Nonlinear thermics

Summary

The operator THER_NON_LINE [U4.54.02] allows to solve the problems of transitory thermics in the solids in the presence of non-linearities of the properties of materials (heat-storage capacity and conductivity), or of the boundary conditions (heat exchange of standard radiation). One presents here the formulation and the algorithm employed, this last being close to that related to the operator STAT_NON_LINE [R5.03.01]. The various options of calculation necessary were presented in the elements of structure plans, axisymmetric and three-dimensional [U3.22.01], [U3.23.01] and [U3.24.01].

Contents

1 Expression of the equation of heat in nonlinear thermics

1.1 Equation of heat for a motionless solid

In this document, only it thermal of the solid bodies is treated, even if the liquid/solid phase shift is taken into account. There is thus no heat transfer by convection but only by conduction.

The first principle of thermodynamics connects the temporal variation of total energy dE_{totale} of a system included in a volume of control Ω with the work of the external efforts δW and with heat δQ receipts by this same system:

$$dE_{totale}=d\left(E_{\text{interne}}+E_{\text{cinétique}}\right)=\delta W+\delta Q$$ **éq 1.1-1**

By injecting the theorem of the kinetic energy in this equation, one reveals thus the power of the interior efforts, function of the field speed [bib1]:

$$\dot{E}_{\text{interne}}=\dot{Q}-P_i(u)$$ **éq 1.1-2**

For the resolution of the problem of thermics, the system is supposed without movement. Power of the interior efforts $P_i(u)$ is thus worthless. Indeed, in the majority of the applications concerned, the thermal and mechanical phenomena are uncoupled; density power density dissipated by the plastic deformations, $P_i=\sigma_c.\dot{\epsilon}_{plastique}$, is neglected in front of the heat exchanged on the surface or the other voluminal sources of heat.

The equation [éq 1.1-2] which expresses the variation of heat in volume Ω is written then:

$$\forall s\in\Omega \quad \rho\frac{d}{dt}\int_S e\,d\Omega=\dot{Q}=\int_S \left(r_{vol}-\operatorname{div} q\right)d\Omega$$ **éq 1.1-3**

where one noted:

- e internal energy,
- ρ density,
- r_{vol} the voluminal rate of contribution external of heat,
- $\boldsymbol{q}$ the vector heat flow.

Moreover, since the solid is motionless, for any volume of control $\Omega(t)=\Omega$, one then obtains the local equation of conservation of heat:

$$\rho\ \frac{de}{dt}=r_{vol}-\operatorname{div} q$$ **éq 1.1-4**

If all the system is actuated by a rigid movement of body, an additional term appears in the member of left, utilizing the speed of the solid and the gradient of energy. This situation is treated by the order `THER_NON_LINE_MO` [R5.02.04].

In the case of a reversible transformation, the equation [éq 1.1-4] becomes, with the assistance of the second principle of the thermodynamics which makes it possible to write in our case $dE_{interne}=TdS$:

$$\rho \ T\dot{s}=r_{vol}-\operatorname{div} q$$ **éq 1.1-5**

and finally the equation of heat in its classical form:

$$\rho \ C_P\dot{T}=r_{vol}-\operatorname{div} q$$ **éq 1.1-6**

with the heat-storage capacity with constant pressure defined by: $C_P=T\frac{\partial s}{\partial T}|_P$

As he is explained in chapter 1.4, he can be advantageous to write the term of left of the equation [éq 1.1-6] with the enthalpy β who does not depend whereas temperature:

$$\dot{\beta}=r_{vol}-\operatorname{div} q$$ **éq 1.1-7**

where $\beta(T)=\int_{T_0}^{T}\rho C_P dT$

1.2 Fourier analysis

In thermal conduction, the Fourier analysis provides an equation connecting the heat flow to the gradient of the temperature (normal vector on the isothermal surface). This law reveals, in its most general form, a tensor of conductivity. In the case of an isotropic material, this tensor is reduced to a coefficient λ (being able to depend on the temperature), thermal conductivity:

$$q(x,t)=-\lambda(T)\nabla T(x,t)$$ **éq 1.2-1**

1.3 Equation of heat in the case of the model of transitory thermics non-linear

By combining the equations [éq 1.1-5] and [éq 1.2-1], one obtains:

$$r_{vol}-\operatorname{div}(-\lambda(T)\nabla T)=\frac{d\beta}{dt}$$ **éq 1.3-1**

or, if the heat-storage capacity does not depend on the temperature:

