Viscoplastic behavior with damage of HAYHURST

Summary:

The viscoplastic model coupled with the isotropic damage called of Hayhurst is particularly adapted to carry out structural analyses in creep. The viscoplastic part of the model was proposed by Hayhurst et al. in [bib1] while the law of damage was proposed by Charles Pétry in [bib2]. This model allows a satisfactory prediction of the fall of ductility in creep via the application of limiting criteria on the deformation and the damage. This model has for the moment mainly used with EDF R & D /MMC for predictions of lifetime in creep on the steel of rank 92. Via the identification of specific parameters and the realization of structural analyses, this model also makes it possible to predict in a satisfactory way behaviour in creep and the lifetime of welded junctions [bib3].

This model is established in *Code_Aster* under the name of `HAYHURST` ; the equations of speed are integrated numerically by an explicit diagram of Runge-Kutta of order 2 with automatic cutting under-not buildings according to an estimate of the error of integration (method of Runge-Kutta encased, confer [R5.03.14]) or by a method of integration implicit of Newton.

Test SSNV225 validates the integration of this model and is presented in the document of validation [V6.04.225] which also provides experimental references in keeping with the case test.

Contents

1 Introduction

The applications of the thermal sector to flame and certain nuclear technologies (GENIV, AGR) require to be able to predict the material behavior in creep at the same time in term of viscoplastic deformation but also in term of damage of creep [bib1].

In addition, from a point of view “material”, the mechanisms of creep must be taken into account in the most physical possible way so that the law of behavior is valid at the same time in the field of creep-dislocation (forced high) and in the field of the flow-diffusion (forced weak). In the field of lower constraints, one generally observes a fall of ductility for steels martenistic incomes containing between 9 and 12% of chromium. This fall of ductility can be modelled via an isotropic variable of damage causing the rupture of material before significant plastic deformations did not have time to develop. In these situations, the criteria in maximum deformation cannot apply, just like the laws of evolution of damage strongly related to the deformation like the law of LEMAITRE.

More generally, of the cavities of creep appear in many families of metallic materials and can be observed on site by carrying out extractive counterparts on the surface of the investigated components. These cavities, associated with a reduction of effective surface resisting the efforts in material, can be directly correlated with a damage of the Kachanov type.

In answer to these needs for modeling, while remaining within a simple phenomenologic mechanical framework, a law of behavior of the Hayhurst type (in reference to its viscoplastic flow) was proposed in [bib2] and was applied to calculations of creep to a P92 steel usually used in the modern components of thermo plants with flame.

This model, implemented in *Code_Aster,* is a viscoplastic model of behavior to double isotropic work hardening, viscosity in law sine hyperbolic and coupled to a damage of Kachanov.

It will be noted that the laws `HAYHURST` and `VENDOCHAB` are both of the viscoplastic laws with isotropic damage, however the law of `HAYHURST` have its own advantages detailed in the continuation of this document.

Nota bene:

One will find in the reference [bib3] the application of this model to calculations on joints welded out of P92 steel, and in the reference [bib4] of preliminary work to the extension of this model to take into account the interaction of type fatigue-creep onto this same material.

2 Formulation of the model

2.1 Tally theoretical

In the initial formulation suggested in [bib1], two distinct variables of damage having each one own kinetics are proposed. A variable ϕ is in particular associated with the evolutions of the microstructure depending only on time (i.e static ageing of material), whereas the variable ω described the mechanisms of cavitation developing under the combined influence of the viscoplastic deformation and the triaxiality of the constraints.

In the modeling retained in Code_Aster, the variable ϕ is preserved. In practice, its identification is delicate, and it is possible not to utilize microstructural ageing while putting at zero the coefficient k_c (cf equations in section 2.2). In addition, the variable ω is famous D because its law of evolution is different from that proposed by Hayhurst (cf [bib1]): the law is here in hyperbolic sine. The advantages of this formulation are detailed in [bib2].

