Modal parameters and standard of the clean vectors

Summary:

In this document, one describes:

- various possibilities in *Code_Aster* to normalize the clean modes,
- important modal parameters associated with the clean modes.

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1 Definition of the problem to the eigenvalues

1.1 General information

That is to say the problem with the eigenvalues according to:

To find

$$(\lambda, \mathbf{\Phi}) \in \mathbb{C} \, x \, \mathbb{C}^n / \qquad (\lambda^2 \mathbf{B} + \lambda \mathbf{C} + \mathbf{A}) \mathbf{\Phi} = 0 \qquad \text{éq 1.1-1}$$

where $\mathbf{A}, \mathbf{C}, \mathbf{B}$ are positive symmetrical real matrices of order n.

Two cases are distinguished:

- quadratic problem: $\mathbf{C} \neq 0$,
- generalized problem: $\mathbf{C} = 0$.

λ eigenvalue is called and $\mathbf{\Phi}$ clean vector. In the continuation, one will speak about clean mode for $\mathbf{\Phi}$ and one will introduce the concept of Eigen frequency.

To solve this problem, several methods are available in *Code_Aster* and one returns the reader to the documents [R5.01.01] and [R5.01.02].

1.2 Generalized problem

The generalized problem can be written in the form:

To find

$$(\lambda, \mathbf{\Phi}) \in \mathbb{R} \, x \, \mathbb{R}^n / \qquad (-\lambda^2 \mathbf{B} + \mathbf{A}) \mathbf{\Phi} = 0 \qquad \text{éq 1.2-1}$$

One introduces two other sizes which make it possible to characterize the clean mode:

$$\lambda = \omega = (2 \pi f) \qquad \text{éq 1.2-2}$$

where

ω : own pulsation associated with the clean mode $\mathbf{\Phi}$,
f : Eigen frequency associated with the clean mode $\mathbf{\Phi}$.

It is also shown that the clean modes are $\mathbf{A}$ and $\mathbf{B}$ orthogonal, i.e.:

$$\begin{cases} \mathbf{\Phi}^{iT} \mathbf{A} \mathbf{\Phi}^{\mathrm{j}} = \delta_{ij} \mathbf{\Phi}^{iT} \mathbf{A} \mathbf{\Phi}^{\mathrm{i}} \\ \mathbf{\Phi}^{iT} \mathbf{B} \mathbf{\Phi}^{\mathrm{j}} = \delta_{ij} \mathbf{\Phi}^{iT} \mathbf{B} \mathbf{\Phi}^{\mathrm{i}} \end{cases} \qquad \text{éq 1.2-3}$$

where $(\mathbf{\Phi}^{\mathrm{i}}, \mathbf{\Phi}^{\mathrm{j}})$ are two clean modes.

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1.3 Quadratic problem

The quadratic problem [éq 1.1-1] can be put in another form of double size (one speaks about linear reduction [R5.01.02]):

To find

$$(\lambda, F) \in \mathbb{C} \, x \, \mathbb{C}^n / \quad \left(\lambda \begin{bmatrix} \mathbf{0} & \mathbf{B} \\ \mathbf{B} & \mathbf{C} \end{bmatrix} + \begin{bmatrix} -\mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \right) \begin{pmatrix} \lambda \boldsymbol{\Phi} \\ \boldsymbol{\Phi} \end{pmatrix} = 0 \qquad \text{éq 1.3-1}$$

One poses in the continuation: $\hat{\mathbf{B}} = \begin{bmatrix} \mathbf{0} & \mathbf{B} \\ \mathbf{B} & \mathbf{C} \end{bmatrix} \quad \hat{\mathbf{A}} = \begin{bmatrix} -\mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix}$.

Like the matrices $\mathbf{A}, \mathbf{C}, \mathbf{B}$ are real, the values and clean modes imaginary are combined two to two.

One introduces three other sizes which make it possible to characterize the clean mode:

$$\lambda = a + ib = -\frac{\xi \omega}{\sqrt{1-\xi^2}} + i\omega = -\frac{\xi (2\pi f)}{\sqrt{1-\xi^2}} + i(2\pi f) \qquad \text{éq 1.3-2}$$

where ω : own pulsation associated with the clean mode Φ,
f : Eigen frequency associated with the clean mode Φ,
ξ : reduced damping.

