Contact in small slips with X-FEM

Summary:

This document presents a new approach to deal with the problems of contact in small slips with the eXtended Finite Element Method (X-FEM) [bib1]. One considers the continuous hybrid formulation of problems of contact between solids [bib2] and the strategy of resolution is similar to that already implemented in Code_Aster for the framework classical finite elements [bib3]. A new type of mixed element of contact is introduced, specific to framework X-FEM.

The approach is implemented in *Code_Aster* in 2D and 3D, and treats at the same time interfaces completely cut by a crack as well as interfaces with bottom of crack. It is usable with the order STAT_NON_LINE [U4.51.03]. The friction of the Coulomb type is taken into account.

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Contents

1Introduction....3
2Strong formulation of the problem of rubbing contact....4
2.1Formulation of the equations of the problem general....4
2.2Laws of the contact....4
2.3Laws of friction....6
2.4Cohesive laws....8
3Mixed variational formulation....11
3.1Formalism common to cohesive regularized and contact-friction....11
3.2Method of Lagrangian increased....12
3.3Penalized method....12
3.4Formulation for a regularized cohesive law....13
3.5Formulation for a mixed cohesive law....13
4Discretizations EF....15
4.1Multipliers of contact....15
4.2Semi-multipliers of friction....15
4.3Finite element of contact....16
4.4Zero setting of the inactive degrees of freedom....21
4.5Calculation of the normal with the facet at the points of integration....23
4.6Conditioning for the penalized method....23
5Strategy of resolution....23
5.1Algorithm of resolution....23
5.2Criterion of stop of the loop on the statutes of contact....24
5.3Criterion of stop of the loop on the thresholds of friction....25
5.4Writing of the formulation at the time of an iteration of Newton....25
5.5Linearization of the problem....27
5.6Elementary terms of rubbing contact....29
6LBB condition....36
6.1Description of the algorithm of Moës for the linear and quadratic elements (algo1)....37
6.2Description of the algorithm modified for the linear and quadratic elements (algo2)....38
6.3Description of the algorithm for the quadratic elements (algorithm 3) :....40
6.4Relations imposed between the semi-multipliers of friction....41
6.5Remarks on the relations imposed by algorithm 1 or 2....42
7 Bibliography....45
8Description of the versions....46

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1 Introduction

To take into account the contact rubbing on the lips of the crack with X-FEM, we chose the framework of the method continues [feeding-bottle1], [feeding-bottle2].

Contrary to the discrete approaches where the problem of contact is taken into account by assembly of nodal forces, the equations are discretized here by the finite element method and the problem of contact is taken into account by an assembly of elementary contributions. This approach was developed by many authors, like Alart *et al.* [feeding-bottle3], Laursen *et al.* [feeding-bottle8], Wriggers [feeding-bottle9], Curnier *et al.* [feeding-bottle10], Pietrzak [feeding-bottle11].

In this “continuous” approach, the conditions of contact are seen like a law of interface and not like boundary conditions. With the concept of law of interface between deformable bodies, one can associate at the time of the passage of the continuous formalism with the discrete model, the concept of element of contact. The resolution exact, and thus rigorous, of the laws of contact (nondifferentiable) can be carried out via a hybrid element contact including the efforts of contact in the unknown factors of the problem. Within this framework, the hybrid formulation led to a nonsymmetrical matrix tangent, nondefinite positive, badly conditioned, with zeros on the diagonal. The difficulty of the problem lies in the non-differentiability of the system to solve [feeding-bottle12].

The method suggested by Ben Dia is very similar, but one chooses to eliminate the non-differentiability of the contact by an algorithm from active constraints, and that of friction by a fixed problem of point on friction so as to obtain a succession of regular problems having methods of resolution whose convergence is established. Many alternatives, according to the adopted algorithms and their fitting exist, but the total convergence of the diagram is not assured

It is pointed out that one one starts from formal Lagrangian of the problem of contact between two deformable solids, that one introduces into the principle of virtual work. A variational formulation mixed displacement-pressure is deduced by incorporating weak formulations from the laws from contact. The equations are discretized by the finite element method. The choice of spaces finite elements of discretization as well as the diagrams of integration (terms of contact) is clarified.

With X-FEM, the lips of the crack are treated as only one surfaces geometrical discontinuity which can be interior with the finite elements. The integration of the terms of contact on this surface (nonwith a grid) then called on the quantities carried by the nodes of the elements crossed by this one. In small displacements, no pairing is not necessary because the points of the surface of the crack in opposite are intrinsically dependent (they correspond to the same geometrical entity). The jump of displacement is expressed according to the discontinuous degrees of freedom of enrichment introduced by X-FEM.

This document is articulated around 6 principal sections, of which this introduction which holds place of section 1. The problem of rubbing contact as well as the put equations concerned are introduced in the paragraph [§2]. On the basis of a Lagrangian approach of the contact introduced into the principle of virtual work, the paragraph [§3] to an expression of the variational formulation mixed displacement-pressure leads. The paragraph [§4] evokes the choice of the discretizations finite elements of the sphere of activities of contact. One specifies the strategy of resolution in the paragraph [§5]. The expressions of the elementary terms of contact and friction resulting from approach X-FEM are detailed in the paragraph [§5.6]. The paragraph [§6] is interested in a particular condition of compatibility of the fields of displacements and pressure. An algorithm aiming at determining a space of the intensifiers of adequate contact (i.e. respecting the LBB condition) is clarified.

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2 Strong formulation of the problem of rubbing contact

2.1 Formulation of the equations of the problem general

One will indicate by r^1 and r^2 densities of the efforts due to interactions of contact rubbing the possible between two surfaces. One will indicate by t_c^1 and t_c^2 forces[1] had with the possible interactions of cohesion between two surfaces in the case of the opening of an interface.

Law of behavior	$\sigma = C : \varepsilon \quad \text{dans } \Omega$
Balance	$\nabla \cdot \sigma = f \quad \text{dans } \Omega$
Imposed surface efforts	$\sigma \cdot n_{ext} = t \quad \text{sur } \Gamma_t$
Density of the efforts of contact	$\sigma \cdot n^i = r^i \quad \text{sur } \Gamma^i \quad i = 1,2$
Density of the efforts of cohesion	$\sigma \cdot n^i = t_c^i \quad \text{sur } \Gamma^i \quad i = 1,2$
Imposed displacements	$u = 0 \quad \text{sur } \Gamma_u$

Table 2.1-1 : Equations of the problem general

Note:

> *In spite of appearances, the relations on the densities the effort of contact and the densities the effort of cohesion are not incompatible. Indeed, the first is valid only when the solids are in contact whereas the second is applicable only if the solids are separated.*

2.2 Laws of the contact

That is to say P a point of Γ_c. One notes P^1 and P^2 points coinciding on Γ^1 and Γ^2 respectively. The condition of not-interpenetration enters P^1 and P^2 is written in the direction n, the normal with Γ^1 :

$$d_n = \left(x\left(P^1\right) - x\left(P^2\right) \right) \cdot n \leq 0$$

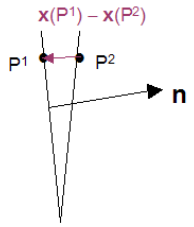

Figure 2.2-1 : Definition of the game

One breaks up the density the effort of contact r in a normal part λ who indicates the normal pressure of contact and another tangential r_τ. Thus, the density the effort of contact is written:

$$r = \lambda n + r_\tau$$

1 Force per unit of area, homogeneous with a constraint

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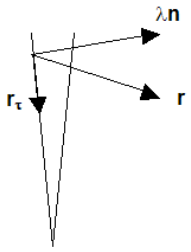

Figure 2.2-2 : Definition of the density the effort of contact

With these notations, the laws of contact (laws of Signorini) are written in the following form:

$$d_n \leq 0,\ \lambda \leq 0,\ \lambda d_n = 0$$

These laws utilize inequations, but those do not lend themselves easily to a weak formulation. For that, one rewrites these laws in another form, by transforming them into only one equivalent equation [feeding-bottle1]:

$$\lambda - \chi(g_n) g_n = 0$$

In this expression, χ is the indicating function of $\Re^-$ defined by

$$\chi(x) = \begin{cases} 1 & \text{si } x < 0 \\ 0 & \text{si } x \geq 0 \end{cases}$$

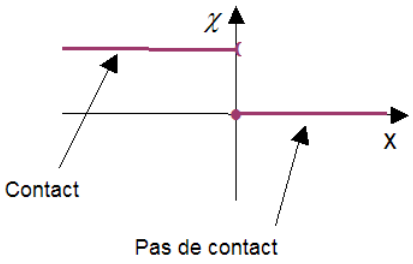

Figure 2.2-3 : Definition of the indicating function of $\Re^-$

and g_n the multiplier (known as of contact increased [feeding-bottle3]) defined by:

$$g_n = \lambda - \rho_n d_n$$

where ρ_n is a strictly positive reality.

The problem of contact posed by the laws of Signorini introduces a not-univocal relation (λ is not a function of d_n), semi-definite positive and nondifferentiable in $\lambda = d_n = 0$ like illustrates it [Figure 2.2-4].

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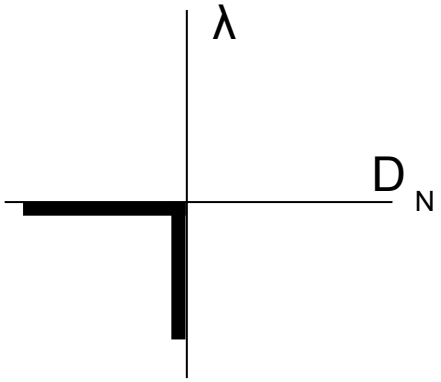

Figure 2.2-4 : Graph of the law of unilateral contact of Signorini

A regularization by penalization of the interpenetration makes it possible to make this relation univocal, to see [Figure 2.2-5].

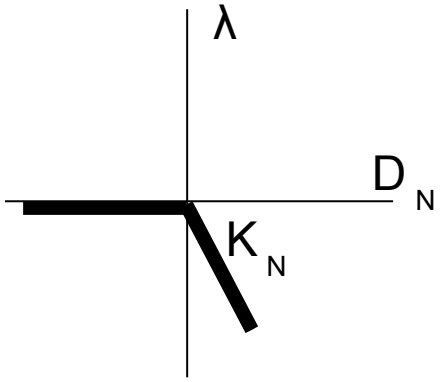

Figure 2.2-5 : Graph of the law of contact regularized

Physically, one authorizes the solids to be interpenetrated at the cost of a stiffness κ_n very high. The contact pressure is then given by $\lambda=-\kappa_n d_n$ and becomes equal to the multiplier of contact g_n. It is noted that more the coefficient of penalization is increased κ_n, more one approaches the classical law of contact.

2.3 Laws of friction

For the phenomena of friction, one uses the laws of Coulomb which are written as follows:

$$\|r_\tau\| \leq \mu |\lambda|$$
$$\text{Si } \|r_\tau\| < \mu|\lambda| \text{ alors } \quad v_\tau = 0$$
$$\text{Si } \|r_\tau\| = \mu|\lambda| \text{ alors } \exists \alpha \geq 0 \ ; \ v_\tau = -\alpha r_\tau$$

where μ is the coefficient of friction of Coulomb and v_τ tangent relative speed.

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One notes thereafter x_τ the projection of x on the tangent level on the surface of contact, defined by $x_\tau=(\mathbf{Id}-n\otimes n)x$, where the symbol $\otimes$ indicate the tensorial product.

Just as for the laws of contact, one can write the laws of friction as follows in an equivalent way:

$$r_\tau=\mu\,\lambda\Lambda$$
$$\Lambda-P_{B(0,1)}(g_\tau)=0$$
$$g_\tau=\Lambda+\rho_\tau v_\tau$$

In these expressions, Λ is a semi-multiplier (vectorial) of friction, g_τ is the semi-multiplier (vectorial) of increased friction, $P_{B(0,1)}$ is projection on the ball unit and ρ_τ a strictly positive parameter. The semi-multiplier of friction Λ , of which the module is always lower or equal to 1, corresponds to the direction of slip when its module is worth 1, and corresponds to the direction of adherence when its module is strictly lower than 1. [Figure 2.3-1] presents in 3D a case of slip and a case of adherence.

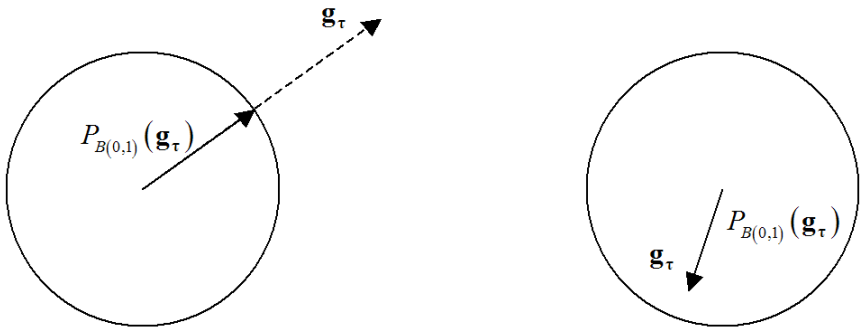

Figure 2.3-1 : Projections on the ball unit in 2D: slip (on the left) and adherence (on the right)

The laws of friction are supplemented by the equation (of standard exclusion) following:

$$d_n\lambda=0 \quad \text{ou} \quad \left(1-\chi(g_n)\right)\lambda=0$$

The problem of friction expressed via the laws of Coulomb introduces a not-univocal relation (R_τ is not a function of v_τ), and nondifferentiable (see [Figure 2.3-2]).

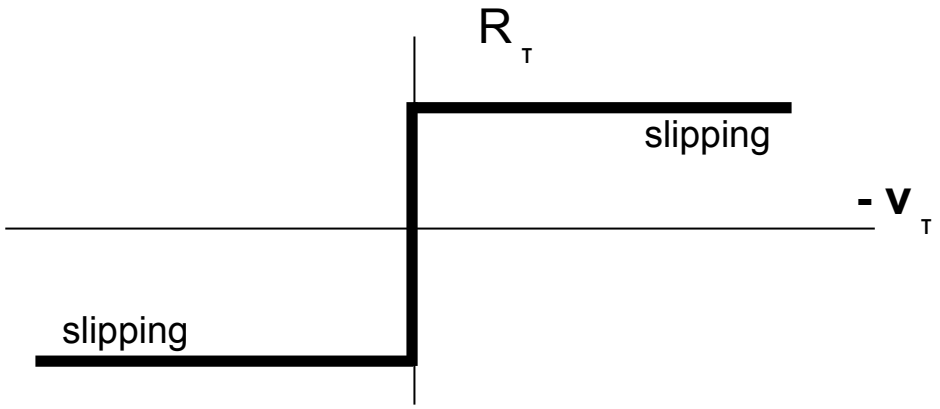

Figure 2.3-2 : Graph of the law of friction of Coulomb

The penalized method makes it possible to make this relation univocal (see [Figure 2.3-3]).

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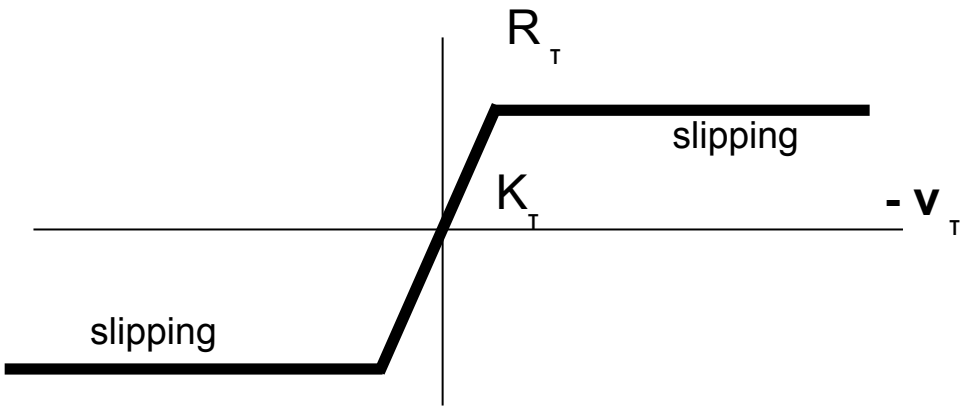

Figure 2.3-3 : Graph of the law of friction regularized

The semi-multiplier of vectorial friction is then given by the relation $\Lambda = P_{B(0,1)}(\kappa_\tau \nu_\tau)$.
It is noted that more the coefficient of penalization is increased κ_τ, more one approaches the classical law of contact.

2.4 Cohesive laws

2.4.1 Regularized cohesive laws

The first type of laws which we can consider is a law in which initial adherence is not perfect: the initial slope is finished. Two cohesive laws of this kind are available in Code_Aster, the laws CZM_EXP_REG and CZM_LIN_REG whose characteristics are detailed in [R7.02.11]. We detail the extension here to XFEM for the law CZM_EXP_REG, while basing itself on the standard commodity [feeding-bottle17]. Extension to the law CZM_LIN_REG is made while following the same paradigm exactly.

We write the jump of displacement such as definite for the algorithm of contact, with the notation defined by the figure 2.2-1:

$$[\![\boldsymbol{u}]\!](P^1) = \boldsymbol{u}(P^1) - \boldsymbol{u}(P^2)$$

We have then $[\![u_n]\!] = [\![\boldsymbol{u}]\!] \cdot \boldsymbol{n}$ negative in opening and positive in interpenetration. Notations of [R7.02.11] pose a jump of displacement δ such as $\delta \cdot \boldsymbol{n}$ that is to say positive in opening and negative in interpenetration. To bring back to us to our notations, we re-use the results of [R7.02.11] while posing $\delta = -[\![\boldsymbol{u}]\!]$. We are satisfied here to recall the principal points of them. The reader can refer to it for a thorough comprehension.