2.2 Equations of the model

The equations of the models are written:

$$
\begin{cases}
\textit{élasticité}: \\
\sigma=(1-D)\,\boldsymbol{C}\,\varepsilon^e \text{ et } \varepsilon^e=\varepsilon-\varepsilon^{th}-\varepsilon^p \\
\\
\textit{viscoplasticité}: \\
\dot{\varepsilon}^p=\frac{3}{2}\dot{p}\frac{\tilde{\sigma}}{\sigma_{eq}} \text{ avec } \dot{p}=\varepsilon_0\sinh\left(\frac{\sigma_{eq}(1-H)}{K(1-D)(1-\phi)}\right) \\
\dot{\phi}=\frac{k_c}{3}(1-\phi)^4 \\
\\
\textit{écrouissage}: \\
H=H_1+H_2 \\
\dot{H}_i=\frac{h_i}{\sigma_{eq}}\left(H_i^*-\delta_i H_i\right)\dot{p} \quad \text{pour } i=1,2 \\
\\
\textit{endommagement}: \\
\text{si } \alpha_\sigma=0 \quad \dot{D}=A_0\sinh\left(\frac{\alpha_D<\sigma_1>_+ +\sigma_{eq}(1-\alpha_D)}{\sigma_0}\right) \\
\text{si } \alpha_\sigma=1 \quad \dot{D}=A_0\sinh\left(\frac{\alpha_D<tr(\sigma)>_+ +\sigma_{eq}(1-\alpha_D)}{\sigma_0}\right)
\end{cases}
$$

where:

ε , ε^e , ε^{th} and ε^p	are respectively the deflections total, elastic, thermal and plastic,
$<x>_+$	is the positive part of x ,
σ_1	is the maximum principal constraint,
$\tilde{\sigma}=\sigma-\frac{1}{3}Tr(\sigma)I$	is the deviatoric part of the tensor of the constraints,
$\sigma_{eq}=\sqrt{\frac{3}{2}\tilde{\sigma}_{ij}\tilde{\sigma}_{ij}}$	is the deviatoric constraint of Von-Put,
$\boldsymbol{C}$	is the elastic tensor of rigidity,
p	is the cumulated plastic deformation,
H , H_1 , H_2	are the variables of viscoplastic isotropic work hardening,
D	is the scalar variable of isotropic damage,
ϕ	is the scalar variable of microstructural damage,
α_σ	is the parameter allowing for choice of to calculate the damage compared to (σ_{eq},σ_1) for $\alpha_\sigma=0$ or $(\sigma_{eq},Tr(\sigma))$ for $\alpha_\sigma=1$,
α_D	is the parameter allowing to adjust the sensitivity to the triaxiality $(\alpha_D=1)$ or sensitivity to the maximum principal constraint $(\alpha_D=0)$ for the calculation of the damage,
δ_i	0 or 1 are worth according to whether one wishes a linear or non-linear isotropic work hardening, respectively.

Nota bene:

Parameters of the model $K,\varepsilon_0,\sigma_0,h_1,h_2,A_0,\alpha_D$, et k_c can be functions of the temperature (in $°C$). In [bib2], the identified parameters vary according to the temperature according to a law of Arrhenius.

In order to prevent the evolution of the damage, the user can employ a value of A_0 worthless. In this case, it acts despite everything to inform one value for σ_0, who intervenes in the hyperbolic sine, nonworthless value and sufficient large.

Note:
The preceding system of equations can be reduced: indeed, those which are relating to the evolution of work hardening are integrated in the following way:

$$H_i = \frac{H_i^*}{\delta_i}\left[1 - \exp\left(\frac{-h_i \delta_i}{\sigma_{eq}} p\right)\right]$$

and the equation relating to the microstructural damage returns to:

$$\phi = 1 - \frac{1}{(1 + k_c t)^{1/3}}$$

It is this expression which will be used in the continuation of the document.

3 Parameters of the law

The parameters material necessary for the use of the model in Code_Aster via the order `DEFI_MATERIAU` (cf Doc. U4.43.01) are the following:

ASTER	Symbol	Definition
`EPS0`	ε_0	Parameter acting on the viscoplastic kinetics of deformation
`K`	K	Parameter governing the behavior in hyperbolic sine of the viscoplastic law
`H1`	h_1	Module of work hardening
`H2`	h_2	Module of work hardening
`DELTA1`	δ_1	Choice of the type of work hardening
`DELTA2`	δ_2	Choice of the type of work hardening
`H1ST`	H_1^*	Value with saturation of work hardening, in the non-linear case
`H2ST`	H_2^*	Value with saturation of work hardening, in the non-linear case
`BIGA`	A_0	Parameter acting on the kinetics of damage
`SIG0`	σ_0	Parameter governing the behavior in hyperbolic sine of the law of damage
`ALPHAD`	α_D	Coefficient exploiting the effective constraint for the calculation of the damage
`KC`	k_c	Parameter governing the kinetics of microstructural damage
`S_EQUI_D`	α_σ	Choice of hydrostatic or principal constraint maximum

Parameters α_D, k_c, and α_σ are optional and have zero values by default.

