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Nonlinear relations of behavior 1D

Summary:

This document describes the quantities calculated by the operator STAT_NON_LINE necessary to the implementation of the quasi static nonlinear algorithm describes in [R5.03.01] in the case of the elastoplastic or viscoplastic behaviors monodimensional. These behaviors, except contrary mention, are applicable to the elements of BAR, with the elements of beam and multifibre beams (direction axial only) and with the elements of concrete reinforcement (modeling GRID).

The behaviors described in this document are:
- the behavior of Von Mises with linear isotropic work hardening: VMIS_ISOT_LINE, and unspecified VMIS_ISOT_TRAC,
- the behavior of Von Mises with linear kinematic work hardening: VMIS_CINE_LINE,
- the behavior of Von Mises with linear, nonsymmetrical work hardening in traction and compression: with restoration of the center of the elastic range: VMIS_ASYM_LINE. This last was developed to model the action of the ground on the Cables with Gas Insulation,
- the behavior of PINTO-MENEGOTTO who allows to represent the uniaxial elastoplastic behavior of the reinforcements of the reinforced concrete. This model translates for it not linearity of the work hardening of the bars under cyclic loading and takes into account the Bauschinger effect. It makes it possible of more than simulate the buckling of the reinforcements in compression. This relation is available in *Code_Aster* for the elements of bar and the elements of grid,
- viscoplastic behaviors with effect of the irradiation: VISC_IRRA_LOG, GRAN_IRRA_LOG.
- the behavior of MAZARS in its version $1D$. The version $1D$ model of MAZARS allows to give an account of the restoration of rigidity in the event of refermeture of the cracks.
- the behavior to model the relieving of the prestressed cables.

The resolution is made case by a method of integration implicit, except contrary mention, as from the moment of preceding calculation, one calculates the stress field resulting from an increment of deformation, and the tangent behavior which makes it possible to build the tangent matrices.

One describes finally a method, similar to the method due to R.de Borst [R5.03.03] allowing to use all the behaviors available in 3D in the elements 1D.

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1 Use of the relations of behavior 1D

1.1 Relations of behavior 1D in *Code_Aster*

The relations treated in this document are:

VMIS_ISOT_LINE	Von Mises with symmetrical linear isotropic work hardening
VMIS_ISOT_TRAC	Von Mises with unspecified isotropic work hardening
VMIS_CINE_LINE	Von Mises with symmetrical linear kinematic work hardening.
ECRO_CINE_1D	Von Mises with symmetrical linear kinematic work hardening.
GRILLE_ISOT_LINE	Von Mises with symmetrical linear isotropic work hardening
GRILLE_CINE_LINE	Von Mises with symmetrical linear kinematic work hardening
PINTO_MENEGOTTO	Behavior of the reinforced concrete reinforcements
GRILLE_PINTO_MEN	Behavior of the reinforced concrete reinforcements
VMIS_ASYM_LINE	Von Mises with asymmetrical linear work hardening and restoration
VISC_IRRA_LOG, GRAN_IRRA_LOG	Viscoplastic behaviour of the fuel assemblies: Models resulting from the tests REFLECTION and FLETANR
MAZARS	Behavior of MAZARS in its version $1D$.
RELAX_ACIER	Behavior to model the relieving of the prestressed cables.

These relations of behavior (incremental) are given in the operator STAT_NON_LINE [U4.51.03] under the keyword factor BEHAVIOR, by the keyword RELATION [U4.51.03]. They are valid only in small deformations. N describes for each relation of behavior the calculation of the stress field starting from an increment of deformation given (cf algorithm of Newton [R5.03.01]), the calculation of the forces **nodal** R and of the tangent matrix K_i^n.

1.2 General notations

All the quantities evaluated at the previous moment are subscripted by $-$.

Quantities evaluated at the moment $t+\Delta t$ are not subscripted.

The increments are indicated by Δ. One has as follows:

$$\mathbf{Q}=\mathbf{Q}(t+\Delta t)=\mathbf{Q}^-(t)+\Delta \mathbf{Q}=\mathbf{Q}^- + \Delta \mathbf{Q}$$

σ	tensor of the constraints (in 1D, one is interested only in the single uniaxial nonworthless component).
$\tilde{\sigma}$	deviative operator: $\tilde{\sigma}_{ij}=\sigma_{ij}-\frac{1}{3}\sigma_{kk}\delta_{ij}$.
$(\ )eq$	equivalent value of Von Mises, equalizes in 1D with the absolute value
$\Delta \varepsilon$	increment of deformation.
A	tensor of elasticity, equal in 1D to the Young modulus E
λ, μ, E, K	moduli of the isotropic elasticity.
α	thermal dilation coefficient secant.
T	temperature.
$(\ )_+$	positive part.
P	cumulated plastic deformation
ε^p	plastic deformation

1.3 Change of variables

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Whatever the type of finite element which refers to a law of behavior 1D, it is necessary to carry out a change of variables to pass from the elementary quantities (efforts, displacements) to the constraints and deformations.

1.3.1 Calculation of the deformations (small deformations)

For each finite element of *Code_Aster*, in STAT_NON_LINE, the total algorithm (Newton) provides to the elementary routine, which integrates the behavior, an increase in the field of displacement.
For the elements of bar, one calculates the deformation (only one axial component) by:

$$\varepsilon = \frac{u(l) - u(0)}{l} \, ,$$

and the increase in deformation by:

$$\Delta \varepsilon = \frac{\Delta u(l) - \Delta u(0)}{l} \, ,$$

For the elements of grid (modelings GRID and GRILLE_MEMBRANE), one calculates the membrane deformation as for the elements of hulls DKT. Simply, only one direction corresponds physically to the directions of reinforcements. One thus finds oneself in the presence of a behavior 1D.
In addition, in small deformations, for all the models described in this document, one writes for any moment the partition of the deformations in the form of an elastic contribution, thermal dilation, and plastic deformation:

$$\varepsilon(t) = \varepsilon^{e}(t) + \varepsilon^{th}(t) + \varepsilon^{p}(t) \text{ with}$$

$$\varepsilon^{e}(t) = \mathbf{A}^{-1}(T(t))\sigma(t) = \frac{1}{E(T)}\sigma(t)$$

$$\varepsilon^{th}(t) = \alpha(T(t))(T(t) - T_{ref})\mathbf{Id}$$

1.3.2 Calculation of the generalized efforts (forced integrated)

For integration of the behavior 1D, it is necessary to integrate the component of constraints obtained, to provide to the total algorithm (Newton) a vector containing the generalized efforts.
For the elements of bar, one calculates the effort (uniform in the element, by supposing that the section is constant) by: $N = S\sigma$,

and the vector forces nodal equivalent (as for the elements of beam, [R3.08.01]) by:

$$F = \begin{bmatrix} -N \\ N \end{bmatrix}$$

For the elements of GRID , one calculates the efforts as for the elements of hulls DKT (membrane efforts) by integration of the constraints in the thickness (only one sleeps and only one point of integration).

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2 Behavior of Von-Put at linear isotropic work hardening: VMIS_ISOT_LINE or VMIS_ISOT_TRAC

2.1 Equations of the model VMIS_ISOT_LINE

They are the restriction of the behavior 3D [R5.03.02] on the uniaxial case:

$$
\begin{cases}
\dot{\bar{\varepsilon}}^p = \dfrac{3}{2}\, \bar{\dot{p}}\, \dfrac{\tilde{\sigma}}{\sigma_{eq}} = \bar{\dot{p}}\, \dfrac{\sigma}{|\sigma|} \\[2ex]
\dfrac{\sigma}{E} = \varepsilon - \varepsilon^p - \varepsilon^{th} \\[2ex]
\sigma_{eq} - R(p) = |\sigma| - R(p) \leq 0 \\[2ex]
\bar{\dot{p}} = 0 \; si \; \sigma_{eq} - R(p) < 0 \\[1ex]
\bar{\dot{p}} \geq 0 \; si \; \sigma_{eq} - R(p) = 0
\end{cases}
$$

with:

- $\dot{\bar{\varepsilon}}^p$ speed of plastic deformation,
- p cumulated plastic deformation,
- $\varepsilon^{th} = \alpha\left(T - T_{ref}\right)$ thermal deformation,
- $R(p) = \dfrac{E\,E_T}{E - E_T}\, p + \sigma_y$ function of linear work hardening isotropic, or $R(p)$ well refine per pieces, deduced from the traction diagram.

In the case VMIS_ISOT_LINE, the data of the material characteristics are those provided under the keyword factor ECRO_LINE or ECRO_LINE_FO of the operator DEFI_MATERIAU [U4.43.01].

/ ECRO_LINE = (D_SIGM_EPSI = E_T , SY = σ_y)

/ ECRO_LINE_FO = (D_SIGM_EPSI = E_T , SY = σ_y)

In the case VMIS_ISOT_TRAC, the data of the characteristics of materials are provided under the keyword factor TRACTION of the operator DEFI_MATERIAU [U4.43.01].

TRACTION = _F (SIGM = courbe_traction)

courbe_traction represent the traction diagram, point by point. The first point makes it possible to define the elastic limit σ_y and it Young modulus E [R5.03.02].
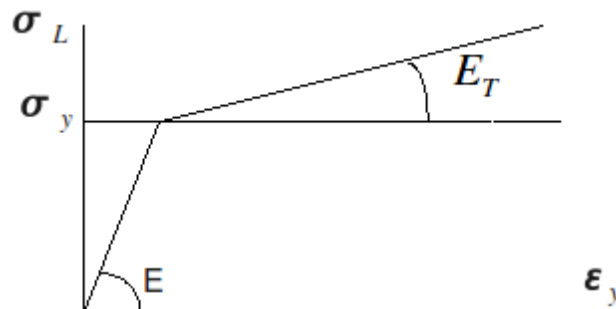
ECRO_LINE_FO corresponds if E_T and σ_y depend on the temperature and are then calculated for the temperature of the point of current Gauss. The Young modulus E and the Poisson's ratio v are

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those provided under the keywords factors `ELAS` or `ELAS_FO`. In this case the traction diagram is the following one:

$$\begin{cases} \sigma_L = E\,\varepsilon_L & si\,\varepsilon_L < \dfrac{\sigma_y}{E} \\[2mm] \sigma_L = \sigma_y + E_T\left(\varepsilon_L - \dfrac{\sigma_y}{E}\right) & si\,\varepsilon_L \geq \dfrac{\sigma_y}{E} \end{cases}$$

When the criterion is reached one a:

$$\sigma_L - R(p) = 0 \text{ , therefore } \sigma_L - R\left(\varepsilon_L - \frac{\sigma_L}{E}\right) = 0 \text{ from where:}$$

$$R(p) = \frac{E_T E}{E - E_T} p + \sigma_y = H\,p + \sigma_y$$

In the case of a traction diagram, the approach is identical to [R5.03.01].

