# Dynamic analysis of structures with viscoelastic materials having frequency depend properties

**Overview:**

This reference document tackles the way how advanced capabilities are introduced in *Code_Aster* to take into account frequency depends properties of viscoelastic materials. Computing of harmonic answer and modes of vibration (real gold complex) is addressed. Having frequency depend is modes has step forward for the modal projection method and for model updating with modal experimental year basis ace has reference. Year iterative method is used in order to compute frequency depends modes.

# Content

# 1 Introduction

ViscoElastic Materials (VEM) are intensively used to solve NVH resulting. Damping pads and anti flutter products reduce the structure limits noise. Sealants and absorbing materials prevent the air limits transmission. Adhesive are common solutions to assembles shares. Whereas VEM are well known, the finite element analysis of to their dynamic behavior is not straightforward. Most of the time, the uses of has finite element software requires to simplify VEM ace elastic materials. This simplification is not always the obvious thing to C and may lead to inaccurate results.

This reference document addresses the finite element analysis of structures comprising VEM with frequency depend dynamic properties. Conventional approaches are discussed and has method is proposed to overcome to their limitations. The method computes frequency depend modes using year iterative algorithm. The way how thesis modes edge modal Be used to improve the projection method is described. Implementation in *Code_Aster* is explained.

# 2 Finite element analysis

## 2.1 State-of-tea-art

From has simulation not of view, the frequency answer of has structure comprising At least one VEM is obtained by solving the dynamic equilibrium equation:

$$\left([K^*(\omega)]-\omega^2[M]\right)\{u(\omega)\}=\{F(\omega)\}\ , \quad \textbf{(2.1-1)}$$

with $[K^*(\omega)]$ , the complex stiffness matrix ace:

$$[K^*(\omega)]=[K'(\omega)]+i[K''(\omega)]\ . \quad \textbf{(2.1-2)}$$

The frequency dependence of the matrix comes from the uses of frequency depend complex moduli to re-press viscoelastic behaviors [1].
In many finite element codes, solving the system of Eq. (2.1-1) is not conventional because it needs to realize the stiffness matrix for each frequency step and to cuts computing procedures which are whitebait to deal with frequency depend matrices.
Direct The answer approach consists in solving Eq. (2.1-1) for each frequency step ace:

$$\{u(\omega)\}=\left([K^*(\omega)]-\omega^2[M]\right)^{-1}\{F(\omega)\} \qquad \forall\,\omega\in[0,\omega_{max}]\ . \quad \textbf{(2.1-3)}$$

This method has the exact advantage of computing the answer of the system. Goal, ace it is necessary to compute and opposite has complex matrix At each frequency step, the computing time edge become prohibitory for industrial structures with several million dismantle of freedom.
With few studies [2][4] cut shown the interest to solve Eq. (2.1-1) with dedicated modal answer methods in order to improve the computational efficiency while maintaining the accuracy of the results. Modal answer methods compute frequency answers by projecting the system of Eq. (2.1-3) there is modal basis $[T]$ , with the assumption:

$$\{u(\omega)\}=[T]\{q(\omega)\}\ . \quad \textbf{(2.1-4)}$$

The projection of the model one the considered basis leads to has low order model, that decreases significantly the number of dismantle of freedom and consequently the computing time of frequency answers:

$$\{q(\omega)\}=\left([T]^T[K^*(\omega)][T]-\omega^2[T]^T[M][T]\right)^{-1}[T]^T\{F(\omega)\}\ . \quad \textbf{(2.1-5)}$$