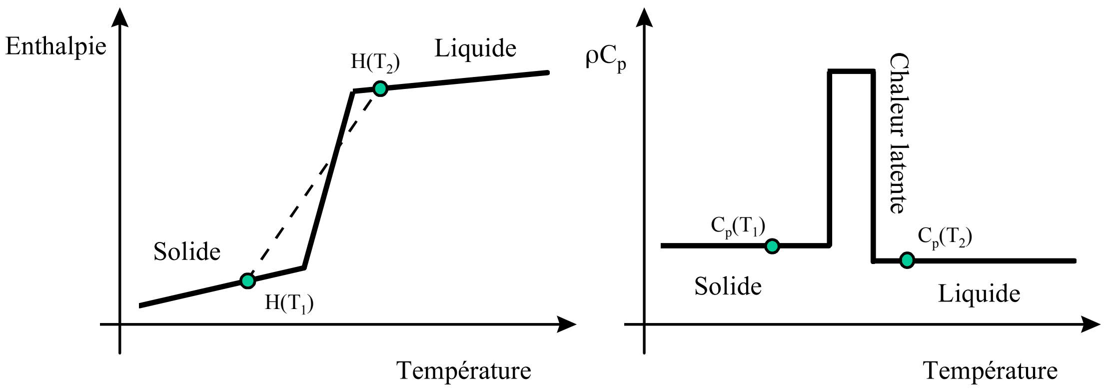
$$r_{vol}-\operatorname{div}(-\lambda(T)\nabla T)=\rho C_p\frac{dT}{dt}$$ **éq 1.3-2**

1.4 Digital advantage of the formulation in enthalpy for the problems with phase shift.

The relation between enthalpy and heat-storage capacity is:

$$\beta(T)=\int_{T_0}^{T}\rho C_p(u)du$$

When this function enthalpy presents abrupt variations, it is more precise to handle $\beta(T)$ that its derivative. Indeed, the paces characteristic of these functions in the vicinity of the melting point are the following ones:



During an iteration, either because the thermal transient is violent, or because the beach of phase shift is very small (pure substance), two the reiterated successive ones of the temperature can be located on both sides of discontinuity. The evaluation of the slope of the function enthalpy in the vicinity of the melting point will be very false if one considers $C_p(T_1)$, $C_p(T_2)$ or a weighted average of both. On the other hand, the slope of the right-hand side in dotted lines is always a correct approximation of $d\beta/dT$ at the melting point.

2 Boundary conditions, loading and initial condition

One will refer to [R5.02.01] for the boundary conditions thermal and the loadings leading to linear equations in temperature like for the initial condition.

2.1 Non-linear normal flow

It is of the conditions of the Neumann type, defining flow entering the field.

$$-q(x,t).\boldsymbol{n}=g(x,T) \quad \text{on the border } \Gamma \qquad \textbf{éq 2.1-1}$$

where $g(x,T)$ is a function of the temperature and possibly of the variable of space x and/or of time t and $\boldsymbol{n}$ indicate the normal external with the border Γ, q is the vector heat flow (directed according to the decreasing temperatures).