2.2 Integration of the relation `VMIS_ISOT_LINE`

By direct implicit discretization of the relations of behavior, in a way similar to integration 3D [R5.03.02] one obtains :

$$\begin{cases} \left|\sigma^- + \Delta\sigma\right| - R(p^- + \Delta p) \leq 0 \\[2mm] E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right) - \left(\sigma^- + \Delta\sigma\right) + \dfrac{E}{E^-}\sigma^- = E\Delta p\dfrac{\sigma^- + \Delta\sigma}{\left|\sigma^- + \Delta\sigma\right|} \\[2mm] \Delta p \geq 0 \;\; si \;\; \left|\sigma^- + \Delta\sigma\right| = R(p^- + \Delta p) \\[2mm] \Delta p = 0 \;\; si \;\; \left|\sigma^- + \Delta\sigma\right| < R(p^- + \Delta p) \end{cases}$$

Two cases arise:

- $\left|\sigma^- + \Delta\sigma\right| < R(p^- + \Delta p)$

  in this case $\Delta p = 0$ that is to say $\sigma = E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right) + \dfrac{E}{E^-}\sigma^-$

  thus $\left|\sigma^- \dfrac{E}{E^-} + E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right)\right| < R(p^-)$

- $\left|\sigma^- + \Delta\sigma\right| = R(p^- + \Delta p)$

  in this case $\Delta p \geq 0$

  thus $\left|\dfrac{\sigma^- E}{E^-} + E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right)\right| \geq R(p^-)$.

One from of deduced the algorithm from resolution:

let us pose $\sigma^e = \dfrac{E\,\sigma^-}{E^-} + E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right)$

if $\left|\sigma^e\right| \leq R(p^-)$ then $\Delta p = 0$ and $\Delta\sigma = E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right)$

if $\left|\sigma^e\right| > R(p^-)$ then it is necessary to solve:

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$$\sigma^e = \sigma^- + \Delta\sigma + E\Delta p \frac{\sigma^- + \Delta\sigma}{|\sigma^- + \Delta\sigma|}$$

$$\sigma^e = \left(1 + \frac{E\Delta p}{|\sigma^- + \Delta\sigma|}\right)(\sigma^- + \Delta\sigma)$$

thus by taking the absolute value :

$$|\sigma^e| = \left(1 + \frac{E\Delta p}{|\sigma^- + \Delta\sigma|}\right)(\sigma^- + \Delta\sigma)$$

maybe, while using $\begin{array}{l}|\sigma^- + \Delta\sigma| = R(p^- + \Delta p) \\ |\sigma^e| = R(p^- + \Delta p) + E\Delta p\end{array}$.

One from of thus deduced:

- in the case of a linear work hardening: $\Delta p = \dfrac{|\sigma^e| - (\sigma_y + Hp^-)}{E + H}$

- and in the case of an unspecified work hardening, the curve $R(p)$ being refined per pieces, one solves the equation directly in Δp : in the same way $E\Delta p + R(p^- + \Delta p) = |\sigma^e|$ that in 3D [R5.03.02].

Let us notice on the way that: $\dfrac{\sigma^e}{|\sigma^e|} = \dfrac{\sigma}{R(p)}$

then $\sigma = (\sigma^- + \Delta\sigma) = \dfrac{\sigma^e}{|\sigma^e|} R(p) = \dfrac{\sigma^e}{1 + \dfrac{E\Delta p}{R(p)}}$

Moreover, the option `FULL_MECA` allows to calculate the tangent matrix $\mathbf{K}_i^n$ with each iteration. The tangent operator who is used for building it is calculated directly on the preceding discretized system. One obtains directly:

if $|\sigma^e| > R(p^-)$ $\dfrac{\delta\sigma}{\delta\varepsilon} = E_T$

if not $\dfrac{\delta\sigma}{\delta\varepsilon} = E$

Note:

The option `RIGI_MECA_TANG` who allows to calculate the tangent matrix $\mathbf{K}_i^0$ used in the phase of prediction of the algorithm of Newton, account of the indicator of plasticity takes at the previous moment:

if $\chi = 1$ $\dfrac{\delta\sigma}{\delta\varepsilon} = E_T$ if $\chi = 0$ $\dfrac{\delta\sigma}{\delta\varepsilon} = E$

2.3 Internal variables

The relation of behavior `VMIS_ISOT_LINE` product two internal variables : p and χ

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3 Behaviour of Von Mises, linear kinematic work hardening 1D: `VMIS_CINE_LINE`

3.1 Equation of the model `VMIS_CINE_LINE`

For reasons of performances the relation is written in 1D. They are the restriction of the behavior 3D ([R5.03.02] and [R5.03.16]) on the uniaxial case. The behavior 3D is written:

$$\sigma = \mathbf{K}\left(\varepsilon - \varepsilon^{\mathbf{p}} - \varepsilon^{\mathbf{th}}\right) \text{ with } \mathbf{K} \text{ operator of elasticity}$$

$$\mathbf{X} = C\,\varepsilon^{\mathbf{p}}$$

$$F(\sigma, \mathbf{R}, \mathbf{X}) = (\tilde{\sigma} - \mathbf{X})_{eq} - \sigma_y \text{ with } \mathbf{A}_{eq} = \sqrt{\frac{3}{2}\tilde{\mathbf{A}}\cdot\tilde{\mathbf{A}}}$$

$$\dot{\varepsilon}^{\mathbf{p}} = \dot{p}\frac{\partial F}{\partial \sigma} = \frac{3}{2}\mathbf{p}\frac{\tilde{\sigma} - \mathbf{X}}{(\tilde{\sigma} - \mathbf{X})_{eq}} \quad \begin{cases} \text{si } F < 0 \quad \dot{p} = 0 \\ \text{si } F = 0 \quad \dot{p} \geq 0 \end{cases}$$

In the uniaxial case, the tensors are written :

$$\tilde{\sigma} = \sigma\,\mathbf{D} \qquad \mathbf{X} = X\,\mathbf{D} \qquad \varepsilon^{\mathbf{p}} = \frac{3}{2}\varepsilon^p\,\mathbf{D} \text{ with } \mathbf{D} = \begin{bmatrix} 2/3 & & \\ & -1/3 & \\ & & -1/3 \end{bmatrix}$$

As long as the loading is monotonous, the following relations immediately are obtained:

$$p = \varepsilon^p \qquad X = \frac{3}{2}C\,\varepsilon^p \qquad \sigma = \frac{3}{2}C\,\varepsilon^p + \sigma_y \qquad \sigma = F(\varepsilon) = \sigma_y + \frac{E \cdot E_T}{E - E_T}p$$

C is determined by: $C = \frac{2}{3}\frac{E\,E_T}{E - E_T}$. One poses: $H = \frac{E\,E_T}{E - E_T} = \frac{3}{2}C$

The relation of behavior 1D is written then:

$$\begin{vmatrix} \left|\sigma^{-} + \Delta\sigma - X^{-} - \Delta X\right| \\[6pt] E\Delta\varepsilon^p = E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right) - \left(\sigma^{-} + \Delta\sigma\right) + \frac{E}{E^{-}}\sigma^{-} \\[6pt] X = \frac{3}{2}C\,\varepsilon^p = H\,\varepsilon^p \\[6pt] \left|\sigma - X\right| - \sigma_y \leq 0 \\[6pt] \begin{cases} \dot{p} = 0\,si\,\left|\sigma - X\right| - \sigma_y < 0 \\ \dot{p} \geq 0\,si\,\left|\sigma - X\right| - \sigma_y = 0 \end{cases} \end{vmatrix}$$

The data of the material characteristics are those provided under the keyword factor `ECRO_LINE` or `ECRO_LINE_FO` of the operator `DEFI_MATERIAU` [U4.43.01]:

/ `ECRO_LINE` = (`D_SIGM_EPSI` = E_T , `SY` = σ_y)

/ `ECRO_LINE_FO` = (`D_SIGM_EPSI` = E_T , `SY` = σ_y)

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3.2 Integration of the relation `VMIS_CINE_LINE`

By direct implicit discretization of the relations of behavior, in a way similar to integration 3D ([R5.03.02] and [R5.03.16]) one obtains:

$$\begin{cases} \left|\sigma^- + \Delta\sigma - X^- - \Delta X\right| - \sigma_y \leq 0 \\[2mm] E\Delta\varepsilon^p = E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right) - \left(\sigma^- + \Delta\sigma\right) + \dfrac{E}{E^-}\sigma^- \\[2mm] \Delta\varepsilon^p = \Delta p \dfrac{\sigma^- + \Delta\sigma - X^- - \Delta X}{\left|\sigma^- + \Delta\sigma - X^- - \Delta X\right|} \\[2mm] \dfrac{X}{H} - \dfrac{X^-}{H^-} = \Delta\varepsilon^p \\[2mm] \Delta p \geq 0 \quad si \quad \left|\sigma^- + \Delta\sigma - X^- - \Delta X\right| = \sigma_y \\[2mm] \Delta p = 0 \quad si \quad \left|\sigma^- + \Delta\sigma - X^- - \Delta X\right| < \sigma_y \end{cases}$$

with $\Delta\varepsilon^{th} = \alpha\left(T - T_{ref}\right) - \alpha^-\left(T^- - T_{ref}\right)$

Two cases arise:

- $\left|\sigma^- + \Delta\sigma - X^- - \Delta X\right| < \sigma_y$ in this case $\Delta p = 0$ that is to say

  $\sigma = E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right) + \dfrac{E}{E^-}\sigma^- - \dfrac{H}{H^-}X^-$ thus $\left|\sigma^- \dfrac{E}{E^-} - X^- \dfrac{H}{H^-} + E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right)\right| < R(p^-)$,

- if not $\Delta p \geq 0$.

To simplify the writings one will pose: $\sigma^e = \dfrac{E}{E^-}\sigma^- - \dfrac{H}{H^-}X^- + E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right)$.

One from of deduced the algorithm from resolution:

1) if $\left|\sigma^e\right| \leq \sigma_y$ then $\Delta p = 0$, $X = X^- \dfrac{H}{H^-}$, $\sigma = E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right) + \dfrac{E}{E^-}\sigma^-$

2) if not it is necessary to solve:

$$\begin{cases} E\Delta\varepsilon^p = E\left(\Delta\varepsilon - \Delta\varepsilon^{th}\right) - \Delta\sigma = \sigma^e - \left(\sigma^- + \Delta\sigma\right) + X^- \dfrac{H}{H^-} \\[2mm] \Delta\varepsilon^p = \Delta p \dfrac{\sigma^- + \Delta\sigma - X^- - \Delta X}{\left|\sigma^- + \Delta\sigma - X^- - \Delta X\right|} = \Delta p \dfrac{\sigma - X}{\left|\sigma - X\right|} \\[2mm] X - \dfrac{H}{H^-}X^- = H\Delta\varepsilon^p \\[2mm] \left|\sigma^- + \Delta\sigma - X^- - \Delta X\right| - \sigma_y = 0 \end{cases}$$

Let us notice that: $\dfrac{H}{H^-}X^- = X - H\Delta\varepsilon^p$.

