Method IMPLEX

Summary:

This document presents a method of resolution of the nonlinear problem, due to *Oliver et al.* [1], substituent with the method of Newton for certain laws of behavior of damage and plasticity (ENDO_FRAGILE [3], ENDO_ISOT_BETON [4] and VMIS_ISOT_LINE [5] at present). It is based on an explicit extrapolation of the internal variables to determine the degrees of freedom (displacement) from which the behavior is integrated implicitly. The nullity of the residue is not checked. It introduces of this fact an approximation of the resolution but makes it possible to guarantee the robustness of calculation.

It thus belongs to the user to have a critical glance on the got results, those being not converged with the classical direction of term, and potentially being able to be far away from the exact solution; one consequently advises to carry out several calculations with different increments of load to make sure that the variation of got results is weak.

In cases of brutal expansion of the damaged zone resulting in a fort snap-back total answer forces/displacement, the method, if it makes it possible to cross instability, could not be reliable in term of result.

1 Introduction

One presents here a method of resolution, robust but approximate, incremental problem of nonlinear quasi-static mechanics, for certain behaviors of damage and plasticity. It is activated by the keyword `METHOD = 'IMPLEX'` of the operator `STAT_NON_LINE`, and replaces the classical method of `NEWTON` (cf [2]).

2 Position of the problem

One places oneself within the framework general of the resolution of a standard problem of non-linear mechanics discretized in space $(K \in \Omega)$ and in time $(t \in [0,T])$, and written in displacement. Its resolution, at the moment t_{n+1}, consists in determining displacements $\boldsymbol{U}_{n+1}$ (thus deformations $\boldsymbol{\varepsilon}(\boldsymbol{U}_{n+1})$), internal variables α_{n+1} and constraints $\boldsymbol{\sigma}_{n+1}$ checking:
(I) At the total level, the balance of the forces:

$$\boldsymbol{F}_{\text{ext}}(t_{n+1}) - \boldsymbol{F}_{\text{int}}(\boldsymbol{\sigma}_{n+1}(\boldsymbol{U}_{n+1}), t_{n+1}) \tag{1}$$

(II) At the local level, equations constitutive of the law of behavior considered for material:
Law of state: $\boldsymbol{\sigma}_{n+1} = \sum (\boldsymbol{\varepsilon}(\boldsymbol{U}_{n+1}), \alpha_{n+1})$

$$\text{Law of evolution: } \begin{cases} f(\alpha_{n+1}, \boldsymbol{\sigma}_{n+1}) \leqslant 0 & \text{Convexe de réversibilité} \\ \dot{\alpha}_{n+1} = \dot{\lambda}_{n+1} \geqslant 0 & \text{Evolution des variables internes} \\ \dot{\lambda}_{n+1} f(\alpha_{n+1}, \boldsymbol{\sigma}_{n+1}) = 0 & \text{Condition de Kuhn-Tucker} \end{cases} \tag{2}$$

with $\boldsymbol{F}_{\text{ext}}$ and $\boldsymbol{F}_{\text{int}}$ external and internal forces, and $\dot{\lambda}_{n+1}$ the multiplier (of damage or plasticity). One considers in the continuation a behavior independent of physical time (not of viscosity nor of dynamic effect); the "pseudo-moment" t_{n+1} represent consequently the load factor applied.

In the majority of the cases, each equation (1) and (2) is of relatively easy resolution.

At the total level, the method of Newton leads to a succession of linear problems of the type:

$$\boldsymbol{K}^{i}_{n+1} \delta \boldsymbol{U}^{i+1}_{n+1} = \boldsymbol{F}_{\text{ext}}(t_{n+1}) - \boldsymbol{F}_{\text{int}}(\boldsymbol{\sigma}^{i}_{n+1}(\boldsymbol{U}^{i}_{n+1})) \tag{3}$$

where the high index represents the iteration of Newton considered, $\delta \boldsymbol{U}^{i+1}_{n+1}$ is the increment of displacement between two successive iterations of Newton and $\boldsymbol{K}^{i}_{n+1}$ is the total tangent matrix. Let us note that this total tangent matrix is written (cf [2])

$$\begin{aligned} \boldsymbol{K}_{n+1} &= \frac{\partial \boldsymbol{F}_{\text{int}}(\boldsymbol{\sigma}_{n+1}(\boldsymbol{U}_{n+1}))}{\partial \boldsymbol{U}_{n+1}} \\ &= \mathbf{A}^{p}_{e=1}\left(\int_{\Omega_{\varepsilon}} \nabla N_e . \boldsymbol{C}_{e_{n+1}} . \nabla N_e \, d\Omega\right) \end{aligned} \tag{4}$$

