Viscoplastic behavior with effect of memory and restoration of Chaboche

Summary:

This document describes the integration of the model of behavior élasto-visco-plastic of Chaboche with an isotropic work hardening comprising a ratchet effect of work hardening and two work hardenings nonlinear kinematics, with taking into possible account of the restoration and viscosity. This model is usable by the relation `VISCOCHAB` keyword `BEHAVIOR`. The established model has an effect of work hardening on the tensorial variables of recall and takes into account all the variations of the coefficients with the temperature[1]. This law of behavior is integrated by the resolution of a system of nonlinear equations. This model is available in 3D, plane deformation, axisymetry. Modeling in plane constraint is taken into account.

1 except for the calculation of the jacobienne

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1 Model élasto-visco-plastic of Chaboche with ratchet effect and restoration

The model elastoviscoplastic of J.L.Chaboche [bib1] most complete superimposes on isotropic work hardening two variables of kinematic work hardening, and makes it possible to take into account the effects of cyclic hardening, creep, the restoration and the memory of largest work hardening. It is thus well adapted to the cyclic loadings.

Under certain conditions of loading, the structures can be prone to a phenomenon of progressive deformation, being able to harm the functional capacities of the device. In its original form, the model largely over-estimates the progressive deformation; to improve its representativeness, it was modified by introducing terms of radial evanescence, in order to better model this phenomenon [bib2]. The resulting model, named VISCOCHAB, is described in this document.

Several studies consisted in testing this law of behavior with respect to its capacity to model the progressive deformations [bib3, bib4], while comparing with tests [bib5, bib6, bib7, bib8] and with other models [bib12] in particular that of Taheri [R5.03.05]. In particular, identifications of steels 316L and 304L were made (resp. in [bib4] and [bib9]). Recently, the studies of thermal tiredness required the use of VISCOCHAB, in particular to take into account the effect of memory.

The model comprises 25 parameters (+ 2 elastic parameters) introduced into the order DEFI_MATERIAU:

VISCOCHAB (VISCOCHAB_FO) = _F (

work hardening isotropic
♦ K = k ,
◊ B = b ,
◊ A_R = α_R , (defect: 1.)

work hardening kinematic
♦ C1 = C_1 ,
♦ C2 = C_2 ,
♦ G1_0 = γ_1^0 ,
♦ G2_0 = γ_2^0 ,
◊ A_I = a_∞ , (defect: 1.)

viscosity
♦ K_0 = K_0 ,
♦ NR = n ,

exponential flow
◊ A_K = α_K , (defect: 0.)
◊ ALP = α , (defect: 0.)

effect of memory
◊ ETA = η , (defect: 0.5)
♦ DRIVEN = μ , (defect: 0.)
♦ Q_M = Q_m ,
♦ Q_0 = Q_0 ,

progressive deformations (Burlet)

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```
◊ D1 = δ1 , (defect: 1.)
◊ D2 = δ2 , (defect: 1.)