One deduces then from the first equation: $\sigma^e = \sigma - X + (E + H)\Delta\varepsilon^p$

One thus obtains, while eliminating $\sigma - X$ second equation:

$$\Delta\varepsilon^p = \sigma^e \dfrac{\Delta p}{(E + H)\Delta p + \sigma_y}$$

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While replacing $\Delta \varepsilon^p$ the relation enters σ^e and $\sigma - X$, one obtains:

$$\sigma - X = \sigma^e \left(\frac{\sigma_y}{(E+H)\Delta p + \sigma_y} \right)$$

By taking the absolute value of the two members of the preceding equation, one finds Δp :

$$(E+H)\Delta p + \sigma_y = |\sigma^e|$$

Once Δp determined, one can calculate:

$$\Delta \varepsilon^p = \Delta p \frac{\sigma^e}{|\sigma^e|}$$

$$X = X^- + \Delta X = \frac{H X^-}{H^-} + H \Delta p \frac{\sigma^e}{|\sigma^e|}$$

and while using: $\dfrac{\sigma - X}{\sigma_y} = \dfrac{\sigma^e}{|\sigma^e|}$, one obtains directly: $\sigma = \sigma_y \dfrac{\sigma^e}{|\sigma^e|} + X$

Moreover, the option `FULL_MECA` allows to calculate the tangent matrix $\mathbf{K}_i^n$ with each iteration. The tangent operator who is used for building it is calculated directly on the preceding discretized system. One obtains directly:

if $|\sigma^e| > R(p^-)$ $\dfrac{\delta \partial}{\delta \varepsilon} = E_T$

if not $\dfrac{\delta \sigma}{\delta \varepsilon} = E$

The option `RIGI_MECA_TANG` who allows to calculate the tangent matrix $\mathbf{K}_i^0$ used in the phase of prediction of the algorithm of Newton is obtained using the indicator of plasticity χ^- previous moment:

- if $\chi^- = 1$ then $\dfrac{\delta \sigma}{\delta \varepsilon} = E_T$

- if $\chi^- = 0$ then $\dfrac{\delta \sigma}{\delta \varepsilon} = E$

3.3 Internal variables

The relation of behavior `VMIS_CINE_LINE` product two internal variables : X .et χ

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4 Behaviour of Von Mises, linear kinematic work hardening 1D: `VMIS_CINE_GC`

4.1 Equation of the model `VMIS_CINE_GC`

For reasons of performances the relation is also written in 1D for a use with finite elements of standard multifibre beam. The equations result from the restriction of the behavior 3D ([R5.03.02] and [R5.03.16]) on the uniaxial case.
The equations of the model are the same ones as those of the § 3.1.

The data of materials are those provided under the keyword factor `ECRO_LINE` of the operator `DEFI_MATERIAU` [U4.43.01] :
/ ECRO_LINE = _F(

♦	D_SIGM_EPSI	= E_T	[Reality]
♦	SY	= σ_y	[Reality]
◊	SIGM_LIM	= sigmlim	[Reality]
◊	EPSI_LIM	= epsilim	[Reality]

)

Operands `SIGM_LIM` and `ESPI_LIM` allow to define the terminals which correspond to the limiting states of service and ultimate, classically used at the time of study in civil engineer.

◊ SIGM_LIM = sigmlim
Definition of the ultimate stress.

◊ EPSI_LIM = epslim
Definition of the limiting deformation.

These terminals are obligatory when the behavior is used `VMIS_CINE_GC` (Cf [U4.51.11] non-linear Behaviors, [U4.42.07] `DEFI_MATER_GC`). In the other cases they are not taken into account.

4.2 Integration of the relation `VMIS_CINE_GC`

The method of integration identical to that is presented to the § 3.2.

4.3 Internal variables

Supported modeling is 1D, numbers of internal variables is of 6.
- $V1$: This variable represents the constraint divided by the ultimate stress `sigmlim`.
- $V2$: This variable represents the total deflection divided by the limiting deformation `epslim`.
- $V3$: Kinematic work hardening: `XCINXX`. In 1D only a scalar is necessary.
- $V4$: Plastic indicator: `INDIPLAS`. Indicate if the material exceeded the elastic criterion.
- $V5$: nonrecoverable dissipation: `DISSIP`. During seismic calculations it can be useful for the user to know nonrecoverable dissipated energy. The variable `DISSIP` represent the nonrecoverable office plurality of energy. The nonrecoverable increment of energy is written in the form:

$$\Delta\, Eg = \frac{1}{2}\left(E^+\Delta\,\varepsilon - \left(\sigma^+ - \sigma^-\right)\right)\Delta\,\varepsilon$$

- $V6$: thermodynamic dissipation: `DISSTHER`. The thermodynamic increment of dissipation is written in the form: $\Delta\, Eg = \sigma_y\,\dot{p}$.

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5 Behavior of Von Mises with asymmetrical linear work hardening: `VMIS_ASYM_LINE`

5.1 Equations of the model `VMIS_ASYM_LINE`

5.1.1 Asymmetrical behaviour in traction and compression

It is a behavior uncoupled in traction and compression, built from `VMIS_ASYM_LINE`, but with elastic and different module limits of work hardening in traction and compression. We adopt an index T for traction and C for compression. Elastic behaviour in traction and compression identical and is characterized by the same Young modulus. There are two fields of isotropic work hardening defined by R_T and R_C. The two fields are independent one of the other.

YT	elastic limit in traction. In absolute value.
YC	elastic limit in compression. In absolute value.
p_T	Variable interns in traction. Algebraic value.
p_C	Variable interns in compression. Algebraic value.
E_{TT}	Slope of work hardening in traction.
E_{TC}	Slope of work hardening in compression.

The equations of the model of behavior are:

$$
\begin{cases}
\dot{\varepsilon}^p = \dot{\varepsilon} - E^{-1}\dot{\sigma} - \dot{\varepsilon}^{th} \\
\dot{\varepsilon}^p = \dot{\varepsilon}_C^p + \dot{\varepsilon}_T^p \\
\dot{\varepsilon}_C^p = \dot{\varepsilon}_C \dfrac{\sigma}{|\sigma|} \\
\dot{\varepsilon}_C^p = \dot{\varepsilon}_T \dfrac{\sigma}{|\sigma|} \\
\sigma - R_T(p_T) \leq 0 \\
-\sigma - R_C(p_C) \leq 0
\end{cases}
\quad avec \quad
\begin{cases}
\dot{p}_C = 0 & si \quad -\sigma - R_C(p_C) < 0 \\
\dot{p}_C \geq 0 & si \quad -\sigma = R_C(p_C) \\
\dot{p}_T = 0 & si \quad \sigma - R_T(p_T) < 0 \\
\dot{p}_T \geq 0 & si \quad \sigma = T_T(p_T)
\end{cases}
$$

$\dot{\varepsilon}_C^p$: speed of plastic deformation in compression,

$\dot{\varepsilon}_T^p$: speed of plastic deformation in traction,

ε^{th} : thermal deformation of origin : $\varepsilon^{th} = \alpha(T - T_{ref})$

It is noticed that one cannot have simultaneously plasticization in traction and compression: that is to say $\dot{p}_C = 0$, that is to say $\dot{p}_C = 0$, that is to say both are worthless.

The data of the material characteristics are those provided under the keyword factor `ECRO_ASYM_LINE` of the operator `DEFI_MATERIAU` [U4.43.01].

```
ECRO_ASYM_LINE = _F (
    DT_SIGM_EPSI = E_TT ,   SY_T = σ_yT ,
    DC_SIGM_EPSI = E_TC ,   SY_C = σ_yC ,)
```

The Young modulus E is provided under the keywords factors `ELAS` or `ELAS_FO`.

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$$R_T(p)=\frac{E_{TT}E}{E-E_{TT}}\,p_T+\sigma_{yT}=H_T\cdot p_T+\sigma_{yT}$$

One calculates the functions of work hardening by:

$$R_C(p)=\frac{E_{TC}E}{E-E_{TC}}\,p_C+\sigma_{yC}=H_C\cdot p_C+\sigma_{yC}$$

5.2 Integration of the behavior `VMIS_ASYM_LINE`

By direct implicit discretization of the asymmetrical relation of behavior, in a way similar to the preceding one, one obtains:

$$\Delta\varepsilon^p=\Delta\varepsilon_T^p+\Delta\varepsilon_C^p$$

$$\Delta\varepsilon^p=\Delta\varepsilon-\Delta\varepsilon^{th}-\frac{\Delta\sigma}{E}$$

$$\Delta\varepsilon_T^p=\Delta p_T\frac{\sigma^-+\Delta\sigma}{|\sigma^-+\Delta\sigma|}$$

$$\Delta p_T\geq 0\quad si\quad (\sigma^-+\Delta\sigma)-R_T(\bar{p}_T+\Delta\bar{p}_T)\ \leq\ 0$$

$$\Delta p_T=0\quad si\quad (\sigma^-+\Delta\sigma)-R_T(\bar{p}_T+\Delta\bar{p}_T)\ <\ 0$$

$$\Delta\varepsilon_C^p=\Delta p_C\frac{\sigma^-+\Delta\sigma}{|\sigma^-+\Delta\sigma|}$$

$$-(\sigma^-+\Delta\sigma)-R_C(\bar{p}_C+\Delta\bar{p}_C)\leq 0$$

$$\Delta p_C\geq 0\quad si\quad -(\sigma^-+\Delta\sigma)-R_C(\bar{p}_C+\Delta\bar{p}_C)\ =\ 0$$

$$\Delta p_C=0\quad si\quad -(\sigma^-+\Delta\sigma)-R_C(\bar{p}_C+\Delta\bar{p}_C)\ <\ 0$$

Integration is similar to that of `VMIS_ISOT_LINE` for each direction of traction and compression. It should well be seen that the centers of the fields of elasticity are data (calculated explicitly with the preceding step) for the incremental problem to solve.

Four cases arise:

- $\Delta\varepsilon-\Delta\varepsilon^{th}>0$ one poses $\sigma_T^e=\sigma^-+E(\Delta\varepsilon-\Delta\varepsilon^{th})$

  ◦ if $\sigma_T^e<R_T(\bar{p}_T)$ in this case $\Delta p_T\ =\ 0$ thus $\sigma=\sigma_T^e$ and $\frac{\delta\sigma}{\delta\varepsilon}\ =\ E$

  ◦ if not: $\Delta p_T=\dfrac{|\sigma_T^e|-(\sigma_{yT}+H_T p_T^-)}{E+H_T}$, $\Delta p_C=0$

  $$\sigma=\frac{\sigma_T^e}{1+\dfrac{E\Delta p_T}{R_T(p_T)}}=\frac{\sigma_T^e}{|\sigma_T^e|}R_T(p_T)$$

  $$\frac{\delta\sigma}{\delta\varepsilon}=E_{TT}$$

- $\Delta\varepsilon-\Delta\varepsilon^{th}<0$ one poses $\sigma_C^e=\sigma^-+E(\Delta\varepsilon-\Delta\varepsilon^{th})$

  ◦ if $-\sigma_C^e<R_C(\bar{p}_C)$ in this case $\Delta p_C\ =\ 0$ thus $\sigma=\sigma_C^e$ and $\frac{\delta\sigma}{\delta\varepsilon}\ =\ E$

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◦   if not: $\Delta\,p_C = \dfrac{\left|\sigma_C^{\,e}\right| - \left(\sigma_{yC} + H_C\,p_C^{\;-}\right)}{E + H_C}$, $\Delta\,p_T = 0$

$$\sigma = \frac{\sigma_C^{\,e}}{1 + \dfrac{E\,\Delta\,p_C}{R_C\!\left(p_C\right)}} = \frac{\sigma_C^{\,e}}{\left|\sigma_C^{\,e}\right|}\,R_C\!\left(p_C\right)$$

$$\frac{\delta\,\sigma}{\delta\,\varepsilon} = E_{TC}$$

Note:
> *The initial tangent matrix (option* `RIGI_MECA_TANG`*) is taken equal to the elastic matrix.*

5.3   Internal variables

The relation of behavior `VMIS_ASYM_LINE` product 2 internal variables : p_C p_T .
It is not usable for the elements of grid.