where $\mathbf{A}$ is the operator of assembly, p the number of elements of the grid, $\boldsymbol{C}_{e_{n+1}}$ the local tangent operator (resulting from the behavior) and N_e functions of form (for the element considered, noted e). Quantity $\boldsymbol{F}_{\text{ext}}(t_{n+1}) - \boldsymbol{F}_{\text{int}}(\boldsymbol{\sigma}^{i}_{n+1}(\boldsymbol{U}^{i}_{n+1}))$ represent the residue of balance, which it is necessary in any rigour to cancel. To determine $\delta \boldsymbol{U}^{i+1}_{n+1}$ thus limits itself to the inversion of the matrix $\boldsymbol{K}^{i}_{n+1}$. If thus the local elements resulting from the behavior (forced, tangent matrix) are known and fixed, and generate a matrix $\boldsymbol{K}$ conditioned well, the resolution is easy.

Moreover more locally, if displacements $\boldsymbol{U}_{n+1}$, and consequently local deformations $\boldsymbol{\varepsilon}(\boldsymbol{U}_{n+1})$, are known, the resolution of the majority of the laws of behavior, i.e. the determination of constraints $\boldsymbol{\sigma}_{n+1}$, internal variables α_{n+1} and of the local tangent matrix $\boldsymbol{C}_{e_{n+1}}$, is relatively easy.

It is thus in the simultaneity of these two laws that resides the implicit difficulty of resolution. The most classical method of resolution is the iterative algorithm of Newton (cf [2]). This iterative diagram always does not converge. Although that remains prone to discussion (there exist other possible factors of loss of convergence, such as the exit of the basin of attraction of Newton or the presence of junctions of the solution), the authors allot mainly this loss of robustness to the singular character of the matrices, in particular for the laws of damage, whose internal variables impact the matrices of rigidity of the behavior: in the cases of important damage, near or reaching the rupture, the softening led to tangent matrices locally very "weak", which can to the extreme not be definite positive more. Through the process of assembly (4), when the damage progresses in the structure, the total matrix $\boldsymbol{K}$ is deteriorated: it becomes too much "flexible", and its minimal eigenvalue tends towards zero. It becomes singular then and the algorithm diverges. The robustness of calculation is not then assured any more.

To increase the robustness of calculation in these situations, Oliver *et al.* [1] propose a special method of resolution, baptized IMPLEX. Total balance is then checked roughly using a tangent matrix (secant in the case of laws of damage) local extrapolated explicitly and assembled, and the laws of behaviors are solved implicitly starting from the field of approximate displacement. The method is presented hereafter.

3 Method IMPLEX

The elements generals of method IMPLEX are presented here. For more details, one can refer in particular to [1].

The method suggested is based on two successive stages carried out to determine the whole of the unknown factors at the "pseudo-moment" (charges) t_{n+1} .

The first stage consists of one *explicit extrapolation* internal variables, then constraints, according to the quantities calculated previously (with the load t_n) and of the step of load Δt_{n+1} . Thanks to this extrapolation, the local tangent matrix is evaluated and solidified; the resolution of the equilibrium equation (3) makes it possible to determine displacements, which one considers right and which are thus fixed in their turn.

The second stage consists in realizing *implicit integration* behavior, according to the degrees of freedom evaluated at the preceding stage.

The first stage is of explicit type, whereas second is of implicit type, from where the name of IMPLEX. At the conclusion of these two stages, the equilibrium equation is not checked (it is it in term of the fields extrapolated at the stage (1) only); however, more the increment of load is small, more the made mistake must be weak; one advises of this fact of carrying out calculations with several different steps of loads in order to check that the difference between the answers obtained is weak (answer converged in term of step of load). In the continuation, the two stages are described. An assessment of the key points is made.

3.1 A method in 2 stages

We here will successively present the 2 fundamental stages of method IMPLEX, synthesized in Table 1.