It is also shown that the clean modes are $\begin{bmatrix} \mathbf{0} & \mathbf{B} \\ \mathbf{B} & \mathbf{C} \end{bmatrix}$ and $\begin{bmatrix} -\mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix}$ orthogonal, i.e.:

$$\begin{cases} -\lambda_i \lambda_j \boldsymbol{\Phi}^{iT} \mathbf{B} \boldsymbol{\Phi}^{\mathbf{j}} + \boldsymbol{\Phi}^{iT} \mathbf{A} \boldsymbol{\Phi}^{\mathbf{j}} = \boldsymbol{\delta}_{ij} (-\lambda_i^2 \boldsymbol{\Phi}^{iT} B \boldsymbol{\Phi}^{\mathbf{i}} + \boldsymbol{\Phi}^{iT} A \boldsymbol{\Phi}^{\mathbf{i}}) \\ (\lambda_i + \lambda_j) \boldsymbol{\Phi}^{iT} \mathbf{B} \boldsymbol{\Phi}^{\mathbf{j}} + \boldsymbol{\Phi}^{iT} \mathbf{C} \boldsymbol{\Phi}^{\mathbf{j}} = \boldsymbol{\delta}_{ij} (2\lambda_i \boldsymbol{\Phi}^{iT} B \boldsymbol{\Phi}^{\mathbf{i}} + \boldsymbol{\Phi}^{iT} C \boldsymbol{\Phi}^{\mathbf{i}}) \end{cases} \qquad \text{éq 1.3-3}$$

where (λ_i, λ_j) are the eigenvalues respectively associated with the clean modes $(\boldsymbol{\Phi}^{\mathbf{i}}, \boldsymbol{\Phi}^{\mathbf{j}})$.

Notice :

the clean modes are thus not $\mathbf{A}, \mathbf{B} \, \text{ou} \, \mathbf{C}$ *orthogonal.*

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2 Normalizes clean modes of the generalized problem

One supposes to have calculated a couple $(\lambda, \mathbf{\Phi})$ solution of the problem [éq 1.2-1]: λ is the eigenvalue associated with the clean mode $\mathbf{\Phi}$. One considers for the moment only the case of the generalized problem.

In *Code_Aster*, the order `NORM_MODE` [U4.52.11] allows to impose a kind of standardisation for the whole of the modes.

2.1 Components of a clean mode

That is to say a clean mode $\mathbf{\Phi}$ components $(\mathbf{\Phi_j})_{j=1,n}$.

Among these components, one distinguishes:

- the components or degrees of freedom called "physics" (they are for example the degrees of freedom of displacement (DX, DY, DZ), degrees of freedom of rotation (DRX, DRY, DRZ), potential characterizing an irrotational fluid (PHI),...),
- the components of Lagrange (the parameters of Lagrange are additional unknown factors which are added with the "physical" problem initial so that the boundary conditions are checked [R3.03.01]).

In *Code_Aster*, one has three families of standards:

- euclidian norm,
- normalizes: "larger component with 1" among a group of degrees of freedom defined,
- mass or unit generalized rigidity normalizes.

They successively are described.

Previously, one defines L a family of indices which contains m terms:

$$L=\{l_k, k=1,m \,\text{avec}\, 1\leq l_k\leq n\} \text{ et } 1\leq m\leq n.$$

2.2 Euclidian norm

The following standard is defined: $\|\mathbf{\Phi}\|_2=\left(\sum_{k=1}^{m}\left(\mathbf{\Phi_{l_k}}\right)^2\right)^{1/2}$

The normalized vector is then obtained $\hat{\mathbf{\Phi}}$: $\hat{\mathbf{\Phi}}=\frac{1}{\|\mathbf{\Phi}\|_2}\mathbf{\Phi} \qquad (\hat{\mathbf{\Phi}}_j=\frac{1}{\|\mathbf{\Phi}\|_2}\mathbf{\Phi}_j \qquad j=1,n)$.

In *Code_Aster* , two standards of this family are available:

- `NORME=' EUCL'` : L corresponds to the whole of the indices which characterize a physical degree of freedom,
- `NORME=' EUCL_TRAN'` : L corresponds to the whole of the indices which characterize a physical degree of freedom of displacement in translation (DX, DY, DZ).