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6   Model of PINTO_MENEGOTTO

The model presented in this chapter describes the behavior 1D steels reinforcing of the reinforced concrete [feeding-bottle 1]. The law constitutive of these steels is made up of two distinct parts: the monotonous loading composed of three successive zones (linear elasticity, plastic stage and work hardening) and the cyclic loading whose analytical formulation was proposed by A. Giuffré and P. Pinto in 1973 [feeding-bottle 2] and was then developed by Mr. Menegotto [feeding-bottle 3].

During cycles, the way of loading between two points of inversion (semi-cycle) is described by an analytical curve of expression of the type $\sigma = f(\varepsilon)$. The interest of this formulation is that the same equation controls the discharge and load diagrams (see for example the figures [Figure 6.1.1-a] and [Figure 6.1.2-a]). Parameters attached to the function f are reactualized after each inversion of loading. The reactualization of these parameters depends on the way carried out in the plastic zone during the half - preceding cycle.

In addition, this model can treat the inelastic buckling of the bars (G. Monti and C. Nuti [feeding-bottle 4]). The introduction of new parameters into the equation of the curves then makes it possible to simulate the softening of the answer stress-strain in compression.

6.1   Formulation of the model

6.1.1   Monotonous loading

This chapter describes the first loading which the bar undergoes, i.e. the part preceding activation by the curve of Giuffré [Figure 6.1.1-a].
The monotonous traction diagram of steel is typically followed by the three following successive zones:

- The linear elasticity, defined by the Young modulus E and elastic limit σ_y . $\sigma = E\varepsilon$ (zone 1, [Figure 6.1.1-a])

- The plastic stage, ranging between the limiting elastic strain ε_y^0 and deformation of work hardening ε_h , higher limit of the plate in deformation. During the stage the constraint remains constant. $\sigma = \sigma_y^0$ (zone 2, [Figure 6.1.1-a])

- Work hardening, following the traction diagram up to the ultimate point of constraint and deformation, $(\varepsilon_u, \sigma_u)$. This part is represented by a polynomial of the fourth degree:

$$\sigma = \sigma_u - \left(\sigma_u - \sigma_y^0\right)\left(\frac{\varepsilon_u - \varepsilon}{\varepsilon_u - \varepsilon_h}\right)^4 \quad \text{(zone 2, [Figure 6.1.1-a])}$$

The slope of work hardening (used thereafter, for the cyclic behavior) is defined here by:

$$E_h = \frac{\sigma_u - \sigma_y^0}{\varepsilon_u - \varepsilon_y^0}$$. It is the average slope of zones 2 and 3 of the following figure.

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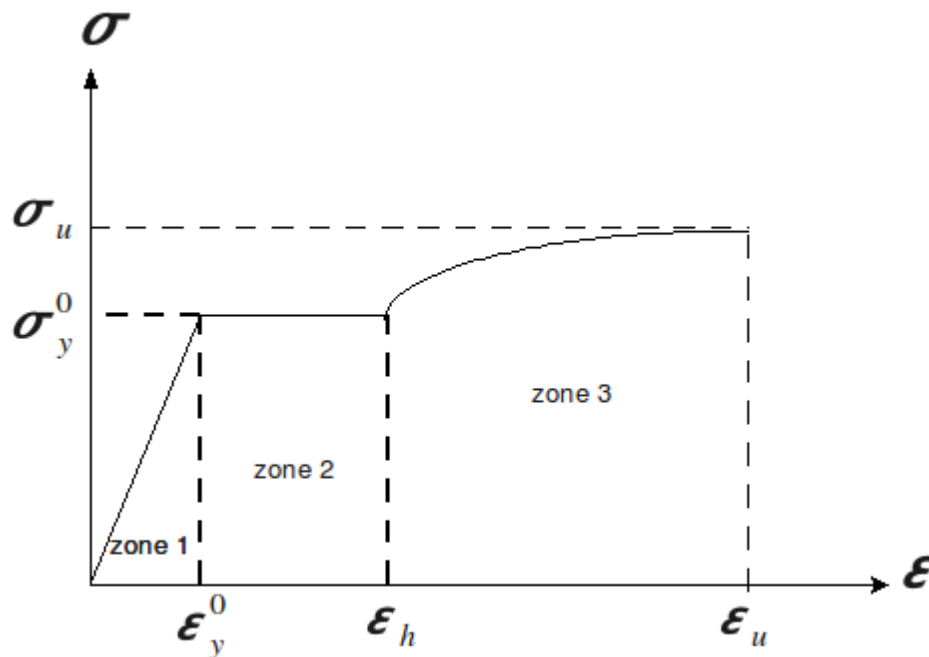*Clé : R5.03.09   Révision :*
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Figure 6.1.1-a : Curve of behavior.

6.1.2 Cyclic loading

One places oneself now if the bar undergoes a consecutive discharge with the first loading. Two cases arise then:

- the starting position is in the elastic zone. The discharge remains in this elastic case,
- the starting position is in the plastic zone ($\varepsilon \geq \varepsilon_y^0$). The answer is first of all rubber band, then, for a certain value of the deformation, the discharge becomes nonlinear [Figure 6.1.2-a] (this is true for a discharge starting from zone 2 or of zone 3).

The relation which the deformation must satisfy so that the curve of Giuffré is activated is the following one:

$$\left| \varepsilon_{max} - \varepsilon \right| > \frac{\left| \varepsilon_y^0 \right|}{3.0} \text{ , with } \varepsilon_{max} \text{ maximum deformation reached in load.}$$

As soon as one crossed this limit with the first discharge, it is the cyclic behavior (curve of Giuffré [Figure 6.1.2-a]) which is activated.
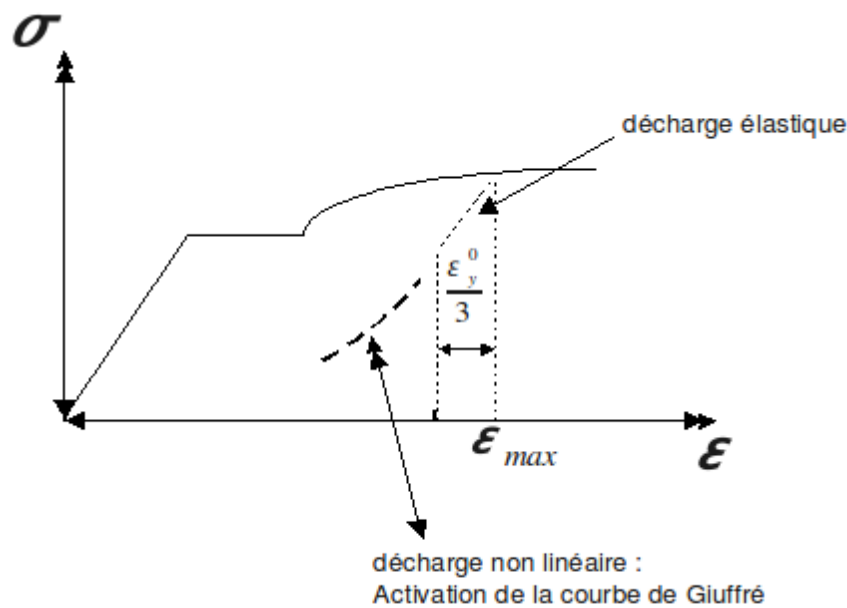
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Figure 6.1.2-a : Curve of behaviour with discharge.

6.1.2.1 Presentation of the nth semi-cycle

The shape of the curve of the nth semi-cycle depends on the plastic excursion carried out during the half - preceding cycle. One defines the following quantities [Figure 6.1.2.1-a]:

- σ_y^n : Elastic limit of the nth semi-cycle. (Calculation clarified with [§ 5.1.2.2])

- σ_r^{n-1} : Constraint at the last point of inversion (forced maximum attack with the n-1ième semi-cycle).

- ε_r^{n-1} : Deformation at the last point of inversion (maximum deformation attack with the n-1ième semi-cycle).

- ε_y^n : Deformation corresponding to σ_y^n : $\varepsilon_y^n = \varepsilon_r^{n-1} + \dfrac{\sigma_y^n - \sigma_r^{n-1}}{E}$

- $f(t)$: Plastic excursion of the nth cycle

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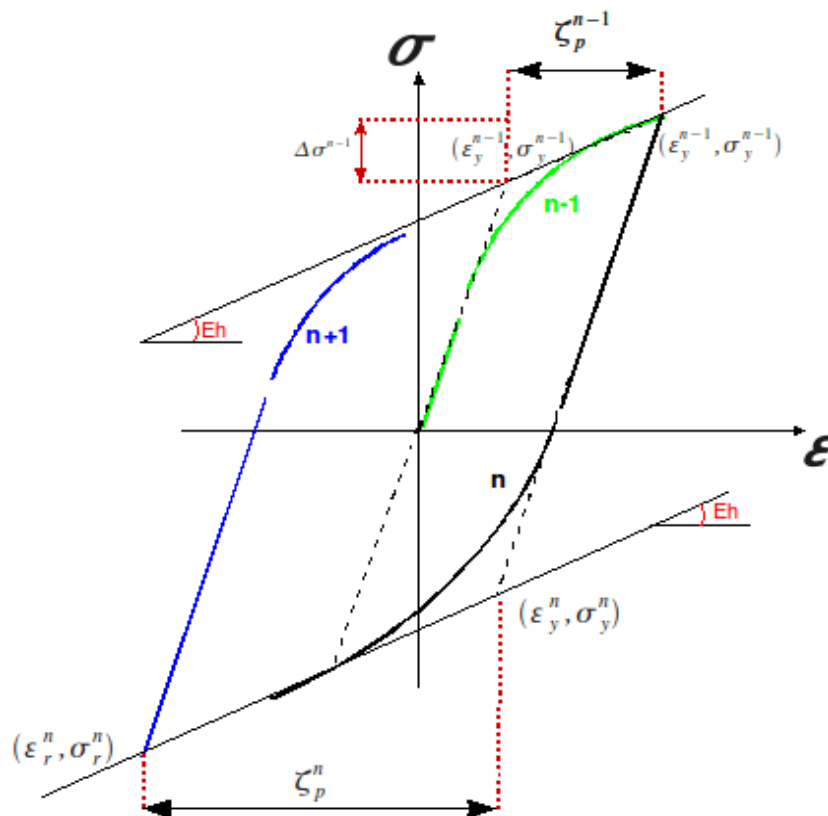*Clé : R5.03.09* *Révision* :
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Figure 6.1.2.1-a : Cyclic behavior.

6.1.2.2 Law of work hardening

The model is based on a kinematic law of work hardening. The branches of the semi-cycles lie between two asymptotes of slope E_h (asymptotic slope of work hardening).

One thus determines σ_y^n in the following way: $\sigma_y^n = \sigma_y^{n-1} . sign\left(-\zeta_p^{n-1}\right) + \Delta\sigma^{n-1}$ where the function $sign(x) = -1$ if $x < 0$ and 1 if $x > 0$ and where $\Delta\sigma^{n-1}$ of constraint of the preceding semi-cycle [Figure is the plastic increment 6.1.2.1-a] which is defined by: $\Delta\sigma^{n-1} = E_h \zeta_p^{n-1}$.