3.1.1 Explicit extrapolation and determination of the degrees of freedom

This first stage relates to in particular the internal variable α , of which the evolution is governed by a law of the type (2).
At the beginning of the step of load t_{n+1} , one has all information resulting from the step from load t_n and of the former steps of load. It is then possible to write the following developments of Taylor (the evolution of the internal variable is considered sufficiently regular to be able to do it):

$$\begin{cases} \alpha_{n+1} & = & \alpha_n + \Delta t_{n+1} \dot{\alpha}_n + O(\Delta t_{n+1}^2) \\ \alpha_n & = & \alpha_{n-1} + \Delta t_n \dot{\alpha}_{n-1} + O(\Delta t_n^2) \\ \Delta t_n \dot{\alpha}_n & = & \Delta t_n \dot{\alpha}_{n-1} + \Delta t_n (\Delta t_n \ddot{\alpha}_{n-1}) + \Delta t_n O(\Delta t_n^2) \quad \Rightarrow \Delta t_n \dot{\alpha}_n = \Delta t_n \dot{\alpha}_{n-1} + O(\Delta t_n^2) \end{cases} \tag{5}$$

$$\Rightarrow \begin{cases} \dot{\alpha}_n & = & \dfrac{\Delta \alpha_n}{\Delta t_n} - O(\Delta t_n^2) \\ \alpha_{n+1} & = & \alpha_n + \dfrac{\Delta t_{n+1}}{\Delta t_n} \Delta \alpha_n - \Delta t_{n+1} O(\Delta t_n^2) + O(\Delta t_{n+1}^2) \end{cases}$$

with $\Delta X_i = X_i - X_{i-1}$. By truncation, by neglecting the terms of order two, one obtains the following prediction $\tilde{\alpha}_{n+1}$ for the variable α_{n+1} :

$$\tilde{\alpha}_{n+1} = \alpha_n + \frac{\Delta t_{n+1}}{\Delta t_n} \Delta \alpha_n \tag{6}$$

Through (6), one notes that $\tilde{\alpha}_{n+1}$ is well obtained explicitly, with the load t_{n+1} , according to the sizes obtained implicitly with the load t_n . Figure 1 schematizes this process of extrapolation.

The error of extrapolation can be defined like the difference between the value of the extrapolated internal variable and its real end value; subject to a sufficiently regular evolution of the internal variable to carry out of them the developments of Taylor to the sufficient order, one can evaluate this error with:

$$e_{\alpha_{n+1}} = |\tilde{\alpha}_{n+1} - \alpha_{n+1}| \approx |\ddot{\alpha}_n| \Delta t_{n+1}^2 \tag{7}$$

This shows that in the case of sufficiently regular evolution of the internal variable, the error decreases in a quadratic way according to the step of load.

Once the extrapolated internal variable, one can determine the constraint $\tilde{\boldsymbol{\sigma}}_{n+1}(\boldsymbol{\varepsilon}_{n+1}, \tilde{\alpha}_{n+1})$ and the local tangent operator $\tilde{\boldsymbol{C}}_{e_{n+1}}$.

In the typical case of the isotropic laws of damage, it is the local secant operator who is used; the extrapolated constraints and this operator are written then:

$$\begin{cases} \tilde{\boldsymbol{\sigma}}_{n+1}(\boldsymbol{\varepsilon}_{n+1}, \tilde{\alpha}_{n+1}) & = & (1-\alpha_{n+1}) \boldsymbol{C}^{\mathbf{elas}} : \boldsymbol{\varepsilon}_{n+1} \\ \tilde{\boldsymbol{C}}_{e_{n+1}} = \dfrac{\partial \tilde{\boldsymbol{\sigma}}_{n+1}}{\partial \tilde{\boldsymbol{\varepsilon}}_{n+1}} & = & (1-\tilde{\alpha}_{n+1}) \boldsymbol{C}^{\mathbf{elas}} \end{cases} \tag{8}$$

These extrapolated local quantities are used for the assembly of the total tangent matrix $\tilde{\boldsymbol{K}}_{n+1}$ *via* (4), and with the determination of the internal force.