The identification of the coefficients of the viscoplastic part of the model is carried out starting from creep tests at various levels of constraint.

Once this identification carried out, the coefficients controlling the damage can be identified starting from long creep tests to low constraint for which a fall of ductility is observed (fall of the deformations with rupture).

A typical example of identification is detailed in [bib2].

4 Establishment in *Code_Aster*

The use of the model `HAYHURST` is possible in modelings 3D, axisymmetric, and deformation planes (`3D` , `AXIS` , `D_PLAN` , respectively)

Possible models of deformation are `SMALL` , `PETIT_REAC` , `GDEF_HYPO_ELAS` , `GDEF_LOG`.

The algorithm used is of the total-room type. The total iterations use the elastic matrix of rigidity calculated starting from the matrix of Hooke damaged: $\underline{\underline{\Delta}}=(1-D)\underline{\underline{\Delta}}^0$

4.1 Explicit integration

On the level of the local iterations (i.e. in each point of GAUSS), the digital integration of the equations of speed is carried out by an explicit diagram of Runge-Kutta of order 2 with automatic cutting under-not buildings according to an estimate of the error of integration (method of Runge-Kutta encased) (cf [R5.03.14]). Test SSNV225A illustrates this method.

4.2 Implicit integration

For implicit integration, one will employ the following notations:
A^- , A and ΔA represent respectively the values of a quantity at the beginning and the step of time considered thus that its increment during the step. The system is discretized according to a theta-method: $\Delta A=\Delta t\, g\left(A^-+\theta\Delta A\right)=\Delta t\, g\left(A^\theta\right)\quad 0<\theta\leq 1$

The problem discretized to solve is then the following: knowing the state at time t^- as well as the increments of deformation $\Delta\boldsymbol{\varepsilon}$ (resulting from the phase from prediction, cf [R5.03.01]) and of temperature ΔT , to determine the state of the internal variables at time t as well as the constraints $\boldsymbol{\sigma}$.

For taking well into account the variation of the parameters of elasticity with the temperature, it is necessary to discretize (in an implicit way) the elastic relation stress-strain in the following way (see for example [R5.03.02]):

$$\boldsymbol{C}^{-1}\boldsymbol{\sigma}=\frac{(1-D)}{(1-D^-)}(\boldsymbol{C}^-)^{-1}\boldsymbol{\sigma}^-+(1-D)\left(\Delta\varepsilon-\Delta\varepsilon^{\mathrm{th}}\right)-(1-D)\Delta\varepsilon^p \quad \text{with} \quad \Delta\varepsilon^p=\Delta p\frac{3}{2}\frac{\tilde{\boldsymbol{\sigma}}}{\boldsymbol{\sigma}_{eq}}=\Delta p\,\boldsymbol{n}$$

To simplify the expressions and to decrease the number of operations at the time of the resolution, IL is also possible to write the first equation according to the elastic strain; that supposes to store the elastic strain or plastic like internal variables. One obtains then:

$$\Delta\boldsymbol{\varepsilon}^{\boldsymbol{e}}-\left(\Delta\varepsilon-\Delta\varepsilon^{\mathrm{th}}\right)+\Delta p\,\boldsymbol{n}^\theta=0 \qquad \text{with} \quad \boldsymbol{n}^\theta=\frac{3}{2}\frac{\tilde{\boldsymbol{\sigma}}^\theta}{\sigma_{eq}^\theta} \qquad (F_e)$$

and the constraints are calculated then using the plastic deformations at time t^- by:

$$\boldsymbol{\sigma}=(1-D)\boldsymbol{\sigma}^{\boldsymbol{nd}}=(1-D)\boldsymbol{C}\,\varepsilon^e=(1-D)\boldsymbol{C}\left((\varepsilon)^--(\varepsilon^{th})^--(\varepsilon^p)^-+\theta\Delta\varepsilon^e\right)$$