The opening of a crack in mixed mode is characterized by a criterion of damage defined by means of the jump of equivalent displacement and internal variable α. The material remains in the elastic range as long as the inequality is checked:

$$f([\![u]\!]_{eq}, \alpha) = [\![u]\!]_{eq} - \alpha \leq 0$$

- $[\![u]\!]_{éq} = \sqrt{\langle [\![u_n]\!] \rangle_-^2 + [\![\boldsymbol{u}_\tau]\!]^2}$ is the jump of equivalent displacement,
- $[\![\boldsymbol{u}_\tau]\!] = [\![\boldsymbol{u}]\!] - [\![u_n]\!]\boldsymbol{n}$ is the jump of tangent displacement,
- $\alpha(t) = max\{\alpha_0, \underset{v \in [0,t]}{max} [\![u(v)]\!]_{éq}\}$ is the internal variable of the cohesive law,

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- α_0 is the initial value of α. This value is given by the user via the parameter material PENA_ADHERENCE so that $\alpha_0 = \frac{G_c}{\sigma_c}$ PENA_ADHERENCE .

The cohesive constraint is written then like summons of an elastic constraint, a dissipative constraint and a constraint of penalization which gives an account of the contact:

$$\boldsymbol{t_c} = H\left(\llbracket u \rrbracket_{\text{èq}} - \alpha\right) \boldsymbol{\sigma_{lin}} + \left(1 - H\left(\llbracket u \rrbracket_{\text{èq}} - \alpha\right)\right) \boldsymbol{\sigma_{dis}} + \boldsymbol{\sigma_{pen}}$$

where H is the indicating function of $\Re^+$

- $\boldsymbol{\sigma}_{pen} = -C \langle \llbracket u_n \rrbracket \rangle_+ \boldsymbol{n}$ is the constraint of penalization.

where C Is a coefficient of penalization clarified in [R7.02.11] given starting from the parameter material PENA_CONTACT.

- $\sigma_{lin} = \sigma_{dis} = -\frac{\sigma_c}{\alpha} \exp\left(-\frac{\sigma_c}{G_c}\alpha\right) \llbracket \boldsymbol{u} \rrbracket$ is the expression common to the linear constraints and dissipative, with $\alpha = \llbracket u \rrbracket_{èq}$ for $\boldsymbol{\sigma}_{dis}$.

where σ_c is the critical stress with the rupture

and G_c is the energy tenacity of material. It corresponds indeed to energy necessary to the complete opening of the interface over a unit length. A fast calculation with the preceding expressions makes it possible to confirm:

$$\int_{-\infty}^{0} \boldsymbol{t}_{c,\tau} \cdot d\llbracket \boldsymbol{u}_\tau \rrbracket + \int_{-\infty}^{0} \left(\boldsymbol{t}_{c,n} \cdot \boldsymbol{n}\right) d\llbracket u_n \rrbracket = G_c$$

One represents on the figure 2.4.1-1 the cohesive constraint for a loading in mode I pure according to the jump of normal displacement.

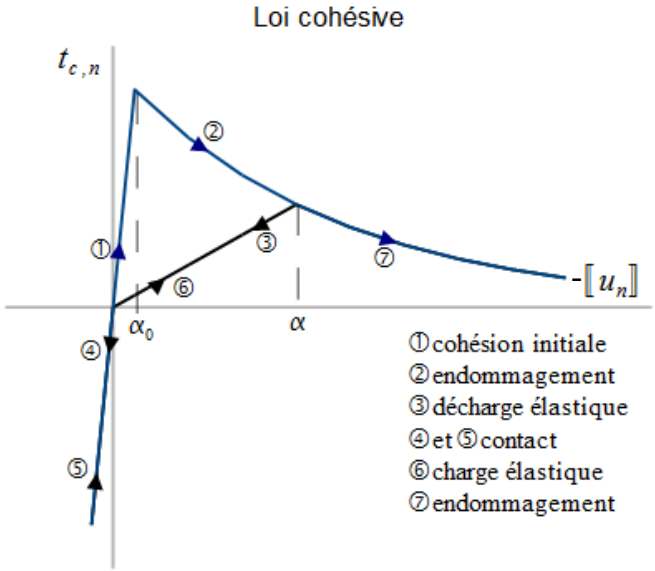

Figure 2.4.1-1: Evolution of the force of normal cohesion according to the jump of displacement

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For $[\![u_n]\!]<0$, it is also usual to define a force of equivalent cohesion $t_{c,eq}$ thanks to the energy condition of equivalence:

$$t_{c,eq}[\![\dot{u}]\!]_{eq}=\boldsymbol{t}_{c,n}\cdot\boldsymbol{n}[\![\dot{u}_n]\!]+\boldsymbol{t}_{c,\tau}\cdot[\![\dot{\boldsymbol{u}}_\tau]\!]$$

To find his value, one derives $[\![u]\!]_{èq}$ compared to time.

$$[\![\dot{u}]\!]_{èq}=\frac{[\![\dot{u}_n]\!][\![u_n]\!]+[\![\dot{\boldsymbol{u}}_\tau]\!]\cdot[\![\boldsymbol{u}_\tau]\!]}{[\![u]\!]_{èq}}$$

From where one identifies: $t_{c,èq}=\boldsymbol{t}_{c,n}\cdot\boldsymbol{n}\frac{[\![u]\!]_{èq}}{[\![u_n]\!]}=\|\boldsymbol{t}_{c,\tau}\|\frac{[\![u]\!]_{èq}}{\|[\![\boldsymbol{u}_\tau]\!]\|}=-\frac{\sigma_c}{\alpha}\exp\left(-\frac{\sigma_c}{G_c}\alpha\right)[\![u]\!]_{èq}$

The figure 2.4.1-2 represent the evolution of the force of cohesion equivalent according to the jump of equivalent displacement according to this law of behavior.

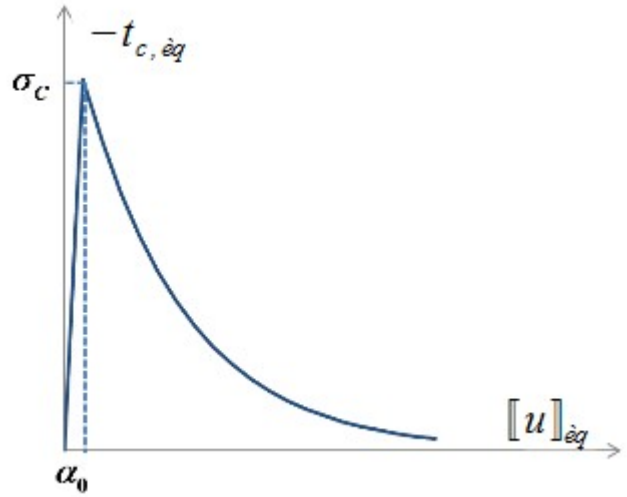

Figure 2.4.1-2: Evolution force of cohesion equivalent according to the jump of displacement are equivalent

Note:

One defines sometimes a jump of equivalent displacement $[\![u]\!]_{èq}=\sqrt{\langle[\![u_n]\!]\rangle_-^2+\beta^2[\![\boldsymbol{u}_\tau]\!]^2}$ *where* β *is an experimental coefficient which represents the report of intensity of the forces of opening in mode* I *and in mode* II*. By taking again the preceding reasoning then with:*

$$t_{c,èq}=-\frac{\sigma_c}{\alpha}\exp\left(-\frac{\sigma_c}{G_c}\alpha\right)[\![u]\!]_{èq}$$

One deduces the expressions from the components:

$$\boldsymbol{t}_{c,n}=\frac{t_{c,eq}[\![u_n]\!]}{[\![u]\!]_{èq}}\boldsymbol{n}=-\frac{\sigma_c}{\alpha}\exp\left(-\frac{\sigma_c}{G_c}\alpha\right)[\![u_n]\!]\boldsymbol{n}\ ,\ \boldsymbol{t}_{c,\tau}=\beta^2\frac{t_{c,eq}[\![\boldsymbol{u}_\tau]\!]}{[\![u]\!]_{èq}}=-\beta^2\frac{\sigma_c}{\alpha}\exp\left(-\frac{\sigma_c}{G_c}\alpha\right)[\![\boldsymbol{u}_\tau]\!]$$

2.4.2 Mixed cohesive laws

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The second type of laws which we can consider is, contrary, a law in which initial adherence is perfect: the initial slope is infinite. Two cohesive laws of this kind are available with XFEM in Code_Aster, laws CZM_OUV_MIX and CZM_TAC_MIX whose characteristics are detailed in [R7.02.11]. We recall here the law CZM_OUV_MIX, represented on the figure 2.4.2-1.

As partly 2.4.1, we introduce the jump of displacement $[\![\boldsymbol{u}]\!]$ such as $[\![u_n]\!]$ that is to say negative in interpenetration and positive in opening, and we are reduced to the notations [R7.02.11] while posing $\delta = -[\![\boldsymbol{u}]\!]$: one returns to this documentation for the expression of the law and his derivative. Let us note that in the same way that for the regularized laws, the material remains in the elastic range as long as:

$$f([\![u]\!]_n, \alpha) = -[\![u]\!]_n - \alpha \leq 0$$

On the other hand, this time, the internal variable has a rigorously worthless initial value, so that the constraint is not explicit any more according to displacement, as shown in the figure 2.4.2-1.

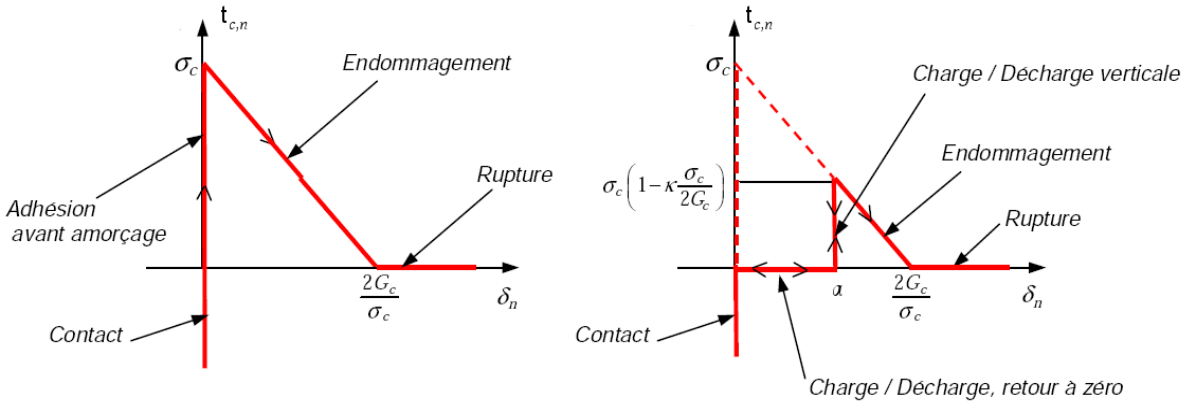

Figure 2.4.2-1: Normal component of the vector forced according to the normal jump for the law CZM_OUV_MIX (threshold α no one on the left and positive on the right).

3 Mixed variational formulation

Let us transform the strong form of the problem into a weak formulation, adapted better to the finite elements. The field u must belong to the unit V_0 fields of displacements kinematically acceptable:

$$V_0 = \{ v \in H^1, v \text{ discontinu à travers } \Gamma_c, v = 0 \text{ sur } \Gamma_u \}$$

Let us start then by giving a unified formulation common to the cases of contact-friction and the cohesive laws **regularized**. The mixed cohesive laws obey when them with an energy formulation according to a different logic, explained partly 3.5.

3.1 Formalism common to cohesive regularized and contact-friction

With this intention we will note $\boldsymbol{r} = \boldsymbol{t}_c$ in the case of regularized laws. We note $H = H^{-1/2}(\Gamma)$ for the cohesive laws, and we indicate by H the subspace of $H^{-1/2}(\Gamma)$ sphere of activities of contact in the case of contact-friction. The weak formulation of the problem of rubbing contact is written as follows:
To find $(u, r^1, r^2) \in V_0 \times H \times H$ such as:

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$$\int_\Omega \sigma(u):\varepsilon(u^*)\,d\Omega=\int_\Omega f\cdot u^*\,d\Omega+\int_{\Gamma_t} t\cdot u^*\,d\Gamma+\int_{\Gamma^1} r^1\cdot u^{*1}\,d\Gamma^1+\int_{\Gamma^2} r^2\cdot u^{*2}\,d\Gamma^2 \quad \forall u^*\in V_0$$

By writing the jump with the following notation

$$[[x]](P^1)=x(P^1)-x(P^2)\ ,$$

and while noting $r=r^1$ and thereafter, the weak formulation of the problem of rubbing contact is written in an equivalent way as follows:

$$\int_\Omega \sigma(u):\varepsilon(u^*)\,d\Omega=\int_\Omega f\cdot u^*\,d\Omega+\int_{\Gamma_t} t\cdot u^*\,d\Gamma+\int_{\Gamma_c} r\cdot[[u^*]]\,d\Gamma_c \quad \forall u^*\in V_0$$

Spaces of the unknown factors of contact are the following:

$$H=\left\{\lambda\in H^{-1/2}(\Gamma_c),\lambda\le 0 \ \text{ sur } \ \Gamma_c\right\}$$
$$H=\left\{r_\tau\in H^{-1/2}(\Gamma_c),\|r_\tau\|\le\mu\lambda \ \text{ sur } \ \Gamma_c\right\}$$

3.2 Method of Lagrangian increased

The weak formulation with three fields is written finally, in the case of an increased Lagrangian formation:

To find $(u,\lambda,\Lambda)\in V_0\times H\times H$

$\forall(u^*,\lambda^*,\Lambda^*)\in V_0\times H\times H$

Equilibrium equation

$$\int_\Omega \sigma(u):\varepsilon(u^*)\,d\Omega-\int_\Omega f\cdot u^*\,d\Omega-\int_{\Gamma_t} t\cdot u^*\,d\Gamma$$
$$-\int_{\Gamma_c}\chi(g_n)\,g_n n\cdot[[u^*]]\,d\Gamma_c-\int_{\Gamma_c}\chi(g_n)\,\mu\lambda_s P_{B(0,1)}(g_\tau)\cdot[[u_\tau^*]]\,d\Gamma_c=0$$

Law of contact

$$\int_{\Gamma c}\frac{-1}{\rho_n}\left(\lambda-\chi(g_n)g_n\right)\lambda^*\,d\Gamma_c=0$$

Law of friction

$$\int_{\Gamma c}\frac{\mu\chi(g_n)\lambda_s\Delta t}{\rho_\tau}\left(\Lambda-P_{B(0,1)}(g_\tau)\right)\Lambda^*\,d\Gamma_c$$
$$+\int_{\Gamma c}\left(1-\chi(g_n)\right)\Lambda\Lambda^*\,d\Gamma_c=0$$

3.3 Penalized method

The weak formulation with three fields is written finally, in the case of a purely penalized formation:

To find $(u,\lambda,\Lambda)\in V_0\times H\times H$

$\forall(u^*,\lambda^*,\Lambda^*)\in V_0\times H\times H$

Equilibrium equation

$$\int_\Omega \sigma(u):\varepsilon(u^*)\,d\Omega-\int_\Omega f\cdot u^*\,d\Omega-\int_{\Gamma_t} t\cdot u^*\,d\Gamma$$
$$-\int_{\Gamma_c}\chi(g_n)\lambda\cdot[[u^*]]\cdot n\,d\Gamma_c$$
$$-\int_{\Gamma_c}\chi(g_n)\,\mu\lambda_s P_{B(0,1)}(\kappa_\tau\nu_\tau)\cdot[[u_\tau^*]]\,d\Gamma_c=0$$

Law of contact

$$\int_{\Gamma c}\frac{-1}{\kappa_n}\left(\lambda+\chi(g_n)\kappa_n d_n\right)\lambda^*\,d\Gamma_c=0$$

Law of friction

$$\int_{\Gamma c}\frac{\mu\chi(g_n)\lambda_s}{\kappa_\tau}\left(\Lambda-P_{B(0,1)}(\kappa_\tau\nu_\tau)\right)\Lambda^*\,d\Gamma_c$$
$$+\int_{\Gamma c}\left(1-\chi(g_n)\right)\Lambda\Lambda^*\,d\Gamma_c=0$$

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It will be noticed that the value of λ obtained by the law of contact is an average when the state is contacting forces of interpenetration for the penalized law. The use of the expressions λ or $-\kappa_n d_n$ in the equilibrium equation should thus lead to the same result if it is not that condition LBB applies only to λ . To have equivalence, it would thus be necessary to defer the treatment of the LBB on the fields of displacement for the penalized method. As that is not done here, the term is used λ for which treatment LBB is made and one reinjects it in the equilibrium equation which thus takes into account this treatment. One could try the same treatment for the law of friction but Λ is obtained like an average for slipping or adherent situations. To reinject this state realised on the level of the equilibrium equation involves nona convergence of the algorithm of Newton. The difference in behavior for λ and Λ comes owing to the fact that one or not integrates discontinuous quantities according to the contacting state contacting for λ and according to the state slipping or adherent for Λ , but that for the state not contacting the contributions on λ are worthless in the law of contact. One can deduce from it that if there exists a formulation penalized satisfying fully conditions LBB for the contact, there does not exist for the moment not formulation penalized satisfying fully conditions LBB for friction. The choice of the coefficient κ_τ in will be all the more important as a result (of strong values being likely to display blockings for the adherent part).