This expression makes it possible to introduce for example conditions of the type exchanges with a coefficient of convectif exchange depend on the temperature:

$$-q(x,t).\boldsymbol{n}=g(x,T)=h(x,T)(T_{ext}(x,t)-T) \qquad \textbf{éq 2.1-2}$$

2.2 Non-linear normal flow - condition of type radiation ad infinitum

A typical case of the boundary conditions preceding is the radiation ad infinitum of gray body which results in a typical case of function $g(x,T)$:

$$-q(x,t).\boldsymbol{n}=\sigma\epsilon\left[(T(x)+273.15)^4-(T_\infty+273.15)^4\right] \qquad \textbf{éq 2.2-1}$$

The characteristics to be defined at the time of the definition of this loading are emissivity ε, the constant of Stefan-Boltzmann $\sigma=5{,}73\,.10^{-8}usi$ and the temperature ad infinitum.

$T(r)$ and T_∞ are then expressed in degrees Celsius. $-273.15\,°C$ is the temperature of the absolute zero.

3 Variational formulation of the problem

We will restrict ourselves here to present the problem with only the boundary conditions of imposed temperature [R5.02.01 §2.1], of imposed normal flow [R5.02.01 §2.3], of exchange [R5.02.01 §2.4], nonlinear flow [§2.1] and radiation [§2.2].

That is to say Ω open of R^3, of border $\Gamma=\Gamma_1\cup\Gamma_2\cup\Gamma_3\cup\Gamma_4\cup\Gamma_5$.
One must solve the equation [éq 1.1-4] in T on $\Omega\times]0,t[$ with the boundary conditions:

$$\begin{cases} T=T^d(r,t) & \text{sur } \Gamma_1 \\ \lambda(T)\dfrac{\partial T}{\partial n} = f(r,t) & \text{sur } \Gamma_2 \\ \lambda(T)\dfrac{\partial T}{\partial n} = h(r,t)(T_{ext}(r,t)-T) & \text{sur } \Gamma_3 \\ \lambda(T)\dfrac{\partial T}{\partial n} = g(r,T) & \text{sur } \Gamma_4 \\ \lambda(T)\dfrac{\partial T}{\partial n} = \sigma\epsilon\left[(T+273.15)^4-(T_\infty+273.15)^4\right] & \text{sur } \Gamma_5 \end{cases} \quad \text{éq 3-1}$$

and with, possibly, of the initial conditions $T(t=0)$. If these last are not specified, one résoud as a preliminary the stationary problem, i.e. the equation [éq 1.3-1] without the term of temporal evolution.

That is to say v a sufficiently regular function cancelling itself on Γ_1 , while noticing:

$$\begin{aligned} &\frac{d}{dt}\left(\int_\Omega \beta(T).v.d\Omega\right)=\int_\Omega \dot\beta(T).v.d\Omega \\ &\int_\Omega \lambda(T)\nabla T.\nabla v.d\Omega = -\int_\Omega div(\lambda(T)\nabla T).v.d\Omega+\int_\Gamma \lambda(T)\frac{\partial T}{\partial n}.v.d\Gamma \end{aligned} \quad \text{éq 3-2}$$

the weak formulation of the equation of heat can then be written:

$$\frac{d}{dt}\left(\int_\Omega \beta(T).v.d\Omega\right)+\int_\Omega \lambda(T)\nabla T.\nabla v d\Omega-\int_\Gamma \lambda(T)\frac{\partial T}{\partial n}.v.d\Gamma=\int_\Omega r_{vol}.v.d\Omega \quad \text{éq 3-3}$$

One from of deduced the variational formulation from the problem:

$$\int_{\Omega} \frac{d\beta(T)}{dt}.v.d\Omega+\int_{\Omega}\lambda(T)\nabla T.\nabla v.d\Omega+\int_{\Gamma_3} hT.v.d\Gamma_3=$$
$$\int_{\Omega} r_{vol}.v.d\Omega+\int_{\Gamma_2} f.v.d\Gamma_2+\int_{\Gamma_3} hT_{ext}.vd\Gamma_3+ \qquad \text{éq 3-4}$$
$$\int_{\Gamma_4} g.v.d\Gamma_4+\int_{\Gamma_5}\sigma\epsilon.\left[(T+273.15)^4-(T_{\infty}+273.15)^4\right].v.d\Gamma_5$$