# restoration
◊ M_R = m_r , (defect: 1.)
◊ G_R = γ_r , (defect: 0.)
◊ M_1 = m_1 , (defect: 1.)
◊ M_2 = m_2 , (defect: 1.)
◊ G_X1 = γ_X1 , (defect: 0.)
◊ G_X2 = γ_X2 , (defect: 0.)
◊ QR_0 = Q*_r , (defect: 0.)
)
```

These parameters are real constants. All these parameters can depend on the temperature (keywords `VISCOCHAB_FO`) and the expected values are of type `function`. There is no value by default in this case. It will be noted that if `C1` or `C2` depends on the temperature, the calculation of the matrix jacobienne is not exact, which can make convergence local difficult if `C1` or `C2` strongly vary with T .

The use of this law of behavior is accessible in the orders `STAT_NON_LINE` or `DYNA_NON_LINE` by the keyword `VISCOCHAB` of `BEHAVIOR`.

This complete model can be “degraded” by cancelling the effect of certain mechanisms (for example the effect of restoration). For example, by cancelling the effect of the restoration and the terms of radial evanescence of kinematic work hardening (parameters materials by default), one finds the model of Chaboche with effect of memory `VISC_CIN2_MEMO` [bib12].

For details concerning the choice of the parameters materials leading to the cancellation of certain effects, one will be able to refer to the documentation of the order `DEFI_MATERIAU`.

In the continuation of this document, one describes the model precisely `VISCOCHAB`. One presents then the detail of his digital integration in link with the construction of the coherent tangent matrix.

2 The model `VISCOCHAB` in *Code_Aster*

2.1 Description of the model

At any moment, the state of material is described by the deformation $\boldsymbol{\varepsilon}$, the temperature T , plastic deformation $\boldsymbol{\varepsilon}^{\mathbf{p}}$, cumulated plastic deformation p and the tensor of recall $\mathbf{X}$. The equations of state then define according to these variables of state the constraint $\boldsymbol{\sigma}=\sigma^H \mathbf{Id}+\tilde{\boldsymbol{\sigma}}$ (broken up into parts hydrostatic and deviatoric), the isotropic share of work hardening R and the kinematic share $\mathbf{X}$:

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$$\sigma^H=\frac{1}{3}tr(\boldsymbol{\sigma})=K\,tr(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{\mathbf{th}}) \quad \textbf{with} \quad \boldsymbol{\varepsilon}^{\mathbf{th}}=\alpha(T-T^{ref})\,\mathbf{Id}$$ **éq 2.1-1**

$$\tilde{\boldsymbol{\sigma}}=\boldsymbol{\sigma}-\sigma^H\,\mathbf{Id}=2\mu(\tilde{\boldsymbol{\varepsilon}}-\boldsymbol{\varepsilon}^{\mathbf{p}})$$ **éq 2.1-2**

$$R=R(p)$$ **éq 2.1-3**

$$\mathbf{X}=(\mathbf{p},\boldsymbol{\varepsilon}^{\mathbf{p}})$$ **éq 2.1-4**

where K,μ,α and coefficients of $\mathbf{X}(p)$ and $R(p)$ are characteristics of material which can depend on the temperature. More precisely, they are respectively the modules of compressibility and shearing, the thermal dilation coefficient, the functions of isotropic and kinematic work hardening. As for T^{ref} , it is the temperature of reference, for which one regards the thermal deformation as being worthless.

2.1.1 Surface of flow - potential viscoplastic

The surface of flow associated with the model VISCOCHAB is represented within the space of principal constraints by:

- its center $\mathbf{X}$, kinematic tensor of work hardening,
- its size $\alpha_R R+k$, k being its initial size and R the isotropic variable of work hardening giving the evolution of this size, modulated by the coefficient α_R ,
- its form given by the criterion of Von Mises steady to $\tilde{\boldsymbol{\sigma}}\text{-}\mathbf{X}$. $\tilde{\boldsymbol{\sigma}}$ is the diverter of the constraints.

The evolution of the plastic deformation is controlled by a law of normal flow to a criterion of plasticity of Von Mises:

$$F(\boldsymbol{\sigma},R,\mathbf{X})=(\tilde{\boldsymbol{\sigma}}-\mathbf{X})_{eq}-\alpha_R R(p)-k \quad \text{avec}\, A_{eq}=\sqrt{\frac{3}{2}\tilde{A}:\tilde{A}}$$ **éq 2.1-5**

The viscoplastic potential of dissipation for the model of Chaboche is written:

$$\Omega^p=\frac{K_0+\alpha_k R}{\alpha(n+1)}\exp\left[\alpha\left\langle\frac{F}{K_0+\alpha_k R}\right\rangle^{n+1}\right]$$ **éq 2.1-6**

where $\langle F\rangle$ is the positive part of F .

One from of deduced the law of evolution from the plastic deformation:

$$\dot{\boldsymbol{\varepsilon}}^p=\frac{\partial\Omega^p}{\partial\boldsymbol{\sigma}}=\dot{p}\,\frac{\partial F}{\partial\boldsymbol{\sigma}}=\frac{3}{2}\dot{p}\,\frac{\tilde{\boldsymbol{\sigma}}-\mathbf{X}}{(\tilde{\boldsymbol{\sigma}}-\mathbf{X})_{eq}}$$ **éq 2.1-7**

while having posed:

$$\dot{p}=\left\langle\frac{F}{K_0+\alpha_k R}\right\rangle^n\exp\left[\alpha\left\langle\frac{F}{K_0+\alpha_k R}\right\rangle^{n+1}\right]$$ **éq 2.1-8**

Classically, $\dot{p}$ is the speed of cumulated plastic deformation. Parameters K_0 , α_k and α are relating to the viscosity of the material (viscosity of Norton). The equation [éq 2.1-7] can be also written in an equivalent way in the following form:

$$\dot{p}=\sqrt{\frac{2}{3}\dot{\boldsymbol{\varepsilon}}^p:\dot{\boldsymbol{\varepsilon}}^p}$$ **éq 2.1-9**

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The unit normal on the surface of flow is noted:

$$\mathbf{n}=\frac{\partial F}{\partial \boldsymbol{\sigma}}\left\|\frac{\partial F}{\partial \boldsymbol{\sigma}}\right\|^{-1}=\sqrt{\frac{2}{3}}\frac{\partial F}{\partial \boldsymbol{\sigma}}=\sqrt{\frac{3}{2}}\frac{\tilde{\boldsymbol{\sigma}}-\mathbf{X}}{\left(\tilde{\boldsymbol{\sigma}}-\mathbf{X}\right)_{eq}} \quad \textbf{éq 2.1-10}$$

The laws of evolution of isotropic work hardening and kinematic work hardening are obtained starting from the thermodynamic potential and from the potential of dissipation.

2.1.2 Formulation of isotropic work hardening

Evolution of the isotropic variable of work hardening R is given by:

$$\dot{R}=b\left(Q-R\right)\dot{p}+\gamma_r\left|Q_r-R\right|^{m_r}\operatorname{sgn}\left(Q_r-R\right) \quad \textbf{éq 2.1-11}$$

This law of evolution of isotropic work hardening utilizes a first linear term according to the speed of cumulated deformation viscoplastic $\dot{p}$. This term is useful to describe the evolution of the loop (softening or hardening) in cyclic loading. This term provides an asymptotic value of R (corresponding at the stabilized state) equal to the variable Q . This variable Q represented cyclic hardening (the effect of memory of the maximum deformations). It is not a constant but it depends on the maximum amplitude of the deformation (effect of memory):

$$Q=Q_0+\left(Q_m-Q_0\right)\left(1-e^{-2\mu q}\right) \quad \textbf{éq 2.1-12}$$

One defines within the space of deformations a surface of flow inside which Q is a constant (field of not-work hardening):

$$f\left(\boldsymbol{\varepsilon}^{\mathbf{p}},\boldsymbol{\xi},q\right)=\frac{2}{3}\left(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi}\right)_{eq}-q\leq 0 \quad \textbf{éq 2.1-13}$$