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2.3 "Larger component with 1 normalizes"

The following standard is defined: $\|\Phi\|_\infty = \max_{k=1,m} |\Phi_{l_k}|$

The normalized vector is then obtained $\hat{\Phi}$: $\hat{\Phi} = \frac{1}{\|\Phi\|_\infty}\Phi \quad (\hat{\Phi}_j = \frac{1}{\|\Phi\|_\infty}\Phi_j \quad j=1,n)$.

In *Code_Aster* , five standards of this family are available:

- NORME=' SANS_CMP=LAGR' : L corresponds to the whole of the indices which characterize a physical degree of freedom,
- NORME=' TRAN' : L corresponds to the whole of the indices which characterize a physical degree of freedom of displacement in translation (DX, DY, DZ) ,
- NORME=' TRAN_ROTA' : L corresponds to the whole of the indices which characterize a physical degree of freedom of displacement in translation and rotation $(DX, DY, DZ, DRX, DRY, DRZ)$,
- NORME=' AVEC_CMP' or 'SANS_CMP' : L is built either by taking all the indices which correspond to types of components stipulated by the user (for example standard displacement following the axis x : 'DX') (NORME=' AVEC_CMP'), that is to say by taking the complementary one to all the indices which correspond to types of components stipulated by the user (NORME=' SANS_CMP'),
- NORME=' NOEUD_CMP': L corresponds to only one index which characterizes a component of a node of the grid. The name of the node and the component are specified by the user (keywords NOM_CMP and NODE order NORM_MODE [U4.52.11]).

By defaults the modes are normalized with the standard 'SANS_CMP=LAGR'.

2.4 Mass or unit generalized rigidity normalizes

That is to say a positive definite matrix of order n . The following standard is defined: $\|\Phi\|_E = (\Phi^T E \Phi)^{1/2}$

The normalized vector is then obtained $\hat{\Phi}$: $\hat{\Phi} = \frac{1}{\|\Phi\|_E}\Phi \quad (\hat{\Phi}_j = \frac{1}{\|\Phi\|_E}\Phi_j \quad j=1,n)$. .

In *Code_Aster* , two standards of this family are available:

- NORME=' MASSE_GENE': $\mathbf{E}=\mathbf{B}$. In a classical problem of vibration, B is the matrix of mass.
- NORME=' RIGI_GENE' : $\mathbf{E}=\mathbf{A}$. In a classical problem of vibration, A is the matrix of rigidity.

Note:

For a mode Φ *rigid body, one a:* $\|\Phi\|_E = \|\Phi\|_A = 0$

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3 Normalizes clean modes of the quadratic problem

3.1 Euclidian norms and “larger component with 1”

For the quadratic problem, one has the same standards as for the generalized problem. The clean modes being complex, one works with the square product. The various “classical” standards become:

- square standard: $\|\mathbf{\Phi}\|_2=\left(\sum_{k=1}^{m}|\mathbf{\Phi}_{\mathbf{l_k}}|^2\right)^{1/2}=\left(\sum_{k=1}^{m}\left(\bar{\mathbf{\Phi}}_{\mathbf{l_k}}\mathbf{\Phi}_{\mathbf{l_k}}\right)\right)^{1/2}$ where $\bar{\mathbf{\Phi}}_{\mathbf{l_k}}$ is combined of $\mathbf{\Phi}_{\mathbf{l_k}}$ (the absolute value in the real field becomes the module in the field complexes),
- “larger component with 1 normalizes”: $\|\mathbf{\Phi}\|_\infty=\max_{k=1,m}|\mathbf{\Phi}_{\mathbf{l_k}}|=\max_{k=1,m}\left(\left(\bar{\mathbf{\Phi}}_{l_k}\mathbf{\Phi}_{\mathbf{l_k}}\right)^{1/2}\right)$.

3.2 Mass or unit generalized rigidity normalizes

With regard to the standard “masses or generalized rigidity”, denomination by analogy with the generalized problem, one uses like matrix associated with the standard, that which intervenes in the writing of the quadratic problem put in the reduced form [éq 1.3-1].

