the steady flow is uniform transversely, but function of the axial position $x$ :

$$\mathbf{u}=\overline{\mathbf{U}}+\tilde{\mathbf{u}}=\overline{U}(x)\,\mathbf{x}+\nabla(\tilde{\varphi})$$

**éq. 5.3.1-1**

Such a field speeds admits a slip on the walls of the cylinders which will make it possible to calculate the viscous constraint by a law of friction.

**H2** The movement of the cylinders does not generate disturbances speed $\tilde{\mathbf{u}}=\nabla(\tilde{\varphi})$ that radially and orthoradialement (assumption of the slim bodies): $\tilde{\mathbf{u}}=\tilde{u}_y\,\mathbf{y}+\tilde{u}_z\,\mathbf{z}$

**H3** The field of pressure is broken up into parts stationary and disturbed according to $P=\overline{P}+\tilde{p}$
The stationary field of pressure depends only on $x$ and its gradient is worth:

$$\frac{d\overline{P}}{dx}(x)=-\rho\overline{U}\frac{d\overline{U}}{dx}(x)-2\rho\frac{C_{fl}}{D_H}|\overline{U}|\overline{U}+\rho\mathbf{g.x}$$

**éq. 5.3.1-2**

where $D_H$ indicate the hydraulic diameter of the beam,

$C_{fl}$ indicate the coefficient of local friction for stationary speed $\overline{U}$. It depends amongst Reynolds, calculated using stationary speed $\overline{U}$, hydraulic diameter of the beam and surface roughness. This coefficient is deduced from the law of Nikuradzé characterizing the flows in control;

$\mathbf{g}$ indicate the field of gravity. Its action on the stationary field of pressure depends on the slope of the beam $(\mathbf{g}.\mathbf{x})$.

### 5.3.2 Determination of the potential disturbed speeds

One seeks an analytical solution for $\tilde{\varphi}\,(r,\theta,x,t)$ in the form of a superposition of elementary singularities which are written:

$$\sum_{n=1}^{N_{tronc}} \left[ C_{nk}(x,t).r_k^{-n}.\cos(n\theta_k) + D_{nk}(x,t).r_k^{-n}.\sin(n\theta_k) \right]$$ **éq. 5.3.2-1**

in the center of each cylinder $k$ and:

$$\sum_{n=1}^{N_{tronc}} \left[ A_n(x,t).r_o^n.\mathbf{cos}(n\theta_o) + B_n(x,t).r_o^n.\mathbf{sin}(n\theta_o) \right]$$ **éq. 5.3.2-2**

in the center of the rigid enclosure when this one is circular where:

| | |
|---|---|
| $N_{tronc}$ | indicate the order of truncation of the series of Laurent $(N_{tronc}=3)$, |
| $r_k,\theta_k$ | the polar coordinates in a plan perpendicular to the axis indicate $\mathbf{x}$, centered in the center of the cylinder K, |
| $r_o,\theta_o$ | the polar coordinates in a plan perpendicular to the axis indicate $\mathbf{x}$, centered in the center of the circular rigid enclosure. |

Coefficients $C_{nk}(x,t), D_{nk}(x,t), A_n(x,t)$ and $B_n(x,t)$ expressions [**éq. 5.3.2-1**] and [**éq. 5.3.2 - 2**] are given by applying the boundary condition of nonpenetration:

- on the contour of each mobile cylinder $k$, this condition is written:

$$\forall\theta_k, \left[\frac{\partial\tilde{\varphi}}{\partial r_k}\right](r_k=R_k) = \frac{Dy_k}{Dt}(x,t)\,\mathbf{cos}(\theta_k) + \frac{Dz_k}{Dt}(x,t)\,\mathbf{sin}(\theta_k)$$

where $y_k(x,t)$ and $z_k(x,t)$ the components of the displacement of neutral fibre of the cylinder K to the X-coordinate indicate $x$ in the reference mark $(\mathbf{y},\mathbf{z})$,

$r_k$ et $\theta_k$ the polar coordinates in the reference mark indicate $(\mathbf{y},\mathbf{z})$ whose origin is taken in the center of the cylinder $k$,

$R_k$ indicate the ray of the cylinder $k$,

$\dfrac{D}{Dt} \equiv \dfrac{\partial}{\partial t} + \overline{U}(x)\dfrac{\partial}{\partial x}$.