This criterion defines a field characterizing the maximum plastic deformations, of which q measurement the ray and $\boldsymbol{\xi}$ the center, calculated according to a law of normality:

$$\dot{\boldsymbol{\varepsilon}}^{p}:\frac{\partial f}{\partial \boldsymbol{\varepsilon}^{\mathbf{p}}}>0 \quad \textbf{éq 2.1-14}$$

The unit normal on the surface of flow is noted:

$$\mathbf{n}^{*}=\frac{\partial f}{\partial \boldsymbol{\sigma}}\left\|\frac{\partial f}{\partial \boldsymbol{\sigma}}\right\|^{-1}=\sqrt{\frac{3}{2}}\frac{\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi}}{\left(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi}\right)_{\mathbf{eq}}} \quad \textbf{éq 2.1-15}$$

Laws of evolution of the variables q and $\boldsymbol{\xi}$ are given in the form:

$$\dot{q}=\eta H\left(f\right)\left\langle \mathbf{n}:\mathbf{n}^{*}\right\rangle \dot{p} \quad \textbf{éq 2.1-16}$$

$$\dot{\boldsymbol{\xi}}=\sqrt{\frac{3}{2}}\left(1-\eta\right)H\left(f\right)\left\langle \mathbf{n}:\mathbf{n}^{*}\right\rangle \dot{p}\ \mathbf{n}^{*} \quad \textbf{éq 2.1-17}$$

The parameter η (which does not exist in the initial formulation 5) allows to partially take into account the effect of memory. If it is equal to 1/2, the initial formulation is found. If it is worth 1, q is equal to the standard of the greatest plastic deformation reached. If it is much lower than 1/2, the effect of memory is taken into account partly only.

Note:

Same expressions for Q , f , $\dot{q}$ and $\dot{\xi}$ are used in the laws `VMIS` */* `VISC_CIN1` */* `2_MEMO` [bib11] *who are versions simplified (and optimized) of* `VISCOCHAB` *cf [R5.03.04].*

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The law of evolution of isotropic work hardening [éq 2.1-11] utilized a second term, allowing to take into account the effect of the restoration. The variable Q_r is given by the equation:

$$Q_r = Q - Q_r^* \left[1 - \left(\frac{Q_m - Q}{Q_m} \right)^2 \right] \quad \text{éq 2.1-18}$$

Let us note that in the initial model of Chaboche [bib1], the coefficient m_r in the equation [éq 2.1-11] 1 is worth.

2.1.3 Formulation of kinematic work hardening

Before giving the expression of the law of work hardening of the model VISCOCHAB, one points out the various stages which allowed its development.

The simplest law is a linear work hardening of the form:

$$\dot{\mathbf{X}} = \frac{2}{3} C \dot{\boldsymbol{\varepsilon}}^p \quad \text{éq 2.1-19}$$

With this law, one can add a non-linear term of recall providing an effect of evanescent memory of the way of loading (initial model of Chaboche):

$$\dot{\mathbf{X}} = \frac{2}{3} C \dot{\boldsymbol{\varepsilon}}^{\mathbf{p}} - \gamma \mathbf{X} \dot{p} \quad \text{éq 2.1-20}$$

with

$$\gamma = \gamma^0 \left[a_\infty + (1 - a_\infty) e^{-bp} \right] \quad \text{éq 2.1-21}$$

It was shown that such a law largely over-estimates the phenomenon of progressive deformation. This brings to introduce a term with radial evanescence (term due to Burlet and Cailletaud):

$$\dot{\mathbf{X}} = \frac{2}{3} C \dot{\varepsilon}^p - \gamma (\mathbf{X} : \mathbf{n}) \mathbf{n} \; \dot{p} \quad \text{éq 2.1-22}$$

In fact, this law underestimates the phenomenon of progressive deformation now. One can then combine the two equations [éq 2.1.20] and [éq 2.1.22] with the parameter of weighting $\delta \in [0,1]$, in order to better consider the deformations progressive:

$$\dot{\mathbf{X}} = \frac{2}{3} C \dot{\boldsymbol{\varepsilon}}^p - \left[\delta (\gamma \mathbf{X} \dot{p}) + (1 - \delta) (\gamma (\mathbf{X} : \mathbf{n}) \mathbf{n} \; \dot{p}) \right] \quad \text{éq 2.1-23}$$

In the initial model of Chaboche, one finds also a term additional which makes it possible to introduce the effects of the restoration into kinematic work hardening, which gives to final the following law:

$$\dot{\mathbf{X}} = \frac{2}{3} C \dot{\boldsymbol{\varepsilon}}^p - \gamma \left[\delta \; \mathbf{X} + (1 - \delta) (\mathbf{X} : \mathbf{n}) \mathbf{n} \right] \dot{p} - \gamma_X \left[(\mathbf{X})_{eq} \right]^{m-1} \mathbf{X} \quad \text{éq 2.1-24}$$

with γ given by the equation [éq 2.1-21].

For an exact taking into account of the dependence of the parameters materials compared to the temperature, it is necessary to add an additional term in $\dot{T}$, which gives:

$$\dot{\mathbf{X}} = \frac{2}{3} C \dot{\boldsymbol{\varepsilon}}^p - \gamma \left[\delta \; \mathbf{X} + (1 - \delta) (\mathbf{X} : \mathbf{n}) \mathbf{n} \right] \dot{p} - \gamma_X \left[(\mathbf{X})_{eq} \right]^{m-1} \mathbf{X} + \frac{1}{C} \frac{\partial C}{\partial T} \mathbf{X} \dot{T} \quad \text{éq 2.1-25}$$

with γ given by the equation [éq 2.1-21].

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The model VISCOCHAB proposed comprises in fact 2 variables of kinematic work hardening $\mathbf{X_1}$ and $\mathbf{X_2}$ whose laws of evolutions are given by the equation [éq 2.1-25].

Assessment:
The law of evolution of work hardening is form:

$$\begin{cases} \mathbf{X}=\mathbf{X_1}+\mathbf{X_2} \\ \dot{\mathbf{X}}_i=\frac{2}{3}C_i\dot{\boldsymbol{\varepsilon}}^p-\gamma_i\left[\delta_i\,\mathbf{X_i}+\left(1-\delta_i\right)\left(\mathbf{X_i}:\mathbf{n}\right)\mathbf{n}\right]\dot{p} \\ \quad -\gamma_{Xi}\left[\left(\mathbf{X_i}\right)_{eq}\right]^{m_i-1}\mathbf{X_i}+\frac{1}{C_i}\frac{\partial C_i}{\partial T}\mathbf{X_i}\dot{T}\quad i=1,2 \end{cases} \qquad \text{éq 2.1-26}$$

with

$$\gamma_i=\gamma_i^0\left[a_\infty+\left(1-a_\infty\right)e^{-bp}\right]\quad i=1,2 \qquad \text{éq 2.1-27}$$

2.2 Implicit integration

To numerically carry out the integration of the law of behavior, one carries out a discretization in time and one adopts a diagram of implicit, famous Euler adapted for elastoplastic relations of behavior. It is the method used by default. It is also possible to carry out an explicit integration (see §2.3) by choosing the keyword ALGO_INTE=' RUNGE_KUTTA'.
Henceforth, the following notations will be employed: A^-, A^+ and ΔA represent respectively the values of a quantity at the beginning and the step of time considered thus that its increment during the step. There is thus the relation: $\Delta A=A^+-A^-$. The problem is then the following: knowing the state at time t^- as well as the increments of deformation $\Delta\boldsymbol{\varepsilon}$ (resulting from the phase of prediction (cf. STAT_NON_LINE [R5.03.01]) and of temperature ΔT , to determine the state of the internal variables at time t as well as the constraints $\boldsymbol{\sigma}$.

2.2.1 System of equations

Implicit discretization of the problem led to a system of 27 equations:

Relation stress-strain	$\Delta\boldsymbol{\sigma}-H\left(\Delta\boldsymbol{\varepsilon}-\Delta\boldsymbol{\varepsilon}^{\mathbf{th}}-\Delta p\frac{\partial F}{\partial\boldsymbol{\sigma}^+}\right)=0$	6 éq	éq 2.2-1