3.4 Formulation for a regularized cohesive law

When a cohesive law is used, the contact is managed by the coefficient of penalization defined in the cohesive law. Knowing the expression of the cohesive law according to $[\![u]\!]$, the formulation is written:

To find $(u,\lambda,\Lambda)\in V_0\times H\times H$ such as:
$\forall(u^*,\lambda^*,\Lambda^*)\in V_0\times H\times H$

Equilibrium equation
$$\int_\Omega \sigma(u):\varepsilon(u^*)\,d\Omega-\int_\Omega f\cdot u^*\,d\Omega-\int_{\Gamma_t} t\cdot u^*\,d\Gamma$$
$$-\int_{\Gamma_c} t_{c,n}\cdot[\![u^*]\!]_n\,d\Gamma_c-\int_{\Gamma_c} t_{c,\tau}\cdot[\![u^*]\!]_\tau\,d\Gamma_c=0$$

Postprocessing left normal
$$\int_{\Gamma c}(\lambda-t_{c,n}\cdot n)\lambda^*\,d\Gamma_c=0$$

Postprocessing left tangential
$$\int_{\Gamma c}(\Lambda-t_{c,\tau})\Lambda^*\,d\Gamma_c=0$$

It is noticed that the multipliers λ and Λ do not intervene in the resolution. They are only used to store the cohesive constraints in an explicit way.

3.5 Formulation for a mixed cohesive law

In opposition to the preceding formulation, the treatment of such a law will require a true formulation with several fields, in the direction where a vectorial dual field $\boldsymbol{\lambda}$ indeed will enter the formulation, instead of being an artifice of postprocessing as in 3.4 . This formulation follows an energy reasoning, explained in detail in documentation [R3.06.13]. Let us summarize in the principal points:

It is written that the opening of the crack costs an energy proportional to surface to be opened, that is to say:

$$E_{\text{fr}}(\boldsymbol{\delta})=\int_\Gamma \Pi(\boldsymbol{\delta})\,dS$$

where $\Pi(\boldsymbol{\delta})$ is the density of cohesive energy. For the law CZM_OUV_MIX , we have for example $\Pi(\boldsymbol{\delta})=\int_0^{\delta_n} t_{c,n}(\delta')\,d\delta'$.

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The field of discontinuity δ appearing in the preceding expressions is then defined like a field except for whole, integrated in the formulation as a new unknown factor. Total energy is written then:

$$E(\boldsymbol{u},\boldsymbol{\delta})=\int_{\Omega\setminus\Gamma}\Phi(\boldsymbol{\varepsilon}(\boldsymbol{u}))\,d\Omega-W_{ext}(\boldsymbol{u})+\int_{\Gamma}\Pi(\boldsymbol{\delta})\,d\Gamma$$

The solution of the problem consists then of the minimization of this total energy under the constraint that δ corresponds, with our conventions of sign, contrary to the jump of displacement. One seeks:

$$\min_{\substack{u,\delta\\ [\![\boldsymbol{u}]\!]=-\boldsymbol{\delta}}} E(\boldsymbol{u},\boldsymbol{\delta})$$

In order to solve this one, we introduce the Lagrangian associated one with the problem, to which we add a term of increase whose utility will appear thereafter:

$$L_r(\boldsymbol{u},\boldsymbol{\delta},\boldsymbol{\lambda})\underset{déf.}{=}E(\boldsymbol{u},\boldsymbol{\delta})+\int_{\Gamma}\boldsymbol{\lambda}\cdot(-[\![\boldsymbol{u}]\!]-\boldsymbol{\delta})\,d\Gamma+\int_{\Gamma}\frac{r}{2}([\![\boldsymbol{u}]\!]+\boldsymbol{\delta})^2\,d\Gamma$$

We can then write the first condition of optimality of this Lagrangian:

$$\forall\ \delta^*\quad\int_{\Gamma}\left[t-\lambda+r\ ([\![u]\!]+\delta)\right]\cdot\delta^*=0\quad\text{avec}\quad t\in\partial\Pi(\delta)$$

This equation fact of intervening the cohesive constraint t_c. However we do not have like expression t_c that of a local law of behavior. This first equation thus must, to have direction, to be discretized in a way which makes it possible to be brought back to a local expression. This is possible if δ is discretized by collocation at the points of Gauss of the interface, of coordinates x_g.
Indeed, with such a discretization, the resolution of the first conditions of optimality amounts satisfying the cohesive law in each point with collocation:

$$t_c(\delta_g,\alpha_g)=\lambda_g-r([\![u_g]\!]+\delta_g)$$

where we noted $\lambda_g=\lambda(x_g)$, for example, values of a field at the points of Gauss, and where $t_c(\delta_g,\alpha_g)$ the law follows 2.4.2-1. The graphic translation of this law of behavior is the following one: the solution corresponds to the intersection of the linear function $\delta\to\lambda_g-r[\![u_g]\!]-r\,\delta$ (with a negative slope given by the coefficient of penalty r) with the graph $t_c(\delta,\alpha)$. We see whereas for r rather large, the solution is single, from where interest to have increased the Lagrangian one.

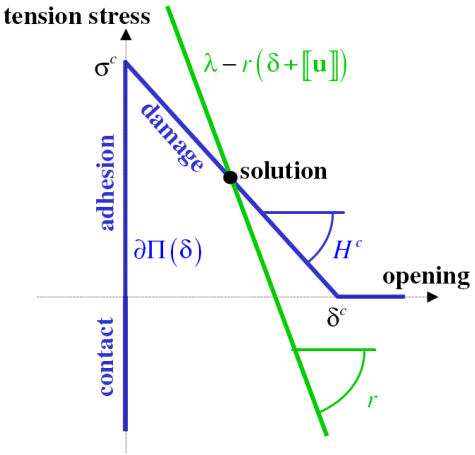

Figure 3.5-a : Solution of the integration of the behavior.

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The field δ is thus written locally like a function of $\lambda - r[\![u]\!]$, that we will call increased multiplier and will note p. Consequently, it disappears from the unknown fields of the problem. The formulation is then given by the two conditions of optimality of Lagrangian remaining:

To find $(u, \lambda) \in V_0 \times H$ such as:

$\forall (u^*, \lambda^*) \in V_0 \times H$

Equilibrium equation

$$\int_\Omega \sigma(u) : \varepsilon(u^*)\, d\Omega - \int_\Omega f \cdot u^*\, d\Omega - \int_{\Gamma_t} t \cdot u^*\, d\Gamma + \int_{\Gamma_c} \left[-\lambda + r\left([\![u]\!] + \delta(p)\right) \right] \cdot [\![u^*]\!]\, d\Gamma = 0 \quad \text{avec} \quad p = \lambda - r[\![u]\!]$$

Law of interface

$$-\int_{\Gamma_c} \left[[\![u]\!] + \delta(p) \right] \cdot \lambda^*\, d\Gamma = 0$$

With regard to the discretization of these two fields of unknown factors, a simple observing stability condition inf-sup, and consistent discretization with the discretization of δ by collocation, consists in discretizing displacement with P2 elements and the multiplier in a P1 way adapted to X-FEM, of which we give the detail in the part 4.1 to follow.

4 Discretizations EF

One details in this part the discretization of the unknown factors of contact-friction, which is done with the nodes tops of the element relative.

4.1 Multipliers of contact

Unknown factors for the contact pressure λ and the semi-multiplier of friction Λ are carried to the nodes tops of the element relative. The approximation of the contact pressure utilizes them Φ_i, function of form of the linear element relative and is written:

$$\lambda^h(x) = \sum_{i=1}^{nno} \lambda_i \Phi_i(x)$$

where nno is the number of nodes of the linear element relative. The pressure is then defined as the trace on the approximated interface of this field λ^h.

To clarify this concept of approximated interface, let us suppose that one cuts out, if need be, the element relative in simpliciaux subelements (i.e. triangles in 2D, tetrahedrons in 3D). By approximated interface, we understand in 2D the broken line connecting between them the points of intersection of the edges of such subelements with the line of the crack. In 3D, the points of intersection of the edges of such subelements with the surface of the crack define a polygon, not necessarily plan within this subelement: there can for example be 4 points of intersection, not inevitably coplanar. The adopted method was that which consists in cutting out this polygon in triangular facets whose tops are these points of intersection ([fig. 4.3.3-1]). The whole of the process describes in this paragraph is partly detailed 4.3.

4.2 Semi-multipliers of friction

Just as for the multipliers of contact, the semi-multipliers of friction are interpolated with Φ_i function of form of the element relative. In 3D, Λ is a vector of the tangent plan on the surface of the crack. The gradients of the level sets make it possible to define a base covariante in the surface of the crack, in which Λ will be expressed. One defines the 2 vectors of the base covariante by:

$$(n^{ls}, \tau^1, \tau^2) = (\nabla lsn, \nabla lst, \nabla lsn \times \nabla lst)$$

where n^{ls} is the local normal resulting from the gradient of the level set normal. Vectors τ^1, τ^2 resulting from the gradients (nodal) level sets, they can be interpolated within the elements so as to

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obtain vectors at the tops i facets of contacts, is τ_i^1 and τ_i^2 , $i=1,3$. The approximation of the semi-multipliers of friction on a facet of contact is written then:

$$\Lambda^h(x)=\sum_{i=1}^{nno}\left(\Lambda_i^1\tau_i^1+\Lambda_i^2\tau_i^2\right)\Phi_i(x)\ ,$$

where vectors τ_i^1,τ_i^2 for a node I correspond to those of the point of intersection of the interface with the edge of the element associated with node I. The associated point of intersection is this node (if it is about a node by where the interface of contact passes), that is to say the intersected edge containing this node (if it is about a cut edge). If the node belongs to several intersected edges, one associates it with the point of intersection of the vital edge.

4.3 Finite element of contact

4.3.1 Case general

The degrees of freedom of contact are carried exclusively by the nodes tops.

By way of an example, the figure 4.3.1 watch bearing nodes degrees of freedom of contact. It is noticed that one distinguishes on the one hand the elements X-FEM cut by the crack which will carry degrees of freedom of contact and on the other hand the not cut elements X-FEM which do not need degrees of freedom of contact. One introduces degrees of freedom of contact only on the nodes tops is on the whole 4 degrees of freedom. Two relations of equalities connect respectively nodes 1 and 3 as well as nodes 2 and 4 in order to satisfy condition LBB (see [§6]). That makes a total of 6 introduced variables.

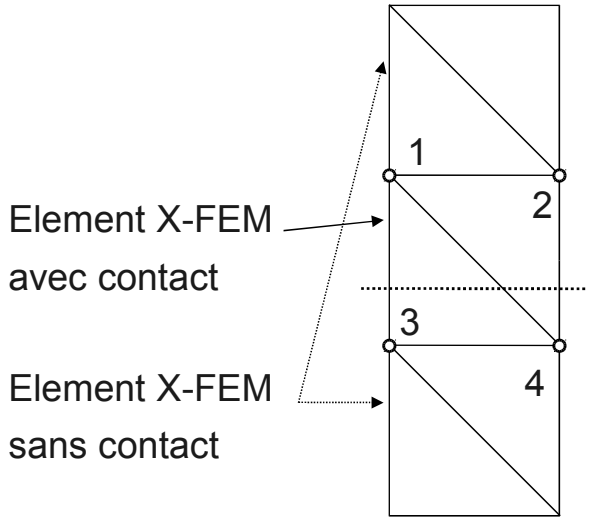

Figure 4.3.1-1 : Nodes carrying the ddl of contact.

FACET OF CONTACT
The surface of contact is used only with ends as integration but it requires the installation of algorithm of research of the points of intersection and the points mediums if the element is quadratic.

The algorithm of research of the points of intersection is presented in the following way:
On an element:

- buckle on the edges of the element

  are E^1 and E^2 two ends of the edge

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if $lsn(E^1)lsn(E^2)\leq 0$ then

•buckle on the two ends

if $lsn(E^j)=0$ and $lst(E^j)\leq 0$ then
the point is added E^j with the list of P_i (with checking of the doubled blooms)
end if
if quadratic element 2D: $lsn(E^3)=0$ and $lst(E^3)\leq 0$ then
the point is added E^3 with the list of P_i (with checking of the doubled blooms)
end if

•fine buckles

if $lsn(E^k)\neq 0\ \forall k\in$ many nodes of the edge, then
interpolation of the coordinates of C
if $lst(C)\leq 0$ then
the point is added C with the list of P_i (with checking of the doubled blooms)
end if

end if

end if

•fine buckles

Details of the interpolation of the coordinates of the point C:

If the element is linear:

$$s=\frac{lsn(E^1)}{lsn(E^1)-lsn(E^2)}$$
$$C=E^1-s(E^2-E^1)$$
$$lst(C)=lst(E^1)-s(lst(E^2)-lst(E^1))$$

If the element is quadratic 2D:
The position of the point on the edge informs about the value of one of its coordinates of reference ξ or η, it is then enough to solve the polynomial equation $\sum_{i=1}^{nno}\Phi_i(\xi,\eta)\,lsn_i=0$ to find the value of the second coordinates of reference. By passage in the real element, one determines his real coordinates (x,y).
end if

If the element is quadratic, it is necessary, besides the points of intersection to interpolate the coordinates of the nodes mediums of the facet of contact. In 2D, the facet of contact is connected with an arc of a circle where one knows only the coordinates of his ends following the preceding algorithm. To determine his point medium, one uses the method of Newton which evaluates the point of intersection between the mediator of the segment connecting the two ends and the izo- zero of the level set normal.

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4.3.2 Case of the elements in bottom of crack

For an element containing the bottom of crack, one needs a particular treatment to determine the points of contact. Indeed, such elements are not entirely cut by the crack. The points of contact are then of two types:

- maybe of the intersections between the surface of the crack and the edges of the element (case general evoked in the preceding paragraph),
- maybe of the intersections between the bottom of crack and the faces of the element (case specific to the elements containing the bottom of crack).

Points of the 1er type are determined by the preceding algorithm, and the points of the 2Nd type by the algorithm of research of the points of the bottom of crack (see the paragraph [§2.4] in [R7.02.12]).

Points of contact of the 2Nd type are associated with no Nœud nor edge, and are thus carried natively by no ddl. They thus do not intervene in the writing of the approximation. This situation corresponds to a P1 approximation of the unknown factors of contact on the facet of contact in bottom, having a zero value in bottom of crack (see diagram from the top of Figure 4.3.2-1). Another solution is to consider a constant connection. In this case, the pressure in bottom of crack is considered equal to the pressure of the point of contact on an edge nearest. It is this solution which was adopted. In the example of Figure 4.3.2-1, the diagram of bottom illustrates the approximation of the contact pressure on the facet ABC :

$$\begin{aligned}\lambda(x) &= \lambda_A\psi_A+\lambda_B\psi_B+\lambda_C\psi_C \\ &= \lambda_D\psi_A+\lambda_C\psi_B+\lambda_C\psi_C\end{aligned}$$

Since $BC<BD$ and $AD<AC$, the unknown factor of pressure in B is carried by the point C and the unknown factor of pressure in A is carried by the point D , there is thus no additional ddl on the faces. This approach could be qualified “P0-P1”, because the approximation is a mixture of P0 and P1. The contact pressure is P1 along the bottom of crack, and P0 along the others with dimensions of the facet.

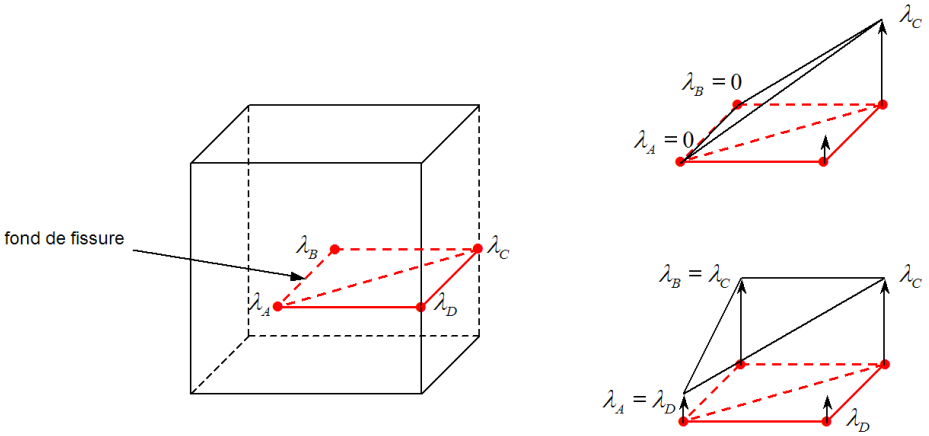

Figure 4.3.2-1 : Various approximations of the contact on the facet in bottom of crack

Another alternative would be to consider a true P1 approximation on the facets of contact in bottom of crack. For that, it would be necessary that them degrees of freedom of contact on the points of the bottom are truths degrees of freedom independent. They could be for example carried by the nodes tops of the opposite edges. On the example of Figure 4.3.2-2, the contact pressure in B would be carried by the node F and the contact pressure in A by the node E . This case is generalizable with any type of configuration. The interest is to have degrees of freedom of independent contact. Such a P1 approximation would improve the precision compared to a constant approximation.