- on the contour of a circular rigid enclosure, she is written:

$\forall\theta_o, \left[\dfrac{\partial\tilde{\varphi}}{\partial r_o}\right](r_o=R_o)=0$ where $R_o$ indicate the ray of the enclosure.

In the case of a rectangular rigid enclosure, this condition is taken into account by a method derived from the method of the "images" [feeding-bottle. 6]; the fluid problem confined by the rectangular enclosure is made equivalent to the problem in infinite medium by creating images of the mobile cylinders of the beam compared to the sides of the enclosure. This method results in introducing new singularities of the form [**éq. 5.3.2-1**], placed at the center of the cylinders "images", in the expression of $\tilde{\varphi}$. She does not add however an unknown factor to the problem since the coefficients for these new singularities are derived from those of the mobile cylinders of the beam by the game of the images.

Finally, the potential disturbed speeds is written:

$$\tilde{\varphi}(r,\theta,x,t)=\sum_{k=1}^{K}\left[f_k(r,\theta)\frac{Dy_k}{Dt}(x,t)+g_k(r,\theta)\frac{Dz_k}{Dt}(x,t)\right]$$

**éq. 5.3.2-3**

Where $K$ indicate the number of mobile cylinders of the beam. Functions $f_k(r,\theta)$ et $g_k(r,\theta)$ are linear combinations of $r^{-n}.\mathbf{cos}(n\theta), r^{-n}.\mathbf{sin}(n\theta)$, $r^{n}.\mathbf{cos}(n\theta)$ and $r^{n}.\mathbf{sin}(n\theta)$ whose coefficients are determined by the boundary conditions preceding. That requires the resolution of linear systems of high natures and with full matrices. The inversions are carried out by implementing the method of Crout.

### 5.3.3 Modeling of the fluid forces

One retains initially the forces due to the disturbances of pressure $\tilde{p}$, connected to the potential the speeds disturbed by:

$$\tilde{p}=-\rho\frac{D\tilde{\varphi}}{Dt}$$ **éq. 5.3.3-1**

The resultant of the field of pressure disturbed around each mobile cylinder is a linear force $\mathbf{f}_{\tilde{p}}$ acting according to $\mathbf{y}$ et $\mathbf{z}$. This force depends linearly on $\frac{D^2y_k}{Dt^2}$ and $\frac{D^2z_k}{Dt^2}$, thus generating terms of mass, damping and stiffness added.

One then takes into account the forces related to the viscosity of the fluid.

In a quasi-static approach, one considers the action of the fluid field speed $(\overline{\mathbf{U}}+\tilde{\mathbf{u}})$ around a cylinder at the moment $t$ : in the reference mark related to the cylinder, the flow, speed $\overline{U}$ to order 0, an incidence compared to the cylinder presents which is function of the disturbances speed and the movement of the cylinder him - even. It results from it a force from trail and a force of bearing pressure. One shows that the components according to $\mathbf{y}$ et $\mathbf{z}$ resulting linear force $\mathbf{f}_v$ are written, for the cylinder $\ell$:

$$\left(\mathbf{f}_v^{\ell}\right)_y=-\rho R_\ell|\overline{U}|\pi C_{fl}\left[\frac{\partial y_\ell}{\partial t}-\overline{\tilde{u}}_y^{\ell}\right]-\rho R_\ell|\overline{U}|\pi C_p\left[\frac{Dy_\ell}{Dt}-\overline{\tilde{u}}_y^{\ell}\right]$$ **éq. 5.3.3-2**

$$\left(\mathbf{f}_v^{\ell}\right)_z=-\rho R_\ell|\overline{U}|\pi C_{fl}\left[\frac{\partial z_\ell}{\partial t}-\overline{\tilde{u}}_y^{\ell}\right]-\rho R_\ell|\overline{U}|\pi C_p\left[\frac{Dz_\ell}{Dt}-\overline{\tilde{u}}_z^{\ell}\right]$$ **éq. 5.3.3-3**

where $C_p$ indicate the slope with worthless incidence of the coefficient of bearing pressure around a cylinder very slightly tilted ( $C_p$ = 0.08).