Kinematic work hardening	$\begin{cases} i=1,2 \\ \Delta\mathbf{X_i}-\frac{2}{3}C_i\Delta\boldsymbol{\varepsilon}^{\mathbf{p}}+\gamma_i^+\left[\delta_i\,\mathbf{X_i^+}+\left(1-\delta_i\right)\left(\mathbf{X_i^+}:\mathbf{n}^+\right)\mathbf{n}^+\right]\Delta p \\ \quad +\gamma_{Xi}\left[\left(\Delta\mathbf{X_i^+}\right)_{eq}\right]^{m_i-1}\Delta\mathbf{X_i^+}\Delta t-\frac{1}{C_i}\frac{\partial C_i}{\partial T}\mathbf{X_i^+}\Delta T=0 \end{cases}$	12 éq	éq 2.2-2
Cumulated plasticity	$\Delta p-\Delta t\left\langle\frac{F^+}{K_0+\alpha_k R^+}\right\rangle^n\exp\left[\alpha\left\langle\frac{F^+}{K_0+\alpha_k R^+}\right\rangle^{n+1}\right]=0$	1 éq	éq 2.2-3
Isotropic work hardening	$\Delta R-b\left(Q^+-R^+\right)\Delta p-\gamma_r\left\|Q_r^+-R^+\right\|^{m_r}\mathrm{sgn}\left(Q_r^+-R^+\right)\Delta t=0$	1 éq	éq 2.2-4
Effect of memory	$\Delta q-\eta H\left(f\right)\left\langle\mathbf{n}:\mathbf{n}^*\right\rangle^+\Delta p=0$	1 éq	éq 2.2-5

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$$\Delta \xi - \sqrt{\frac{3}{2}}(1-\eta) H(f) \langle \mathbf{n} : \mathbf{n}^* \rangle^+ \Delta p \, (\mathbf{n}^*)^+ = 0$$ 6 éq **éq 2.2-6**

In this system, γ_i^+ is given according to p^+ by the equation [éq. 2.1-27]; Q^+ and Q_r^+ are obtained according to q^+ by the equations [éq. 2.1-12] and [éq. 2.1-18].

The 27 unknown factors are: $\Delta \boldsymbol{\sigma}$, $\Delta \mathbf{X_1}$, $\Delta \mathbf{X_2}$, Δp , ΔR , Δq and $\Delta \boldsymbol{\xi}$.

Note:
Contrary to VISC_CIN2_MEMO [R5.03.04], the equation of checking of the threshold of the maximum deformations: $f(\boldsymbol{\varepsilon}^{\mathbf{p}}, \boldsymbol{\xi}, q) = \frac{2}{3}(\boldsymbol{\varepsilon}^{\mathbf{p}} - \boldsymbol{\xi})_{eq} - q = 0$ (see [éq. 2.1-13]) is not taken into account.

2.2.2 Outline general of resolution

One calculates the constraint by making the assumption of a purely elastic increment ($\Delta p = 0$).

$$\Delta \boldsymbol{\sigma} = H \Delta \boldsymbol{\varepsilon} \quad \text{avec} \quad \Delta \boldsymbol{\varepsilon}^{\mathbf{p}} = \Delta \mathbf{X_i} = \Delta \boldsymbol{\xi} = 0 \qquad \Delta R = \Delta p = \Delta q = 0$$ **éq 2.3-1**

The function threshold is calculated $F^+ = (\tilde{\boldsymbol{\sigma}} - \mathbf{X})^+_{eq} - \alpha_R R^+ - k$. If $F^+ \leq 0$ then the increment is purely elastic and integration is finished. If not of the viscoplastic corrections are carried out by solving the system (S1) according to:

Relation stress-strain	$\Delta \boldsymbol{\sigma} - H\left(\Delta \boldsymbol{\varepsilon} - \Delta \boldsymbol{\varepsilon}^{\mathbf{th}} - \Delta p \frac{\partial F}{\partial \boldsymbol{\sigma}^+}\right) = 0$	6 éq	**éq 2.3-2**
Kinematic work hardening	$\begin{cases} i=1,2 \\ \Delta \mathbf{X_i} - \frac{2}{3} C_i \Delta \boldsymbol{\varepsilon}^{\mathbf{p}} + \gamma_i^+ \left[\delta_i \mathbf{X_i^+} + (1-\delta_i)(\mathbf{X_i^+} : \mathbf{n}^+) \mathbf{n}^+\right] \Delta p \\ \quad + \gamma_{Xi} \left[(\mathbf{X_i^+})_{eq}\right]^{m_i - 1} \mathbf{X_i^+} \Delta t - \frac{1}{C_i} \frac{\partial C_i}{\partial T} \mathbf{X_i} \Delta T = 0 \end{cases}$	12 éq	**éq 2.3-3**
Cumulated plasticity	$\Delta p - \Delta t \left(\frac{F^+}{K_0 + \alpha_k R^+}\right)^n \exp\left[\alpha \left(\frac{F^+}{K_0 + \alpha_k R^+}\right)^{n+1}\right] = 0$	1 éq	**éq 2.3-4**
Isotropic work hardening	$\Delta R - b(Q^+ - R^+) \Delta p - \gamma_r \lvert Q_r^+ - R^+ \rvert^{m_r} \operatorname{sgn}(Q_r^+ - R^+) \Delta t = 0$	1 éq	**éq 2.3-5**
Effect of memory	$\Delta q = 0$	1 éq	**éq 2.3-6**
	$\Delta \boldsymbol{\xi} = 0$	6 éq	**éq 2.3-7**

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The 20 unknown factors are: $\Delta\boldsymbol{\sigma}$, $\Delta\mathbf{X_1}$, $\Delta\mathbf{X_2}$, Δp , ΔR . It is noticed that Δq and $\Delta\boldsymbol{\xi}$ in are not part of the unknown factors because their solution is commonplace.

It is also noticed that, at this stage, one can check that $F^{+}>0$, and like $K_0>0$, $\alpha_k>0$ and $R^{+}>0$, one formally replaced the hooks (left positive) of the equation [éq 2.2.3] by simple brackets in the equation [éq 2.3-4].

One calculates then the dual function f "envelope surface of the maximum deformations":

$$f^{+}=\frac{2}{3}\left(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi}\right)^{+}_{eq}-q^{+} \qquad \textbf{éq 2.3-8}$$

If $f^{+}\leq 0$ then integration is finished. If not, one solves the system (S2) according to:

Relation stress-strain	$\Delta\boldsymbol{\sigma}-H\left(\Delta\boldsymbol{\varepsilon}-\Delta\boldsymbol{\varepsilon}^{\mathbf{th}}-\Delta p\frac{\partial F}{\partial\boldsymbol{\sigma}^{+}}\right)=0$	6 éq	**éq 2.3-9**
Kinematic work hardening	$\begin{cases} i=1{,}2 \\ \Delta\mathbf{X_i}-\frac{2}{3}C_i\Delta\boldsymbol{\varepsilon}^{\mathbf{p}}+\gamma_i^{+}\left[\delta_i\,\mathbf{X_i^{+}}+(1-\delta_i)\left(\mathbf{X_i^{+}}:\mathbf{n}^{+}\right)\mathbf{n}^{+}\right]\Delta p \\ \quad +\gamma_{Xi}\left[\left(\mathbf{X_i^{+}}\right)_{eq}\right]^{m_i-1}\mathbf{X_i^{+}}\Delta t-\frac{1}{C_i}\frac{\partial C_i}{\partial T}\mathbf{X_i}\Delta T=0 \end{cases}$	12 éq	**éq 2.3-10**
Cumulated plasticity	$\Delta p-\Delta t\left(\frac{F^{+}}{K_0+\alpha_k R^{+}}\right)^{n}\exp\left[\alpha\left(\frac{F^{+}}{K_0+\alpha_k R^{+}}\right)^{n+1}\right]=0$	1 éq	**éq 2.3-11**
Isotropic work hardening	$\Delta R-b\left(Q^{+}-R^{+}\right)\Delta p-\gamma_r\left\|Q_r^{+}-R^{+}\right\|^{m_r}\mathrm{sgn}\left(Q_r^{+}-R^{+}\right)\Delta t=0$	1 éq	**éq 2.3-12**
Effect of memory	$\Delta q-\eta\langle\mathbf{n}:\mathbf{n}^{*}\rangle^{+}\Delta p=0$	1 éq	**éq 2.3-13**
	$\Delta\boldsymbol{\xi}-\sqrt{\frac{3}{2}}(1-\eta)\langle\mathbf{n}:\mathbf{n}^{*}\rangle^{+}\Delta p\left(\mathbf{n}^{*}\right)^{+}=0$	6 éq	**éq 2.3-14**

The 27 unknown factors are: $\Delta\boldsymbol{\sigma}$, $\Delta\mathbf{X_1}$, $\Delta\mathbf{X_2}$, Δp , ΔR , Δq and $\Delta\boldsymbol{\xi}$.
It is noticed that since $f^{+}>0$, then one could replace $H(f)$ equations [éq 2.2-5] and [éq 2.2-6] by 1 in the equations [éq 2.3-13] and [éq 2.3-14].

The system (S1) (resp. (S2)) with 20 (resp is a non-linear implicit system. 27) equations and 20 (resp. 27) unknown.
One can formally write his systems: $\phi(\Delta Y)=0$, with:

- $\Delta Y=\left(\Delta\boldsymbol{\sigma},\Delta\mathbf{X_1},\Delta\mathbf{X_2},\Delta p,\Delta R\right)^{t}$ for (S1),
- and $\Delta Y=\left(\Delta\boldsymbol{\sigma},\Delta\mathbf{X_1},\Delta\mathbf{X_2},\Delta p,\Delta R,\Delta q,\Delta\boldsymbol{\xi}\right)^{t}$ for (S2).

These non-linear systems are solved by the iterative method of Newton (in environment PLASTI describes for example in [R5.03.10]):

$$\phi(\Delta Y_k)+\left(\frac{\partial\phi}{\partial\Delta Y}\right)_{\Delta Y_k}\left(\Delta Y_{k+1}-\Delta Y_k\right)=0$$

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while reiterating in k until convergence and while starting with an initial solution ensuring this convergence.

2.2.3 Calculation of Jacobienne

According to whether one has to solve the system (S1) or (S2), one breaks up the system $\phi(\Delta Y)=0$ in subsystems:

$$\phi(\Delta Y)=\begin{bmatrix} g(\Delta Y) \\ l(\Delta Y) \\ j(\Delta Y) \\ f(\Delta Y) \\ r(\Delta Y) \end{bmatrix} \text{ for (S1) and } \phi(\Delta Y)=\begin{bmatrix} g(\Delta Y) \\ l(\Delta Y) \\ j(\Delta Y) \\ f(\Delta Y) \\ r(\Delta Y) \\ h(\Delta Y) \\ c(\Delta Y) \end{bmatrix} \text{ for (S2).}$$

Where:

g represent the relation stress-strain
l represent the equations of kinematic work hardening $\mathbf{X_1}$
j represent the equations of kinematic work hardening $\mathbf{X_2}$
f represent the equation defining cumulated plasticity p
r represent the equation defining isotropic work hardening $R(p)$
h represent the equation defining the effect of memory q
c represent the equations defining the effect of memory $\boldsymbol{\xi}$

The Jacobienne matrix of the system is the matrix J written per blocks:

$$J=\begin{bmatrix}
\frac{\partial g}{\partial(\Delta\sigma)} & \frac{\partial g}{\partial(\Delta X_1)} & \frac{\partial g}{\partial(\Delta X_2)} & \frac{\partial g}{\partial(\Delta p)} & \frac{\partial g}{\partial(\Delta R)} & \frac{\partial g}{\partial(\Delta q)} & \frac{\partial g}{\partial(\Delta\xi)} \\
\frac{\partial l}{\partial(\Delta\sigma)} & \frac{\partial l}{\partial(\Delta X_1)} & \frac{\partial l}{\partial(\Delta X_2)} & \frac{\partial l}{\partial(\Delta p)} & \frac{\partial l}{\partial(\Delta R)} & \frac{\partial l}{\partial(\Delta q)} & \frac{\partial l}{\partial(\Delta\xi)} \\
\frac{\partial j}{\partial(\Delta\sigma)} & \frac{\partial j}{\partial(\Delta X_1)} & \frac{\partial j}{\partial(\Delta X_2)} & \frac{\partial j}{\partial(\Delta p)} & \frac{\partial j}{\partial(\Delta R)} & \frac{\partial j}{\partial(\Delta q)} & \frac{\partial j}{\partial(\Delta\xi)} \\
\frac{\partial f}{\partial(\Delta\sigma)} & \frac{\partial f}{\partial(\Delta X_1)} & \frac{\partial f}{\partial(\Delta X_2)} & \frac{\partial f}{\partial(\Delta p)} & \frac{\partial f}{\partial(\Delta R)} & \frac{\partial f}{\partial(\Delta q)} & \frac{\partial f}{\partial(\Delta\xi)} \\
\frac{\partial r}{\partial(\Delta\sigma)} & \frac{\partial r}{\partial(\Delta X_1)} & \frac{\partial r}{\partial(\Delta X_2)} & \frac{\partial r}{\partial(\Delta p)} & \frac{\partial r}{\partial(\Delta R)} & \frac{\partial r}{\partial(\Delta q)} & \frac{\partial r}{\partial(\Delta\xi)} \\
\frac{\partial h}{\partial(\Delta\sigma)} & \frac{\partial h}{\partial(\Delta X_1)} & \frac{\partial h}{\partial(\Delta X_2)} & \frac{\partial h}{\partial(\Delta p)} & \frac{\partial h}{\partial(\Delta R)} & \frac{\partial h}{\partial(\Delta q)} & \frac{\partial h}{\partial(\Delta\xi)} \\
\frac{\partial c}{\partial(\Delta\sigma)} & \frac{\partial c}{\partial(\Delta X_1)} & \frac{\partial c}{\partial(\Delta X_2)} & \frac{\partial c}{\partial(\Delta p)} & \frac{\partial c}{\partial(\Delta R)} & \frac{\partial c}{\partial(\Delta q)} & \frac{\partial c}{\partial(\Delta\xi)}
\end{bmatrix}$$

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Each term of this not-symmetrical matrix is clarified in Appendix [§7.1]. It will be noted that the terms $\frac{\partial l}{\partial(\Delta X_1)}$, $\frac{\partial l}{\partial(\Delta X_2)}$, $\frac{\partial j}{\partial(\Delta X_1)}$ and $\frac{\partial j}{\partial(\Delta X_2)}$ do not take account of the dependence in ΔT (see [éq 2.3-10]).

2.3 Explicit integration

To carry out the explicit integration of the law of behavior, one uses the method of Runge-Kutta [R5.03.14]. One thus integrates directly by this method the system of 27 differential equations according to:

Flow	$\dot{\boldsymbol{\varepsilon}}^p=\frac{\partial\Omega^p}{\partial\boldsymbol{\sigma}}=\dot{p}\,\frac{\partial F}{\partial\boldsymbol{\sigma}}=\frac{3}{2}\dot{p}\,\frac{\tilde{\boldsymbol{\sigma}}-\mathbf{X}}{(\tilde{\boldsymbol{\sigma}}-\mathbf{X})_{eq}}$	6 éq	**éq 2.3-10**
Kinematic work hardening	$\dot{\boldsymbol{\alpha}}_i=\dot{\boldsymbol{\varepsilon}}^p-\gamma_i\left[\delta_i\,\boldsymbol{\alpha}_\mathbf{i}+(1-\delta_i)(\boldsymbol{\alpha}_\mathbf{i}:\mathbf{n})\,\mathbf{n}\right]\dot{p}-\gamma_{Xi}\left[(\mathbf{X}_\mathbf{i})_{eq}\right]^{m_i-1}\boldsymbol{\alpha}_\mathbf{i}\quad i=1,2$	12 éq	**éq 2.3-11**
Cumulated plasticity	$\dot{p}=\left\langle\frac{F}{K_0+\alpha_k R}\right\rangle^n\exp\left[\alpha\left\langle\frac{F}{K_0+\alpha_k R}\right\rangle^{n+1}\right]$	1 éq	**éq 2.3-12**
Isotropic work hardening	$\dot{R}=b(Q-R)\,\dot{p}+\gamma_r\lvert Q_r-R\rvert^{m_r}\,\mathrm{sgn}\,(Q_r-R)$	1 éq	**éq 2.3-13**
Effect of memory	$\dot{q}=\eta\,H(f)\langle\mathbf{n}:\mathbf{n}^*\rangle\,\dot{p}$	1 éq	**éq 2.3-14**
	$\dot{\boldsymbol{\xi}}=\sqrt{\frac{3}{2}}(1-\eta)H(f)\langle\mathbf{n}:\mathbf{n}^*\rangle\,\dot{p}\;\mathbf{n}^*$	6 éq	**éq 2.3-15**

with:

$$\tilde{\boldsymbol{\sigma}}=\boldsymbol{\sigma}-\sigma^H\,\mathbf{Id}=2\mu\,(\tilde{\boldsymbol{\varepsilon}}-\boldsymbol{\varepsilon}^\mathbf{p})$$

$$A_{eq}=\sqrt{\frac{3}{2}\tilde{\mathbf{A}}:\tilde{\mathbf{A}}}$$

$$F(\boldsymbol{\sigma},R,\mathbf{X})=(\tilde{\boldsymbol{\sigma}}-\mathbf{X})_{eq}-\alpha_R R(p)-k$$

$$\mathbf{X}=\mathbf{X_1}+\mathbf{X_2}$$

$$\mathbf{X_1}=\frac{2}{3}C_1\boldsymbol{\alpha}_\mathbf{1}\qquad\mathbf{X_2}=\frac{2}{3}C_2\boldsymbol{\alpha}_\mathbf{2}$$

$$Q=Q_0+(Q_m-Q_0)(1-e^{-2\mu q})$$

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$$Q_r = Q - Q_r^* \left[1 - \left(\frac{Q_m - Q}{Q_m} \right)^2 \right]$$

$$f\left(\boldsymbol{\varepsilon}^{\mathbf{p}}, \boldsymbol{\xi}, q\right) = \frac{2}{3}\left(\boldsymbol{\varepsilon}^{\mathbf{p}} - \boldsymbol{\xi}\right)_{eq} - q$$

$$\mathbf{n}^* = \sqrt{\frac{3}{2}} \frac{\boldsymbol{\varepsilon}^{\mathbf{p}} - \boldsymbol{\xi}}{\left(\boldsymbol{\varepsilon}^{\mathbf{p}} - \boldsymbol{\xi}\right)_{\mathbf{eq}}}$$

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2.4 Significance of the internal variables

Internal variables of the model VISCOCHAB at the points of Gauss (VARI_ELGA) are:

in implicit 3D	in 2D implicit	Runge-Kutta
V1 = X_{1xx}	V1 = X_{1xx}	V1 = ε^{p}_{xx}
V2 = X_{1yy}	V2 = X_{1yy}	V2 = ε^{p}_{yy}
V3 = X_{1zz}	V3 = X_{1zz}	V3 = ε^{p}_{zz}
V4 = X_{1xy}	V4 = X_{1xy}	V4 = ε^{p}_{xy}
V5 = X_{1xz}	V5 = X_{2xx}	V5 = ε^{p}_{xz}
V6 = X_{1yz}	V6 = X_{2yy}	V6 = ε^{p}_{yz}
V7 = X_{2xx}	V7 = X_{2zz}	V7 = α_{1xx}
V8 = X_{2yy}	V8 = X_{2xy}	V8 = α_{1yy}
V9 = X_{2zz}	V9 = p	V9 = α_{1zz}
V10 = X_{2xy}	V10 = R	V10 = α_{1xy}
V11 = X_{2xz}	V11 = q	V11 = α_{1xz}
V12 = X_{2yz}	V12 = ξ_{xx}	V12 = α_{1yz}
V13 = p	V13 = ξ_{yy}	V13 = α_{2xx}
V14 = R	V14 = ξ_{zz}	V14 = α_{2yy}
V15 = q	V15 = ξ_{xy}	V15 = α_{2zz}
V16 = ξ_{xx}	V16 = ζ	V16 = α_{2xy}
V17 = ξ_{yy}		V17 = α_{2xz}
V18 = ξ_{zz}		V18 = α_{2yz}
V19 = ξ_{xy}		V19 = ξ_{xx}
V20 = ξ_{xz}		V20 = ξ_{yy}
V21 = ξ_{yz}		V21 = ξ_{zz}
V22 = ζ		V22 = ξ_{xy}
		V23 = ξ_{xz}
		V24 = ξ_{yz}
		V25 = R
		V26 = q

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		V27 = p
		V27 = 0

- the indicator ζ 1 is worth if the point of Gauss plasticized during the increment or 0 if not
- p represent the cumulated equivalent plastic deformation (positive or worthless)

3 Features and checking