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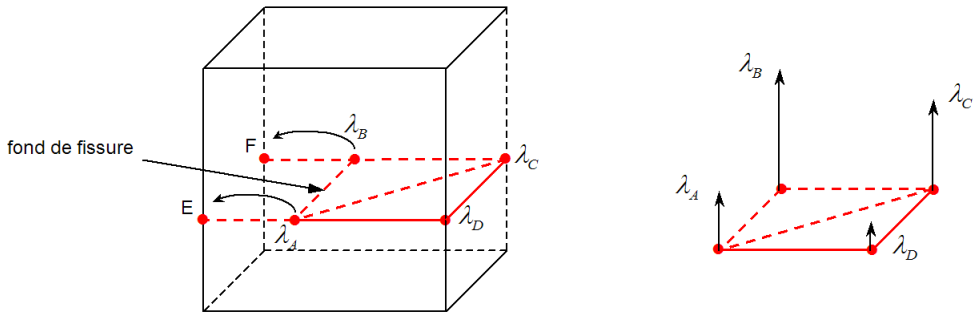

Figure 4.3.2-2 : P1 approximation on the facet of contact in bottom of crack

4.3.3 Under-cutting in triangular facets of contact

For each element, starting from the list of the points of contact P_i, it is necessary to create a under-cutting in triangular facets. For that, one sorts the points of contact per element with the same process as that being used to direct the points of the bottom fissures (see the paragraph [§2.4] in [R7.02.12]). One determines n the average of the normals to the nodes (based on the gradients of the level sets). One determines G the barycentre of the points of contact on this element. One projects the points of contact in the plan of normal n and passing by G. For each one of projected, one determines the angle θ compared to the 1er not list, then one sorts the points of the list according to θ growing.

To illustrate under-cutting in triangular facets, let us take a hexahedron occupying the area $[0,1]^3$. That is to say the plan of Cartesian equation $4x-11y-9z+d=0$. Let us examine the intersections between this plan and the hexahedron, for various values of the parameter d.

For $d=-1$, it 3 points of intersection enters there the plan and the hexahedron. The trace of the plan in the hexahedron is a triangle, which corresponds to the facet of contact. For $d=4$, it 4 points of intersection enters there the plan and the hexahedron. The trace of the plan in the hexahedron is a quadrangle, is cut out in two triangular facets of contact. For $d=6$, it 5 points of intersection enters there the plan and the hexahedron. The trace of the plan in the hexahedron is a pentagon, is cut out in three triangular facets of contact. For $d=8$, it 6 points of intersection enters there the plan and the hexahedron. The trace of the plan in the hexahedron is a hexagon, is cut out in four triangular facets of contact. Figure 4.3.3-1 present the various diagrams for the values of d previously evoked.
Moreover, when the bottom of crack is contained in an element, it may be that adds a point of intersection moreover. For example for $d=8$, if the bottom of crack cuts segments P1P2 and P2P3 then that adds a P2a point located on P1P2 and another P2b point located on P2 P3 and removes the P2 point (2 added points and 1 removed point, is on the whole 1 added point). The maximum number of points of intersection is then 7.

This example illustrates the various cases which can occur. In a general way, one can gather the various cases according to the number of points of intersection met. Table 4.3.3-1 gather the under-cuttings carried out according to number of points of intersection found between the element (that the element is a tetrahedron, a pentahedron or a hexahedron) and surfaces it crack.

	3 points of intersection	4 points of intersection	5 points of intersection	6 points of intersection	7 points of intersection

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Triangular facets	P1 P2 P3	P1 P2 P3 P1 P3 P4	P1 P2 P3 P1 P3 P4 P1 P4 P5	P1 P2 P3 P1 P3 P5 P1 P5 P6 P3 P4 P5	P1 P2 P3 P1 P3 P5 P3 P4 P5 P1 P5 P7 P5 P6 P7

Table 4.3.3-1 : Cutting in triangular facets

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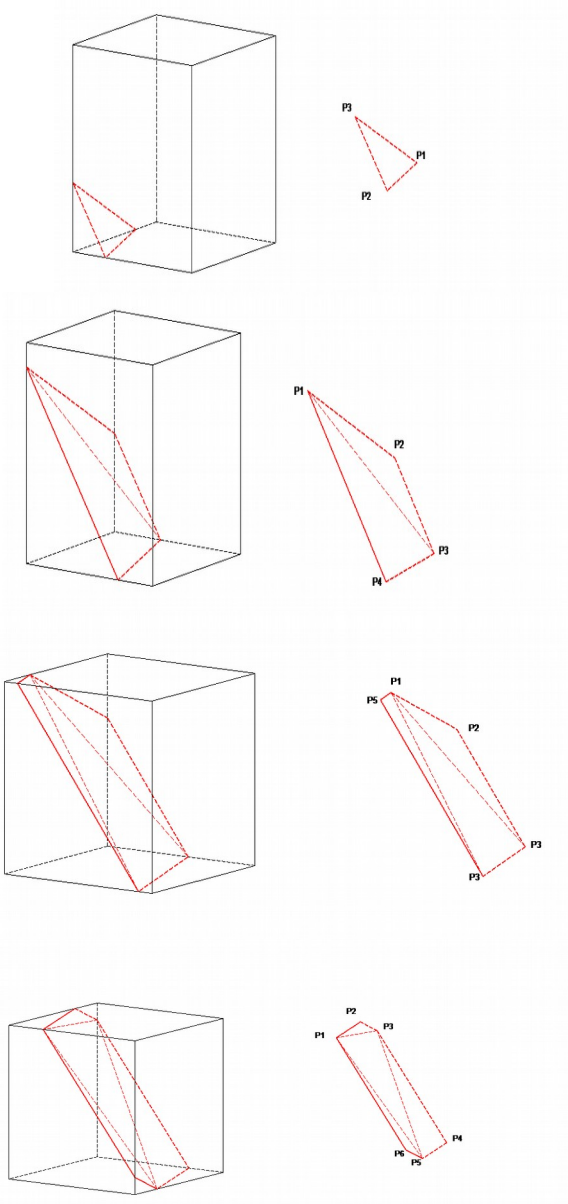

Figure 4.3.3-1 : Intersections and cuttings for D = -1,4,6 and 8 (from top to bottom)

4.4 Zero setting of the inactive degrees of freedom

This procedure is used to put at zero the value of the degrees of freedom of which do not intervene in the equations. Several solutions can be considered.

Notice :

> *The discussion which follows is strongly influenced by the possibilities (and the restrictions) of Code_Aster; the solutions considered could not then be exhaustive.*

4.4.1 Not to introduce

First of all, couldn't one simply avoid introducing these degrees of freedom which for nothing and which are not used will have to be cancelled thereafter? One could for example initially imagine an element without degree of freedom of contact, and once the intersections between the edges of the element

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and the crack determined, to add the degrees of freedom of contact necessary. That would like to say to transform the finite element into finite element having of the degrees of freedom of contact in certain precise nodes (for example with the nodes $N1$, B , C , $N4$ fig. 4.3.3-1). One easily foresees the number of possibilities of that represents, and the number of different finite elements that would imply (21 for a tetrahedron with 3 or 4 150 and points of intersection, more than for a hexahedron with 3,4,5 or 6 points of intersection). This solution is thus isolated. One is thus well obliged to have an element of contact with degrees of freedom of contact on all the nodes top and medium.

4.4.2 Elimination

To eliminate the degrees of freedom from contact which are not used for nothing, the simplest idea is that which consists in withdrawing total system of equations the corresponding lines and columns (on paper, that amounts “striping” the useless lines and columns). The system obtained is thus size lower than the total system, and of the same size than that which one would have obtained with the method of the preceding paragraph. However, the fact of withdrawing total system of the lines and columns is not possible at the present time in *Code_Aster*.

4.4.3 Substitution

If one cannot eliminate from the degrees of freedom, value 0 should be affected to them. For that one can decide to modify the tangent matrix and the second member, by putting an unspecified actual value k_0 (for example 1) on the diagonal from the matrix and 0 in the second member, with the position corresponding to the degree of freedom to be cancelled. There is thus of an the same system cuts, but numerically badly-conditioned because of the unspecified value selected on the diagonal. Indeed, on the level of the calculation of the elementary matrix, one does not know overall the matrix of stiffness, therefore the optimal value of the parameter k_0 (in term of conditioning) is not known.

4.4.4 Dualisation

To mitigate this kind of problem, the keyword DDL_IMPO of the operator AFFE_CHAR_MECA allows to impose on a degree of freedom of a node a preset value. To solve the linear system under constraints thus obtained, the technique of double the multipliers of Lagrange is used [feeding-bottle5], which allows a better conditioning than with the simplistic technique of the preceding paragraph, because the choice of the parameters k_0 is carried out knowing the assembled matrix. The principal disadvantage is that two additional equations are added with the imposed system for each ddl.

4.4.5 Generalized substitution

The method by substitution is generalized with the imposition of ddl to an unspecified value (other that 0) in *Code_Aster* (operator AFFE_CHAR_CINE). However, this operator does not function at the present time with the nonlinear operator of statics (STAT_NON_LINE) used to solve the system (the non-linearity of with the dealt problem is due to contact-friction).

4.4.6 Selected solution

L‘elimination is useful for the bilinear meshs and the funds of crack. One does it by using the method by substitution: however this choice must be the object of a follow-up, even of a study thorough of the robustness, because there are possible impacts on the stability of convergence. For the rest, namely the relations of equality, the solution which was adopted is that of the double multipliers of Lagrange. Let us note that with the use of Mumps as solvor, only one multiplying is taken into account. However, one is return-account that the argument on the bad conditioning which led us not to choose the method by substitution does not hold. Admittedly, the matrix can be badly-conditioned, but that does not have a consequence on the quality of the results because the matrix is diagonal per block (for example, the diagonal matrix $diag(1,10.e16)$ is badly conditioned but does not pose a problem). One thus thinks of using in the future the method by substitution (either in putting at the hand of the 1 on the diagonal,

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that is to say while using AFFE_CHAR_CINE when it is functional , which is equivalent) instead of the dualisation.

Notice :

> *The cancellation of the degrees of Heaviside freedom and ace-tip nouveau riches in excess is made by using the method of substitution. Indeed for problems where the grid is refined in the zone of the crack, the number of equations added to cancel them in the case of the choice of the dualisation would generate considerable additional computing times.*

4.5 Calculation of the normal with the facet at the points of integration

As long as the fields of the level sets are interpolated by linear functions of form, one can admit a normal n single on the facet of contact, exit vector product on the sides of this facet. When one goes up in order, the crack is nonplane and it is necessary to consider a new normal in each point of integration. This one is resulting from the gradient of the level set normal, which results from the approximation to the nodes of the facet of the gradients to the nodes. The gradients with the nodes are themselves resulting from an average to the nodes of the elementary gradients of the elements connected to the node.

4.6 Conditioning for the penalized method

A good conditioning of the equilibrium equation of the penalized formulation imposes a “beach advised” for the definition of the coefficients of penalization, which is left at the discretion of the user. We have:

$K_{\text{méca}} \sim E\,h$ et $A_u \sim \kappa_n h^2$, which imposes κ_n reasonable in front $\frac{E}{h}$.

$K_{\text{méca}} \sim E\,h$ et $B_u \sim \sigma\,\mu\,\kappa_\tau h^2$, which imposes κ_τ reasonable in front $\frac{E}{\mu\,\sigma\,h}$.

In the tests, one takes $\kappa_n \sim 10^5 \frac{E}{h}$ and $\kappa_\tau \sim 10^5 \frac{E}{\mu\,\sigma\,h}$.

5 Strategy of resolution

The strategy of resolution is the same one as that used by the method continues within the classical framework finite elements [feeding-bottle1]. The only difference is that with X-FEM, no pairing is not necessary.

5.1 Algorithm of resolution

5.1.1 Law of contact-friction

With X-FEM, the points in opposite are known *a priori* and this intrinsic “pairing” does not evolve during calculation (assumption of small displacements). Thus, it is not necessary to carry out a phase ofpairing as within the classical framework. The geometrical loop is also removed since there is no reactualization. For each step of time, there remain 3 overlapping loops. The loop on the thresholds of friction makes it possible to solve the problem of friction by a fixed search for point on the thresholds of friction of Tresca. The loop on the statutes of contact (related with the method of the active constraints) makes it possible to determine the space of the effective points of contact. At the major level, the loop on the iterations of Newton makes it possible to solve remaining non-linearity, that due to projection on the ball unit.

For a step of time:

Initialization of the thresholds of friction λ_s .

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- Buckle on the thresholds of friction

Initialization of the statutes of contact χ

- Buckle on the statutes of contact

- Iterations of Newton

Calculation of the tangent matrix and the second member

- End of the iterations of Newton

Actualization of the statutes of contact χ

- End of the loop of the active constraints

Actualization of the thresholds of friction λ_s

- End of the loop on the thresholds of friction

5.1.2 Cohesive law.

In the implementation of the cohesive laws, as well mixed as regularized, we do not introduce fields of statutes as we had been able to do it as for the contact: the various modes are managed by the routine of behavior itself, directly in the method of Newton. The only operation to be realized besides the iterations of Newton is thus the actualization of the internal variable.

For a step of time:

- Iterations of Newton

Calculation of the tangent matrix and the second member

- End of the iterations of Newton

Actualization of the internal variable α

One could legitimately wonder why the internal variable is not brought up to date during iterations of Newton. In fact, as it is about a parameter measuring the irreversibility, and determined by a maximum in the course of time, he should be updated only with each step of converged time. Indeed, in the contrary case, if this parameter exceeds its value of balance at the time of an iteration of Newton, the algorithm of Newton will be then unable to decrease it to find the value of balance.

5.2 Criterion of stop of the loop on the statutes of contact

For the supposed points lack of contact $(\chi=0)$, the condition of noninterpenetration is checked $(d_n \leq 0)$. The test is the following: if there is interpenetration $(d_n > 0)$ then the point is supposed contacting $(\chi=1)$ at the time of the iteration of active constraints following. Numerically, the test is written $d_n > 10^{-16}$.

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For the supposed points contacting $(\chi=1)$, it is checked that the value of the reaction of contact following the normal is negative $(\lambda_n \leq 0)$. The test is the following: if there is separation $(\lambda_n > 0)$ then the point is supposed lack of contact $(\chi=0)$ at the time of the iteration of active constraints following. Numerically, the test is written $\lambda_n > -10^{-3}$.

Notice :

> *The statute of contact being defined independently for each point of integration of each facet of contact, the tests of stop are carried out of each one of these points.*

Notice :

> *The game in these points of integration is calculated thanks to the interpolation of the field of displacement on the element relative (3D) whereas the reaction of contact is calculated thanks to the interpolation of the ddls of contact on the facet of contact (2D). One could also calculate the game by interpolation of displacement at the tops of the facet of contact (this last being given with the functions of form of the element relative).*

Notice :

> *The digital values of the 2 tests of stops are delicate to determine. In certain configurations, oscillations of statute of contact appear and prevent the convergence of the algorithm. This phenomenon should be identified, and if the values in the 2 cases (contacting and lack of contact) are close, one could consider that the convergence of the loop of active constraints is reached.*

5.3 Criterion of stop of the loop on the thresholds of friction

It is considered that the loop on the thresholds of friction converged if the solution does not change too much from one iteration to another. More precisely, that is to say v the mixed vector solution (displacement, multipliers of contact, semi-multipliers of friction), one converged with the iteration i if:

$$\frac{\max|v^i - v^{i-1}|}{\max|v^{i-1}|} < 10^{-3}$$

where $\max(u)$ mean the max on all the components of the vector u .

5.4 Writing of the formulation at the time of an iteration of Newton

Let us rewrite the weak formulation with three fields described in the paragraph [§3] at the time of an iteration of Newton. It is necessary to take account of the loop on the thresholds of friction, of that on the active constraints. Thus, on this level, the thresholds of friction become noted constants λ_s , statutes of contact $\chi(g_n)$ become constants χ . Moreover, the problem remaining being non-linear (because of projection on the ball unit), it is linearized by the method of Newton-Raphson.

In the unidimensional case, the method of Newton is an iterative process making it possible to approach the zeros of a continuous and derivable function. One is reduced to the resolution of $F(x)=0$. A succession of points is built x^k by doing one develop of Taylor of F in the vicinity of x^k , which gives to the first order:

$$F(x^{k+1}) \approx F(x^k) + F'(x^k)(x^{k+1} - x^k)$$

While noting $\delta x^k = x^{k+1} - x^k$ the increment between two successive iterations, the equation linearized with the iteration $k+1$ is then the following one:

$$F'(x^k)\,\delta x^k = -F(x^k)$$

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In the case of the finite element method, $F'(x^k)$ are connected with the tangent matrix, which can be calculated with each iteration so necessary, δx^k is the vector of the increments of the nodal unknown factors, and $F(x^k)$ is the second member. It is noted that $F'(x^k)$ and $F(x^k)$ only quantities of the iteration utilize k , which is thus known quantities.