$\overline{\tilde{u}}_y$ and $\overline{\tilde{u}}_z$ the averages of the disturbances speed indicate along the axes $\mathbf{y}$ and $\mathbf{z}$ around the cylinders, which depend linearly on $\frac{Dy_k}{Dt}$ and $\frac{Dz_k}{Dt}$ (cf [**éq. 5.3.2-3**]).

These forces generate terms of added damping and stiffness.

One finally takes into account the action of the stationary field of pressure on the deformed mobile structures. It is shown that the resulting linear force $\mathbf{f}_p^\ell$ on the cylinder $\ell$ has as components, with order 1:

$$\left(\mathbf{f}_{\bar{p}}^\ell\right)_y = \pi R_\ell^2 \frac{\partial}{\partial x}\left[\bar{P}\frac{\partial y_\ell}{\partial x}\right] \quad \text{éq. 5.3.3-4}$$

$$\left(\mathbf{f}_{\bar{p}}^\ell\right)_z = \pi R_\ell^2 \frac{\partial}{\partial x}\left[\bar{P}\frac{\partial z_\ell}{\partial x}\right] \quad \text{éq. 5.3.3-5}$$

These forces generate only terms of added stiffness and any coupling between cylinders.

Expressions [**éq. 5.3.3-1**], [**éq. 5.3.3-2**] and [**éq. 5.3.3-3**] highlight the need for solving the disturbed fluid problem before estimating the forces fluid-rubber bands.

### 5.3.4 Expression of the terms of the fluid matrix of transfer of the elastic forces -

**Summary of the linear forces**

For each cylinder $\ell$, the forces fluid-rubber bands are written according to $\mathbf{y}$ and $\mathbf{z}$ :

$$\mathbf{f}_\ell = \mathbf{f}_{\tilde{p}}^\ell + \mathbf{f}_v^\ell + \mathbf{f}_{\bar{p}}^\ell \quad \text{éq. 5.3.4-1}$$

and are linear combinations of:

$$\left[\frac{\partial y_k}{\partial t}, \frac{\partial^2 y_k}{\partial t^2}, \frac{\partial^2 y_k}{\partial t \partial x}, \frac{\partial^2 y_k}{\partial x^2}, \frac{\partial z_k}{\partial t}, \frac{\partial^2 z_k}{\partial t^2}, \frac{\partial^2 z_k}{\partial t \partial x}, \frac{\partial^2 z_k}{\partial x^2}\right] (k = 1, K)$$

**Decomposition of the movement on modal basis**

The movement of the beam of cylinders is broken up according to $N$ modes of vibration in air. One notes $\phi_j^k$ ($1 \le \mathrm{k} \le \mathrm{K}$ and $1 \le \mathrm{j} \le \mathrm{N}$) deformations according to $\mathbf{y}$ et $\mathbf{z}$ cylinder $k$ corresponding to J ème mode of the beam. Components of the displacement of neutral fibre of the cylinder $k$ with the X-coordinate $x$ can then be written:

$$y_k(t) = \sum_{j=1}^{N} \mathbf{q}_j(t) \phi_j^k(x).\mathbf{y} \quad \text{éq. 5.3.4-2}$$

$$z_k(t) = \sum_{j=1}^{N} \mathbf{q}_j(t) \phi_j^k(x).\mathbf{z} \quad \text{éq. 5.3.4-3}$$

where $(\mathbf{q}) = (\mathbf{q}_j)_{j=1,N}$ is the vector of generalized displacements.