4 Discretization in time of the differential equation

4.1 Introduction of Θ - method

A classical way to discretize a first order differential equation is Θ - method. Let us consider the following differential equation:

$$\begin{cases} \dot{y}(t)=\phi(t,y(t)) \\ y(0)=y_0 \end{cases}$$ **éq 4.1-1**

Θ - method consists in discretizing the equation [éq 4.1-1] by a diagram with the finished differences

$$\frac{1}{\Delta t}(y_{n+1}-y_n)=\theta.\phi(t_{n+1},y_{n+1})+(1-\theta).\phi(t_n,y_n)$$ **éq 4.1-2**

where y_{n+1} is an approximation of $y(t_{n+1})$, y_n being supposed known and θ is the parameter of the method, $\theta\in[0,1]$.

Note:

if $\theta=0$ *the diagram is known as explicit,*
if $\theta\neq 0$ *the diagram is known as implicit.*

4.2 Application to the equation of heat

Let us use Θ - method in the variational formulation of the equation of heat, where one posed:

$$\begin{array}{llll} T^+=T(r,t+\Delta t) & T^-=T(r,t) & h^+=h(r,t+\Delta t) & h^-=h(r,t) \\ f^+=f(r,t+\Delta t) & f^-=f(r,t) & T^+_{ext}=T_{ext}(r,t+\Delta t) & T^-_{ext}=T_{ext}(r,t) \\ r^+_{vol}=r_{vol}(r,t+\Delta t) & r^-_{vol}=r_{vol}(r,t) & T^{d+}=T^d(r,t+\Delta t) & T^{d-}=T^d(r,t) \\ g^+=g(r,T^+) & g^-=g(r,T^-) & & \end{array}$$

where $T^d(r,t)$ represent the temperature imposed on the border of the field, according to time and of space.

Let us introduce following spaces of functions:

$$V_{t^+}=\left\{v\in H^1(\Omega)\, v_{/\Gamma_1}=T^{d+}\right\}$$
$$V_{t^-}=\left\{v\in H^1(\Omega)\, v_{/\Gamma_1}=T^{d-}\right\}$$
$$V_0=\left\{v\in H^1(\Omega)\, v_{/\Gamma_1}=0\right\}$$

The field $T^-\in V_{t^-}$ being supposed known, one seeks $T^+\in V_{t^+}$ such as $\forall\, v\in V_0$:

$$\begin{aligned}
&\int_\Omega \frac{\beta(T^+)-\beta(T^-)}{\Delta t} v.d\Omega && +\int_\Omega (\theta\lambda(T^+)\nabla T^+.\nabla v+(1-\theta)\lambda(T^-)\nabla T^-.\nabla v)d\Omega \\
&-\int_{\Gamma_2}(\theta f^+ +(1-\theta)f^-)v.d\Gamma_2 && -\int_{\Gamma_3}(\theta h^+ T^+_{ext}+(1-\theta)h^- T^-_{ext}-\theta h^+ T^+ -(1-\theta)h^- T^-)v.d\Gamma_3 \\
&-\int_{\Gamma_4}(\theta g^+ +(1-\theta)g^-)v.d\Gamma_4 && = \\
&\int_\Omega (\theta r^+_{vol}+(1-\theta)r^-_{vol})v.d\Omega && +\int_\Omega (\theta r_v(T^+)+(1-\theta)r_v(T^-))v.d\Omega
\end{aligned}$$

éq 4.2-1

Not to excessively weigh down the writing and insofar as the process is identical to the other terms, one did not make be reproduced the term of radiation in these equations (integral on Γ_5).