Projection on the ball unit is written:

$$P_{B(0,1)}(x)=\begin{cases} x & \text{si } x\in B(0,1) \\ \dfrac{x}{\|x\|} & \text{sinon} \end{cases}$$

Differential of this application, in any point not located on the edge of $B(0,1)$, is defined by:

$$\partial_x P_{B(0,1)}(x)\delta x = K(x)\delta x$$

with

$$K(x)=\begin{cases} I_d & \text{si } x\in B(0,1) \quad (\text{adhérence}) \\ \dfrac{1}{\|x\|}\left(I_d-\dfrac{x\cdot x^T}{\|x\|^2}\right) & \text{sinon} \quad (\text{glissement}) \end{cases}$$

Thus, differential of $P_{B(0,1)}(g_\tau)$ will be:

$$K(g_\tau)\delta g_\tau = K\left(\Lambda+\frac{\rho_\tau}{\Delta t}\Delta[[u]]_\tau\right)\left(\delta\Lambda+\frac{\rho_\tau}{\Delta t}[[\delta u]]_\tau\right)$$

where g_τ is the semi-multiplier of friction resulting from the preceding iteration of Newton and δg_τ the increment of unknown factors. The knowledge of the semi-multiplier of friction resulting from the preceding iteration of Newton makes it possible to know easily if the point is in the state adherent or slipping.

In the same way, in the case of the regularized cohesive law CZM_EXP_REG, for example, differential of $\boldsymbol{t}_c(\llbracket \boldsymbol{u} \rrbracket)$ will be $\dfrac{\partial \boldsymbol{t}_c}{\partial \llbracket \boldsymbol{u} \rrbracket}\cdot\llbracket \delta \boldsymbol{u} \rrbracket$ with:

However $\dfrac{\partial \boldsymbol{t}_c}{\partial \llbracket \boldsymbol{u} \rrbracket}=H(\llbracket u \rrbracket_{\text{èq}}-\alpha)\dfrac{\partial \boldsymbol{\sigma}_{lin}}{\partial \llbracket \boldsymbol{u} \rrbracket}+(1-H(\llbracket u \rrbracket_{\text{èq}}-\alpha))\dfrac{\partial \boldsymbol{\sigma}_{dis}}{\partial \llbracket \boldsymbol{u} \rrbracket}+\dfrac{\partial \boldsymbol{\sigma}_{pen}}{\partial \llbracket \boldsymbol{u} \rrbracket}$

We re-use the expressions from these three derivative partial which are given in [R7.02.11], with $\boldsymbol{\delta}=-\llbracket \boldsymbol{u} \rrbracket$. In the expression of $\boldsymbol{\sigma}_{dis}$, it is necessary to write $\alpha=\llbracket u \rrbracket_{èq}$, which becomes thus a variable to be taken into account in derivation. In practice, in the code, one distinguishes four cases for clearness from reading:

- $\llbracket u \rrbracket_{èq}\geq\alpha$ et $\llbracket u_n \rrbracket<0$ (dissipative lack of contact). We have then:

$$\frac{\partial \boldsymbol{t}_c}{\partial \llbracket \boldsymbol{u} \rrbracket}=-\sigma_c\exp\left(-\frac{\sigma_c}{G_c}\llbracket u \rrbracket_{èq}\right)\left(\frac{\mathbf{Id}}{\llbracket u \rrbracket_{èq}}-\frac{\llbracket \boldsymbol{u} \rrbracket}{\llbracket u \rrbracket_{èq}}\otimes\frac{\llbracket \boldsymbol{u} \rrbracket}{\llbracket u \rrbracket_{èq}}\left(\frac{\sigma_c}{G_c}+\frac{1}{\llbracket u \rrbracket_{èq}}\right)\right)$$

- $\llbracket u \rrbracket_{èq}<\alpha$ et $\llbracket u_n \rrbracket\geq 0$ (contacting rubber band). With $(\boldsymbol{\tau}_1,\boldsymbol{\tau}_2)$ a base of the tangent plan, we have:

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$$\frac{\partial \boldsymbol{t}_c}{\partial [\![\boldsymbol{u}]\!]}=\frac{\partial \boldsymbol{\sigma}_{lin}([\![\boldsymbol{u}_\tau]\!])}{\partial [\![\boldsymbol{u}]\!]}+\frac{\partial \boldsymbol{\sigma}_{pen}([\![\boldsymbol{u}_n]\!])}{\partial [\![\boldsymbol{u}]\!]}=-C\,\boldsymbol{n}\otimes\boldsymbol{n}-\frac{\sigma_c}{\alpha}\exp\left(\frac{-\sigma_c}{G_c}\alpha\right)\left(\boldsymbol{\tau}_1\otimes\boldsymbol{\tau}_1+\boldsymbol{\tau}_2\otimes\boldsymbol{\tau}_2\right)$$

- $[\![u]\!]_{èq}\geq\alpha$ et $[\![u_n]\!]\geq 0$ (dissipative contacting). By a similar reasoning while replacing $\boldsymbol{\sigma}_{lin}$ by $\boldsymbol{\sigma}_{dis}$, we obtain:

$$\frac{\partial \boldsymbol{t}_c}{\partial [\![\boldsymbol{u}]\!]}=-C\,\boldsymbol{n}\otimes\boldsymbol{n}-\exp\left(\frac{-\sigma_c}{G_c}[\![u]\!]_{èq}\right)\left[\frac{\sigma_c}{[\![u]\!]_{èq}}\left(\boldsymbol{\tau_1}\otimes\boldsymbol{\tau_1}+\boldsymbol{\tau_2}\otimes\boldsymbol{\tau_2}\right)-\left(\frac{\sigma_c^2}{G_c}+\frac{\sigma_c}{[\![u]\!]_{èq}}\right)\frac{[\![\boldsymbol{u}]\!]_\tau\otimes[\![\boldsymbol{u}]\!]_\tau}{[\![u]\!]_{èq}{}^2}\right]$$

- $[\![u]\!]_{èq}<\alpha$ et $[\![u_n]\!]<0$ (lack of contact rubber band).

$$\frac{\partial \boldsymbol{t}_c}{\partial [\![\boldsymbol{u}]\!]}=-\frac{\sigma_c}{\alpha}\exp\left(\frac{-\sigma_c}{G_c}\alpha\right)\mathbf{Id}$$

5.5 Linearization of the problem

5.5.1 Integral writing with the method of Lagrangian increased

The linear system of the three equations to the iteration of Newton $k+1$ is written in the following way (by weighing down the writing, the references to the iteration of Newton are omitted, because in an obvious way, the unknown factors are noted with one δ in front of, and the fields tests from now on are noted with a star):

To find $(\delta u\ ,\ \delta\lambda\ ,\ \delta\Lambda)\in V_0\times H\times H$ such as:
$\forall(u^*,\lambda^*,\Lambda^*)\in V_0\times H\times H$

Equilibrium equation

$$\begin{aligned}
&\int_\Omega \sigma(\delta u):\varepsilon(u^*)\,d\Omega\\
&-\int_{\Gamma_c}\chi\delta\lambda[[u^*]]\cdot n d\Gamma_c+\int_{\Gamma_c}\chi\rho_n[[\delta u]]\cdot n\,[[u^*]]\cdot n d\Gamma_c\\
&-\int_{\Gamma_c}\chi\mu\lambda_s K(g_\tau)\delta g_\tau[[u^*]]_\tau d\Gamma_c\\
&=-\int_\Omega\sigma(u):\varepsilon(u^*)d\Omega+\int_\Omega f\cdot u^*\,d\Omega+\int_{\Gamma_t}t\cdot u^*\,d\Gamma_t\\
&+\int_{\Gamma_c}\chi\left(\lambda-\rho_n[[u]]\cdot n\right)[[u^*]]\cdot n d\Gamma_c\\
&+\int_{\Gamma_c}\chi\mu\lambda_s P_{B(0,1)}(g_\tau)\cdot[[u^*]]_\tau d\Gamma_c
\end{aligned}$$

Law of contact

$$\begin{aligned}
&-\int_{\Gamma c}\frac{(1-\chi)}{\rho_n}\delta\lambda\ \lambda^*d\Gamma_c-\int_{\Gamma c}\chi[[\delta u]]\cdot n\lambda^*d\Gamma_c\\
&=\int_{\Gamma c}\frac{(1-\chi)}{\rho_n}\lambda\lambda^*d\Gamma_c+\int_{\Gamma c}\chi[[u]]\cdot n\lambda^*d\Gamma_c
\end{aligned}$$

Law of friction

$$\begin{aligned}
&\int_{\Gamma c}\frac{\chi\mu\lambda_s\Delta t}{\rho_\tau}\left[\delta\Lambda-K(g_\tau)\delta g_\tau\right]\Lambda^*d\Gamma_c+\int_{\Gamma c}(1-\chi)\delta\Lambda\Lambda^*d\Gamma_c\\
&=-\int_{\Gamma c}\frac{\chi\mu\lambda_s\Delta t}{\rho_\tau}\left[\Lambda-P_{B(0,1)}(g_\tau)\right]\Lambda^*d\Gamma_c-\int_{\Gamma c}(1-\chi)\Lambda\Lambda^*d\Gamma_c
\end{aligned}$$

5.5.2 Integral writing with the penalized method

The linear system of the three equations to the iteration of Newton $k+1$ is written in the following way (by weighing down the writing, the references to the iteration of Newton are omitted, because in an

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obvious way, the unknown factors are noted with one δ in front of, and the fields tests from now on are noted with a star):

To find $(\delta u\ ,\ \delta\lambda\ ,\ \delta\Lambda)\in V_0\times H\times H$ such as:
$\forall(u^*,\lambda^*,\Lambda^*)\in V_0\times H\times H$

Equilibrium equation

$$\int_\Omega \sigma(\delta u):\varepsilon(u^*)\,d\Omega$$
$$-\int_{\Gamma_c}\chi\,\delta\lambda\,[[u^*]]\cdot n d\Gamma_c$$
$$-\int_{\Gamma_c}\chi\mu\lambda_s\kappa_\tau K(\kappa_\tau\nu_\tau)\delta\nu_\tau[[u^*]]_\tau d\Gamma_c$$
$$=-\int_\Omega\sigma(u):\varepsilon(u^*)d\Omega+\int_\Omega f\cdot u^*\,d\Omega+\int_{\Gamma_t}t\cdot u^*\,d\Gamma_t$$
$$+\int_{\Gamma_c}\chi\lambda[[u^*]]\cdot n d\Gamma_c$$
$$+\int_{\Gamma_c}\chi\mu\lambda_s P_{B(0,1)}(\kappa_\tau\nu_\tau)\cdot[[u^*]]_\tau d\Gamma_c$$

Law of contact

$$-\int_{\Gamma c}\frac{(1-\chi)}{\kappa_n}\delta\lambda\ \lambda^*d\Gamma_c-\int_{\Gamma c}\chi\left(\frac{\delta\lambda}{\kappa_n}+[[\delta u]]\cdot n\right)\lambda^*d\Gamma_c$$
$$=\int_{\Gamma c}\frac{(1-\chi)}{\kappa_n}\lambda\lambda^*d\Gamma_c+\int_{\Gamma c}\chi\left(\frac{\lambda}{\kappa_n}+[[u]]\cdot n\right)\lambda^*d\Gamma_c$$

Law of friction

$$\int_{\Gamma c}\frac{\chi\mu\lambda_s}{\kappa_\tau}\left[\delta\Lambda-K(\kappa_\tau\nu_\tau)\kappa_\tau\delta\nu_\tau\right]\Lambda^*d\Gamma_c+\int_{\Gamma c}(1-\chi)\delta\Lambda\Lambda^*d\Gamma_c$$
$$=-\int_{\Gamma c}\frac{\chi\mu\lambda_s}{\kappa_\tau}\left[\Lambda-P_{B(0,1)}(\kappa_\tau\nu_\tau)\right]\Lambda^*d\Gamma_c-\int_{\Gamma c}(1-\chi)\Lambda\Lambda^*d\Gamma_c$$

One chose here to solve the problems of contact and implicit friction of way (the semi-multipliers of contact and friction are expressed according to the jump of displacement of the current iteration of Newton). This choice makes the matrix of rigidity nonsymmetrical, the block B_r (see [§5.6.1]) being nonnull because of term in red in the writing of the law of friction whereas the block B_r^T is null.

5.5.3 Integral writing for a formulation with regularized cohesive law

The linear system of the three equations to the iteration of Newton $k+1$ is written in the following way (by weighing down the writing, the references to the iteration of Newton are omitted, because in an obvious way, the unknown factors are noted with one δ in front of, and the fields tests from now on are noted with a star):
To find $(\delta u\ ,\ \delta\lambda\ ,\ \delta\Lambda)\in V_0\times H\times H$ such as:
$\forall(u^*,\lambda^*,\Lambda^*)\in V_0\times H\times H$

Equilibrium equation

$$\int_\Omega\sigma(\delta u):\varepsilon(u^*)\,d\Omega$$
$$-\int_{\Gamma_c}\left[\frac{\partial t_{c,n}}{\partial[[u]]_n}[[\delta u]]_n+\frac{\partial t_{c,n}}{\partial[[u]]_\tau}[[\delta u]]_\tau\right][[u^*]]_n d\Gamma_c$$
$$-\int_{\Gamma_c}\left[\frac{\partial t_{c,\tau}}{\partial[[u]]_n}[[\delta u]]_n+\frac{\partial t_{c,\tau}}{\partial[[u]]_\tau}[[\delta u]]_\tau\right][[u^*]]_\tau d\Gamma_c$$
$$=-\int_\Omega\sigma(u):\varepsilon(u^*)d\Omega+\int_\Omega f\cdot u^*\,d\Omega+\int_{\Gamma_t}t\cdot u^*\,d\Gamma_t$$
$$+\int_{\Gamma_c}\left(t_{c,n}[[u^*]]_n+t_{c,\tau}[[u^*]]_\tau\right)d\Gamma_c$$

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Interface: normal part $\int_{\Gamma c} \lambda^* \left(\lambda+\delta\lambda-t_{c,n}\cdot n\right) d\Gamma_c=0$

Interface: tangential part $\int_{\Gamma c} \Lambda^* \left(\Lambda+\delta\Lambda-t_{c,\tau}\right) d\Gamma_c=0$

5.5.4 Integral writing for a formulation with mixed cohesive law

The linear system of the three equations to the iteration of Newton $k+1$ is written in the following way (by weighing down the writing, the references to the iteration of Newton are omitted, because in an obvious way, the unknown factors are noted with one δ in front of, and the fields tests from now on are noted with a star):

To find $(\delta u\ ,\ \delta\lambda)\in V_0\times H$ such as:

$\forall (u^*,\lambda^*)\in V_0\times H$

Equilibrium equation

$$\int_\Omega \sigma(\delta u):\epsilon(u^*)\,d\Omega-\int_\Gamma \left(Id-r\frac{\partial\delta}{\partial p}\right)\cdot\delta\lambda\cdot[\![u^*]\!]\,d\Gamma$$
$$+\int_\Gamma r\left(Id-r\frac{\partial\delta}{\partial p}\right)\cdot[\![\delta u]\!]\cdot[\![u^*]\!]\,d\Gamma$$
$$=-\int_\Omega \sigma(u):\epsilon(u^*)\,d\Omega+\int_\Omega f\cdot u^*\,d\Omega+\int_{\Gamma_t} t\cdot u^*\,d\Gamma$$
$$+\int_{\Gamma_c}\left[\lambda-r([\![u]\!]+\delta(p))\right]\cdot[\![u^*]\!]\,d\Gamma$$

Law of interface

$$-\int_{\Gamma_c}\left(1-r\frac{\partial\delta}{\partial p}\right)\cdot[\![u]\!]\cdot\lambda^*\,d\Gamma-\int_\Gamma \frac{\partial\delta}{\partial p}\cdot\delta\lambda\cdot\lambda^*\,d\Gamma$$
$$=\int_\Gamma([\![u]\!]+\delta)\cdot\lambda^*\,d\Gamma$$

5.6 Elementary terms of rubbing contact

5.6.1 Matric writing of the linearized problem

By taking again the notations of [feeding-bottle1], and by considering the unified writing adopted for the laws of contact friction and cohesive regularized, the system linearized such as it is solved with the iteration $k+1$ of Newton can put itself in form matriciit:

Equilibrium equation

$$\{u^*\}[K_{méca}](\delta u)$$
$$+\{u^*\}[A]^T(\delta\lambda)+\{u^*\}[A_u](\delta u)$$
$$+\{u^*\}[B_r]^T(\delta\Lambda)+\{u^*\}[B_u](\delta u)$$
$$+\{u^*\}[D_u](\delta u)$$
$$=\{u^*\}(L^1_{méca})$$
$$+\{u^*\}(L^1_{cont})$$
$$+\{u^*\}(L^1_{frott})$$
$$+\{u^*\}(L^1_{coh})$$

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Law of contact
$$\{\lambda^*\}[C]\{\delta\lambda\}+\{\lambda^*\}[A]\{\delta u\} = \{\lambda^*\}\{L^2_{cont}\}+\{\lambda^*\}\{L^2_{coh}\}$$

Law of friction
$$\{\Lambda^*\}[F_r]\{\delta\Lambda\}+\{\Lambda^*\}[B_r]\{\delta u\} = \{\Lambda^*\}\{L^3_{frott}\}+\{\Lambda^*\}\{L^3_{coh}\}$$

where the vectors column are noted $\{x\}$ and the vectors line $\{x\}=\{x\}^T$. This system can be put in the following matric form:

$$\begin{bmatrix} K_{méca}+A_u+B_u+D_u & A^T & B_r^T \\ A & C & 0 \\ B_r & 0 & F_r \end{bmatrix} \begin{Bmatrix} \delta u \\ \delta\lambda \\ \delta\Lambda \end{Bmatrix} = \begin{Bmatrix} L^1_{méca}+L^1_{cont}+L^1_{frott}+L^1_{coh} \\ L^2_{cont}+L^2_{coh} \\ L^3_{frott}+L^3_{coh} \end{Bmatrix}$$

The unknown factor is the increment compared to the preceding iteration of Newton. One voluntarily omitted reference to the number of the iteration of Newton.