**Projection of the forces on modal basis**

- One notes $\mathbf{F}_i(t)$ the projection of the forces fluid-rubber bands according to [i ème] mode of the beam.

$$\mathbf{F}_i(t) = \sum_{k=1}^{K} \int_0^L \mathbf{f}_k(x,t).\phi_i^k(x)dx$$ **éq. 5.3.4-4**

$\mathbf{F}_i(t)$ is a linear combination of $(\mathbf{q}_j, \dot{\mathbf{q}}_j, \ddot{\mathbf{q}}_j)_{j=1,N}$

- One notes $\mathbf{F}(t)$ the vector of the modal forces fluid-rubber bands: $\mathbf{F}(t) = (\mathbf{F}_i(t))_{i=1,N}$ who is written:

$$\mathbf{F}(t) = -\lfloor \mathbf{M}_a \rfloor (\ddot{\mathbf{q}}(t)) - \lfloor \mathbf{C}_a \rfloor (\dot{\mathbf{q}}(t)) - \lfloor \mathbf{K}_a \rfloor (\mathbf{q}(t))$$ **éq. 5.3.4-5**

Where $\lfloor \mathbf{M}_a \rfloor$ indicate the matrix of the terms of mass added by the fluid,
$\lfloor \mathbf{C}_a \rfloor$ indicate the matrix of the terms of damping added by the fluid,
$\lfloor \mathbf{K}_a \rfloor$ indicate the matrix of the terms of stiffness added by the fluid.

These matrices are square real of order $N$ and their terms are independent of the movement of the structures. The matrix $\lfloor \mathbf{M}_a \rfloor$ is symmetrical; matrices $\lfloor \mathbf{C}_a \rfloor$ and $\lfloor \mathbf{K}_a \rfloor$ are not it necessarily.

- The projection of the equations of the movement on modal basis provides:

$$(\lfloor \mathbf{M}_{ii} \rfloor + \lfloor \mathbf{M}_a \rfloor)(\ddot{\mathbf{q}}(t)) + (\lfloor \mathbf{C}_{ii} \rfloor + \lfloor \mathbf{C}_a \rfloor)(\dot{\mathbf{q}}(t)) + (\lfloor \mathbf{K}_{ii} \rfloor + \lfloor \mathbf{K}_a \rfloor)(\mathbf{q}(t)) = (0)$$ **éq. 5.3.4-6**

where $\lfloor \mathbf{M}_{ii} \rfloor, \lfloor \mathbf{C}_{ii} \rfloor \text{et} \lfloor \mathbf{K}_{ii} \rfloor$ the matrices of masses, depreciation and stiffnesses of structure in air indicate; these matrices are of order $N$ and diagonals.

In the field of Laplace, the relation [**éq. 5.3.4-6**] becomes:

$$([\mathbf{M}_{ii}] + [\mathbf{M}_a])s^2 + ([\mathbf{C}_{ii}] + [\mathbf{C}_a])s + ([\mathbf{K}_{ii}] + [\mathbf{K}_a])(\mathbf{q}(s)) = (0)$$ **éq. 5.3.4-7**

- One introduces then the matrix of transfer of the forces fluid-rubber bands $[B(s)]$ defined by:

$$[\mathbf{B}(s)] = -[\mathbf{M}_a]s^2 - [\mathbf{C}_a]s - [\mathbf{K}_a]$$ **éq. 5.3.4-8**

And one finds the relation [**éq. 1.2-1**] paragraph [§ 1.2]:

$$([\mathbf{M}_{ii}]s^2 + [\mathbf{C}_{ii}]s + [\mathbf{K}_{ii}] - [\mathbf{B}(s)])(\mathbf{q}(s)) = (0)$$

## 5.4 Resolution of the modal problem under flow

The modal problem under flow is formulated by the relation [**éq. 5.3.4-7**] preceding paragraph.

This problem is solved after rewriting in the form of a standard problem to the vectors and the eigenvalues of the type $\lfloor \mathbf{A} \rfloor (\mathbf{X}) = \lambda (\mathbf{X})$.