One has then:

- normalizes generalized mass:

$$\|\mathbf{\Phi}\|_{\hat{B}}=\left(\lambda\mathbf{\Phi}^{\mathbf{T}},\mathbf{\Phi}^{\mathbf{T}}\right)\hat{\mathbf{B}}\begin{pmatrix}\lambda\mathbf{\Phi}\\ \mathbf{\Phi}\end{pmatrix}=\left(\lambda\mathbf{\Phi}^{\mathbf{T}},\mathbf{\Phi}^{\mathbf{T}}\right)\begin{bmatrix}\mathbf{0} & \mathbf{B}\\ \mathbf{B} & \mathbf{C}\end{bmatrix}\begin{pmatrix}\lambda\mathbf{\Phi}\\ \mathbf{\Phi}\end{pmatrix}=2\lambda\mathbf{\Phi}^{\mathbf{T}}\mathbf{B}\mathbf{\Phi}+\mathbf{\Phi}^{\mathbf{T}}\mathbf{C}\mathbf{\Phi}\,,$$

$$\hat{\mathbf{\Phi}}=\frac{1}{\|\mathbf{\Phi}\|_{\hat{\mathbf{B}}}}\mathbf{\Phi}\ ,$$

- normalizes generalized rigidity:

$$\|\mathbf{\Phi}\|_{\hat{B}}=\left(\lambda\mathbf{\Phi}^{\mathbf{T}},\mathbf{\Phi}^{\mathbf{T}}\right)\hat{\mathbf{A}}\begin{pmatrix}\lambda\mathbf{\Phi}\\ \mathbf{\Phi}\end{pmatrix}=\left(\lambda\mathbf{\Phi}^{\mathbf{T}},\mathbf{\Phi}^{\mathbf{T}}\right)\begin{bmatrix}-\mathbf{B} & \mathbf{0}\\ \mathbf{0} & \mathbf{A}\end{bmatrix}\begin{pmatrix}\lambda\mathbf{\Phi}\\ \mathbf{\Phi}\end{pmatrix}=-\lambda^2\mathbf{\Phi}^{\mathbf{T}}\mathbf{B}\mathbf{\Phi}+\mathbf{\Phi}^{\mathbf{T}}\mathbf{A}\mathbf{\Phi}\,,$$

$$\hat{\mathbf{\Phi}}=\frac{1}{\|\mathbf{\Phi}\|_{\hat{\mathbf{A}}}}\mathbf{\Phi}\ .$$

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4 Modal parameters associated for the generalized problem

One in the case of places a classical generalized problem of vibration. One a:

- $\mathbf{A}=\mathbf{K}$ is the matrix of rigidity,
- $\mathbf{B}=\mathbf{M}$ is the matrix of mass.

That is to say a couple $(\lambda, \mathbf{\Phi})$ solution of the problem:

$$\left(-\lambda^2\mathbf{M}+\mathbf{K}\right)\mathbf{\Phi}=0 \quad \text{éq 4-1}$$

In the continuation, one defines successively the following sizes:

- generalized sizes,
- effective modal mass and unit effective modal mass,
- factor of participation.

4.1 Generalized sizes

4.1.1 Definition

Two generalized sizes are defined:

- Generalized mass of the mode $\mathbf{\Phi}$: $m_{\Phi}=\mathbf{\Phi}^{\mathbf{T}}\mathbf{M}\mathbf{\Phi}$,
- Generalized rigidity of the mode $\mathbf{\Phi}$: $k_{\Phi}=\mathbf{\Phi}^{\mathbf{T}}\mathbf{K}\mathbf{\Phi}$.

These quantities depend on the standardisation on F . These sizes are accessible in the concept `RESULT` of type `mode_meca` under the names `MASS_GENE, RIGI_GENE`.

Notice 1:

One has the following relation between the pulsation (or the frequency) of the mode and the mass and rigidity generalized of the mode:

$$\lambda^2=\omega^2=(2\pi f)^2=\frac{\mathbf{\Phi}^{\mathbf{T}}\mathbf{K}\mathbf{\Phi}}{\mathbf{\Phi}^{\mathbf{T}}\mathbf{M}\mathbf{\Phi}}=\frac{k_{\Phi}}{m_{\Phi}} .$$

Notice 2:

From the physical point of view, the generalized mass (which is a positive value) can be interpreted as mass moving:

$m_{\Phi}=\mathbf{\Phi}^{\mathbf{T}}\mathbf{M}\mathbf{\Phi}=\int \rho\,\mathbf{\Phi}^{\mathbf{2}}$ *where* ρ *is the density of the structure.*

Kinetic energy of the structure vibrating according to the mode $\mathbf{\Phi}$ *is equal then to:*

$$E_c=\frac{1}{2}\omega^2 m_{\Phi}=\frac{1}{2}\omega^2\mathbf{\Phi}^{\mathbf{T}}\mathbf{M}\mathbf{\Phi} .$$