The law of behavior is defined by the keyword VISCOCHAB (keyword factor BEHAVIOR orders STAT_NON_LINE, DYNA_NON_LINE, SIMU_POINT_MAT,…). It is associated with materials VISCOCHAB and VISCOCHAB_FO (order DEFI_MATERIAU).

The law VISCOCHAB is checked in particular by the cases following tests:

COMP002I	[V6.07.102]	Elementary test of robustness and reliability viscoplastic behaviors.
COMP010I	[V6.07.110]	Elementary validation of the taking into account of the temperature in the viscoplastic behaviors.
HSNV125D	[V7.22.125]	Element of volume in traction/shearing and temperature variables (comparison with other codes)
SSND105B	[V6.08.105]	Law of behavior visco-élasto-plastic with effect of memory. Comparison with VISC_CIN2_MEMO
SSND111A	[V6.08.111]	Effect of memory in a cyclic test. Comparison with VISC_CIN2_MEMO
SSNV118	[V6.04.118]	Tensile test shearing with the viscoplastic model of Chaboche. Comparison with the software SIDOLO

4 Identification of the parameters of the model

The model suggested being very close to that of Chaboche, one will be able to refer to [bib4] for the identification of the parameters of the initial model of Chaboche.

For the identification of the additional parameters, the reference [bib4] presents the tests used to supplement the identification of steel 316 SPH.

More recently, the identification of steel 304L was carried out without taking account of the phenomena of restoration and progressive deformation [bib10].

5 Bibliography

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6 Description of the versions of the document

Version Aster	Author (S) or contributor (S), organization	Description of the modifications
9.4	S. Geniaut EDF/R & D /AMA	Initial text, law VISCOCHAB
10.5	J.M.Proix EDF/R & D /AMA	Correction of the § significance of the internal variables and addition of the § features and checking

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7 Appendices

7.1 Expression of the terms of the Jacobienne matrix

In calculations presented here, one will omit the exhibitor "+" to indicate the quantities at the current moment (fine of the step of time).

The Jacobienne matrix of the system is the matrix $\mathbf{J}$ written per blocks:

$$\mathbf{J}=\begin{bmatrix}
\frac{\partial g}{\partial(\Delta\boldsymbol{\sigma})} & \frac{\partial g}{\partial(\Delta\mathbf{X_1})} & \frac{\partial g}{\partial(\Delta\mathbf{X_2})} & \frac{\partial g}{\partial(\Delta p)} & \frac{\partial g}{\partial(\Delta R)} & \frac{\partial g}{\partial(\Delta q)} & \frac{\partial g}{\partial(\Delta\boldsymbol{\xi})} \\
\frac{\partial l}{\partial(\Delta\boldsymbol{\sigma})} & \frac{\partial l}{\partial(\Delta\mathbf{X_1})} & \frac{\partial l}{\partial(\Delta\mathbf{X_2})} & \frac{\partial l}{\partial(\Delta p)} & \frac{\partial l}{\partial(\Delta R)} & \frac{\partial l}{\partial(\Delta q)} & \frac{\partial l}{\partial(\Delta\boldsymbol{\xi})} \\
\frac{\partial j}{\partial(\Delta\boldsymbol{\sigma})} & \frac{\partial j}{\partial(\Delta\mathbf{X_1})} & \frac{\partial j}{\partial(\Delta\mathbf{X_2})} & \frac{\partial j}{\partial(\Delta p)} & \frac{\partial j}{\partial(\Delta R)} & \frac{\partial j}{\partial(\Delta q)} & \frac{\partial j}{\partial(\Delta\boldsymbol{\xi})} \\
\frac{\partial f}{\partial(\Delta\boldsymbol{\sigma})} & \frac{\partial f}{\partial(\Delta\mathbf{X_1})} & \frac{\partial f}{\partial(\Delta\mathbf{X_2})} & \frac{\partial f}{\partial(\Delta p)} & \frac{\partial f}{\partial(\Delta R)} & \frac{\partial f}{\partial(\Delta q)} & \frac{\partial f}{\partial(\Delta\boldsymbol{\xi})} \\
\frac{\partial r}{\partial(\Delta\boldsymbol{\sigma})} & \frac{\partial r}{\partial(\Delta\mathbf{X_1})} & \frac{\partial r}{\partial(\Delta\mathbf{X_2})} & \frac{\partial r}{\partial(\Delta p)} & \frac{\partial r}{\partial(\Delta R)} & \frac{\partial r}{\partial(\Delta q)} & \frac{\partial r}{\partial(\Delta\boldsymbol{\xi})} \\
\frac{\partial h}{\partial(\Delta\boldsymbol{\sigma})} & \frac{\partial h}{\partial(\Delta\mathbf{X_1})} & \frac{\partial h}{\partial(\Delta\mathbf{X_2})} & \frac{\partial h}{\partial(\Delta p)} & \frac{\partial h}{\partial(\Delta R)} & \frac{\partial h}{\partial(\Delta q)} & \frac{\partial h}{\partial(\Delta\boldsymbol{\xi})} \\
\frac{\partial c}{\partial(\Delta\boldsymbol{\sigma})} & \frac{\partial c}{\partial(\Delta\mathbf{X_1})} & \frac{\partial c}{\partial(\Delta\mathbf{X_2})} & \frac{\partial c}{\partial(\Delta p)} & \frac{\partial c}{\partial(\Delta R)} & \frac{\partial c}{\partial(\Delta q)} & \frac{\partial c}{\partial(\Delta\boldsymbol{\xi})}
\end{bmatrix}$$

<u>terms related to the elastic relation stress-strain:</u>

$$g(\Delta Y)=\Delta\boldsymbol{\sigma}-H\left(\Delta\boldsymbol{\varepsilon}-\Delta\boldsymbol{\varepsilon}^{\mathbf{th}}-\Delta p\frac{\partial F}{\partial\boldsymbol{\sigma}^{+}}\right)$$

$$\frac{\partial g}{\partial(\Delta\boldsymbol{\sigma})}=I_d+H\left(\frac{\partial^2 F}{\partial^2\boldsymbol{\sigma}}\right)\Delta p$$

$$\frac{\partial g}{\partial(\Delta\mathbf{X_i})}=H\left(\frac{\partial^2 F}{\partial\mathbf{X_i}\partial\boldsymbol{\sigma}}\right)\Delta p$$

$$\frac{\partial g}{\partial(\Delta p)}=H\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}\right)$$

$$\frac{\partial g}{\partial(\Delta R)}=0,\quad \frac{\partial g}{\partial(\Delta q)}=0,\quad \frac{\partial g}{\partial(\Delta\boldsymbol{\xi})}=0$$

-

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terms related to kinematic work hardening:

$$k_i(\Delta Y)=\Delta \mathbf{X_i}-\frac{2}{3}C_i\Delta \boldsymbol{\varepsilon}^{\mathbf{p}}+\gamma_i^+\left[\delta_i\,\mathbf{X_i^+}+(1-\delta_i)(\mathbf{X_i^+}:\mathbf{n}^+)\mathbf{n}^+\right]\Delta p+\gamma_{Xi}\left[(\mathbf{X_i^+})_{eq}\right]^{m_i-1}\mathbf{X_i^+}\Delta t \quad i=1,2$$

with:

$$n=\sqrt{\frac{2}{3}}\frac{\partial F}{\partial \boldsymbol{\sigma}}=\sqrt{\frac{3}{2}}\frac{\tilde{\boldsymbol{\sigma}}-\mathbf{X}}{(\tilde{\boldsymbol{\sigma}}-\mathbf{X})_{eq}} \text{ and } \Delta \boldsymbol{\varepsilon}^{\mathbf{p}}=\Delta p\frac{\partial F}{\partial \boldsymbol{\sigma}}$$

$$\frac{\partial k_i}{\partial(\Delta \boldsymbol{\sigma})}=-\frac{2}{3}C_i\Delta p\frac{\partial^2 F}{\partial \boldsymbol{\sigma}^2}+\gamma_i\frac{2}{3}(1-\delta_i)\Delta p\left[\left(X_i:\frac{\partial F}{\partial \boldsymbol{\sigma}}\right)\frac{\partial^2 F}{\partial \boldsymbol{\sigma}^2}+\left(\frac{\partial^2 F}{\partial \boldsymbol{\sigma}^2}:\mathbf{X_i}\right)\otimes\frac{\partial F}{\partial \boldsymbol{\sigma}}\right]$$

$$\begin{aligned}\frac{\partial k_i}{\partial(\Delta \mathbf{X_i})}=&\mathbf{I_d}-\frac{2}{3}C_i\Delta p\frac{\partial^2 F}{\partial \mathbf{X_i}\partial \boldsymbol{\sigma}}\\&+\gamma_i\Delta p\left\{\delta_i\mathbf{I_d}+\frac{2}{3}(1-\delta_i)\left[\left(\mathbf{X_i}:\frac{\partial F}{\partial \boldsymbol{\sigma}}\right)\frac{\partial^2 F}{\partial \mathbf{X_i}\partial \boldsymbol{\sigma}}+\frac{\partial F}{\partial \boldsymbol{\sigma}}\otimes\frac{\partial F}{\partial \boldsymbol{\sigma}}+\left(\frac{\partial^2 F}{\partial \mathbf{X_i}\partial \boldsymbol{\sigma}}:\mathbf{X_i}\right)\otimes\frac{\partial F}{\partial \boldsymbol{\sigma}}\right]\right\}\\&+\gamma_{Xi}\Delta t\left[(\mathbf{X_i})_{eq}\right]^{m_i-1}\mathbf{I_d}+\gamma_{Xi}\Delta t(m_i-1)\frac{\partial(\mathbf{X_i})_{eq}}{\partial \mathbf{X_i}}\left[(\mathbf{X_i})_{eq}\right]^{m_i-2}\mathbf{X_i}\end{aligned}$$