The following equations result from the expressions of the derivative of the internal variables:

$$\Delta p-\Delta t\,\varepsilon_0\sinh\left(\frac{\sigma_{eq}^{\theta}(1-H_1^{\theta}-H_2^{\theta})}{K\left(1-D^{\theta}\right)(1-\phi)}\right)=0 \qquad (F_p)$$

$$\Delta H_i-\frac{h_i}{\sigma_{eq}}\left(H_i^{*}-\delta_i(H_i^{-}+\theta\Delta H_i)\right)\Delta p=0 \quad \text{, for i=1,2} \qquad (F_{H_i})$$

$$\Delta D-\Delta t\,A_0\sinh\left(\frac{\alpha_D<\sigma_p^{\theta}>_{+}+\sigma_{eq}^{\theta}(1-\alpha_D)}{\sigma_0}\right)=0 \text{ with } \sigma_p^{\theta}=\max_I \sigma_I^{\theta}\,\text{ou}\,tr(\sigma^{\theta}) \qquad (F_D)$$

One can formally write this system: $F(\Delta Y)=0$, with $\Delta Y=\left(\Delta\boldsymbol{\varepsilon}^{e},\Delta p,\Delta H_1,\Delta H_2,\Delta D\right)^{t}$ and $F(\Delta Y)=\left(F_e,F_p,F_{H_1},,F_{H_2},,F_D\right)^{t}$

This non-linear system is solved by the iterative method of Newton [R5.03.14]):

$$F(\Delta Y_k)+\left(\frac{\partial F}{\partial\Delta Y}\right)_k\left(\Delta Y_{k+1}-\Delta Y_k\right) \text{ while reiterating in } k \text{ until convergence.}$$

The matrix jacobienne of the system, necessary to the resolution by the method of Newton, can be calculated either numerically (`ALGO_INTE=' NEWTON_PERT'`, cf test `SSNV225B`), that is to say analytically.
In this last case the expression of the derivative is:

$$\left(\frac{\partial F_e}{\partial\Delta\boldsymbol{\varepsilon}^{e}}\right)=\boldsymbol{I}_d+\Delta p\frac{\partial\boldsymbol{n}^{\theta}}{\partial\Delta\boldsymbol{\varepsilon}^{e}} \quad \text{with} \quad \frac{\partial\boldsymbol{n}^{\theta}}{\partial\Delta\boldsymbol{\varepsilon}^{e}}=2\mu\theta\frac{(1-D^{\theta})}{\sigma_{eq}}[\boldsymbol{I}_{dev}-\boldsymbol{n}\otimes\boldsymbol{n}] \quad \text{and}$$

$$\boldsymbol{I}_{dev}=\frac{3}{2}(\boldsymbol{I}_4-\frac{1}{3}I_2\otimes I_2)$$

$$\left(\frac{\partial F_e}{\partial\Delta p}\right)=\boldsymbol{n}^{\theta} \qquad \left(\frac{\partial F_e}{\partial\Delta D}\right)=0$$

$$\left(\frac{\partial F_p}{\partial\Delta\boldsymbol{\varepsilon}^{e}}\right)=-\Delta t\,\varepsilon_0\cosh\left(\frac{\sigma_{eq}^{\theta}(1-H_1^{\theta}-H_2^{\theta})}{K\left(1-D^{\theta}\right)(1-\phi)}\right)\frac{(1-H_1^{\theta}-H_2^{\theta})}{K\left(1-D^{\theta}\right)(1-\phi)}\frac{\partial\sigma_{eq}^{\theta}}{\partial\Delta\boldsymbol{\varepsilon}^{e}}$$

$$\left(\frac{\partial F_p}{\partial\Delta p}\right)=1$$

$$\left(\frac{\partial F_p}{\partial\Delta H_i}\right)=\Delta t\,\varepsilon_0\cosh\left(\frac{\sigma_{eq}^{\theta}(1-H_1^{\theta}-H_2^{\theta})}{K\left(1-D^{\theta}\right)(1-\phi)}\right)\theta\frac{\sigma_{eq}^{nd}}{K(1-\phi)}$$

$$\left(\frac{\partial F_p}{\partial\Delta D}\right)=0 \quad \text{because : } \frac{\boldsymbol{\sigma}^{\theta}}{1-D^{\theta}}=\boldsymbol{\sigma}^{nd} \text{ is independent of } \Delta D$$

$$\left(\frac{\partial F_{H_i}}{\partial\Delta\boldsymbol{\varepsilon}^{e}}\right)=\frac{h_i}{\sigma_{eq}^{2}}\Delta p\left(H_i^{*}-\delta_i H_i^{\theta}\right)\frac{\partial\sigma_{eq}^{\theta}}{\partial\Delta\boldsymbol{\varepsilon}^{e}}$$