For each semi-cycle one thus determines σ_y^n according to σ_y^{n-1} and ζ_p^{n-1}, one from of deduced, ε_y^n then the following semi-cycle is calculated (by the law of behavior below). The maximum deformation (in absolute value) reached before changing direction will make it possible to calculate the plastic excursion $\zeta_p^n = \varepsilon_r^n - \varepsilon_y^n$.

6.1.2.3 Analytical description of the curves $\sigma = f(\varepsilon)$

The expression chosen in the model to follow the curves of loading is the following one:

$$\sigma^* = b\,\varepsilon^* + \left(\frac{1-b}{\left(1+\left(\varepsilon^*\right)^R\right)^{1/R}}\right)\varepsilon^*$$

With $b = \dfrac{E_h}{E}$ report of the slope of work hardening on the slope of elasticity.

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$$\varepsilon^* = \frac{\varepsilon - \varepsilon_r^{n-1}}{\varepsilon_y^n - \varepsilon_r^{n-1}}$$

$$\sigma^* = \frac{\sigma - \sigma_r^{n-1}}{\sigma_y^n - \sigma_r^{n-1}}$$

$$\xi_p^{n-1} = \frac{\zeta_p^{n-1}}{\varepsilon_y^n - \varepsilon_r^{n-1}}$$

Size R allows to describe the pace of the curve of the branches. It is function of the plastic way carried out during the preceding semi-cycle:

$$R(\xi) = R_0 - g(\xi) \quad \text{where} \quad g(\xi) = \frac{A_1 . \xi}{A_2 + \xi}$$

Parameters R_0, A_1 and A_2 are constants without unit depending on the mechanical properties of steel. Their values are obtained in experiments and Menegotto [feeding-bottle 3] proposes:

$$R_0 = 20.0 \quad A_1 = 18.5 \quad A_2 = .015$$

6.1.3 Case of inelastic buckling

Monti and Nuti [feeding-bottle 4] show that for a relationship between the length L and the diameter D bar lower than 5, the curve of compression is identical to that of traction. On the other hand, when $L/D > 5$ a buckling of the bar is observed. In this case the curve of compression in the plastic zone has a lenitive behavior. The model available in *Code_Aster* allows to also describe this phenomenon.

One defines the following variables [Figure 6.1.3-a]:
- E_0 : Initial elastic Young modulus (correspondent with E without buckling).
- b_c : Report of the slope of work hardening on the elastic slope in compression.
- b_t : Report of the slope of work hardening on the elastic slope in traction (refill after compression with buckling).
- E_r : Modulus Young reduced in traction (slope of the curve of refill after compression with buckling).
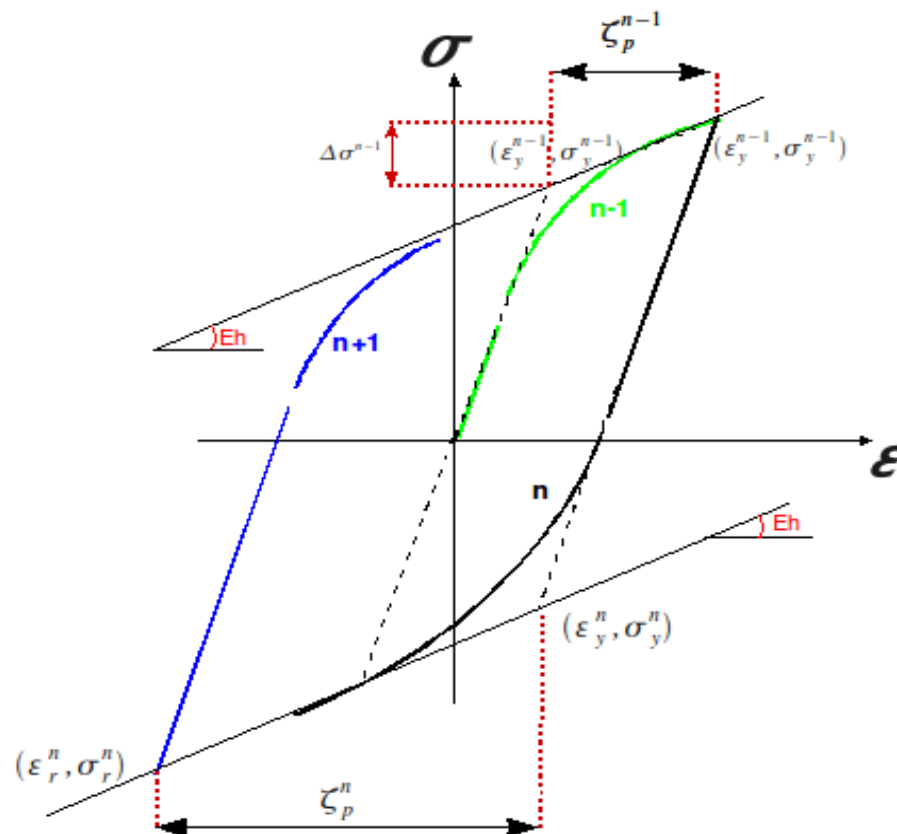
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Figure 6.1.3-a : Cyclic curve of behavior.

6.1.3.1 Compression

A negative slope is introduced $b_c \times E$, where b_c is defined by:

$$b_c = a(5.0 - L/D) e\left(b\zeta' \frac{E}{\sigma_y^0 - \sigma^\infty}\right)$$

With $\sigma_\infty = 4.0 \frac{\sigma_y^0}{L/D}$ and $\zeta' = \max\left(\left|\zeta_p^n\right|\right)$ the greatest plastic way carried out during the loading.

It is necessary then, as in the model without buckling, to determine σ_y^n. The method is identical, but a complementary constraint is added σ_s^* in order to correctly position the curve compared to the asymptote [Figure 6.1.3-a].

$$\sigma_s^* = \gamma_s b E \frac{b - b_c}{1 - b_c} \qquad \text{where } \gamma_s \text{ is given by : } \gamma_s = \frac{11.0 - L/D}{10\left(e^{cL/D} - 1.0\right)}$$

And one thus has: $\sigma_y^n = \left(\sigma_y^n\right)_{\text{sans flambage}} + \sigma_s^*$

This modifies also the value of $\varepsilon_y^n = \varepsilon_r^{n-1} + \frac{\sigma_y^n * \sigma_r^{n-1}}{E}$

6.1.3.2 Traction

At the time of the semi-cycle in traction according to one adopts a reduced Young modulus defines by:

$$E_r = E_0\left(a_5 + (1.0 - a_5) e^{\left(-a_6 \zeta_{pl}^2\right)}\right) \text{ with } a_5 = 1.0 + (5.0 - L/D)/7.5$$

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Note:

> Parameters a, c and a_6 are constants (without unit) depend on the mechanical properties of
> steel and are in experiments given. Values adopted by Monti and Nuti [feeding-bottle 4] are:
> $a = 0.006$ $c = 0.500$ $a_6 = 620.0$

6.2   Establishment in *Code_Aster*

This model is accessible in *Code_Aster* starting from the keyword BEHAVIOR
(RELATION = 'PINTO_MENEGOTTO') or (RELATION = 'GRILLE_PINTO_MEN') order
STAT_NON_LINE [U4.51.03]. The whole of the parameters of the model are given via the order
DEFI_MATERIAU (keyword factor PINTO_MENEGOTTO) [U4.43.01]. One indexes the parameters here
intervening in the model:

Parameters of the model	Intervenes in	adopted value by default in *Aster*
σ_y^0	First loading	–
ε_u	First loading	–
σ_u	First loading	–
ε_h	First loading	–
$b = \dfrac{E_h}{E}$	Cycles	If no value entered one takes the computed value with the first loading
R_0	Cycles	20
a_1	Cycles	18.5
a_2	Cycles	0.15
L/D	Cycles with buckling (if $L/D > 5$)	4 (to be by default except buckling)
a_6	Buckling	620
c	Buckling	0.5
a	Buckling	0,006

Parameters R_0, a_1, a_2, a_6, c and a depend on the mechanical properties of steel and are in
experiments given. Adopted values by default in *Code_Aster* are those proposed in the literature
[feeding-bottle 1].

One gives in [Figure 6.2-a] a comparison of the model following the value of $b = \dfrac{E_h}{E}$ for two values:

$b = 0.01$ and $b = 0.001$

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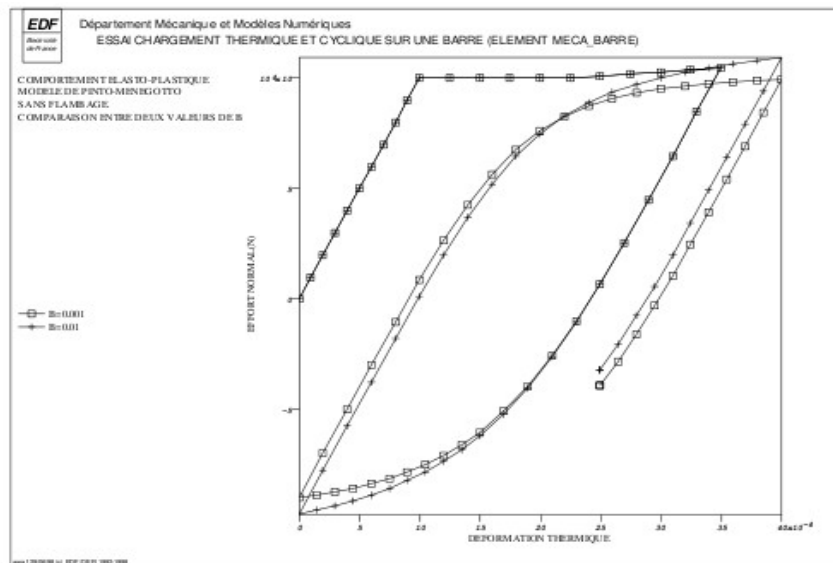*Clé : R5.03.09   Révision :*
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Figure 6.2-a : Comparison of 2 sets of parameters.

One gives in [Figure 6.2-b] a comparison of the model without buckling and the model and buckling.
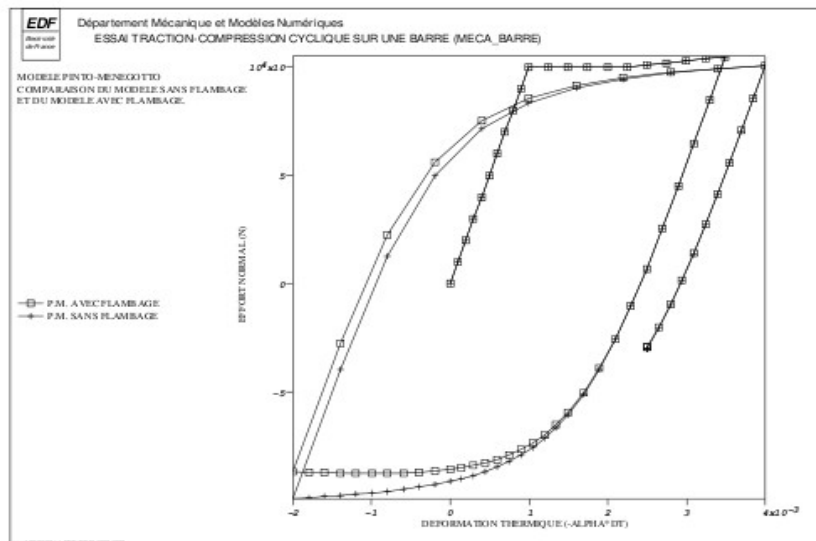

Figure 6.2-b : Comparison with and without buckling.