Of course, terms of cohesion D_u , L^1_{coh} , L^2_{coh} and L^3_{coh} appear only in the formulation for regularized cohesive law. If such is the case, all the other terms are worthless, except for the terms $K_{méca}$ and $L^1_{méca}$ of course, but also except for C , F_r , L^2_{cont} and L^3_{frott} who are used for postprocessing, and who are deduced from the expressions detailed to follow while considering $\chi=0$ and $\rho_n=1$.

$K_{méca}$ is the mechanical matrix of rigidity defined in the paragraph [§3.2] of [R7.02.12].
A_u is the matrix of increased rigidity due to the contact.
B_u is the matrix of increased rigidity due to friction.
D_u is the matrix of rigidity due to the forces of cohesion.
A is the matrix binding the terms of displacement to those of contact (matrix of the law of contact).
B_r is the matrix binding the terms of displacement to those of friction (matrix of the laws of friction). This matrix is noted B in feeding-bottle1, but not to confuse it with the matrix of the derivative of the functions of form, we will note it B_r .
C is the matrix allowing to determine contact pressures in the lack of contact case.
F_r is the matrix making it possible to determine the multipliers of friction in the case of not-rubbing contact.
$L^1_{méca}$ is the second member representing the internal forces and the increments of loadings.
L^1_{cont} and L^2_{cont} are the second members due to the contact.
L^1_{frott} and L^3_{frott} are the second members due to friction.
L^1_{coh} , L^2_{coh} and L^3_{coh} are the second members due to the forces of cohesion.

Note:

> *It is pointed out that the system solved by Code_Aster is not of the type $[K][U]=[F]$ but of the type $[K][U]+[F]=0$. There thus exists a minus sign between the second members given in this document and those coded in files FORTRAN.*

5.6.2 Form of the elementary matrices of contact

5.6.2.1 Method of Lagrangian increased

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Taking into account the discretizations of the fields evoked in the paragraphs [§3.2] of [R7.02.12] and [§4.1] this document, the continuous matric system above is replaced by a discrete system. More precisely, the matrix A has the following form:

$$[A]_{ij} = \begin{matrix} \delta a_j & \delta b_j & \delta c_j^1 & \delta c_j^2 & \delta c_j^3 & \delta c_j^4 \end{matrix} \\ \begin{bmatrix} 0 & x & x & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \lambda_i^* \end{matrix}$$

Indeed, this is due to the fact that the terms of contact are cancelled for the ddls whose function of form associated is continuous. Indeed,

$$\begin{aligned}
&\left(\lambda^*\right)_i [A]_{ij} (\delta u)_j = -\int_{\Gamma^1} \chi \psi_i \lambda_i^* \varphi_j \left(\delta a_j + H_j \delta b_j + F^1 \delta c_j^1 + F^2 \delta c_j^2 + F^3 \delta c_j^3 + F^4 \delta c_j^4\right) \cdot n \, d\Gamma \\
&+ \int_{\Gamma^2} \chi \psi_i \lambda_i^* \varphi_j \left(\delta a_j + H_j \delta b_j + F^1 \delta c_j^1 + F^2 \delta c_j^2 + F^3 \delta c_j^3 + F^4 \delta c_j^4\right) \cdot n \, d\Gamma \\
&= -\int_{\Gamma^1} \chi \psi_i \lambda_i^* \varphi_j \left(\delta a_j + H_j(x^-) \delta b_j - \sqrt{r}\, \delta c_j^1\right) \cdot n \, d\Gamma + \int_{\Gamma^2} \chi \psi_i \lambda_i^* \varphi_j \left(\delta a_j + H_j(x^+) \delta b_j + \sqrt{r}\, \delta c_j^1\right) . n \, d\Gamma \\
&= \int_{\Gamma} \chi \psi_i \lambda_i^* \varphi_j \left(2 \delta b_j + 2 \sqrt{r}\, \delta c_j^1\right) \cdot n \, d\Gamma
\end{aligned}$$

Here, one clearly sees appearing the product of the functions of form ψ_i triangle with the functions of form Φ_j voluminal element relative.

Note:

One notes following this calculation the expression of the jump of displacement according to the degrees of freedom nouveau riches X-FEM:

$$\begin{aligned}
[[u]]_j &= \left(a_j + H_j b_j + F^1 c_j^1 + F^2 c_j^2 + F^3 c_j^3 + F^4 c_j^4\right)_1 \\
&\quad - \left(a_j + H_j b_j + F^1 c_j^1 + F^2 c_j^2 + F^3 c_j^3 + F^4 c_j^4\right)_2 \\
&= \left(a_j + H_j(x^-) b_j - \sqrt{r}\, c_j^1\right) - \left(a_j + H_j(x^+) b_j + \sqrt{r}\, c_j^1\right) \\
&= -\left(2 b_j + 2 \sqrt{r}\, c_j^1\right)
\end{aligned}$$

Indeed by construction (cf [R7.02.12]), the coefficient of jump for the functions of selection of field is worth:

$$H_j(x^+) - H_j(x^-) = 2$$

In the same way, the matrix of increased rigidity due to the contact has the following form:

$$[A_u]_{ij} = \begin{matrix} \delta a_j & \delta b_j & \delta c_j^1 & \delta c_j^2 & \delta c_j^3 & \delta c_j^4 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \times & \times & 0 & 0 & 0 \\ 0 & \times & \times & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} a_i^* \\ b_i^* \\ c_i^{1*} \\ c_i^{2*} \\ c_i^{3*} \\ c_i^{4*} \end{matrix}$$

$$\left(u^*\right)_i [A_u]_{ij} (\delta u)_j = \int_{\Gamma} \chi \rho_n \Phi_i \left\{\left(2 b_i^* + 2 \sqrt{r}\, c_i^{1*}\right) \cdot n\right\} \Phi_j \left(2 \delta b_j + 2 \sqrt{r}\, \delta c_j^1\right) \cdot n \, d\Gamma$$

The form of the matrix C do not change compared to the classical case without X-FEM:

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$$\left\{\lambda^*\right\}_i[C]_{ij}(\delta\lambda)_j=-\int_\Gamma \frac{1}{\rho_n}(1-\chi)\psi_i\lambda_i^*\psi_j\delta\lambda_j d\Gamma$$

5.6.2.2 Penalized method

The matrix C has as an expression:

$$\left\{\lambda^*\right\}_i[C]_{ij}(\delta\lambda)_j=-\int_\Gamma \frac{1}{\kappa_n}\psi_i\lambda_i^*\psi_j\delta\lambda_j d\Gamma$$

The matrix A_u is worthless.

The matrix A has as an expression:

$$\left\{\lambda^*\right\}_i[A]_{ij}(\delta u)_j=\int_\Gamma \chi\,\psi_i\lambda_i^*\Phi_j\left(2\,\delta b_j+2\sqrt{r}\,\delta c_j^1\right)\cdot n\,d\Gamma$$

5.6.2.3 Formulation for regularized cohesive law

A matrix C necessary to the postprocessing is used, whose expression is that of the Lagrangian one increased where one wrote $\chi=0$ and $\rho_n=1$. We obtain as follows:

$$\left\{\lambda^*\right\}_i[C]_{ij}(\delta\lambda)_j=\int_\Gamma \psi_i\lambda_i^*\psi_j\delta\lambda_j d\Gamma$$

5.6.3 Expression of the second members of contact

5.6.3.1 Method of Lagrangian increased

These expressions utilize sizes with the preceding iteration of Newton (iteration $k-1$). Therefore one made appear the reference to the index explicitly $k-1$:

$$\left\{u^*\right\}_i\left(L_{cont}^1\right)_i=-\int_\Gamma \chi\,\Phi_i\left\{\left(2\mathrm{b}_i^*+2\sqrt{r}\,c_i^{1^*}\right)\cdot n\right\}\left(\lambda^{k-1}-\rho_n d_n^{k-1}\right)d\Gamma$$

The expression of the vector L_{cont}^2 do not change compared to the classical case without X-FEM:

$$\left\{\lambda^*\right\}_i\left(L_{cont}^2\right)_i=\int_\Gamma \psi_i\lambda_i^*\left(\frac{1-\chi}{\rho_n}\lambda^{k-1}+\chi d_n^{k-1}\right)d\Gamma$$

5.6.3.2 Penalized method

These expressions utilize sizes with the preceding iteration of Newton (iteration $k-1$). Therefore one made appear the reference to the index explicitly $k-1$:

$$\left\{u^*\right\}_i\left(L_{cont}^1\right)_i=-\int_\Gamma \chi\,\Phi_i\left\{\left(2\mathrm{b}_i^*+2\sqrt{r}\,c_i^{1^*}\right)\cdot n\right\}\lambda^{k-1}d\Gamma$$

$$\left\{\lambda^*\right\}_i\left(L_{cont}^2\right)_i=\int_\Gamma \frac{1}{\kappa_n}\psi_i\lambda_i^*\lambda^{k-1}d\Gamma+\int_\Gamma \chi\,\psi_i\lambda_i^*d_n^{k-1}d\Gamma$$

5.6.3.3 Formulation for regularized cohesive law

A vector L_{cont}^2 necessary to the postprocessing is used, whose expression is that of the Lagrangian one increased where one wrote $\chi=0$ and $\rho_n=1$. We obtain as follows:

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$$\left\{\lambda^*\right\}_i\left(L^2_{cont}\right)_i=\int_\Gamma \psi_i\,\lambda_i^*\,\lambda^{k-1}\,d\Gamma$$

5.6.4 Form of the matrices of friction

5.6.4.1 Method of Lagrangian increased

In order to express the quantities in the tangent plan, one uses the expression of the paragraph [§2.3], that one writes in matric form:

$$u_\tau=(\mathrm{Id}-n\otimes n)\,u=[P]\,u$$

In this expression, the matrix P appoint the operator of projection as regards normal n . The matrix of this symmetrical operator has as an expression:

$$[P]=\begin{bmatrix}1-n_x^2 & -n_x n_y & -n_x n_z\\ -n_x n_y & 1-n_y^2 & -n_y n_z\\ -n_x n_z & -n_y n_z & 1-n_z^2\end{bmatrix}$$

where n_x , n_y , n_z are the coordinates of the normal n as defined in [the §4.5]. With the choice of a constant normal per facet, this matrix, depending only on the normal, has the same value in each point of Gauss and can be calculated only once for each facet.

The matrix of increased rigidity due to friction is written in the following way:

$$\left\{u^*\right\}_i\left[B_u\right]_{ij}(\delta u)_j=-\int_\Gamma \chi\mu\lambda_s\frac{\rho_\tau}{\Delta t}\Phi_i\left\{2\mathrm{b}_i^*+2\sqrt{r}\,c_i^{1^*}\right\}[P]^T[K_n]\Phi_j\left(2\,\delta b_j+2\sqrt{r}\,\delta c_j^1\right)[P]\,d\Gamma$$

where the matrix K_n represent the tangent matrix of projection on the ball unit of the semi-multiplier of friction increased with the preceding iteration of Newton: $K_n=K\left(g_\tau\right)$. It is a known matrix.

The matrix B_r has as an expression:

$$\left\{\Lambda^*\right\}_i\left[B_r\right]_{ij}(\delta u)_j=\int_\Gamma \chi\mu\lambda_s\psi_i\left\{\Lambda_i^{1^*}\tau_i^1+\Lambda_i^{2^*}\tau_i^2\right\}[K_n][P]\Phi_j\left(2\mathrm{b}_j+2\sqrt{r}\,c_j^1\right)d\Gamma$$

The matrix F_r has as an expression:

$$\left\{\Lambda^*\right\}_i\left[F_r\right]_{ij}(\delta\Lambda)_j=\int_\Gamma\frac{\chi\mu\lambda_s\,\Delta t}{\rho_\tau}\psi_i\left\{\Lambda_i^{1^*}\tau_i^1+\Lambda_i^{2^*}\tau_i^2\right\}\left[I_d-K_n\right]\psi_j\left(\Lambda_j^1\tau_j^1+\Lambda_j^2\tau_j^2\right)d\Gamma$$
$$+\int_\Gamma(1-\chi)\psi_i\left\{\Lambda_i^{1^*}\quad\Lambda_i^{2^*}\right\}\cdot\begin{bmatrix}\tau_i^1\tau_j^1 & \tau_i^1\tau_j^2\\ \tau_i^2\tau_j^1 & \tau_i^2\tau_j^2\end{bmatrix}\cdot\psi_j\begin{pmatrix}\Lambda_j^1\\ \Lambda_j^2\end{pmatrix}d\Gamma$$

5.6.4.2 Penalized method

The matrix F_r has as an expression:

$$\left\{\Lambda^*\right\}_i\left[F_r\right]_{ij}(\delta\Lambda)_j=\int_\Gamma\left((1-\chi)+\chi\frac{\mu\lambda_s}{\kappa_\tau}\right)\psi_i\left\{\Lambda_i^{1^*}\quad\Lambda_i^{2^*}\right\}\cdot\begin{bmatrix}\tau_i^1\tau_j^1 & \tau_i^1\tau_j^2\\ \tau_i^2\tau_j^1 & \tau_i^2\tau_j^2\end{bmatrix}\cdot\psi_j\begin{pmatrix}\Lambda_j^1\\ \Lambda_j^2\end{pmatrix}d\Gamma$$

The matrix B_u has as an expression:

$$\left\{u^*\right\}_i\left[B_u\right]_{ij}(\delta u)_j=-\int_\Gamma\chi\,\mu\lambda_s\frac{\kappa_\tau}{\Delta t}\Phi_i\left\{2\mathrm{b}_i^*+2\sqrt{r}\,c_i^{1^*}\right\}[P]^T[K_n]\Phi_j\left(2\,\delta b_j+2\sqrt{r}\,\delta c_j^1\right)[P]\,d\Gamma$$

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where the matrix K_n represent the tangent matrix of projection on the ball unit of the semi-multiplier of friction increased with the preceding iteration of Newton: $K_n = K\left(\kappa_\tau v_\tau\right)$. It is a known matrix.

The matrix B_r has as an expression:

$$\left\{\Lambda^*\right\}_i\left[B_r\right]_{ij}\left(\delta u\right)_j = \int_\Gamma \chi\mu\lambda_s\psi_i\left\{\Lambda_i^{1*}\tau_i^1 + \Lambda_i^{2*}\tau_i^2\right\}\left[K_n\right]\left[P\right]\Phi_j\left(2\mathrm{b}_j + 2\sqrt{r}\,c_j^1\right)d\Gamma$$

In penalized method, the matrix of rigidity is not symmetrical. One does not have $B_r^T = B_r$ but a worthless matrix instead of B_r^T .

5.6.4.3 Formulation for regularized cohesive law

A matrix F_r necessary to the postprocessing is used, whose expression is that of the Lagrangian one increased where one wrote $\chi = 0$. We obtain as follows:

$$\left\{\Lambda^*\right\}_i\left[F_r\right]_{ij}\left(\delta\Lambda\right)_j = \int_\Gamma \psi_i\left\{\Lambda_i^{1*} \quad \Lambda_i^{2*}\right\}\cdot\begin{bmatrix}\tau_i^1\tau_j^1 & \tau_i^1\tau_j^2\\ \tau_i^2\tau_j^1 & \tau_i^2\tau_j^2\end{bmatrix}\cdot\psi_j\begin{pmatrix}\Lambda_j^1\\ \Lambda_j^2\end{pmatrix}d\Gamma$$

5.6.5 Expression of the second members of friction

5.6.5.1 Method of Lagrangian increased

The second members of friction have the following expressions:

$$\left\{u^*\right\}_i\left(L_{frott}^1\right)_i = -\int_\Gamma \chi\mu\lambda_s\Phi_i\left\{2\mathrm{b}_i^* + 2\sqrt{r}\,c_i^{1*}\right\}\left[P\right]^T P_{B(0,1)}\left(g_\tau^{k-1}\right)d\Gamma$$

$$\left\{\Lambda^*\right\}_i\left(L_{frott}^3\right)_i = -\int_\Gamma \frac{\chi\mu\lambda_s\Delta t}{\rho_\tau}\psi_i\left\{\Lambda_i^{1*}\tau_i^1 + \Lambda_i^{2*}\tau_i^2\right\}\left(\Lambda^{k-1} - P_{B(0,1)}\left(g_\tau^{k-1}\right)\right)d\Gamma$$

$$-\int_\Gamma (1-\chi)\psi_i\left\{\Lambda_i^{1*}\tau_i^1 + \Lambda_i^{2*}\tau_i^2\right\}\Lambda^{k-1}d\Gamma$$

where $k-1$ represent the index of the preceding iteration of Newton.

5.6.5.2 Penalized method

The second members of friction have the following expressions:

$$\left\{\Lambda^*\right\}_i\left(L_{frott}^3\right)_i = -\int_\Gamma \psi_i\left\{\Lambda_i^{1*}\tau_i^1 + \Lambda_i^{2*}\tau_i^2\right\}\left((1-\chi)\Lambda^{k-1} + \chi\frac{\mu\,.\,\lambda_k}{\kappa_\tau}\left(\Lambda^{k-1} - P_{B(0,1)}(\kappa_\tau\nu_\tau^{k-1})\right)\right)d\Gamma$$

$$\left\{u^*\right\}_i\left(L_{frott}^1\right)_i = -\int_\Gamma \chi\,\mu\lambda_s\Phi_i\left\{2\mathrm{b}_i^* + 2\sqrt{r}\,c_i^{1*}\right\}\left[P_T\right]P_{B(0,1)}\left(\kappa_\tau\nu_\tau^{k-1}\right)d\Gamma$$

where $k-1$ represent the index of the preceding iteration of Newton.