Potential energy of deformation associated with the mode $\mathbf{\Phi}$ *is equal to:*

$$E_p=\frac{1}{2}k_{\Phi}=\frac{1}{2}\mathbf{\Phi}^{\mathbf{T}}\mathbf{K}\mathbf{\Phi} .$$

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4.1.2 Use

During a calculation by modal recombination [R5.06.01], one seeks a solution of the equation of dynamics:

$$\mathbf{M}\,\ddot{\mathbf{x}}+\mathbf{C}\,\dot{\mathbf{x}}+\mathbf{K}\,\mathbf{x}=f(t)\ ,$$

in the form $x=\sum_{i=1,m}\boldsymbol{\alpha}_\mathbf{i}(t)\,\boldsymbol{\Phi}^\mathbf{i}$ where $\boldsymbol{\Phi}^\mathbf{i}$ real associate with the eigenvalue is the clean mode λ_i , solution of the generalized problem (in general one has $m\leq n$ (n is the number of degree of freedom) because one does not take into account that part of the modal base):

$$\left(-\mathbf{M}\lambda_i^2+\mathbf{K}\right)\boldsymbol{\Phi}^\mathbf{i}=0$$

The generalized vector $\boldsymbol{\alpha}=\left(\boldsymbol{\alpha}_\mathbf{i}\right)_{i=1,m}$ is solution of:

$\tilde{\mathbf{M}}\,\ddot{\boldsymbol{\alpha}}+\tilde{\mathbf{C}}\,\dot{\boldsymbol{\alpha}}+\tilde{\mathbf{K}}\,\boldsymbol{\alpha}=\tilde{f}$ (problem of order m) with:

$$\begin{array}{ll}\tilde{\mathbf{M}}=\left(\tilde{\mathbf{M}}_{ij}\right)=\left(\boldsymbol{\Phi}^{iT}\mathbf{M}\boldsymbol{\Phi}^\mathbf{j}\right) & \tilde{\mathbf{C}}=\left(\tilde{\mathbf{C}}_{ij}\right)=\left(\boldsymbol{\Phi}^{iT}\mathbf{C}\boldsymbol{\Phi}^\mathbf{j}\right)\\ \tilde{\mathbf{K}}=\left(\tilde{\mathbf{K}}_{ij}\right)=\left(\boldsymbol{\Phi}^{iT}\mathbf{K}\boldsymbol{\Phi}^\mathbf{j}\right) & \tilde{f}=\left(\tilde{f}_i\right)=\left(\boldsymbol{\Phi}^{iT}f\right)\end{array}\ .$$

The modes of vibration of the generalized problem are $\mathbf{K}$ and $\mathbf{M}$ orthogonal [R5.01.01]. Matrices $\tilde{\mathbf{M}}$ and $\tilde{\mathbf{K}}$ are then diagonal and are made up by rigidities and generalized masses of each mode. The matrix $\tilde{\mathbf{C}}$ is usually full if one does not make additional assumptions on $\mathbf{C}$ [R5.05.04].

4.2 Effective modal masses and unit effective modal masses

4.2.1 Effective modal masses

That is to say $\mathbf{U_d}$ an unit vector in the direction d . In each node of the vector $\mathbf{U_d}$ having the components of displacement $(DX\ ,DY\ ,DZ)$ one a:

$(\mathrm{DX}=x_d,\mathrm{DY}=y_d,\mathrm{DZ}=z_d)$ where (x_d,y_d,z_d) are the cosine Directors of the direction d (one thus has: $x_d^2+y_d^2+z_d^2=1$).

For example, if d is the direction x , the vector $\mathbf{U_d}$ has all its components DX equal to 1 and its other components equal to 0.