While posing:

$$\begin{aligned}
&(hT_{ext})^\theta=\theta h^+ T^+_{ext}+(1-\theta)h^- T^-_{ext}\\
&f^\theta=\theta f^+ +(1-\theta)f^-\\
&r^\theta=\theta r^+_{vol}+(1-\theta)r^-_{vol}
\end{aligned}$$

one obtains finally:

$$\begin{aligned}
&\int_\Omega \frac{\beta(T^+)}{\Delta t}v.d\Omega+\theta\int_\Omega \lambda(T^+)\nabla T^+.\nabla v.\,d\Omega+\theta\int_{\Gamma_3} h^+ T^+ v.d\Gamma_3\\
&-\theta\int_{\Gamma_4} g(T^+).v.d\Gamma_4-\theta\int_\Omega r_v(T^+).v.d\Omega=L_1(v,T^-)\\
&\forall v\in V_0
\end{aligned}$$

éq 4.2-2

where one posed:

$$\begin{aligned}
L_1(v,T^-)=&\int_\Omega \frac{\beta(T^-)}{\Delta t}v.d\Omega-\int_\Omega (1-\theta)\lambda(T^-)\nabla T^-.\nabla v.d\Omega+\int_{\Gamma_2} f^\theta v.d\Gamma_2\\
&+\int_{\Gamma_3}((hT_{ext})^\theta-(1-\theta)h^- T^-)v.d\Gamma_3+\int_\Omega r^\theta v.d\Omega\\
&+(1-\theta)\int_{\Gamma_4} g(T^-)v.d\Gamma_4+(1-\theta)\int_\Omega r_v(T^-)v.d\Omega
\end{aligned}$$

éq 4.2-3

At one moment of calculation given, this term is known. Indeed, only the temperature at the previous moment, T^- , as well as the values at the moment running of function *known* time, intervenes.

If the distribution of temperature in the system at the initial moment is not provided, the stationary problem is solved. The terms of evolution disappear, $\Theta=1$; the field of temperature at the initial moment is given by:

$$\begin{aligned}
&\int_\Omega \lambda(T^{t=0})\nabla T^{t=0}.\nabla v.d\Omega+\int_{\Gamma_3} h^{t=0}T^{t=0}v.d\Gamma_3-\int_{\Gamma_4} g(T^{t=0})v.d\Gamma_4\\
&=\int_{\Gamma_2} f^{t=0}v.d\Gamma_2+\int_{\Gamma_3} h^{t=0}T^{t=0}_{ext}v.d\Gamma_3+\int_\Omega r^{t=0}v.d\Omega\\
&\forall v\in V_0
\end{aligned}$$

éq 4.2-4

The problem is written finally in the condensed form:

$$\begin{cases} Soit\ T^- \in V_{t^-}\ connu\ ,trouver\ T^+ \in V_{t^+}\ tel\ que \\ \forall v \in V_0 : a(v, T^+) = L_1(v, T^-) \end{cases}$$ **éq 4.2-5**

5 Space discretization and adaptation of the algorithm of Newton to the problem

The principle of the method of Newton is very detailed in [R5.03.01], one will expose here only the adaptations specific to the nonlinear algorithm of thermics.

5.1 Space discretization

That is to say P_h a space division Ω , let us indicate by N the number of nodes of the grid, p_i the function of form associated with the node i . One indicates by J the whole of the nodes belonging to the border G_1.

Are:

$$V^h_{t^+} = \left\{ v = \sum_{i=1,N} v_i\, p_i(x)\ ;\ v_j = T^d(x_j, t^+) \quad j \in J \right\}$$
$$V^h_{t^-} = \left\{ v = \sum_{i=1,N} v_i\, p_i(x)\ ;\ v_j = T^d(x_j, t^-) \quad j \in J \right\}$$
$$V^h_0 = \left\{ v = \sum_{i=1,N} v_i\, p_i(x)\ ;\ v_j = 0 \quad j \in J \right\}$$ **éq 5.1-1**

The problem [éq 4.2-5] can be replaced by the problem discretized with many unknown factors finished according to:

That is to say $T^- \in V^h_t$ known, to find $T^+ \in V^h_{t^+}$ such as

$v_h \in V^h_0 a(v_h, T^+) = L_1(v_h, T^-)$

éq 5.1-2

that one can also write, with the same formalism as `STAT_NON_LINE` [R5.03.01], in vectorial form:

$v^T R(T^+, t^+) = v^T L(T^-, t^+) \quad \forall v$ such as $Bv = 0$

$BT^+ = T^d(t^+)$

éq 5.1-3

where the operator $\boldsymbol{B}$ express the boundary condition of imposed temperature $T^+ \in V^h_{t^+}$. It is defined by:

$$(\mathbf{Bv})_j = \begin{cases} 0\ si\ j \notin J \\ v_j\ si\ j \in J \end{cases}$$ **éq 5.1-4**

The case where the application R is linear is treated by the order `THER_LINEAIRE` [R5.02.01].

The dualisation of the boundary conditions, detailed in [R3.03.01], led to the nonlinear problem in T^+ :

$$\begin{cases} R(T^+, t^+) + B^T \lambda^+ = L(T^-, t^+) \\ BT^+ = T^d(t^+) \end{cases} \quad \textbf{éq 5.1-5}$$

The unknown factors are the couple (T^+, λ^+), where λ^+ represent the "multipliers of Lagrange" of the boundary conditions of Dirichlet.

To solve the system [éq 5.1-5] amounts cancelling in (T_i^+, λ_i^+) the vector $F(T^+, \lambda^+)$, called residue, defined by:

$$F(T^+, \lambda^+) = \begin{pmatrix} L(T^-, t^+) - R(T^+, t^+) - B^T \lambda^+ \\ T^d(t^+) - BT^+ \end{pmatrix} \quad \textbf{éq 5.1-6}$$

The method of Newton consists in building a vector series $\{x^n\}_n$ converging towards the solution of $F(x)=0$ using the tangent linear application of F .

5.2 Stationary calculation

The variational problem is that of the equation [éq 4.2-4]. To note: in stationary calculation, the enthalpy does not intervene in the application R .

One introduces the matrix of the tangent linear application of the function $R(T^n)$:

$$K^n = \frac{\partial R}{\partial T}\Big|_{T_n}$$

That of the function $F(T^n, \lambda^n)$ is then:

$$\begin{bmatrix} K^n & B^T \\ B & 0 \end{bmatrix}$$

In the case of stationary calculation, one must reiterate starting from a value of initialization of the field of temperature which is given by the keyword `ETAT_INIT`. The first iteration of calculation, known as iteration of prediction, consists in solving the following system:

$$\begin{bmatrix} K(T_0) & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} T_1 - T_0 \\ \lambda_1 - \lambda_0 \end{bmatrix} = \begin{bmatrix} L - R(T_0) - B^T \lambda_0 \\ T^d - BT_0 \end{bmatrix} \quad \textbf{éq 5.2-1}$$

As one can see it in the equation of the stationary problem [éq 4.2-4], the temperature does not appear to the second member: one writes L and not $L(T_0)$.

If the problem is linear, $R(T_0) = K(T_0) T_0 = K.T_0$. All terms in T_0 disappear by simplification. The solution is obtained in an iteration by inversion of a system identical to that described in [R5.02.01 §6].

The following iterations are iterations of Newton, with reactualization or not of the tangent matrix K .

$$\begin{bmatrix} K(T_{(i)}) & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} T_{i+1} - T_i \\ \lambda_{i+1} - \lambda_i \end{bmatrix} = \begin{bmatrix} L - R(T_i) - B^T \lambda_i \\ 0 \end{bmatrix} \quad \text{éq 5.2-2}$$

For the iteration of prediction, the writing of the lower subsystem of the equation [éq 5.2-1], after simplification, ensures us that $BT_1 = T^d$. The reiterated first and all the following thus check the conditions of Dirichlet.