$$\left(\frac{\partial F_{H_i}}{\partial\Delta p}\right)=-\frac{h_i}{\sigma_{eq}}\left(H_i^{*}-\delta_i H_i^{\theta}\right)$$

$$\left(\frac{\partial F_{H_i}}{\partial \Delta H_i}\right)=1+\frac{h_i}{\sigma_{eq}}\delta_i\theta\Delta p$$

$$\left(\frac{\partial F_{H_i}}{\partial \Delta D}\right)=\frac{-h_i}{\sigma_{eq}^2}\Delta p\left(H_i^*-\delta_i H_i^\theta\right)\theta\,\sigma_{eq}^{nd}$$

$$\left(\frac{\partial F_D}{\partial \Delta \boldsymbol{\varepsilon}^{\mathbf{e}}}\right)=-\Delta t\frac{A_0}{\sigma_0}\cosh\left(\frac{\alpha_D<\sigma_p^\theta>_+ +\sigma_{eq}^\theta(1-\alpha_D)}{\sigma_0}\right)\left(\alpha_D\frac{\partial<\sigma_p^\theta>_+}{\partial\Delta\boldsymbol{\varepsilon}^{\mathbf{e}}}+(1-\alpha_D)\frac{\partial\sigma_{eq}^\theta}{\partial\Delta\boldsymbol{\varepsilon}^{\mathbf{e}}}\right)$$

$$\left(\frac{\partial F_D}{\partial \Delta D}\right)=1+\frac{\Delta t A_0\theta}{\sigma_0}\cosh\left(\frac{\alpha_D<\sigma_p^\theta>_+ +\sigma_{eq}^\theta(1-\alpha_D)}{\sigma_{nd}}\right)\left[\alpha_D<\sigma_p^0>+(1-\alpha_D)\sigma_{eq}^{nd}\right]$$

with: $\frac{\partial\sigma_{eq}^\theta}{\partial\Delta\boldsymbol{\varepsilon}^{\mathbf{e}}}=2\mu\theta(1-D^\theta)\boldsymbol{n}^{\boldsymbol{\theta}}$

and

$$\frac{\partial<tr\,\sigma^\theta>}{\partial\Delta\boldsymbol{\varepsilon}^{\mathbf{e}}}=\frac{<tr\,\sigma^\theta>}{tr\,\sigma^\theta}(3\lambda+2\mu)\theta(1-D^\theta)\boldsymbol{I_d} \quad \text{if} \quad \sigma_p^\theta=tr(\sigma^\theta)$$

$$\frac{\partial<\sigma_1^\theta>}{\partial\Delta\boldsymbol{\varepsilon}^{\mathbf{e}}}=\frac{<\sigma_1^\theta>}{\sigma_1^\theta}\boldsymbol{I_H} \quad \text{in the principal reference mark, if} \quad \sigma_p^\theta=\sigma_1=\max_I\sigma_I^\theta$$

with $\boldsymbol{I_H}=\begin{bmatrix}\lambda+2\mu & \lambda & \lambda & 0 & 0 & 0\\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0\\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$

5 Significance of the internal variables

Internal variables of the model at the points of Gauss (keyword VARI_ELGA) are accessible by:

1) $V1=\varepsilon_{vp}^{11}$
2) $V2=\varepsilon_{vp}^{22}$
3) $V3=\varepsilon_{vp}^{33}$
4) $V4=\varepsilon_{vp}^{12}$
5) $V5=\varepsilon_{vp}^{13}$
6) $V6=\varepsilon_{vp}^{23}$
7) $V7=p$, cumulated plastic deformation
8) $V8=H_1$ the first variable of isotropic work hardening viscoplastic
9) $V9=H_2$, the second variable of isotropic work hardening viscoplastic
10) $V10=\phi$, the variable of microstructural damage

11) $V11=D$, the variable of damage
12) $V12=0$, variable interns not used into explicit, indicator of plasticity into implicit.

6 Bibliography

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3. PETRY , C. “Behaviour in creep of the welded junctions of P92 steel: experimental characterization and modeling”, H-T24-2010-00225-FR Notes

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7 Description of the versions of the document

Version Aster	**Author (S)** **Organization (S)**	**Description of the modifications**
11.2	F. LATOURTE EDF R & D /MMC	Initial text
11.4	J.M.PROIX EDF R & D /AMA	Addition of implicit integration