One defines the effective modal masses in the direction d by:

$$m_{\Phi,d}=\frac{\left(\boldsymbol{\Phi}^\mathbf{T}\mathbf{M}\,\mathbf{U_d}\right)^2}{\left(\boldsymbol{\Phi}^\mathbf{T}\mathbf{M}\boldsymbol{\Phi}\right)}\ .$$

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4.2.2 Property

Statement:

The sum of the effective modal masses in a direction d is equal to the mass of the structure according to this direction. That is written:

$\sum_{i=1,n} \frac{\left(\mathbf{\Phi}^{\mathbf{i}\,\mathbf{T}} \mathbf{M}\, \mathbf{U}_{\mathbf{d}}\right)^2}{\left(\mathbf{\Phi}^{\mathbf{i}\,\mathbf{T}} \mathbf{M} \mathbf{\Phi}^{\mathbf{i}}\right)} = \sum_{i=1,n} m_{\Phi^i,d} = \mathbf{U}_{\mathbf{d}}^{\mathbf{T}} \mathbf{M}\, \mathbf{U}_{\mathbf{d}} = m_{totale,d}$ where n is the full number of modes associated with the problem [éq 4-1]

Notice :

In the majority of the cases, the mass is isotropic. The preceding relation is put then in the following form:

$\sum_{i=1,n} \frac{\left(\mathbf{\Phi}^{\mathbf{i}\,\mathbf{T}} \mathbf{M}\, \mathbf{U}_{\mathbf{d}}\right)^2}{\left(\mathbf{\Phi}^{\mathbf{i}\,\mathbf{T}} \mathbf{M} \mathbf{\Phi}^{\mathbf{i}}\right)} = \sum_{i=1,n} m_{\Phi^i,d} = m_{totale,d} = m_{totale}$ *where* m_{totale} *is the mass of the structure*

4.2.3 Unit effective modal masses

By using the preceding property, one defines the unit effective modal masses in the direction D:

$$\tilde{m}_{\Phi^i,d} = \frac{m_{\Phi^i,d}}{m_{totale,d}} \text{ ,}$$

and one a: $\sum_{i=1,n} \tilde{m}_{\Phi^i,d} = 1$.

Modal masses $\tilde{m}_{\Phi^i,d}$ and $m_{\Phi^i,d}$ are independent of the standardisation of the mode Φ^i of vibration.

4.2.4 Use

“Empirical” relation :

At the time of a study “seismic request of a structure in a direction d “by a method of modal recombination, one must preserve the modes of vibration which have an important unit effective mass and it is of use in France to consider that one so has a good modal representation for the unit of the preserved modes one a:

$$\sum_{i=1,n} \tilde{m}_{\Phi^i,d} \geq 0{,}9$$

This empirical relation for example is stated in the RCC-G (Rules of design and construction applicable to Génie Civil).

Notice :

The sum of the effective modal masses is worth in fact the total mass which works on the selected modal basis. In other words, this working total mass is worth the total mass minus the contributions in mass which are carried by embedded degrees of freedom (which thus do not work on the modal basis). Thus, for example, on a system with 1 degree of freedom mass-arises with a mass $M1$ *at the top and another mass* $M2$ *at the level to erase it, then the working mass will be worth* $M1$ *and total mass* $M1+M2$ *. Consequently, the unit*

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effective modal mass for the only mode of the system will be worth $M1/(M1+M2)$ *. total office plurality will thus have the same value and, according to the ratio in* $M1$ *and* $M2$ *, one will not be able inevitably to thus reach 90% of the total mass* $(M1+M2)$ *, even by considering all the modes (there is only one only mode on this example). In practice, more the model with the finite elements will be fine and realistic, more the difference between the working mass and the total mass will be weak.*

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4.2.5 Directions privileged in *Code_Aster*

In *Code_Aster*, one has three directions which are those of the reference mark of definition of the grid:

- d = direction X,
- d = direction Y,
- d = direction Z.

The effective modal masses and the unit effective modal masses are accessible in the concept `RESULT` of type `mode_meca` under the names `MASS_EFFE_DX, MASS_EFFE_DY, MASS_EFFE_DZ, MASS_EFFE_UN_DX, MASS_EFFE_UN_DY, MASS_EFFE_UN_DZ`.

4.3 Factors of participation

4.3.1 Definition

One defines other parameters called factor of participation:

$$p_{F,d}^{i}=\frac{(\mathbf{\Phi}^{\mathbf{iT}}\mathbf{M}\mathbf{U}_{\mathbf{d}})}{(\mathbf{\Phi}^{\mathbf{iT}}\mathbf{M}\mathbf{\Phi}^{\mathbf{i}})}.$$

This parameter depends on the standardisation of the mode of vibration $\mathbf{\Phi}^{\mathbf{i}}$.

As for the effective masses, one has three directions d who are those of the reference mark of definition of the grid.