The new formulation is the following one:

$$\begin{bmatrix} [0] & [Id] \\ -([\mathbf{M}_{ii}] + [\mathbf{M}_a])^{-1}([\mathbf{K}_{ii}] + [\mathbf{K}_a]) & -([\mathbf{M}_{ii}] + [\mathbf{M}_a])^{-1}([\mathbf{C}_{ii}] + [\mathbf{C}_a]) \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ s\mathbf{q} \end{Bmatrix} = s \begin{Bmatrix} \mathbf{q} \\ s\mathbf{q} \end{Bmatrix}$$

**éq. 5.4- 1**

**Note:**

1) *One doubles the dimension of the problem compared to that of initial problem.*
2) *Properties of the matrices* $\lfloor \mathbf{M}_{ii} \rfloor$ *and* $\lfloor \mathbf{M}_a \rfloor$ *allow the inversion.*

The resolution of this problem is done by means of algorithm QR. Modules implemented by the operator `CALC_FLUI_STRU` are the same ones as those used by `CALC_MODES`.

The problem with the clean elements that one solves is a complex problem. One thus obtains an even number of combined complex eigenvalues two to two. One preserves only those of which the imaginary part is positive or worthless.

The clean vectors are complex, defined except for constant a complex multiplicative. As one takes into account only real modes, it is initially a question of determining, for each clean vector, the constant which minimizes the imaginary part of the vector compared to its real part, within the meaning of the euclidian norm. The clean vectors are then redefined compared to this standard. Taking into account the standardisation used, it is then possible not to preserve in the concept `mode_meca` that the real part of the clean vectors. One restores however, in the file `MESSAGE`, indicators on the relationship between imaginary part and real part of the clean vectors thus normalized, so that the user can consider skew introduced by not taken into account of the imaginary part of the normalized vectors.

## 5.5 Taking into account of the presence of the grids of the tube bundle

Modeling described previously, of the forces induced by an axial flow on a beam of cylinders, does not take into account the presence of the grids of the beam (for example, grids of mixture and maintenance of the fuel assemblies). A comparison between this model and tests carried out on the model CHAIR (in the configuration of a beam of nine flexible tubes comprising a grid) is presented in a note of synthesis [feeding-bottle. 8]: it is noted that the coupling fluid-rubber band between the grid and the axial flow is not negligible and that it generates an increase in the reduced modal damping of the tubes. The object of this paragraph is the description of the additional effects due to the grids and their taking into account in model MEFISTEAU.

### 5.5.1 Description of the configuration of the grids

One restricts here the study with two types of grids:

- the grids of maintenance which are located at the ends of the beam,
- the grids of mixture which are distributed between the grids of maintenance.

The grids all are positioned perpendicular to the beam of cylinders and are appeared as a prismatic network to square base on side $d_g$ and height $h_g$ (along the axis $x$ cylinders). The grids of the same type are characterized by identical dimensions.

### 5.5.2 Additional stages of calculation

The first additional stage relates to the specification of the type of configuration of the grids with the operator DEFI_FLUI_STRU , then the checking of the good provision of the grids ones compared to the others, and the ends of the beam.

The second phase relates to the resolution of the modal problem under flow. In the loop on the rates of flow, the matrix of transfer of the forces fluid-rubber bands in the modal base in air is supplemented by the calculation of a matrix of added damping and a matrix of added stiffness, been dependent on the grids.

### 5.5.3 Modeling of the fluid forces exerted on the grids

**Calculation of the jump of pressure**

First of all, the presence of grids disturbs the stationary field of pressure $\overline{P}(x)$ ; one regards each grid as a singularity involving a jump of pressure, whose expression is put in the form:

$$\Delta\overline{P}(x_g)=\frac{1}{2}\rho_g(x_g)\overline{U}_g^2(x_g)K_g(x_g)$$

**éq. 5.5.3-1**

where $K_g$ indicate the loss ratio of load due to the grid,

$\overline{U}_g$ indicate the stationary speed of the flow on the level of the grid,

$\rho_g$ indicate the density of the flow on the level of the grid,

$x_g$ indicate the axial position of the medium of the grid along the beam.