6.3   Internal variables

They 8, and are defined by:

$$V1 = \varepsilon_r^{n-1}$$

$$V2 = \varepsilon,_r^{n}$$

$$V3 = \sigma_r^{n}$$

$$V4 = \varepsilon^- + \Delta \varepsilon - \alpha\left(T - T^-\right) V5 = \Delta \varepsilon + \alpha\left(T - T^-\right)$$

$$V6 = \text{cycl} \qquad\qquad = 0 \text{ si le comportement cyclique n'est pas activé}$$
$$= 1 \text{ dans le cas contraire}$$

$$V7 = \chi \qquad\qquad = 0 \text{ si le pas de temps correspond à une évolution linéaire}$$
$$= 1 \text{ dans le cas contraire (indicateur de plasticité}$$

$$V8 = \text{indicateur de flambage}$$

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7 Behaviors `VISC_IRRA_LOG` and `GRAN_IRRA_LOG`

The model presented in this chapter describes the viscoplastic behaviors 1D `VISC_IRRA_LOG` and `GRAN_IRRA_LOG` (creep and growth under irradiation of the alloys M5 and Zircaloy-4) for the modeling of the fuel assemblies, and applicable to the elements of bars and multifibre beams.

7.1 Formulation of the model

The equations are the following ones:

$$\begin{cases} \dot{\varepsilon}^{vp} = \dot{\varepsilon}\,\dfrac{\sigma}{|\sigma|} \\[2mm] \dot{\varepsilon} = |\sigma|.\left(e^{\frac{-Q}{T}}\right).\dot{\Phi}\left(\dfrac{A\omega}{1+\omega\Phi} + B - C\omega e^{-\omega t}\right) \\[2mm] \dfrac{\dot{\sigma}}{E} = \dot{\varepsilon} - \dot{\varepsilon}^{vp} - \dot{\varepsilon}^{g} - \dot{\varepsilon}^{th} \end{cases}$$

These relations are deduced from the creep tests `HALIBUT` and `REFLECTION` [8] for various values of neutron flux.

The coefficients are provided under the keyword `VISC_IRRA_LOG` or `GRAN_IRRA_LOG` of `CHALLENGE_MATERIAL` and Φ is the neutron fluence (integral flow compared to time).

ε^{g} represent the deformation of growth under flow. She is taken into account only in the behavior `GRAN_IRRA_LOG` and expresses itself in the form:

$\varepsilon^{g}(t) = f\left(T, \Phi_{t}(x,y,z)\right)$

Note:

1) *Neutron fluence $\Phi_{t}(x,y,z)$ express yourself obligatorily in $10^{20}\,n/cm^{2}$. By convention in `DEFI_MATERIAU` [U4.43.01], if the value provided under the keyword `FLUX_PHI` is equal to 1, it is the field of fluence which is used for the behavior. In the contrary case, the value provided in `DEFI_MATERIAU` is used as constant neutron flow.*
2) *It is a field with the nodes defined as variable of order in the order `AFFE_MATERIAU`.*
3) *Caution : The exposure field is incremental and corresponds to the history of irradiation (stored in internal variable – cf below) to which one adds the increment of the field of fluence coming from the variable of order.*

7.2 Internal variables

Three internal variables:
- $V1$: cumulated viscoplastic deformation: ε_{p} ;
- $V2$: memorizing of the history of irradiation (fluence).
- $V3$: deformation of growth : ε^{g} .

7.3 Implicit integration

By direct implicit discretization of the relations of behavior, one obtains:

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$$\left|\begin{array}{l} \Delta \varepsilon^{vp} = \Delta p \dfrac{\sigma(t^- + \Delta t)}{|\sigma(t^- + \Delta t)|} \\[2ex] \Delta p = |\sigma(t^- + \Delta t)| \left(e^{\frac{-Q}{T}}\right) \cdot \left(\dfrac{A\,\omega}{1 + \omega\,\Phi(t^- + \Delta t)} + B - C\,\omega\,e^{-\omega t}\right) \Delta\Phi \\[2ex] \dfrac{\sigma}{E} - \dfrac{\sigma^-}{E^-} = \Delta\varepsilon - \Delta\varepsilon^{vp} - \Delta\varepsilon^g - \Delta\varepsilon^{th} \\[2ex] avec \\[1ex] \Delta\varepsilon^{th} = \alpha(T)(T - T_{ref}) - \alpha(T^-)(T - T_{ref}) \\[1ex] \Delta\varepsilon^g = f(T^+, \Phi_t^+) - f(T^-, \Phi_t^-) \end{array}\right.$$

One can solve these equations explicitly while posing: $\sigma^e = \dfrac{E}{E^-}\sigma^- + E(\Delta\varepsilon - \Delta\varepsilon^g - \Delta\varepsilon^{th})$

then the system is reduced to : $\sigma = \sigma^e - E\sigma\left(e^{\frac{-Q}{T}}\right) \cdot \left(\dfrac{A\,\omega}{1 + \omega\,\Phi} + B - C\,\omega\,e^{-\omega t}\right)\Delta\Phi$

thus the solution is obtained immediately: $\sigma = \dfrac{\sigma^e}{1 + E\left(e^{\frac{-Q}{T}}\right) \cdot \left(\dfrac{A\,\omega}{1 + \omega\,\Phi} + B - C\,\omega\,e^{-\omega t}\right)\Delta\Phi}$

and the tangent operator is written: $\dfrac{\partial\sigma}{\partial\varepsilon} = \dfrac{E}{1 + E\left(e^{\frac{-Q}{T}}\right) \cdot \left(\dfrac{A\,\omega}{1 + \omega\,\Phi} + B - C\,\omega\,e^{-\omega t}\right)\Delta\Phi}$

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8 Model of MAZARS in 1D

8.1 Equations of the model

The objective of this modeling is to give an account of refermeture of the cracks. This model is used only with the multifibre beams. Equations presented in the document [R7.01.08] "Model of damage of MAZARS"are taken again and rewritten in $1D$.

$$\begin{cases} \sigma_{xx} = (1-D_t) E \langle \varepsilon_{xx}^e \rangle_+ \\ \sigma_{xx} = (1-D_c) E \langle \varepsilon_{xx}^e \rangle_- \end{cases} \qquad \textbf{[éq 8.1-1]}$$

with :

- E : Young modulus,
- D_t : the variable of damage in traction.
- D_c : the variable of damage in compression.
- ε_{xx}^e : elastic strain $\varepsilon_{xx}^e = \varepsilon - \varepsilon^{th}$
- $\varepsilon^{th} = \alpha (T - T_{ref})$: thermal dilation

The only modification is to have a damage of traction and compression. Coupling $\alpha_t^\beta D_t + (1-\alpha_t)^\beta D_C$ do not exist any more. The damage remains always controlled by the extensions.

The damages of traction and compression are defined by the following equations if $\varepsilon_{eq} \geq \varepsilon_{d0}$:

$$D_c = 1 - \frac{\varepsilon_{d0}(1-A_c)}{\varepsilon_{eq}} - \frac{A_c}{\exp[B_c(\varepsilon_{eq}-\varepsilon_{d0})]} \qquad D_c \in [0,1[\qquad \textbf{[éq 8.1-2]}$$

$$D_t = 1 - \frac{\varepsilon_{d0}(1-A_t)}{\varepsilon_{eq}} - \frac{A_t}{\exp[B_t(\varepsilon_{eq}-\varepsilon_{d0})]} \qquad D_t \in [0,1[\qquad \textbf{[éq 8.1-3]}$$

where $A_c, A_t, B_c, B_t, \varepsilon_{d0}$ are parameters materials to be identified.

The damage is controlled by the equivalent deformation ε_{eq}. Lbe extensions are paramount in the phenomenon of cracking of the concrete, the introduced equivalent deformation is defined starting from the positive values of the deformations, that is to say :

$$\begin{cases} si\, \varepsilon_{xx}^e \geq 0\, alors\, \varepsilon_{eq} = |\varepsilon_{xx}^e| \\ si\, \varepsilon_{xx}^e \leq 0\, alors\, \varepsilon_{eq} = \sqrt{2}\, \nu\, |\varepsilon_{xx}^e| \end{cases} \qquad \textbf{[éq 8.1-4]}$$

Note:

If $\varepsilon_{xx}^e \leq 0$, in $1D$ the principal deformations in the other directions are $\varepsilon_{yy}^e = \varepsilon_{zz}^e = -\nu\, \varepsilon_{xx}^e$. By using the formula $\varepsilon_{eq} = \sqrt{\langle \varepsilon_1 \rangle_+^2 + \langle \varepsilon_2 \rangle_+^2 + \langle \varepsilon_3 \rangle_+^2}$ the preceding expression well is obtained.

The tangent matrix with for expression: $\frac{d\sigma_{xx}}{d\varepsilon_{xx}^e} = (1-\tilde{D}) E - \frac{d\tilde{D}}{d\varepsilon_{xx}^e} E \varepsilon_{xx}^e$ with:

$$si\, \varepsilon_{xx}^e \geq 0\, et\, \varepsilon_{eq} \geq \varepsilon_{d0} \quad \frac{d\tilde{D}}{d\varepsilon_{xx}^e} = \frac{dD_t}{d\varepsilon_{xx}^e} = \left(\frac{\varepsilon_{d0}(1-A_t)}{\varepsilon_{eq}^2} + \frac{A_t B_t}{\exp[B_t(\varepsilon_{eq}-\varepsilon_{d0})]} \right)$$

$$si\, \varepsilon_{xx}^e < 0\, et\, \varepsilon_{eq} \geq \varepsilon_{d0} \quad \frac{d\tilde{D}}{d\varepsilon_{xx}^e} = \frac{dD_c}{d\varepsilon_{xx}^e} = -\sqrt{2}\, \nu \left(\frac{\varepsilon_{d0}(1-A_c)}{\varepsilon_{eq}^2} + \frac{A_c B_c}{\exp[B_c(\varepsilon_{eq}-\varepsilon_{d0})]} \right)$$