Brackets around the index of iteration in the expression $K(T_{(i)})$ mean that one can reactualize or not the tangent matrix with the wire of the iterations.

Note:

The temperature of initialization T_0 has of influence only for one nonlinear stationary calculation. While being of about size of the expected temperatures, it would make it possible “to leave” less far from the solution that a null field everywhere; and thus the iteration count would decrease.

5.3 Transitory calculation

For the first iteration of the step of time, known as iteration of prediction, one “makes as if” the problem describes in [éq 5.1-5] were linear. This formulation must make it possible to directly obtain the solution to a linear problem of thermics. But here, the situation is a little different from stationary calculation because of the formulation in enthalpy. The linearization of [éq 5.1-5] gives:

$$\begin{cases} R(T^-, t^+) + K(T^-, t^+)(T^+ - T^-) + B^T \lambda^+ = L(T^-, t^+) \\ BT^+ = T^d(t^+) \end{cases} \quad \text{éq 5.3-1}$$

What amounts solving, for the problem presented in matric form:

$$\begin{bmatrix} K(T^-) & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} T_1^+ \\ \lambda_1^+ \end{bmatrix} = \begin{bmatrix} L(T^-, t^+) + K(T^-)T^- - R(T^-) \\ T^d(t^+) \end{bmatrix} \quad \text{éq 5.3-2}$$

The function enthalpy is known with a constant of integration close which appears in the relation flexible $R(T^-)$ with $K(T^-)T^-$. This same constant is found in the expression of $L(T^-, t^+)$. One can then eliminate it while leading to the system from equations according to:

$$\begin{bmatrix} K(T^-) & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} T_1^+ \\ \lambda_1^+ \end{bmatrix} = \begin{bmatrix} \tilde{L}(T^-, t^+) \\ T^d(t^+) \end{bmatrix} \quad \text{éq 5.3-3}$$

where $\tilde{L}(T^-, t^+)$ is the second member calculated with the heat-storage capacity and not the enthalpy (option `CHAR_THER_EVOLNI` [§6.2]).

Lastly, as for the stationary case seen in the preceding chapter, the following iterations are iterations of Newton:

$$\begin{bmatrix} K(T_{(i)}, t^+) & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} T_{i+1}^+ - T_i^+ \\ \lambda_{i+1}^+ - \lambda_i^+ \end{bmatrix} = \begin{bmatrix} L(T^-, t^+) - R(T_i, t^+) - B^T \lambda_i \\ 0 \end{bmatrix} \quad \text{éq 5.3-4}$$

This time, on the other hand, $L(T^-,t^+)$ is calculated with the enthalpy and not the heat-storage capacity to be coherent with $R(T_i^+)$.

5.4 Convergence

Since time intervenes in the form of the tangent matrix, and also the step of time, one prefers systematically to bring up to date this one at the beginning of each step not to degrade the speed of convergence too much. On the other hand, freedom is left to the user control his frequency of calculation during a step of time.

With each iteration, one can carry out the search for an optimum step of progression towards the solution by some iterations (2 or 3) of linear research. This method is described in detail in [R5.03.01].

Calculation famous is converged when the vector residue is null [éq 5.1-6]:

$$F(T_i^+,\lambda_i^+,t^+)=\begin{pmatrix} L(T^-,t^+)-R(T_i^+,t^+)-B^T\lambda_i^+ \\ T^d(t^+)-BT_i^+ \end{pmatrix} \quad \text{éq 5.4-1}$$

The lower part of the vector is always worthless (conditions of Dirichlet). One thus checks:

$$\frac{\|L(T^-,t^+)-R(T_i^+,t^+)-B^T\lambda_i^+\|_2}{\|L(T^-,t^+)-B^T\lambda_i^+\|_2}\leq\epsilon \quad \text{éq 5.4-2}$$

The user also has the possibility of stopping the iterations on an absolute criterion:

$$\|L(T^-,t^+)-R(T_i^+,t^+)-B^T\lambda_i^+\|_\infty\leq\epsilon \quad \text{éq 5.4-3}$$

6 Principal options of non-linear thermics calculated in *Code_Aster*

6.1 Boundary conditions and loadings

One will refer to [R5.02.01] for the boundary conditions and the loadings linear.