Density $\rho_g(x_g)$ is calculated by linear interpolation of the profile of density $\rho(x)$ flow in the absence of grid. Stationary speed $\overline{U}_g(x_g)$ is calculated pursuant to the conservation of the mass throughput, which results in the following equation:

$$\rho_g(x_g)\overline{U}_g(x_g)A_{Fg}=\rho_o\overline{U}_o A_F$$

**Calculation of the specific fluid forces exerted on each grid**

According to the same quasi-static approach as that carried out in the paragraph [§5.3.3], one shows that the action of the fluid field speed $(\overline{\mathbf{U}}+\tilde{\mathbf{u}})$ around a grid implies a force of trail and a force of bearing pressure, according to the incidence flow compared to the grid. Components **there** and **Z** resulting specific force **F** $_g$ are thus written, for each basic cell $k$ of one

where $\rho_o$ and $\overline{U}_o$ respectively indicate the profile of density and stationary speed of the flow in foot of beam,

$A_F$ indicate the fluid section of the beam in the absence of grid,

$A_{Fg}$ indicate the fluid section of the beam on the level of the grid: $A_{Fg}=A_F-A_g$ with $A_g$ section solid of the grid.

One from of deduced the expression:

$$\overline{U}_g(x_g)=\frac{1}{\left(1-\frac{A_g}{A_F}\right)}\frac{1}{\rho_g(x_g)}\rho_o\overline{U}_o$$

The loss ratio of load $K_g$ is calculated starting from the expression of the total hydrodynamic force which applies to the grid, and we obtain:

$$K_g = \frac{1}{A_F}\left[A_g C_{dg}(x_g) + \left(1 - \frac{A_g}{A_F}\right)^2 h_g P_m C_{fl}(x_g)\right]$$ **éq. 5.5.3-2**

The 1er term (in $A_g C_{dg}$ ) comes from the effort of trail; $C_{dg}(x_g)$ is the coefficient of drag of the grid. 2 ème term (in $P_m C_{fl}$ ) is an effort corrector term of friction applied to the beam alone to the altitude of the grid ( $P_m$ is the wet perimeter of the beam in the absence of grid).
By introducing the expression [**éq. 5.5.3-2**] in the relation [**éq. 5.5.3-1**], one thus obtains the expression of the jump of pressure $\Delta\overline{P}(x_g)$ for each grid of altitude $x_g$ . This jump of pressure is taken into account on the level of the calculation of the stationary field of pressure $\overline{P}(x)$ , in the following way:

$$\overline{P}(x_{i+1}) = \overline{P}(x_i) - \Delta\overline{P}(x_g) \qquad \forall x_g \in [x_{i+1}, x_i]$$

grid:

$$\left(\mathbf{f}_g\right)_y = -\frac{1}{2}\rho_g\left|\overline{U}_g\right|\frac{A_g}{K}\left[C_{dg}\left(\frac{\partial y_k}{\partial t} - \tilde{\overline{u}}_y^k\right) + C_{pg}\left(\frac{Dy_k}{Dt} - \tilde{\overline{u}}_y^k\right)\right]$$

$$\left(\mathbf{f}_g\right)_z = -\frac{1}{2}\rho_g\left|\overline{U}_g\right|\frac{A_g}{K}\left[C_{dg}\left(\frac{\partial z_k}{\partial t} - \tilde{\overline{u}}_z^k\right) + C_{pg}\left(\frac{Dz_k}{Dt} - \tilde{\overline{u}}_z^k\right)\right]$$

where $C_{pg}$ indicate the slope with worthless incidence of the coefficient of bearing pressure around a grid very slightly tilted.
$\frac{A_g}{K}$ indicate the solid section of the basic cell $k$ grid (which understands some $K$ ).

These forces will thus generate additional terms of added damping and stiffness, that one obtains after modal decomposition of the movement and projection of these forces on the modal basis.

## 5.6 Catch in depreciation account in fluid at rest

Until now, the damping brought to a tube bundle by the presence of a fluid at rest was not taken into account in modeling. One thus proposes here a model of damping in fluid at rest, whose appendix 1 of the note of synthesis of the tests CHAIR [bib8] constitutes the reference material.