The equilibrium equation in term of extrapolated field $\boldsymbol{F}_{\mathbf{ext}}(t_{n+1}) - \boldsymbol{F}_{\mathbf{int}}(\boldsymbol{U}_{n+1}, \tilde{\boldsymbol{\sigma}}_{n+1}, t_{n+1}) = \mathbf{0}$ is then solved with this extrapolated matrix and the field of displacement $\tilde{\boldsymbol{U}}_{n+1}$ obtained. It is solved by a method of Newton-Raphson classical; it is thus linearized by $\tilde{\boldsymbol{K}}_{n+1} \delta \tilde{\boldsymbol{U}}_{n+1} = \boldsymbol{F}_{\mathbf{ext}}(t_{n+1}) - \boldsymbol{F}_{\mathbf{int}}\left(\tilde{\sigma}_{n+1}, t_{n+1}\right)$. In the case of the laws of behavior treated here (isotropic damage with secant operator and linear isotropic plasticity), the operator of behavior is constant during a step of time (i.e. independent of the current state of deformation $\boldsymbol{\varepsilon}_{n+1}$) ; the process of resolution of Newton-Raphson becomes linear by step of time and thus converges in an iteration. This stage is however iterative in certain cases (plasticity not linear for example), because of the nonconstant character of the tangent operator. With the first iteration, one would take then $\boldsymbol{\varepsilon}^{0}_{n+1}=\boldsymbol{\varepsilon}_{n}$.

At the end of this first stage, the field of displacement $\boldsymbol{U}_{n+1}$ is regarded as equal to the field obtained by the resolution of the equilibrium equation written in term of the extrapolated fields: $\boldsymbol{U}_{n+1}=\tilde{\boldsymbol{U}}_{n+1}$. This field of displacement will not be modified any more thereafter.

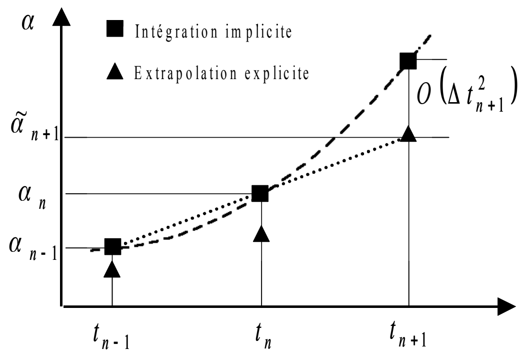

Figure 1: Schematization of the method of extrapolation

3.1.2 Determination implicit elements of the law of behavior

Following the first phase, the field of displacement is known. It is fixed for the step of load and will thus not be modified.

The deformation then is determined $\boldsymbol{\varepsilon}_{n+1}(\boldsymbol{U}_{n+1})$, then the equations (2) of the law of behavior are implicitly solved in order to obtain the stress fields $\boldsymbol{\sigma}_{n+1}$ and of internal variable α_{n+1} .

This stage is identical to the standard resolution of the equations of the laws of behavior. The only difference is the need for storing $\frac{\Delta \alpha_{n+1}}{\Delta t_{n+1}}$ to carry out the extrapolation of the variable interns with the following step.

At the end of this stage, the field of displacement $\boldsymbol{U}_{n+1}$ and stress fields $\boldsymbol{\sigma}_{n+1}$ and of internal variable α_{n+1} are thus known. A major difference compared to an implicit classical calculation is the fact that the real equilibrium equation is not checked; it is it only in term of the extrapolated fields.

Stage 1: Explicit extrapolation	**Stage 2: Implicit integration**
Extrapolation of the internal variables: $\tilde{\alpha}_{n+1}=\alpha_n+\frac{\Delta t_{n+1}}{\Delta t_n}\Delta \alpha_n$	Fixed displacements: $\dot{\boldsymbol{U}}_{n+1}=\mathbf{0}$
Calculation of the extrapolated constraint: $\tilde{\boldsymbol{\sigma}}_{n+1}(\boldsymbol{\varepsilon}_{n+1}, \tilde{\alpha}_{n+1})$	Implicit resolution of the laws of behavior:

Resolution of the equilibrium equation in extrapolated fields: $\boldsymbol{F}_{\mathrm{ext}}(t_{n+1})-\boldsymbol{F}_{\mathrm{int}}(\boldsymbol{U}_{n+1},\tilde{\boldsymbol{\sigma}}_{n+1},t_{n+1})=\boldsymbol{0}$	Law of state: $\boldsymbol{\sigma}_{n+1}=\sum(\boldsymbol{\varepsilon}(\boldsymbol{U}_{n+1}),\alpha_{n+1})$ Law of evolution: $\begin{cases} f(\alpha_{n+1},\boldsymbol{\sigma}_{n+1})\leqslant 0 \\ \dot{\alpha}_{n+1}=\dot{\lambda}_{n+1}\geqslant 0 \\ \dot{\lambda}_{n+1} f(\alpha_{n+1},\boldsymbol{\sigma}_{n+1})=0 \end{cases}$
Exit of stage 1: $\boldsymbol{U}_{n+1}$	**Exit of stage 2:** $\boldsymbol{U}_{n+1}$, $\boldsymbol{\sigma}_{n+1}$, α_{n+1}