The factors of participation are accessible in the concept `RESULT` of type `mode_meca` under the names `FACT_PARTICI_DX, FACT_PARTICI_DY, FACT_PARTICI_DZ`.

4.3.2 Property

Statement:

Factors of participation associated with a direction d check the following relation:

$$m_{totale}=\sum_{i=1,n}\frac{\left(\mathbf{\Phi}^{iT}\mathbf{M}\mathbf{U}_{\mathbf{d}}\right)^{2}}{\left(\mathbf{\Phi}^{iT}\mathbf{M}\mathbf{\Phi}^{\mathbf{i}}\right)}=\sum_{i=1,n}\left(\frac{\mathbf{\Phi}^{iT}\mathbf{M}\mathbf{U}_{\mathbf{d}}}{\mathbf{\Phi}^{iT}\mathbf{M}\mathbf{\Phi}^{\mathbf{i}}}\right)^{2}\left(\mathbf{\Phi}^{iT}\mathbf{M}\mathbf{\Phi}^{\mathbf{i}}\right)=\sum_{i=1,n}\left(p_{\Phi^{i},d}\right)^{2}m_{\Phi^{i}}$$

where n is the full number of modes associated with the problem [éq 4-1].

This result is obtained easily by expressing the factor of participation according to the effective modal mass and by using the stated result in [§ 4.2.3].

4.3.3 Use

These parameters are used in particular to calculate the answer of a structure subjected to an earthquake by spectral method. One returns the reader to the document [R4.05.03].

4.4 Unit vector displacement

In what precedes, a unit vector of displacement was considered $\mathbf{U}_{\mathbf{d}}$ who relates to only the degrees of freedom of translation (DX, DY, DZ). This concept can be wide with rotations by considering the following definition. A matrix is defined $\mathbf{U}$ of dimension $(n\times 6)$. If all the nodes of the grid support 3 degrees of freedom of translation and 3 others of rotation, the matrix $\mathbf{U}$ is formed by the stacking of the matrices $\mathbf{u}_{trd}^{\mathbf{k}}(6\times 6)$ following (the index k corresponds to the node of number k):

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$$\mathbf{u}_{tr}^{\mathbf{k}}=\begin{bmatrix} 1 & 0 & 0 & 0 & (z_k-z_c) & -(y_k-y_c) \\ 0 & 1 & 0 & -(z_k-z_c) & 0 & (x_k-x_c) \\ 0 & 0 & 1 & (y_k-y_c) & -(x_k-x_c) & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where (x_k, y_k, z_k) are the coordinates of the node and (x_c, y_c, z_c) are the coordinates of the instantaneous center of rotation.

One can thus define effective modal masses, factors of participation associated with degrees of freedom with rotation.

For the moment, the calculation of these parameters is not available in Code_Aster.

4.5 Typical case of the factors of participations on modes themselves expressed in generalized coordinates

There exist cases where the modes are expressed according to a first modal base. An example is the calculation of the fluid masses added on a modal basis in air, classical technique in Code_Aster. If one wants to then calculate the parameters such as the factors of modal participation or the effective modal masses, useful in particular for seismic calculations, one then does not have access to the vectors of unit displacement. The factors of modal participation of the initial modal base then play the part of the vectors of unit displacement. This approach is explained in this paragraph.

4.5.1 Kinetic energy

$T=\frac{1}{2}V^t M\,V$ where V speed of the points of the structure and M matrix of mass of the structure.

Description of the movement broken up on the basis as of modes of the structure:

$$V=\sum_i \dot{q}_i \varphi_i$$

Calculation moving relative compared to the ground:

$V=\sum_i \dot{q}_i \varphi_i+\dot{s}\,D$ where s is the position of the ground and D direction of the earthquake.

From where kinetic energy of the structure expressed in the reference mark of the ground:

$T=\frac{1}{2}V^t M\,V=\frac{1}{2}\sum_{ij} m_{ij}\dot{q}_i\dot{q}_j+\dot{s}\sum_i \dot{q}_i\varphi_i^t M\,D$ (by using the symmetry of M)

Contribution of the kinetic energy to the equations of Euler-Lagrange:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)=m_i\ddot{q}_i+\ddot{s}\,\varphi^t M\,D$$

One from of deduced the expression from the factor of modal participation:

$$d_i=\frac{\varphi^t M\,D}{m_i}$$

And by definition the modal mass is given by:

$$me_i=m_i d_i^2$$

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4.5.2 Case of the movement describes on a basis of generalized modes

One can write the speed of the structure on the new basis:

$$V=\sum_i \dot{Q}_i \Phi_i$$

with generalized modes Φ_i defined on the first modal basis $\Phi_i=\sum_k \psi_{ik}\varphi_k$

N.B.: these modes can describe a modal deformation different from the initial modes.