### 5.6.1 Modeling of the fluid force at rest exerted on a tube bundle

The method of calculating of damping in fluid at rest which is put in work here, is a generalization of the method of CHEN [feeding-bottle. 9].

It is a question of calculating the force resulting on each tube from the constraints due to shearing in the boundary layer. It is a nonlinear problem because the fluid damping coefficient depends on the frequency One thus introduces following simplifications:

- the problem is written using the water frequencies at rest calculated without taking into account fluid damping,
- one neglects the coupling between modes.

The linear force $f_i^k$ being exerted on the tube $k$ subjected to a harmonic movement of the beam following the mode $i$ at the frequency $f_i$ is given by the following relation:

$$f_i^k = \rho \left| U_i^k \right| U_i^k R_k C_{Dki} \qquad \textbf{éq. 5.6.1-1}$$

where $U_i^k$ indicate the speed of slip between the tube $k$ and fluid at rest, on both sides of the boundary layer, defined by:

$$U_i^k = U_{im}^k \dot{\mathbf{q}}_i(t) \qquad \textbf{éq. 5.6.1-2}$$

with $q_i(t) = \sin(2\pi f_i t)$ and

$U_{im}^k$ depends on the averages $\overline{\tilde{u}}_y$ and $\overline{\tilde{u}}_z$ disturbances speed around the cylinders, calculated beforehand by the model.

$C_{Dki}$ indicate the coefficient of drag of a cylinder of ray $R_k$ , subjected to a harmonic flow of amplitude ad infinitum $\left| U_i^k \right|_{\max} = \left| U_{im}^k \right| 2\pi f_i$ , and is defined by:

$$C_{Dki} = \frac{f_i 2R_k}{\left| U_i^k \right|_{\mathbf{max}}} \frac{3\pi^3}{2} \sqrt{\frac{\nu}{\pi f_i (2R_k)^2}} \qquad \textbf{éq. 5.6.1-3}$$

where $\nu$ indicate the kinematic viscosity of the fluid.

The relation obtained while replacing [**éq. 5.6.1-2**] and [**éq. 5.6.1-3**] in the equation [**5.6.1-1**] is linearized by a development in Fourier series (term $\left|\dot{\mathbf{q}}_i(t)\right| \dot{\mathbf{q}}_i(t)$ ) whose one retains only the first term; it comes:

$$f_i^k \approx 2\pi (2R_k) \rho U_{im}^k \sqrt{\pi f_i}\, \dot{\mathbf{q}}_i(t)$$

**Projection on modal basis**

By projection on modal basis and by neglecting the coupling between modes, one obtains the generalized force being exerted on the beam of tube following the mode $i$ :

$$\mathbf{F}_i(t) = \sum_{k=1}^{K} \int_0^L \vec{f}_i^k . \vec{\phi}_i^k(z) dz \cong \sum_{k=1}^{K} 2\pi (2R_k) \sqrt{\pi f_i} \left[ \int_0^L \rho \sqrt{\nu}\, \vec{U}_{im}^k . \vec{\phi}_i^k(z) dz \right] \dot{\mathbf{q}}_i(t)$$

$\mathbf{F}_i(t)$ is thus proportional to $\dot{\mathbf{q}}_i(t)$ and the vector of modal force associated $\mathbf{F}(t) = (\mathbf{F}_i(t))_{i=1,N}$ puts itself in the form:

$$\mathbf{F}(t) = -\lfloor \mathbf{C}_a \rfloor (\dot{\mathbf{q}}(t))$$

where $\lfloor \mathbf{C}_a \rfloor$ indicate the matrix of damping added by the fluid at rest.

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# 7 Description of the versions of the document

| Version Aster | Author (S) Organization (S) | Description of the modifications |
|---|---|---|
| 8.4 | T.Kestens EDF-R&D/MFEE | Initial text |
| 9.2 | A.Adobes, E. Longatte EDF-R&D/MFEE | Addition of precise details concerning the way of calculating of instability fluid-rubber band by the method of Connors and by method of the correlations fluid-rubber bands. |