Nonlinear flow	CHAR_THER_FLUNL	$\int_{\Gamma_4}(1-\theta)g(T^-)v.d\Gamma_4$
Radiation	CHAR_THER_RAYO_R CHAR_THER_RAYO_F	$\int_{\Gamma_4}\sigma\epsilon\left[(T+273.15)^4-(1-\theta)(T^-+273.15)^4\right].v.d\Gamma_4$
Non-linear source	CHAR_THER_SOURNL	$\int_{\Omega}(1-\theta)r_v(T^-)v.d\Omega$

6.2 Calculation of the elementary matrices and transitory term

Thermal inertia, conductivity	MTAN_RIGI_MASS	$\int_{\Omega}\frac{\rho\ Cp}{\Delta t}v.v.d\Omega+\int_{\Omega}\theta\lambda(T^+)\nabla v.\nabla v.d\Omega$
Radiation	MTAN_THER_RAYO_R MTAN_THER_RAYO_F	$\int_{\Gamma_4}\theta.4.\sigma.\epsilon(T^++273.15)^3 v.v.d\Gamma_4$
Coefficient of exchange	MTAN_THER_COEF_R MTAN_THER_COEF_F	$\int_{\Gamma_4}\theta h.v.vd\Gamma_4$
Nonlinear flow	MTAN_THER_FLUXNL	$-\int_{\Gamma_4}\theta\frac{dg}{dT}(T^+)v.v.d\Gamma_4$
Nonlinear source	MTAN_THER_SOURNL	$-\int_{\Omega}\theta\frac{d r_v}{dT}(T^+)v.v.d\Omega$
Transitory term	CHAR_THER_EVOLNI	$\int_{\Omega}\frac{\beta(T^-)}{\Delta t}.v.d\Omega-\int_{\Omega}(1-\theta)\lambda(T^-)\nabla T^-.\nabla v.d\Omega$
		$\int_{\Omega}\frac{\rho\ CpT^-}{\Delta t}.v.d\Omega-\int_{\Omega}(1-\theta)\lambda(T^-)\nabla T^-.\nabla v.d\Omega$

6.3 Calculation of the residue

	RESI_RIGI_MASS	$\int_{\Omega}\frac{1}{\Delta t}\beta(T^i)v.d\Omega+\int_{\Omega}\theta\lambda(T^i)\nabla T^i.\nabla v.d\Omega$
Radiation	RESI_THER_RAYO_R RESI_THER_RAYO_F	$\int_{\Gamma_4}\theta\sigma\epsilon(T^i+273.15)^4 v.d\Gamma_4$

Coefficient of exchange	RESI_THER_COEF_R RESI_THER_COEF_F	$\int_{\Gamma_3} (\theta h^+ T^i) v.d\Gamma_3$
Nonlinear flow	RESI_THER_FLUXNL	$-\int_{\Gamma_3} \theta g(T^i) v.d\Gamma_3$
Nonlinear source	RESI_THER_SOURNL	$-\int_{\Omega} \theta r_v(T^i) v.d\Omega$

7 Bibliography

- SALENCON. Mechanics of the continuous mediums. Ellipses. 1988.
- RUUP, PENIGUEL. SYRTHES - Conduction and radiation, theoretical Handbook of version 3.1. HE-41/98/048/A

8 History of the versions of the document

Version Aster	Author (S) or contributor (S), organization	Description of the modifications
5	C. Durand EDF R & D MN	
13	M.Abbas	Possibility of using characteristic materials for a stationary calculation at a temperature different from zero (card 22397)