$\tilde{M}$ is the matrix of generalized mass expressed on Φ_i, not systematically the projection of M ; for example in the case of a system with added mass: $\tilde{M}=M+M_a$

Contribution of the movement of training to the equations of Euler-Lagrange:

$$\ddot{s}\Phi_i^t \tilde{M}\,D=\ddot{s}\sum_k \psi_{ik}\varphi_k^t \tilde{M}\,D$$

However $D=\sum_j d_j\varphi_j$ where the vector $[d]$ is the vector of the factors of participation of the modal base $[\varphi]$

Thus $\ddot{s}\Phi_i^t \tilde{M}\,D=\ddot{s}\sum_k \psi_{ik}\varphi_k^t \tilde{M}\sum_j d_j\varphi_j=\ddot{s}\sum_{kj}\psi_{ik}\varphi_k^t \tilde{M}\varphi_j d_j=\ddot{s}[\psi_i]^t[\varphi^t \tilde{M}\varphi][d]$

From where, by identification, the factor of participation of the generalized mode Φ_i :

$\tilde{d}_i=\dfrac{[\psi_i]^t[\varphi^t \tilde{M}\varphi][d]}{\tilde{m}_i}$ where $\tilde{m}_i$ is the modal mass of the generalized mode.

5 Modal parameters associated for the quadratic problem

One writes the quadratic problem in the form: $(\lambda^2\mathbf{M}+\lambda\mathbf{C}+\mathbf{K})\mathbf{\Phi}=0$.

For the quadratic problem, one calculates only three parameters which correspond to the following generalized sizes:

- generalized mass (real quantity): $m_\Phi=\bar{\mathbf{\Phi}}^{\mathbf{T}}\mathbf{M}\mathbf{\Phi}$,
- generalized rigidity (real quantity): $k_\Phi=\bar{\mathbf{\Phi}}^{\mathbf{T}}\mathbf{K}\mathbf{\Phi}$,
- generalized damping (real quantity): $c_\Phi=\bar{\mathbf{\Phi}}^{\mathbf{T}}\mathbf{C}\mathbf{\Phi}$.

Attention, if one normalizes the clean mode with the standard “masses generalized”, one does not have in the quadratic case: $m_\Phi=1$. One can pass the same remark concerning generalized rigidity.

By using the relations of orthogonality and the fact that the clean elements appear per combined pairs, one can write the following relations:

$$\frac{\bar{\mathbf{\Phi}}^{\mathbf{T}}\mathbf{C}\mathbf{\Phi}}{\bar{\mathbf{\Phi}}^{\mathbf{T}}\mathbf{M}\mathbf{\Phi}}=\frac{c_\Phi}{m_\Phi}=2\,\mathrm{Re}(\lambda)=-\frac{2\xi\omega}{\sqrt{1-\xi^2}}=-\frac{2\mathrm{x}(2\pi f)}{\sqrt{1-\xi^2}},$$

$$\frac{\bar{\mathbf{\Phi}}^{\mathbf{T}}\mathbf{K}\mathbf{\Phi}}{\bar{\mathbf{\Phi}}^{\mathbf{T}}\mathbf{M}\mathbf{\Phi}}=\frac{k_\Phi}{m_\Phi}=|\lambda|^2=\frac{\omega^2}{1-\xi^2}=\frac{(2\pi f)^2}{1-\xi^2}.$$

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6 Bibliography

1) J.R. LEVESQUE, L. VIVAN, Fe WAECKEL: Seismic answer by spectral method [R4.05.03].

2) D. SELIGMANN, B. QUINNEZ: Algorithms of resolution for the generalized problem [R5.01.01].

3) D. SELIGMANN, R. MICHEL: Algorithms of resolution for the quadratic problem [R5.01.02].

4) Operator NORM_MODE [U4.52.11].

7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
01/04/00	*B. QUINNEZ J.R. LEVESQUE (EDF/IMA/MMNR)*	Initial text

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