Table 1: Summary of method IMPLEX

3.2 Automatic management of the step of time

Method `IMPLEX` introduced, like all the explicit methods, an intrinsic error which must decrease in a quasi-quadratic way with the step of load. The solution can thus depend slightly on the step of load selected by the user.
This one can, if it wishes it, use an automatic management of the step of step of load *via* the order `DEFI_LIST_INST`, while specifying `METHODE= 'CAR'` and `MODE_CALCUL_TPLUS=' IMPLEX'` (cf [9]). This method makes it possible to control the error while optimizing the computing time provided the user chose a first step of well gauged time.
The goal is thus to minimize the error defined by the equation (7). For that one will maximize the increment of variable extrapolated by a noted size Tol :

$$\Delta\tilde{\alpha}_{n+1}=\frac{\Delta t_{n+1}}{\Delta t_n}\Delta\alpha_n\leqslant Tol \quad (9)$$

From where:

$$\Delta t_{n+1}\leqslant\frac{Tol.\Delta t_n}{\Delta\alpha_n} \quad (10)$$

As the increments of internal variable depend on the point considered, the step of time will be selected like the minimal value of the expression (10) on the whole of the structure, that is to say finally:

$$\Delta t_{n+1}=Tol.\Delta t_n MIN_{x\in\Omega}\left(\frac{1}{\Delta\alpha_n(x)}\right) \quad (11)$$

One adds conditions then limiting acceleration and deceleration, as well as terminals minimal and maximum for the increment of time. Conditions of acceleration (μ) and deceleration are not modifiable and were gauged on concrete cases, whereas the terminals minimal and maximum can be modified by the user (one gives the values by default here).

$$\begin{gathered}\Delta t_{n+1}\leqslant\mu\Delta t_n=1,2\Delta t_n\\ \Delta t_{n+1}\geqslant 0.5\Delta t_n\\ \Delta t_{n+1}\leqslant 10\Delta t_0\\ \Delta t_{n+1}\geqslant 0.001\Delta t_0\end{gathered} \quad (12)$$

Thus, the first step of time Δt_0 , provided by the user, defines the terminals of the increment of time. To choose it, one advises with the user to have carried out preliminary calculations with the method of Newton and to know the yield stress of the structure; it would seem that the choice of a first step of time equal to half of the yield stress allows a good effectiveness of the method.

Let us show whereas the made mistake is well controlled. For that one makes the assumption that equation 9 is checked. One thus has $\frac{\Delta t_{n+1}}{\Delta t_n}\Delta\alpha_n\leqslant Tol$, and if it is considered that the factors limits acceleration and deceleration make it possible to write $\frac{\Delta t_{n+1}}{\Delta t_n}\approx 1$, one has $\Delta\alpha_n\leqslant Tol$; and gradually:

$$\forall n\in\mathbb{N},\Delta\alpha_n\leqslant Tol \quad (13)$$

What finally makes it possible to write:

$$e_{\alpha_{n+1}}\approx|\ddot{\alpha}_n|\Delta t_{n+1}^2\leqslant Tol\,\mu^2(1+\mu) \quad (14)$$

The error is thus controlled.

3.3 Key points of the method

After this summary presentation, some remarks are passed here to understand the interest of the method and also its limitations.

First of all, it is based on an extrapolation of the internal variables, determined starting from development of Taylor. The developments of Taylor being valid only for sufficiently regular functions, the method will be it too. Thus, the crossing of the yield stress, or the passage of a state of load to discharge, are points for which the method is not in any not adapted rigour: the derivative of the damage is worthless side of the discharge or yield stress, and nonworthless on the side of the load. However, if the steps of load are sufficiently small, the approximation can be made, insofar as the implicit correction takes place and thus that the errors of extrapolations are partly gummed. In situations unstable however, for example when the damaged zone grows brutally (what is characterized in general by an important snap-back of the total answer forces/displacement), the method, although robust, cannot guarantee a reliable answer, whatever the increment of load used: it is not adapted to this kind of situation.