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The cases test [V6.02.120], [V6.02.119], [V5.02.130] put in work law of behavior of MAZARS in its version $1D$.
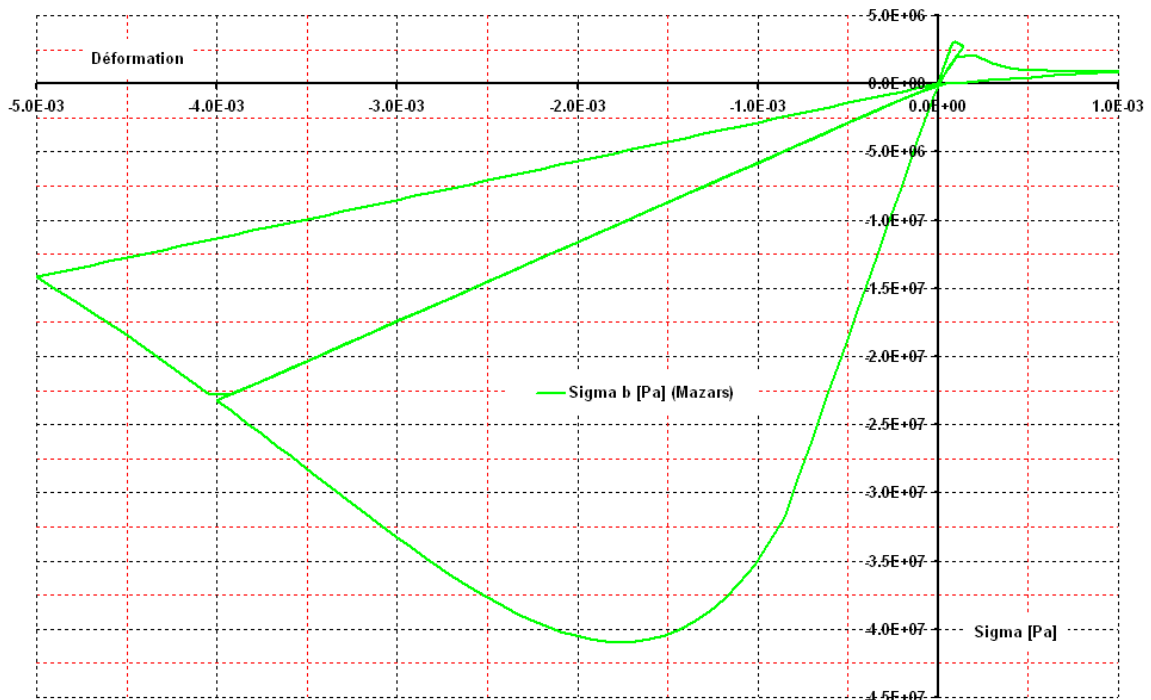

Figure 8.1-a : Behavior of Mazars in its version $1D$.

8.2   Internal variables

The law of behavior is written by uncoupling the damages from traction and of compression, the 2 damages are not any more of the internal variables [R7.01.08].
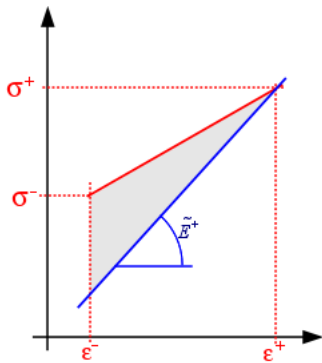
This law is dedicated to calculations of civil engineer. To facilitate interpretations of the results 2 variables are created to describe the state "limit" material concrete, in accordance with what this fact in the regulations of reinforced concrete calculation to the limiting states.
- The variable CRITSIG give information compared to the state of stress. This variable represents the constraint divided by the ultimate stress of the concrete given by the user SIGM_LIM.
- The variable CRITEPS give information compared to the state of deformation. This variable represents the equivalent deformation ε_{eq} divided by the deformation limits given by the user using the key word EPSI_LIM.
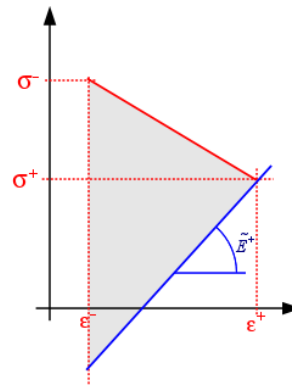
Values of the ultimate stress SIGM_LIM and of the limiting deformation EPSI_LIM are modifiable by the user at the time of the definition of material: DEFI_MATERIAU [U4.43.01], DEFI_MATER_GC [U4.42.0 7].

The writing of the law of MAZARS does not allow to calculate an intrinsic dissipation with the model. But, during seismic calculations it can be useful for the user to know nonrecoverable dissipated energy. The variable DISSIP represent the nonrecoverable office plurality of energy. The

nonrecoverable increment of energy is written $\Delta Eg = \frac{1}{2} \left(E\left(1 - D^+\right) \Delta\varepsilon - \left(\sigma^+ - \sigma^-\right) \right) \Delta\varepsilon$.

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Internal variables for the law of MAZARS in $1D$:

$V1$	CRITSIG	: Criterion in constraint
$V2$	CRITEPS	: Criterion in deformation.
$V3$	ENDO	: Endommagement [R7.01.08].
$V4$	EPSEQT	: Equivalent deformation of traction
$V5$	EPSEQC	: Equivalent deformation of compression
$V6$	RSIGMA	: Report of tri-axialité.
$V7$	TEMP_MAX	Maximum temperature attack in material
$V8$	DISSIP	: Energy nonrecoverable.

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9 Law of behavior `RELAX_ACIER`

The phenomenon of relieving of steels used in prestressed is regulated. The principal regulations are: `BPEL83`, `NF-EN-1992-1-1` October 2005, `AFCEN-ETCC-2010`,

One wishes to be able to model deformations who will vary slowly with the court of time, in particular for the taking into account of the creep of the concrete and temperature variations . One also wishes to take into account the influence of the temperature on the phenomenon of relieving.
In the regular manner, it would be possible to take account of the effect of the creep of the concrete, of the thermal deformation by making a linear combination of the various phenomena (cf regulations for more details). This approach is incompatible with a calculation with the finite elements .

Cbe regulations allow also to correct relieving after a "sauT " of deformation, being able to be due to a variation of the constraint resulting from an external loading (for example during a decennial test on an enclosure). To in no case this "jump" cannot evolve in time, it can be only one Dirac. The method used in this case is that of the equivalent time which, by construction, is incompatible with a code with the finite elements.

9.1 Formulation of the model

So that the law of relieving is usable in a code with the finite elements for structural analyses with variations of loadings such as: creep of the concrete, resumption of tension of the cables, taken into account of the influence of thermics,… it must be incremental and thermodynamically correct.

Several formulations for the modeling of the deformation related to relieving exist in the literature. formulation adopted is based on that proposed by J.Lemaitre :

$$\sigma = E \cdot \varepsilon^e \qquad \varepsilon = \varepsilon^e + \varepsilon^{an}$$

$$\dot{\varepsilon}^{an} = \left\langle \frac{\sigma - R(\varepsilon^{an})}{f_{prg} \cdot k} \right\rangle^n \text{ with } R(\varepsilon^{an}) = \frac{f_{prg} \cdot c \cdot \varepsilon^{an}}{\left(1 + (b \cdot \varepsilon^{an})^{nr}\right)^{\frac{1}{nr}}} \qquad \text{[éq 9.1-1]}$$

The law of behavior is 1D, and only available for modelings of type `BAR`, who are used to model the cables of prestressed in the operator `DEFI_CABLE_BP`.
To take account of the influence of the temperature on Lhas relieving, all coefficients of the law can be functions of the temperature.

Note:

Parameters c , b , n and nr are without unit, and thus independent of the units used during the study. k is adimensionné compared to f_{prg} thus compared to the constraints, but not compared to time. Indeed $\dot{\varepsilon}^{an}$ is homogeneous with $[s]^{-1}$, if the unit of time is in second. Thus if one knows k for a speed of deformation in a unit of time, it is necessary to convert its value compared to the unit of time used at the time of the study.

9.2 Internal variables

Two internal variables:
- $V1$: deformation unelastic cumulated: ε^{an} .
- $V2$: memorizing of tangent stiffness with the behavior.

9.3 Integration explicit

Integration is done by a method of the Runge-Kutta type of order five with variable steps.

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9.4   Identification of the parameters

Several solutions are possible:
- An analytical work makes it possible to determine relations between the coefficients present in the formulas lawful and that of the law of behavior: tangent at the beginning of relieving, asymptote for an "infinite" time,…
- Simulation using the formulas lawful several "tests of relieving". For a kind of cable given which corresponds to a set of parameters (k1, k2, ρ_{1000}, f_{prg}). Différentes value of μ go permettRE to obtain a beam "of tests" on which an identification of the parameters can be realized.
- Bibliographical study to find the experimental and theoretical work completed on the relieving of the cables of prestressing, in order to use the tests directly and to identify the parameters.

9.5   Result of an Identification

This identification was realized compared to simulation ofa test of relieving obtained with the formulas lawful.
One places oneself in the case $\rho_{1000}=2.5\%$ ($k1=6.0E-03$, $k2=1.1$) with $f_{prg}=1800\,MPa$, $E=190000\,MPa$. The initial deformation corresponds toa rate of loading of $\mu=0.75$. Simulation as well as the identification are carried out on [0h, 4000h].

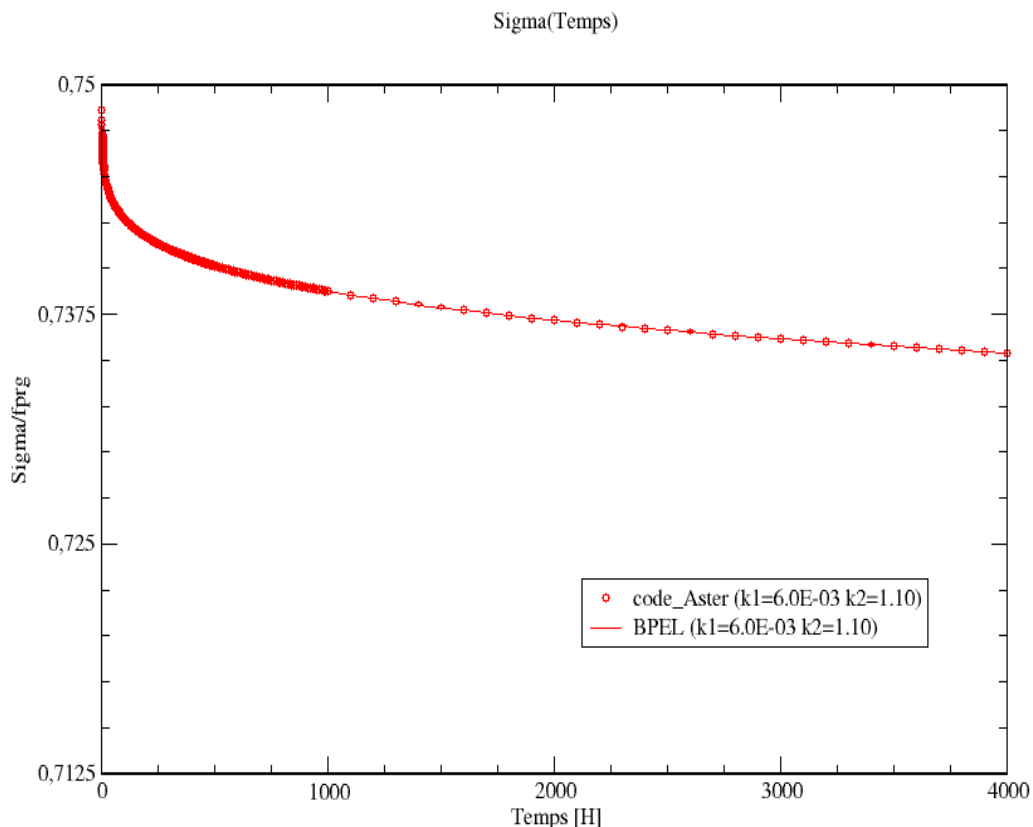The figures below compare the result of the identification with the curve lawful.

Figure 9.5-a : Evolution of the constraint in the course of time: reference and code_aster.