5.6.5.3 Formulation for cohesive law

A vector L_{frott}^3 necessary to the postprocessing is used, whose expression is that of the Lagrangian one increased where one wrote $\chi = 0$. We obtain as follows:

$$\left\{\Lambda^*\right\}_i\left(L_{frott}^3\right)_i = \int_\Gamma \psi_i\left\{\Lambda_i^{1*}\tau_i^1 + \Lambda_i^{2*}\tau_i^2\right\}\Lambda^{k-1}d\Gamma$$

5.6.6 Form of the matrices of cohesion

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Are two directions of the fixed base I and J, unit vectors $\boldsymbol{e}_I$ and $\boldsymbol{e}_J$. Let us introduce the tangent matrix of the cohesive law into the fixed base $[K_{gl}]$ coefficients $[K_{gl}]_{IJ}=\boldsymbol{e}_I\cdot\dfrac{\partial\boldsymbol{\delta}}{\partial[\![\boldsymbol{u}]\!]}\cdot\boldsymbol{e}_J$. With the expression of $\dfrac{\partial\boldsymbol{t}_c}{\partial[\![\boldsymbol{u}]\!]}$ data with [the §5.4], we have the tangent matrix of the law cohesive $[K_{loc}]$ in the local base (see Doc. [R7.02.11]). We obtain then $[K_{gl}]$ by $[K_{gl}]=[Q]^T[K_{loc}][Q]$, where $[Q]$ is an orthonormal matrix of passage defined by:

$$[Q]=\begin{bmatrix} n_x & n_y & n_z \\ \tau_x^1 & \tau_y^1 & \tau_z^1 \\ \tau_x^2 & \tau_y^2 & \tau_z^2 \end{bmatrix}$$

Are i and j two nodes nouveau riches. That is to say Γ the intersection of the supports of i and j. The matrix $[D_u]$ is then given by:

$$\{u^*\}_i[D_u]_{ij}\{\delta u\}_j=-\int_\Gamma 2b_i^*\phi_i[K_{gl}]2b_j\phi_j d\Gamma$$

5.6.7 Expression of the second members of cohesion

The second members of cohesion have the following expressions:

$$\{u_i\}^*\left(L_{coh}^1\right)_i=-\int_\Gamma 2b_i^*\phi_i\boldsymbol{t}_c^{k-1}d\Gamma$$

$$\{\lambda^*\}_i\left(L_{coh}^2\right)_i=-\int_\Gamma \psi_i\lambda_i^*\left(\boldsymbol{t}_{c,n}^{k-1}\cdot\boldsymbol{n}\right)d\Gamma$$

$$\{\Lambda^*\}_i\left(L_{coh}^3\right)_i=-\int_\Gamma \psi_i\left\{\Lambda_i^{1*}\tau_i^1+\Lambda_i^{2*}\tau_i^2\right\}\boldsymbol{t}_{c,\tau}^{k-1}d\Gamma$$

where $k-1$ represent the index of the preceding iteration of Newton.
To express $\boldsymbol{t}_c$ in the total base, one can use $[Q]$.

5.6.8 Ematric criture of the problem with mixed cohesive law

The matric system such as it is solved with the iteration $k+1$ of Newton can put itself in following matric form:

$$\begin{bmatrix} K_{méca}+A_u & A^T \\ A & C \end{bmatrix}\begin{pmatrix}\delta u\\ \delta\lambda\end{pmatrix}=\begin{pmatrix} L_{méca}^1+L_{coh}^1 \\ L_{coh}^2\end{pmatrix}$$

5.6.8.1 Form of the elementary matrices of cohesion:

Are two directions of the fixed base I and J, unit vectors $\boldsymbol{e}_I$ and $\boldsymbol{e}_J$. We introduce as previously the tangent matrix of the cohesive law into the fixed base $[K_{gl}]$ coefficients $[K_{gl}]_{IJ}=\boldsymbol{e}_I\cdot\dfrac{\partial\boldsymbol{\delta}}{\partial\boldsymbol{p}}\cdot\boldsymbol{e}_J$. We lay out, by the cohesive law of behavior, of the tangent matrix $[K_{loc}]$ in the local base (see Doc. [R7.02.11]). We obtain then $[K_{gl}]$ by $[K_{gl}]=[Q]^T[K_{loc}][Q]$, where $[Q]$ is an orthonormal matrix of passage defined by:

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$$[Q]=\begin{bmatrix} n_x & n_y & n_z \\ \tau_x^1 & \tau_y^1 & \tau_z^1 \\ \tau_x^2 & \tau_y^2 & \tau_z^2 \end{bmatrix}$$

Having introduced these notations, we have:

$$\{u^*\}_i[A_u]_{ij}\{\delta u\}_j=\int_\Gamma 2\phi_i b_i^* r([Id]-r[K_{gl}])2\phi_j b_j d\Gamma$$

In addition, we choose to discretize $\boldsymbol{\lambda}$ in local base. The coefficient $(1,J)$ matrix $\{\lambda_n^*\}[A_u]_{ij}\{\delta u\}_j$ is then $\int_\Gamma 2\phi_i\lambda_i^*\boldsymbol{n}\cdot\left(Id-r\frac{\partial\delta}{\partial p}\right)\cdot\boldsymbol{e_j}2\phi_j b_j d\Gamma$. For the coefficients $(2,J)$ and $(3,J)$, the formula is the same one while replacing $\boldsymbol{n}$ by $\boldsymbol{\tau_1}$ and $\boldsymbol{\tau_2}$, respectively. By exploiting the notations introduced into the preceding paragraph, we deduce some:

$$\{\lambda^*\}_i[A_u]_{ij}\{\delta u\}_j=\int_\Gamma \varphi_i\lambda_i^*([Id]-r[K_{loc}])\cdot[Q]2\varphi_j b_j d\Gamma$$

As for the matrix $[C]$, she is written simply:

$$\{\lambda^*\}_i[C]_{ij}\{\delta\lambda\}_j=-\int_\Gamma \phi_i\lambda_i^*[K_{loc}]\phi_j\lambda_j d\Gamma$$

5.6.8.2 Expression of the elementary vectors of cohesion:

The coefficient I vector $\{u\}_i^*\left(L_{coh}^1\right)_i$ has as an expression $\int_\Gamma 2b_i^*\phi_i\left(-\boldsymbol{\lambda}\cdot\boldsymbol{e_I}+r(\llbracket u\rrbracket+\delta)\cdot\boldsymbol{e_I}\right)d\Gamma$. With the notations indroduites in the preceding ones part, we deduce:

$$\{u_i\}^*\left(L_{coh}^1\right)_i=\int_\Gamma 2b_i^*\phi_I\left(-[Q]^T\cdot\{\lambda\}+r\ \left(\{\llbracket u\rrbracket\}+[Q]^T\cdot\{\delta\}\right)\right)d\Gamma$$

where $\{\delta\}$ and $\{\lambda\}$ are given in local base, and $\{\llbracket u\rrbracket\}$ in fixed base.

The coefficient 1 vector $\{\lambda\}_i^*\left(L_{coh}^2\right)_i$ is written $\int_\Gamma\lambda_i^*(\llbracket u\rrbracket\cdot n+\delta\cdot n)d\Gamma$. For the coefficients 2 and 3, the formula is the same one while replacing $\boldsymbol{n}$ by $\boldsymbol{\tau_1}$ and $\boldsymbol{\tau_2}$, respectively. We deduce some:

$$\{\lambda\}_i^*\left(L_{coh}^2\right)_i=\int_\Gamma\lambda_i^*([Q]\{\llbracket u\rrbracket\}+\{\delta\})d\Gamma$$

where $\{\delta\}$ is given in local base, and $\{\llbracket u\rrbracket\}$ in fixed base.

6 LBB condition

Selected approximations on the one hand for the dieplacement and in addition for contact pressures do not seem by satisfying the condition *inf-sup* in all the cases. The non-observance of the LBB condition generates oscillations of contact pressures, phenomenon comparable to that met of incompressibility [feeding-bottle13]. Physically, in the case of the Lagrangian contact, that amount wanting to impose the contact in too many points of the interface (overstrained), making the system hyperstatic. To slacken it, it is necessary to restrict the space of the multipliers of Lagrange, as that is done in [feeding-bottle 14] for the conditions of Dirichlet with X-FEM. The algorithm proposed by Moës [feeding-bottle14] to reduce the oscillations is extended to the case 3D. Its goal is to impose relations of equality between multipliers of Lagrange. This algorithm was the object of an improvement to make it more physical and more effective.
In the case of the penalized contact one finds the same oscillations. For a grid triangle for example, one can show that there does not exist combination of the degrees of freedom of the Heaviside which allow a rotation of the average surface of the crack without generating oscillations of the jump of displacement. For a stiffness of raised interface, as it is the case in penalization, this generates

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oscillations of pressure. To cure it, it is necessary to recover the explicit pressure in the degree of freedom λ , to apply the condition of LBB to him and of to make go up in balance, which explains the formulation given to the § 3.3 . The actualization of the statutes of contact is done as in the Lagrangian case, where λ and not d_n is tested to pass from a state contacting to a state not contacting, this in order to avoid the oscillations on the statutes of contact.

6.1 Description of the algorithm of Moës for the linear and quadratic elements (algo1)

The algorithm introduced by Moës [feeding-bottle14] is presented there with an aim of imposing conditions of Dirichlet on an interface within the framework of X-FEM. It shows that the technique of the multipliers of Lagrange to impose conditions of Dirichlet must be used carefully, because the condition *inf-sup* is not always respected. Paper is restricted with the case 2D, but the algorithm presented is easily generalizable with the case 3D. The first phase is a phase of selection of the nodes, in which the selected nodes are those which are "important" for the approximation of the multipliers of Lagrange. The other nodes are superabundant and bring to oscillations of the multipliers of Lagrange. Once the selected "important" nodes, of the relations of equalities are imposed between the multipliers of Lagrange, in order to restrict the space of the multipliers. Thus, the multipliers of emanating Lagrange of the edges of the same selected node are equal.
In a more formal way, are E and N units containing all the edges and all the nodes of the grid. Two ends of an edge $e\in E$ are noted $\left(v_1(e), v_2(e)\right)\in N^2$. First of all, one starts with a phase of initialization (iteration $k=0$ algorithm). One determines first of all S_e^0 , the whole of the edges which are cut by the interface. The interface being represented by the level set normal lsn , an edge $e\in E$ is crossed by the interface if and only if $lsn\left(v_1(e)\right)\cdot lsn\left(v_2(e)\right)\leq 0$. Let us note that if the interface coincides with the node $v_1(e)$ or the node $v_2(e)$, the edge e east belongs to S_e^0 :

$$S_e^0=\left\{ e\in E,\ lsn\left(v_1(e)\right)\cdot lsn\left(v_2(e)\right)\leq 0\right\}$$

That is to say N_e the whole of the nodes connected by the elements of S_e^0 :

$$N_e=\left\{ n\in N,\ \exists e\in S_e^0\ \ n=v_1(e)\ \text{ ou }\ n=v_2(e)\right\}$$

That is to say S_n^0 the whole of the nodes selected with the iteration $k=0$ (initialization). These nodes are those which coincide with the interface (this unit can be empty):

$$S_n^0=\left\{ n\in N_e,\ lsn(n)=0\right\}$$

After this phase of initialization, the algorithm reiterates $k=1,\text{nmax\_iter}$.
With each iteration, the following stages are carried out:

- Update of the whole of the whole of the edges: one removes those which are connected to a node selected with the preceding iteration

$$S_e^k=S_e^{k-1}\backslash\left\{ e\in S_e^{k-1},\ v_1(e)\in S_n^{k-1}\ \text{ ou }\ v_2(e)\in S_n^{k-1}\right\}$$

- Calculation of the score of the nodes: for each node in N_e , one calculates a score made up of 2 digits: the first figure corresponds to the numbers of edges in S_e^k connected, and the second corresponds to the absolute value of the level set normal in this node. This score sc_no is thus a matrix with two columns whose lines represent the node.

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$$\forall n \in N_e \quad \begin{cases} sc\_no^k(n,1) = \text{nombre d'arêtes connectées au noeud } n \\ sc\_no^k(n,2) = |lsn(n)| \end{cases}$$

•Calculation of the score of the edges: for each edge in S_e^k , one calculates a score made up of 2 digits: the first figure corresponds to the absolute value of the difference of the 1er figure of the score of the 2 nodes ends, and the second corresponds to a relationship between the values of the 2ème figure of the 2 nodes ends (i.e. a value ratio of lsn . This score sc_ar is thus a matrix with two columns whose lines represent the edge.

$$\forall e \in S_e^k, \quad \forall j \in (1,2), \quad s_j = sc\_no^k(v_j(e),1), \quad l_j = sc\_no^k(v_j(e),2)$$

$$\begin{cases} sc\_ar^k(e,1) = |s_1 - s_2| \\ sc\_ar^k(e,2) = \begin{cases} \dfrac{l_1}{l_1+l_2} & \text{si } s_1 < s_2 \\ \dfrac{l_1}{l_1+l_2} & \text{si } s_1 > s_2 \\ \dfrac{\min(l_1, l_2)}{l_1+l_2} & \text{si } s_1 = s_2 \end{cases} \end{cases}$$

•Research of the “best edge” b_e : it is the edge including the 1er figure of its score is largest. In the event of equality between 2 edges, it is that including 2ème figure of its score is largest.

•Research of the “best node” b_n : it is the node end of b_e including the 1er figure of its score is largest. In the event of equality, it is the node including 2ème figure of the score is smallest (the node nearest to the interface). The node b_n is the only node selected with this iteration:

$$S_n^k = (b_n)$$

The algorithm stops so at the time of an iteration, the unit S_e^k becomes the empty set. The final whole of the selected nodes will be then:

$$W = \bigcup_k S_n^k$$

After this phase of selection of the nodes, the algorithm builds the space of the multipliers of Lagrange, whose size is equal to that of W . Thus, the space of the multipliers is:

$$S_\lambda = \{ \lambda^i, i \in (1, card(W)) \}$$

With this algorithm any Nœud of N_e / W is connected by an edge of S_e^0 with Nœud of W . One then imposes a relation of equality between these two Nœuds. In the event of conflict (connections with several Nœuds of W by as many edges), one discriminates Nœuds of W by criterion 2 of Nœuds (weaker level-set normal).

It should be noted that this algorithm can also be used in great slips.

6.2 Description of the algorithm modified for the linear and quadratic elements (algo2)

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Based on similar ideas, a new algorithm was proposed. Thus, in the new version, one leaves the whole of the edges on which the level set normal is cancelled at least in a point. These edges connect points on both sides of the interface (or possibly of the points on the interface). The algorithm seeks the minimal subset of edges making it possible to connect all the points ends of the edges. Then, groups of connected edges are extracted from it. The imposed relations are then the following ones:

•multipliers on Nœuds tops of each group is imposed equal,

In a more formal way, are E and N units containing all the edges and all the nodes of the grid. Two ends of an edge $e\in E$ are noted $\left(v_1(e),v_2(e)\right)\in N^2$. One determines first of all S_e , the whole of the edges which are strictly cut by the interface. The interface being represented by the level set normal lsn , an edge $e\in E$ is strictly crossed by the interface if and only if $lsn\left(v_1(e)\right)\cdot lsn\left(v_2(e)\right)<0$. Let us note that if the interface coincides with the node $v_1(e)$ or the node $v_2(e)$, the edge e is not in S_e :

$$S_e=\left\{e\in E,\ \ lsn\left(v_1(e)\right)\cdot lsn\left(v_2(e)\right)<0\right\}$$

That is to say N_e the whole of the nodes connected by the elements of S_e . One separates N_e in two parts: nodes “below” and “above” the crack, according to the sign of lsn :

$$N_e=\left\{n\in N,\ \ \exists e\in S_e\ \ n=v_1(e)\ \ \text{ou}\ \ n=v_2(e)\right\}$$
$$N_e^+=\left\{n\in N_e,\ \ lsn(n)>0\right\}\ \ \text{et}\ \ N_e^-=\left\{n\in N_e,\ \ lsn(n)<0\right\}$$

The algorithm searches S_{ve} , the minimal subset of S_e who allows to connect the nodes in N_e^+ with the nodes in N_e^- . Each node in N_e^+ must be connected to at least a node dan N_e^- S, and each node in N_e^- must be connected to at least a node in N_e^+ . Edges in S_{ve} are called “vital edges”, because if one of these edges disappears, at least a node in N_e will be orphan. This whole of vital edges is not necessarily single. In the presence of choice, the vital edge shortest is privileged. As it thereafter will be seen, that amounts minimizing the area of P0 approximation. For the research of the unit S_{ve} , we chose an algorithm based on the concepts of scores of nodes and edge, concept that one finds in the algorithme1. The algorithm will remove with all not-vital edges, until there remain nothing any more but vital edges. More precisely, one associates a score with each node, which corresponds to the number of edges connected to this node. With each edge, one associates a score, which corresponds at least of the scores of the two nodes ends. That is to say e the edge having the score more raised (with identical score, the longest edge is privileged). If the score of e is equal to 1, then all the edges which remain are vital edges. S_{ve} is determined and the algorithm stops. If the score of e is strictly higher than 1, the edge e is a symbolically removed not-vital edge, and it list of the edges S_e . The algorithm starts again, with a new calculation of the score of the nodes, and so on until there remain nothing any more but vital edges.