$$\frac{\partial k_i}{\partial(\Delta \mathbf{X_j})_{(j\neq i)}}=-\frac{2}{3}C_i\Delta p\frac{\partial^2 F}{\partial \mathbf{X_j}\partial \boldsymbol{\sigma}}+\gamma_i\frac{2}{3}(1-\delta_i)\Delta p\left[\left(\mathbf{X_i}:\frac{\partial F}{\partial \boldsymbol{\sigma}}\right)\frac{\partial^2 F}{\partial \mathbf{X_j}\partial \boldsymbol{\sigma}}+\left(\frac{\partial^2 F}{\partial \mathbf{X_j}\partial \boldsymbol{\sigma}}:\mathbf{X_i}\right)\otimes\frac{\partial F}{\partial \boldsymbol{\sigma}}\right]$$

$$\frac{\partial k_i}{\partial(\Delta p)}=-\frac{2}{3}C_i\frac{\partial F}{\partial \boldsymbol{\sigma}}+\left(\gamma_i'(p)\Delta p+\gamma_i(p)\right)\left[\delta_i\mathbf{X_i}+\frac{2}{3}(1-\delta_i)\left(\mathbf{X_i}:\frac{\partial F}{\partial \boldsymbol{\sigma}}\right)\frac{\partial F}{\partial \boldsymbol{\sigma}}\right]$$

$$\frac{\partial k_i}{\partial(\Delta R)}=0,\quad \frac{\partial k_i}{\partial(\Delta q)}=0,\quad \frac{\partial k_i}{\partial(\Delta \boldsymbol{\xi})}=0$$

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terms related to cumulated plasticity:

$$f(\Delta Y)=\Delta p-\Delta t\left(\frac{F^+}{K_0+\alpha_k R^+}\right)^n \exp\left[\alpha\left(\frac{F^+}{K_0+\alpha_k R^+}\right)^{n+1}\right]$$

$$\frac{\partial f}{\partial(\Delta\boldsymbol{\sigma})}=-\Delta t\left(\frac{F}{K_0+\alpha_k R}\right)^{n-1}\exp\left[\alpha\left(\frac{F}{K_0+\alpha_k R}\right)^{n+1}\right]\frac{1}{K_0+\alpha_k R}\left[n+\alpha(n+1)\left(\frac{F}{K_0+\alpha_k R}\right)^{n+1}\right]\frac{\partial F}{\partial\boldsymbol{\sigma}}$$

$$\frac{\partial f}{\partial(\Delta\mathbf{X_i})}=-\Delta t\left(\frac{F}{K_0+\alpha_k R}\right)^{n-1}\exp\left[\alpha\left(\frac{F}{K_0+\alpha_k R}\right)^{n+1}\right]\frac{1}{K_0+\alpha_k R}\left[n+\alpha(n+1)\left(\frac{F}{K_0+\alpha_k R}\right)^{n+1}\right]\frac{\partial F}{\partial\mathbf{X_i}}$$

$$\frac{\partial f}{\partial(\Delta R)}=\Delta t\left(\frac{F}{K_0+\alpha_k R}\right)^{n-1}\exp\left[\alpha\left(\frac{F}{K_0+\alpha_k R}\right)^{n+1}\right]\frac{1}{K_0+\alpha_k R}$$
$$\times\left[n+\alpha(n+1)\left(\frac{F}{K_0+\alpha_k R}\right)^{n+1}\right]\left(\alpha_R+\alpha_k\frac{F}{K_0+\alpha_k R}\right)$$

$$\frac{\partial f}{\partial(\Delta p)}=1,\quad \frac{\partial g}{\partial(\Delta q)}=0,\quad \frac{\partial g}{\partial(\Delta\boldsymbol{\xi})}=0$$

terms related to isotropic work hardening:

$$r(\Delta Y)=\Delta R-b(Q^+-R^+)\Delta p-\gamma_r|Q_r^+-R^+|^{m_r}\operatorname{sgn}(Q_r^+-R^+)\Delta t$$

$$\frac{\partial r}{\partial(\Delta\boldsymbol{\sigma})}=\mathbf{0}$$

$$\frac{\partial r}{\partial(\Delta\mathbf{X_i})}=\mathbf{0}$$

$$\frac{\partial r}{\partial(\Delta p)}=-b(Q-R)$$

$$\frac{\partial r}{\partial(\Delta R)}=1+b\Delta p+\gamma_r m_r|Q_r-R|^{m_r-1}\Delta t$$

$$\frac{\partial r}{\partial(\Delta q)}=-b\Delta p\,Q'(q)-\gamma_r m_r|Q_r-R|^{m_r-1}\,Q_r'(q)\Delta t$$

$$\text{avec}\quad\begin{cases}Q'(q)=2\mu(Q_m-Q_0)e^{-2muq}\\ Q_r'(q)=Q'(q)\left[1-2\mathrm{Q}_r^*\left(\dfrac{Q_m-Q}{Q_m^2}\right)\right]\end{cases}$$

$$\frac{\partial r}{\partial(\Delta\boldsymbol{\xi})}=\mathbf{0}$$

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dependent terms in keeping with field relating to the effect of memory:

$$\boxed{h(\Delta Y)=\Delta q-\eta\langle \mathbf{n}:\mathbf{n}^*\rangle^+\Delta p} \quad \text{with} \quad \mathbf{n}^*=\sqrt{\frac{3}{2}}\frac{\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi}}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}$$

Here, one makes an approximation, by considering that $\langle \mathbf{n}:\mathbf{n}^*\rangle^+=(\mathbf{n}:\mathbf{n}^*)^+$.

$$\frac{\partial h}{\partial(\Delta\boldsymbol{\sigma})}=-\eta\Delta p\frac{1}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}[\frac{\partial^2 F}{\partial\boldsymbol{\sigma}^2}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})+\Delta p\frac{\partial^2 F}{\partial\boldsymbol{\sigma}^2}:\frac{\partial F}{\partial\boldsymbol{\sigma}}$$

$$-\frac{3}{2}\frac{\Delta p}{[(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}]^2}\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})\right)\left(\frac{\partial^2 F}{\partial\boldsymbol{\sigma}^2}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})\right)]$$

$$\frac{\partial h}{\partial(\Delta\mathbf{X_i})}=-\eta\Delta p\frac{1}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}[\frac{\partial^2 F}{\partial\mathbf{X_i}\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})+\Delta p\frac{\partial^2 F}{\partial\mathbf{X_i}\partial\boldsymbol{\sigma}}:\frac{\partial F}{\partial\boldsymbol{\sigma}}$$

$$-\frac{3}{2}\frac{\Delta p}{[(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}]^2}\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})\right)\left(\frac{\partial^2 F}{\partial X_i\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})\right)]$$

$$\frac{\partial h}{\partial(\Delta p)}=-\eta\frac{1}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}[\frac{\partial F}{\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})+\Delta p\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}:\frac{\partial F}{\partial\boldsymbol{\sigma}}\right)-\frac{3}{2}\frac{\Delta p}{[(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}]^2}\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})\right)^2]$$

$$\frac{\partial h}{\partial(\Delta R)}=0$$

$$\frac{\partial h}{\partial(\Delta q)}=1$$

$$\frac{\partial h}{\partial(\Delta\boldsymbol{\xi})}=\frac{\eta}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}\Delta p\left[\frac{\partial F}{\partial\boldsymbol{\sigma}}+\frac{3}{2}(\mathbf{n}:\mathbf{n}^*)\frac{\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi}}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}\right]$$

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terms related to the center of the field relating to the effect of memory:

$$c(\Delta Y)=\Delta\boldsymbol{\xi}-\sqrt{\frac{3}{2}}(1-\eta)\langle\mathbf{n}:\mathbf{n}^*\rangle^+\Delta p\,(\mathbf{n}^*)^+$$

Here, one makes an approximation, by considering that $\langle\mathbf{n}:\mathbf{n}^*\rangle^+=(\mathbf{n}:\mathbf{n}^*)^+$.

$$\frac{\partial c}{\partial(\Delta\boldsymbol{\sigma})}=-\frac{3}{2}(1-\eta)\frac{\Delta p}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}[\left(\frac{\partial^2 F}{\partial\boldsymbol{\sigma}^2}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})\right)\otimes\frac{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}+\Delta p(\mathbf{n}:\mathbf{n}^*)\frac{\partial^2 F}{\partial\boldsymbol{\sigma}^2}$$