According to the equation (8), in the case of isotropic laws of damage, because of limitation of α to 1 and symmetrical and definite character positive of the local elastic matrix $\boldsymbol{C}^{\text{elas}}$, the local secant matrix $\tilde{\boldsymbol{C}}_{e_{n+1}}$ is always symmetrical definite positive. So by assembly, the total tangent matrix $\tilde{\boldsymbol{K}}_{\boldsymbol{n+1}}$ remain conditioned well: problems of robustness mentioned in introduction, dependent on the increasingly singular character of $\boldsymbol{K}$, must thus be eliminated.

Moreover, for a step of load given and the laws of behavior developed here (`ENDO_FRAGILE` confer [3], `ENDO_ISOT_BETON` confer [4] and `VMIS_ISOT_LINE` confer [5]), the local tangent matrix is known by extrapolation clarifies and remains constant during all the step of load. The linearization of the equilibrium equation (1) by a method of Newton-Raphson leads to a total tangent matrix constant $\tilde{\boldsymbol{K}}_{\boldsymbol{n+1}}$; in other words, the equilibrium equation becomes linear with each step of load, and its resolution requires only one iteration.

Then, and to the risk to be redundant, the method leads to external forces and interns not balanced at the end of each step of load; they are it only at the end of the first phase, i.e. only in term of extrapolated fields:

$$\begin{cases}\boldsymbol{F}_{\mathbf{ext}}(t_{n+1})-\boldsymbol{F}_{\mathbf{int}}(\boldsymbol{\sigma}_{\boldsymbol{n+1}}(\boldsymbol{U}_{\boldsymbol{n+1}}),t_{n+1}) & \neq & \mathbf{0}\\ \boldsymbol{F}_{\mathbf{ext}}(t_{n+1})-\boldsymbol{F}_{\mathbf{int}}(\boldsymbol{U}_{\boldsymbol{n+1}},\tilde{\boldsymbol{\sigma}}_{n+1},t_{n+1}) & = & \mathbf{0}\end{cases} \quad (15)$$

Consequently, during calculation, one should not ask the algorithm to check the residue except for a tolerance, as it is usual to do it (into implicit). From this point of view still, the robustness is guaranteed, since the classical criterion of stop is without object.

To finish, this method has the role only to increase the robustness of calculation, and not the quality of the answer obtained. Thus, it introduces an intrinsic error by the means of extrapolation. At best, one can get only results almost as good as those obtained by an implicit classical method of local resolution. This error, must decrease in a quasi-quadratic way according to the step of load imposed, subject to a sufficiently regular evolution of the internal variable (what excludes in fact the unstable propagations of damaged zones).

The use of this method thus requires a certain critical glance. In order to secure strong errors, one recommends to carry out calculations with increments of loads different and small, in order to check that the solution does not differ too much from one increment to another (in other words, that the solution is close to convergence in term of increments of load). Moreover, in the case of crossing of unstable situations, the got results should be considered only for their qualitative aspect.

4 Practical aspects of use

This method of resolution is activated while specifying some, under the simple keyword `METHOD` of the operator `STAT_NON_LINE`, `METHODE= 'IMPLEX'` and by specifying the law of behavior used under `BEHAVIOR`.
Only certain laws of behavior are currently available with the method `IMPLEX`. Table 2 recapitulates the laws of behavior available following the type of elements considered.

Surface or voluminal elements	**Elements of bar**	**Elements of membrane and grille_membrane**
ELAS	ELAS	ELAS
VMIS_ISOT_LINE	VMIS_ISOT_LINE	
ENDO_ISOT_BETON		
ENDO_FRAGILE		

TABLE 2: laws of behavior available with the method IMPLEX

For each law of behavior, the last internal variable is modified and corresponding in this case to the ratio $\frac{\Delta\alpha}{\Delta t}$.

The method imposes a reactualization of the matrix on each increment (`REAC_INCR = 1`) and only one iteration. The residue of balance is calculated, but no criterion is associated for him. One will be able to realize of possible an important error by seeing that the relative residue is high.

For more information on the practical aspects, one will refer to [6] and [7]. Method `IMPLEX` is illustrated through the case test SSNP140 [8] and SSLS132.

5 Bibliography

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[2] Handbook of reference R5.03.01 Aster, quasi-static nonlinear Algorithm

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[8] Handbook of validation V06.03.140 Aster, Plate perforated in traction with method IMPLEX

[9] Operator, instruction manual U4.34.03 Aster `DEFI_LIST_INST`

6 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
10.2	R.BARGELLINI- S.FAYOLLE R & D /AMA	Initial text
10 4	R.BARGELLINI	Addition of the automatic management of the step of time