It is important to note that S_{ve} is composed of certain disconnected edges, and certain edges connected between them. These groups of connected vital edges are extracted from S_{ve} . Let us note that in such a group, all the edges are connected by a single node (see Figure 6.2-1). That is to say G_{cve}^i the group of vital edges connected by the node i . Then G_{cve}^i is defined by:

$$G_{cve}^i=\left\{e\in S_{ve},\ \ i=v_1(e)\ \ \text{ou}\ \ i=v_2(e)\right\}$$

Now, one imposes relations between the multipliers of Lagrange. All multipliers carried by Nœuds tops of the same group is imposed equal.

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We illustrate these algorithms on the case 2D of Figure 6.2-1. Groups of Nœuds bound by relations between equality and version 1 is marked by the blue circles. Groups of Nœuds bound by relations between equality and version 2 is marked by the full edges connected between them.

It should be noted that this algorithm can also be used in great slips.

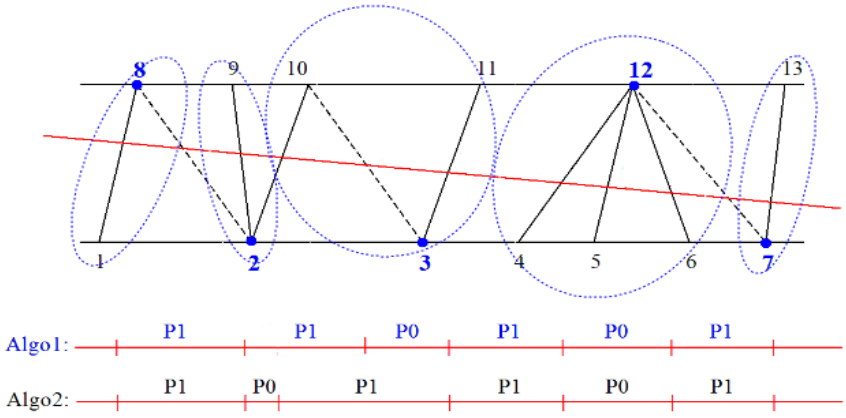

Figure 6.2-1 : Example of edges cut by an interface and resulting approximation

6.3 Description of the algorithm for the quadratic elements (algorithm 3) :

This algorithm makes it possible to force the minimal number of relations of equality in order to reach the optimal rate of convergence for the quadratic elements. Determination of the edges carrying them relations of equality between the “Lagrange” is held in two times:

– Study of the related components:

A related component is a set of edges cut dependent between them continuously (by one of the two tops of each edge). The algorithm analyzes the whole of the edges cut in order to determine the related components, then it puts a relation of equality on the first edge of each component.

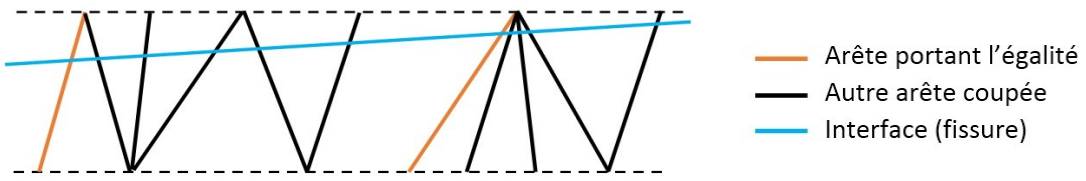

Figure 6.3-1: Relations of equality on related components

– Study of the criterion of proximity:

It is a question of analyzing, for each cut edge, the proximity of the nodes compared to the interface. The idea is to bind the “Lagrange” of the nodes to “weak” influence. Concretely one starts by eliminating the nonvital edges in order to lead to a subset of cut edges binding all the nodes on both sides of the interface. Then the relations of equality are put on each edge where the criterion of proximity is checked. Finally the nonvital edges are re-studied: if one among them connects two edges of which only one checks the criterion of proximity, the nonvital edge in question carries an equality; in the other cases one does nothing.

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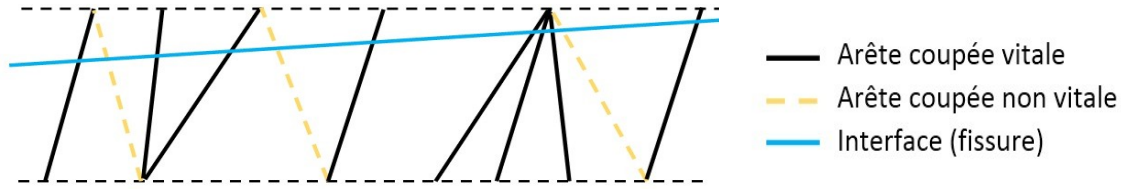

Figure 6.3-2: Selection of the group of vital edges

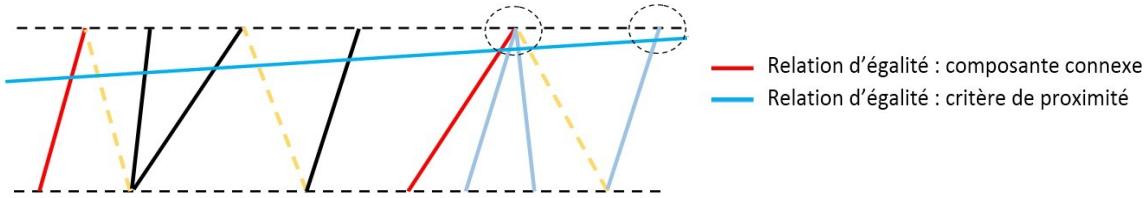

Figure 6.3-3: Relations of equality on related components and criterion of proximity

In a more formal way, and by taking again the notations used with the §6.2, one considers S_e the whole of the edges strictly cut by the interface.
The first stage is of partitionner of a union of disjoined related components: $S_e = U_i S_{ei}$
A relation of equality is imposed on the first edge of each subset S_{ei} .
Then one builds S_{ve} , the group of the "vital" edges (within the meaning of algorithm 2), to study the criterion of proximity. S_{ve} is composed of two types of edges: independent edges, and edges connected between them via a common top. One notes S_{ind} and S_{cte} subsets of S_{ve} corresponding respectively to these two types, such as: $S_e = S_{ind} U S_{cte}$
The criterion of proximity is studied on the following nodes:
– for the edges of S_{ind} one seeks the node nearest to the interface and if the distance (relative) is lower than the distance threshold fixed by the criterion, the edge carries a relation of equality;
– for the edges of S_{cte} one calculates the minimal distance compared to the common top, and if this minimal distance is lower than the threshold one imposes relations of equality on all the edges of the package.
Thus one manages to bind the "Lagrange" to "weak" influence.

It should be noted that this algorithm cannot be used within the framework of great slips, various possible pairings changing as slip.

6.4 Relations imposed between the semi-multipliers of friction

When friction is taken into account, one observes the same phenomenon of oscillations on the semi-multipliers of friction as in the preceding case on the multipliers of contact. So that the reaction of contact does not oscillate any more, it is necessary to remove the oscillations of the normal reaction (contact pressure) and of tangential reaction (thus of the semi-multipliers of friction). For that, it is thus necessary also to activate the algorithm of restriction of spaces of the multipliers for the semi-multipliers of friction.

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The relations are to be imposed on the tangential reactions, and utilize the unknown factors of friction Λ_1 and Λ_2 as well as the vectors of the base covariante τ_1 and τ_2 .

In the case of imposition of a relation of equality between the nodes A and B , the relation is written:

$$\Lambda_1^A \tau_1^A + \Lambda_2^A \tau_2^A = \Lambda_1^B \tau_1^B + \Lambda_2^B \tau_2^B$$

Both unknown factors to be determined (Λ_1^A and Λ_2^A for example) being scalar, it is necessary to transform the preceding vectorial relation into two scalar relations. This is done by projection on the basis $\left(\tau_1^A, \tau_2^A\right)$. The two relations to be imposed are thus finally:

$$\begin{cases} \Lambda_1^A \left(\tau_1^A \cdot \tau_1^A\right) + \Lambda_2^A \left(\tau_2^A \cdot \tau_1^A\right) = \Lambda_1^B \left(\tau_1^B \cdot \tau_1^A\right) + \Lambda_2^B \left(\tau_2^B \cdot \tau_1^A\right) \\ \Lambda_1^A \left(\tau_1^A \cdot \tau_2^A\right) + \Lambda_2^A \left(\tau_2^A \cdot \tau_2^A\right) = \Lambda_1^B \left(\tau_1^B \cdot \tau_2^A\right) + \Lambda_2^B \left(\tau_2^B \cdot \tau_2^A\right) \end{cases} \quad \textbf{éq 6.4-1}$$

This choice is called into question by the introduction of the great slips [R5.03.53]: indeed the base of contact changes with each geometrical iteration. The preceding relations introduced into hard by using home base thus do not have any more a direction. To solve the conflict, one rather introduces the two relations of equalities on the components:

$$\begin{cases} \Lambda_1^A = \Lambda_1^B \\ \Lambda_2^A = \Lambda_2^B \end{cases} \quad \textbf{éq 6.4-2}$$

By considering that bases $\left(\tau_1^A, \tau_2^A\right)$ and $\left(\tau_1^B, \tau_2^B\right)$ are almost identical because of proximity of the points A and B , equations 6.4-1 and 6.4-2 are almost equivalent.

6.5 Remarks on the relations imposed by algorithm 1 or 2

6.5.1 On the multipliers of contact

That is to say the linear relation between the multipliers of Lagrange of contact λ_1 , λ_2 and λ_3 :

$$\lambda_2 = \alpha \lambda_1 + (1-\alpha)\lambda_3$$

The relation relates to the value of the pressure and not the vector contact pressure. In the event of structure a relation curves on the vectors pressure of the type:

$$\lambda_2 n_2 = \alpha \lambda_1 n_1 + (1-\alpha)\lambda_3 n_3$$

because the vector is not inevitably possible n_2 is not an unknown factor.

6.5.2 On the semi-multipliers of friction

The unknown factor for friction is vectorial. One could imagine to bind the vectors tangent reactions between them:

$$r_{\tau_2} = \alpha r_{\tau 1} + (1-\alpha) r_{\tau 3}$$

however

$$r_{\tau_i} = \mu \lambda_i \Lambda_i$$

what gives

$$\Lambda_2 = \frac{\alpha \lambda_1 \Lambda_1 + (1-\alpha)\lambda_3 \Lambda_3}{\alpha \lambda_1 + (1-\alpha)\lambda_3} = \beta \Lambda_1 + (1-\beta)\Lambda_3$$

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with $\beta = \frac{\alpha\lambda_1}{\alpha\lambda_1 + (1-\alpha)\lambda_3} \approx \alpha$ if the grid is rather fine

This choice of relation is impossible because in 3D, if items 1 and 3 are in slipping contact, then item 2 will not be it! Indeed, the standard of Λ_2 will be strictly lower than 1 if the directions of slip are not colinéaires (see Figure 6.5.2-1).

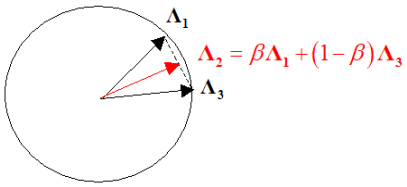

Figure 6.5.2-1 : Case of a point adhering between two slipping points

The suggested solution is to impose a linear relation between the standards of the vectors tangent reactions, which is equivalent to impose a relation between the standards of the semi-multipliers of friction:

$$\|r_{\tau 2}\| = \alpha\|r_{\tau 1}\| + (1-\alpha)\|r_{\tau 3}\| \Leftrightarrow \|\Lambda_2\| = \beta\|\Lambda_1\| + (1-\beta)\|\Lambda_3\| \quad \textbf{éq. 6.5.2-1}$$

This relation is non-linear because of standard. The method of Newton makes it possible to be reduced to the successive imposition of the linear relations. With the iteration of Newton I , the relation is:

$$\delta\Lambda_2^i \cdot \frac{\Lambda_2^{i-1}}{\|\Lambda_2^{i-1}\|} - \beta\delta\Lambda_1^i \cdot \frac{\Lambda_1^{i-1}}{\|\Lambda_1^{i-1}\|} - (1-\beta)\delta\Lambda_3^i \cdot \frac{\Lambda_3^{i-1}}{\|\Lambda_3^{i-1}\|} = \beta\|\Lambda_1^{i-1}\| + (1-\beta)\|\Lambda_3^{i-1}\| - \|\Lambda_2^{i-1}\|$$

This kind of linear relation is currently not available in *Code_Aster* , where only the linear relations whose coefficients are constant throughout all calculation are authorized.

Currently, the relation between the semi-multipliers of friction established is the following one:

$$\Lambda_2 = \alpha\Lambda_1 + (1-\alpha)\Lambda_3$$

Lorsque the slip or adherence is one-way, one finds the equation [éq well. 4.6 - 1] while having substituted α with β .

6.5.3 Nonsimpliciaux elements (quadrangles, hexahedrons…)

In the case of meshs *quad* in 2D, *penta* or *hexa* in 3D, one notes certain configurations where nodes of the mesh do not belong with a cut edge. On the two examples of the figure 6.5.3-1 , the red nodes are connected to no other node. In order to satisfy the LBB, the idea is to connect these nodes to the related nodes but while trying not to introduce into this case of the linear relations.

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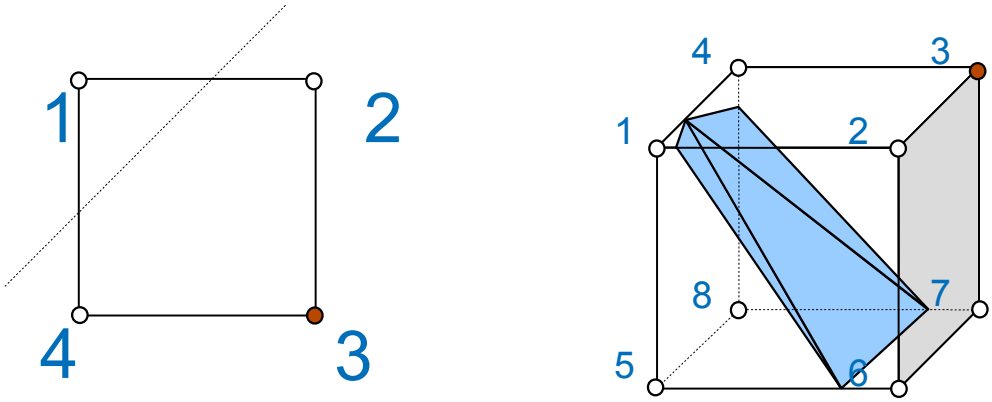

Figure 6.5.3-1 : The red nodes do not belong to a cut edge.

To avoid adding linear relations, the idea is to eliminate from the interpolation the nodes who do not belong to a cut edge. To satisfy the partition with the unit of the contributions of contact, one distributes the functions of forms of the nodes eliminated on the other nodes by using the following uniform distribution:

$$\tilde{\Phi}_{i\in N_{actif}} = \Phi_i + \sum_{j\in N_{elim}} \Phi_j / N_{actif} \qquad \text{éq. 6.5.3-1}$$

Where N_{actif} is the whole of the directly cut nodes of the element or pertaining to a cut edge. N_{elim} is the whole of the nodes to be eliminated. For the quadrangle of the figure 6.5.3-1 , the modified functions of form are written then $\tilde{\Phi}_{i=1,2,4} = \Phi_i + \Phi_3/3$. For the hexahedron they are written $\tilde{\Phi}_{i=1,2,4,5,6,7,8} = \Phi_i + \Phi_3/7$.
It is noticed that other choices of distribution are possible: one for example could have chosen to distribute a node eliminated on his related active nodes. For the quadrangle of the figure 6.5.3-1 , the functions of form modified would be written then $\tilde{\Phi}_1 = \Phi_1$ and $\tilde{\Phi}_{i=2,4} = \Phi_i + \Phi_3/2$.
The elimination of the degrees of freedom of contact in excess is discussed [§4.4]), substitution is chosen (by putting 1 on the diagonal and 0 in the second member).

One can generalize this approach for the funds of crack. Indeed on the two examples of the figure 6.5.3-2, the edge {1-4} is cut whereas {2-3} is not it: nodes 2 and 3 are thus to eliminate. By using the approach described previously, the modified functions of form are written $\tilde{\Phi}_{i=1,4} = \Phi_i + (\Phi_2 + \Phi_3)/2$.
That amounts making P0 integration described in the paragraph [§4.3.2] on the elements crossed containing the point.

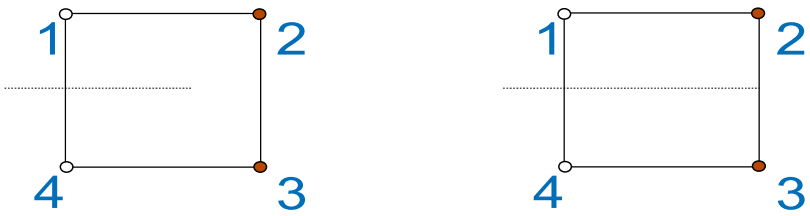

Figure 6.5.3-2 : The red nodes do not belong to a cut edge.

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8 Description of the versions

Index document	Version Aster	Author (S) Organization (S)	Description of the modifications
With	11	S.GENIAUT, P.MASSIN EDF/R & D AMA	Initial text

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