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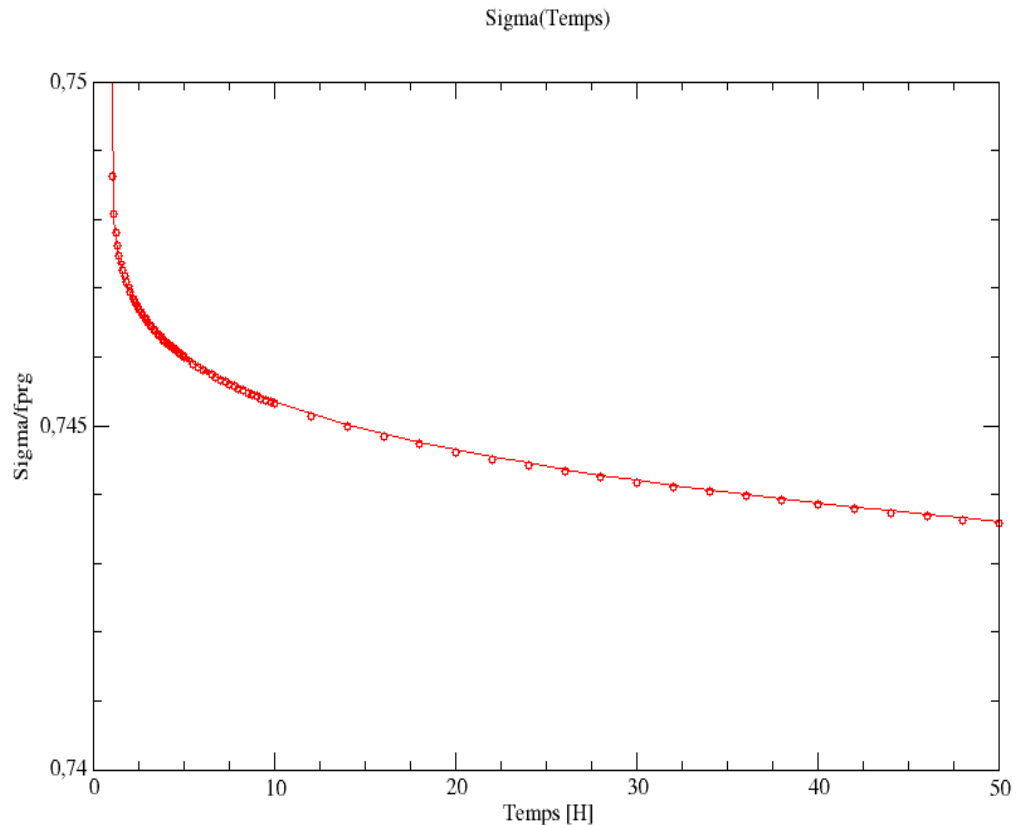*Clé : R5.03.09* *Révision :*
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Sigma(Temps)

Figure 9.5-b : Zoom over the first 10 hours, of the é volution of the constraint in the course of time.

Observations with respect to the identification:
• Very good correspondence enters the identified law and the law lawful.
• Very good identification of the slope in the beginning and the asymptote.

The law suggested is thus ready to correctly describe the relieving of a cable of prestressing, if its relieving follows the lawful curves.

9.6 Use in Code_hasster

The definition of the parameters materials is done classically in the order DEFI_MATERIAU, key word RELAX_ACIER, all the parameters can depend on the temperature.

f_{prg}	Constraint with rupture cable. This size is optional, because it can be also defined in materials BPEL_ACIER or ETCC_ACIER and in this case f_{prg} is a constant. The value/function given under RELAX_ACIER_CABL is priority.
ECOU_K ECOU_NR	Corresponds to the coefficient K, in the equation 9.1-1. Corresponds to coefficient N, in the equation 9.1-1.
ECRO_NR ECRO_B ECRO_C	Corresponds to the coefficient NR, in the equation 9.1-1. Corresponds to the coefficient B, in the equation 9.1-1. Corresponds to the coefficient C, in the equation 9.1-1.

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10   Method to use in 1D all the behaviors 3D

As for the treatment of the plane constraints [R5.03.03] , it is possible to profit for modelings 1D behaviors available in 3D . One extends for that the method due to R.de Borst to the case 1D , by treating this condition (unidimensional stress field) not with the level of the law of behavior but with the level of balance. One obtains thus during iterations of the algorithm of STAT_NON_LINE stress fields which tend towards an one-way field. One checks, with convergence of the total iterations of Newton, that the stress fields are indeed one-way, except for a precision, if not one continues the iterations. The method consists in breaking up the fields of strains and stresses into a purely one-way part (direction X) and a part relative to the other directions, and to carry out a static condensation by writing that the components of the constraints relative to the other directions are worthless. One does not consider in the tensors (order 2) only the diagonal terms, written in the form of vectors with 3 components. Direction X corresponds to the direction of the element (bars, multifibre beam) or to the direction of the reinforcements of grid. At one unspecified moment of the resolution of the incremental behavior, the tangent operator D connect the increase in constraints to the increase in deformation by:

$$d\sigma = \left[\frac{\partial \sigma}{\partial \epsilon}\right] d\epsilon = D\, d\epsilon \quad \text{that one rewrites} \quad \begin{bmatrix} d\sigma_x \\ d\sigma_y \\ d\sigma_z \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} d\epsilon_x \\ d\epsilon_y \\ d\epsilon_z \end{bmatrix}.$$

By writing these increases like the difference between the iterations n and $n+1$ of Newton, one obtains:

$$d\sigma = \sigma^{n+1} - \sigma^n = \Delta\sigma^{n+1} - \Delta\sigma^n \;,\; d\epsilon = \Delta\epsilon^{n+1} - \Delta\epsilon^n$$

With convergence, this variation must tend towards zero.

By introducing the conditions $\sigma_y^{n+1} = 0$ and σ_z^{n+1} (one-way behavior), one obtains, for the iteration $n+1$:

$$\begin{bmatrix} d\sigma_x \\ d\sigma_y \\ d\sigma_z \end{bmatrix} = \begin{pmatrix} \sigma_x^{n+1} - \sigma_x^n \\ \sigma_y^{n+1} - \sigma_y^n \\ \sigma_z^{n+1} - \sigma_z^n \end{pmatrix} = \begin{pmatrix} \sigma_x^{n+1} - \sigma_x^n \\ -\sigma_y^n \\ -\sigma_z^n \end{pmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} d\epsilon_x \\ d\epsilon_y \\ d\epsilon_z \end{bmatrix}$$

The two last equations make it possible to express $d\epsilon_y$ and $d\epsilon_z$ according to $d\epsilon_x$:

$$\begin{cases} d\epsilon_y = \dfrac{1}{D_{22}} \left(-\sigma_y^n - D_{21} d\epsilon_x - D_{23} d\epsilon_z \right) \\[2ex] d\epsilon_z = \dfrac{1}{D_{33}} \left(-\sigma_z^n - D_{31} d\epsilon_x - D_{32} d\epsilon_y \right) \end{cases}$$

that is to say

$$d\epsilon_y = \frac{1}{\Delta} \left(-D_{33}\sigma_y^n + D_{23}\sigma_z^n + D_y d\epsilon_x \right)$$

$$d\epsilon_z = \frac{1}{\Delta} \left(-D_{32}\sigma_y^n + D_{22}\sigma_z^n + D_z d\epsilon_x \right)$$

with $\Delta = D_{33} D_{22} - D_{23} D_{32,}$ $D_y = D_{23} D_{31} - D_{21} D_{33,}$ $D_z = D_{32} D_{21} - D_{31} D_{22}$

by deferring these expressions in the first equation, one obtains:

$$\sigma_x^{n+1} = \sigma_x^n + \left(D_{11} + \frac{D_{12} D_y + D_{13} D_z}{\Delta} \right) d\epsilon_x + \frac{D_{12} D_{23} - D_{22} D_{13}}{\Delta} \sigma_z^n + \frac{D_{12} D_{32} - D_{12} D_{33}}{\Delta} \sigma_y^n$$

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Balance with the iteration $n+1$ is written:

$$\int D^T \sigma^{n+1} dv = \int B^T \sigma_x^{n+1} dv = \int B^T \left(D_{11} + \frac{D_{12} D_y + D_{13} D_z}{\Delta} \right) d\varepsilon_x$$

$$+ \int B^T \left(\sigma_x^n + \frac{D_{12} D_{23} - D_{22} D_{13}}{\Delta} \sigma_z^n + \frac{D_{12} D_{32} - D_{12} D_{33}}{\Delta} \sigma_y^n \right) dv$$

$$= K^n du^{n+1} + \int B^T \left(\sigma_x^n + \frac{D_{12} D_{23} - D_{22} D_{13}}{\Delta} \sigma_z^n + \frac{D_{12} D_{32} - D_{12} D_{33}}{\Delta} \sigma_y^n \right) dv$$

It is noted that the taking into account of the unidimensional behavior intervenes on two levels:
* in the tangent matrix, by the corrective term:

$$\int B^T \frac{D_{12} D_y + D_{13} D_z}{\Delta} B \ dv$$

* in the writing of the second member, by the corrective term:

$$\frac{\int B^T}{\Delta} \left((D_{12} D_{23} - D_{22} D_{13}) \sigma_z^n + (D_{12} D_{32} - D_{12} D_{33}) \sigma_y^n \right) dv$$

To implement this method, it is enough to calculate these corrective terms and to add them to the constraints and tangent matrix obtained of the resolution 3D of the behavior. For that it is necessary to store information of an iteration of Newton to the other, by the means of 4 additional internal variables. The stages of the resolution are:
* with the iteration $n+1$, the data are: $\Delta u^{n+1}, \sigma^-, \alpha^-$ and 4 internal variables (calculated with the iteration n):

$$V1 = \Delta \varepsilon_y^n + \frac{1}{\Delta} \left(D_{23} \sigma_z^n - D_{33} \sigma_y^n - D_y \Delta \varepsilon_x^n \right), V2 = \frac{D_y}{\Delta}$$

$$V3 = \Delta \varepsilon_z^n + \frac{1}{\Delta} \left(D_{32} \sigma_z^n - D_{22} \sigma_z^n - D_z \Delta \varepsilon_x^n \right), V4 = \frac{D_z}{\Delta}$$

* before carrying out the integration of the behavior (carried out into axisymmetric) one calculates:

$$\Delta \varepsilon_y^{n+1} = \Delta \varepsilon_y^n + \frac{1}{\Delta} \left(-D_{33} \sigma_y^n + D_{23} \sigma_z^n + D_y d\varepsilon_x \right)$$

$$\Delta \varepsilon_z^{n+1} = \Delta \varepsilon_z^n + \frac{1}{\Delta} \left(-D_{32} \sigma_y^n + D_{22} \sigma_z^n + D_z d\varepsilon_x \right)$$

* the integration of the behavior provides the constraints σ^{n+1} and the tangent operator D,
* one modifies the second member and the tangent matrix as indicated above,
* the new internal variables are stored and it is checked if $\left| \sigma_z^{n+1} \right| < \eta$ and $\left| \sigma_y^{n+1} \right| < \eta$, with

$$\eta = \xi \left| \sigma_x^{n+1} \right|, \ \xi = \texttt{RESI\_INTE\_RELA}$$

Notice :
> *four additional internal variables are added after the internal variables of the law of behavior.*

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