$$-3\frac{\Delta p}{\left[(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}\right]^2}(\mathbf{n}:\mathbf{n}^*)\left(\frac{\partial^2 F}{\partial\boldsymbol{\sigma}^2}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})\right)\otimes(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})+\Delta p\frac{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}\otimes\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}:\frac{\partial^2 F}{\partial\boldsymbol{\sigma}^2}\right)]$$

$$\frac{\partial c}{\partial(\Delta\mathbf{X_i})}=-\frac{3}{2}(1-\eta)\frac{\Delta p}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}[\left(\frac{\partial^2 F}{\partial\mathbf{X_i}\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})\right)\otimes\frac{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}+\Delta p(\mathbf{n}:\mathbf{n}^*)\frac{\partial^2 F}{\partial\mathbf{X_i}\partial\boldsymbol{\sigma}}$$
$$-3\frac{\Delta p}{\left[(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}\right]^2}(\mathbf{n}:\mathbf{n}^*)\left(\frac{\partial^2 F}{\partial\mathbf{X_i}\partial\boldsymbol{\sigma}}:(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})\right)\otimes(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})+\Delta p\frac{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}\otimes\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}:\frac{\partial^2 F}{\partial\mathbf{X_i}\partial\sigma}\right)]$$

$$\frac{\partial c}{\partial(\Delta p)}=-\frac{3}{2}(1-\eta)\frac{1}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}[(\mathbf{n}:\mathbf{n}^*)(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})+\frac{\Delta p}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}\left(\frac{\partial F}{\partial\boldsymbol{\sigma}}:\frac{\partial F}{\partial\boldsymbol{\sigma}}\right)(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})$$
$$+\Delta p(\mathbf{n}:\mathbf{n}^*)\frac{\partial F}{\partial\boldsymbol{\sigma}}-3\frac{\Delta p}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}(\mathbf{n}:\mathbf{n}^*)^2(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})]$$

$$\frac{\partial c}{\partial(\Delta R)}=0$$

$$\frac{\partial c}{\partial(\Delta q)}=0$$

$$\frac{\partial c}{\partial(\Delta\boldsymbol{\xi})}=\mathbf{I_d}+\frac{3}{2}(1-\eta)\frac{\Delta p}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}\left[\frac{\partial F}{\partial\boldsymbol{\sigma}}\otimes\frac{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}+(\mathbf{n}:\mathbf{n}^*)\mathbf{I_d}-3(\mathbf{n}:\mathbf{n}^*)\frac{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}\otimes\frac{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}\right]$$
$$=\mathbf{I_d}+\frac{3}{2}(1-\eta)\frac{\Delta p}{(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi})_{eq}}\left[(\mathbf{n}\otimes\mathbf{n}^*)+(\mathbf{n}:\mathbf{n}^*)I_d-3(\mathbf{n}:\mathbf{n}^*)(\mathbf{n}^*\otimes\mathbf{n}^*)\right]$$

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7.2 Calculation of tangent rigidity

The iterative diagram of total Newton (to ensure balance) requires to know the tangent operator of the system assembled at the end of each increment. The tangent operator is noted $\mathbf{M_c}=\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}}$. It is possible to determine this operator starting from the terms of the jacobienne J local system, already previously calculated (see §7.1).

Indeed, the system $\phi(\Delta Y)=0$ is checked at the end of the increment and for a small variation of ϕ , by considering this time ε like variable, the system remains with balance and thus one checks $d\,\phi=0$.
By differentiation, one obtains:

For the system (S1):

$$\frac{\partial\phi}{\partial(\Delta\varepsilon)}d(\Delta\varepsilon)+\frac{\partial\phi}{\partial(\Delta\sigma)}d(\Delta\sigma)+\frac{\partial\phi}{\partial(\Delta \mathrm{X_i})}d(\Delta \mathrm{X_i})+\frac{\partial\phi}{\partial(\Delta p)}d(\Delta p)+\frac{\partial\phi}{\partial(\Delta R)}d(\Delta R)=0$$

For the system (S2):

$$\frac{\partial\phi}{\partial(\Delta\varepsilon)}d(\Delta\varepsilon)+\frac{\partial\phi}{\partial(\Delta\sigma)}d(\Delta\sigma)+\frac{\partial\phi}{\partial(\Delta \mathrm{X_i})}d(\Delta \mathrm{X_i})+\frac{\partial\phi}{\partial(\Delta p)}d(\Delta p)+\frac{\partial\phi}{\partial(\Delta R)}d(\Delta R)+\frac{\partial\phi}{\partial(\Delta q)}d(\Delta q)+\frac{\partial\phi}{\partial(\Delta\xi)}d(\Delta\xi)=0$$

In the continuation, the presentation is limited to the system (S1), but the approach is identical for the system (S2).

One rewrites the system by putting the ends in ε in the member of right-hand side:

$$\frac{\partial\phi}{\partial(\Delta\sigma)}d(\Delta\sigma)+\frac{\partial\phi}{\partial(\Delta \mathrm{X_i})}d(\Delta \mathrm{X_i})+\frac{\partial\phi}{\partial(\Delta p)}d(\Delta p)+\frac{\partial\phi}{\partial(\Delta R)}d(\Delta R)=-\frac{\partial\phi}{\partial(\Delta\varepsilon)}d(\Delta\varepsilon)$$

With the notations defined previously, this system is written in matric form:

$$J\,.\,d(\Delta Y)=-\frac{\partial\phi}{\partial(\Delta\varepsilon)}d(\Delta\varepsilon)\ .$$

However $\frac{\partial\phi}{\partial(\Delta\varepsilon)}=\begin{pmatrix}\frac{\partial g}{\partial(\Delta\varepsilon)}\\ \frac{\partial k_i}{\partial(\Delta\varepsilon)}\\ \frac{\partial f}{\partial(\Delta\varepsilon)}\\ \frac{\partial r}{\partial(\Delta\varepsilon)}\end{pmatrix}=\begin{pmatrix}-\mathbf{H}\\0\\0\\0\end{pmatrix}$ where $\mathbf{H}$ represent the elastic module.

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From where $\mathbf{J}.d(\Delta \mathrm{Y})=\begin{pmatrix}\mathbf{H}\,d(\Delta\varepsilon)\\0\\0\\0\end{pmatrix}$.

The idea then consists in writing this system per blocks, while separating $d(\Delta\boldsymbol{\sigma})$ other variables $\mathbf{Z}=\left(d(\Delta\mathbf{X_i}),d(\Delta p),d(\Delta R)\right)^t$, which gives:

$$\begin{bmatrix}\mathbf{J}_{\sigma\sigma} & \mathbf{J}_{\sigma\mathbf{Z}}\\ \mathbf{J}_{\sigma\mathbf{Z}} & \mathbf{J}_{\mathbf{ZZ}}\end{bmatrix}\begin{pmatrix}d(\Delta\boldsymbol{\sigma})\\ \mathbf{Z}\end{pmatrix}=\begin{pmatrix}\mathbf{H}\,\mathbf{d}(\Delta\boldsymbol{\varepsilon})\\ \mathbf{0}\end{pmatrix}$$

By calculating the complement of Schur of $\mathbf{J}_{\mathbf{ZZ}}$, it is found that:

$$\left[\mathbf{J}_{\sigma\sigma}-\mathbf{J}_{\sigma\mathbf{Z}}(\mathbf{J}_{\mathbf{ZZ}})^{-1}\mathbf{J}_{\mathbf{Z}\sigma}\right]d(\Delta\boldsymbol{\sigma})=\mathbf{H}\,d(\Delta\boldsymbol{\varepsilon}) ,$$

from where the expression of the tangent operator:

$$\mathbf{M_c}=\frac{\partial\boldsymbol{\sigma}}{\partial\boldsymbol{\varepsilon}}=\frac{d(\Delta\boldsymbol{\sigma})}{d(\Delta\varepsilon)}=\left[\mathbf{J}_{\sigma\sigma}-\mathbf{J}_{\sigma\mathbf{Z}}(\mathbf{J}_{\mathbf{ZZ}})^{-1}\mathbf{J}_{\mathbf{Z}\sigma}\right]^{-1}\mathbf{H}$$

Note:

Since $\mathbf{J}$ is not symmetrical, the tangent operator $\mathbf{M_c}$ is not it either. It however is symmetrized in Code_Aster.

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