Using properties of elastic models (real and frequency independent stiffness matrix), the standard basis of the popular spectral decomposition method combine normal modes solving of the classical eigenvalue problem:

$$\left([K]-\omega_r^2[M]\right)\{\Phi_r\}=\{0\} \quad \textbf{(2.1-6)}$$

and has static correction to ensure has correct representation of the low frequency contribution of truncated high frequency normal modes [5]. Classically, $[T]$ is composed by the normal between modes $0$ and minimum of has $1.5\times\omega_{max}$.
However, frequency dependence of spectral the VEM dynamic properties prevents from using the decomposition method, because the modal basis $[\{\Phi_{r=1,N}\}]$ is only valid At the frequency $\omega_{ref}$, for which it has been computed. In practice, it has been demonstrated [3] the validity domain of the basis edge Be extended in has arranges around the frequency $\omega_{ref}$. Yew give the VEM of the model increasing moduli with respect to the frequency, the uses of $\omega_{ref}=\omega_{max}$ will lead to has larger validity domain than for any other frequencies. Unfortunately, the validity domain of the basis may not extend over all the frequency arranges of interest. Inaccurate results may Be obtained. To tackle this difficulty, static correction associated to the VEM edge also Be taken into account. Technical This, presented in [U2.06.04], leads to very accurate results, goal its implementation for year industrial box is tricky.

## 2.2 Computation of frequency depend mode

For has frequency depend real stiffness matrix, $[K(\omega)]$, the eigenvalue problem of Eq. (2.1-6) is written ace:

$$\left([K(\omega)]-(\omega_r(\omega))^2[M]\right)\{\Phi_r(\omega)\}=\{0\}\ , \quad \textbf{(2.2-1)}$$

where the eigenfrequency, $\omega_r(\omega)$, and the eigenvector, $\{\Phi_r(\omega)\}$, are frequency depend too. For has fixed, given frequency, $\omega=\omega_p$, such eigensolutions edge Be obtained by solving the standard problem of Eq. (2.1-6). Hence, the proposed method for solving the frequency depends problem of Eq. (2.2-1) consists in using year iterative algorithm searching for:

$$|\omega_r(\omega_p)-\omega_p|<\epsilon.\omega_p \qquad \forall\,\omega_p\in[\omega_1,\omega_2]\ , \quad \textbf{(2.2-2)}$$

where $\omega_1$ and $\omega_2$ are respectively the lower and the upper cut-off frequencies of the eigenproblem and $\epsilon$ is the iterative convergence criterion of the algorithm. First, for $\omega_p=\omega_1$, the stiffness matrix is realized and the eigenproblem of Eq. (2.1-6) is solved. Then, $\omega_p$ is updated by taking the been worth of the $n^{th}$ eigenfrequency for which:

$$|\omega_n-\omega_p|=min|\omega_{r=1,N}-\omega_p|\ . \quad \textbf{(2.2-3)}$$

Next, the stiffness matrix is realized for the new been worth of $\omega_p$ and the eigenproblem of Eq. (2.1-6) is solved again. The iterations will not stop while

$$|\omega_n-\omega_p|\geqslant\epsilon.\omega_p\ . \quad \textbf{(2.2-4)}$$

When the procedure converge, $\omega_n$ and $\{\Phi_n\}$ are extracted to forms the frequency depend eigensolutions of Eq. (2.2-1) and the iterative algorithm will continues using $\omega_p=\omega_{n+1}$ ace the guess been worth to compute the next eigensolution. Finally, the algorithm will stop when $\omega_p>\omega_2$. It means all the frequency depend eigensolutions cuts been computed between $\omega_1$ and $\omega_2$.

The computing time of the proposed method is classically driven by the number of frequency depend modes to Be computed, the number of dismantle of freedom of the model and the been worth of the convergence criterion $\epsilon$. It is also depend one the number of normal eigenvalues which are computed ace solutions of Eq. (2.1-6) At each iteration. Having all the eigenvalues between $\omega_1$ and $\omega_2$ for each prohibitory iteration is not necessary and could lead to computing times. In theory, minimum of two eigenvalues may Be has sufficient to run iterations: $\omega_n$ to update $\omega_p$ ace described in Eq. (2.2-3) and $\omega_{n+1}$ to continues when the procedure converge. In practice, this minimum number of eigenvalues will Be determined by the capabilities of the numerical solver for Eq. (2.1-6). It will Be discussed for *Code_Aster* in section 3.

The proposed method edge Be naturally extended to the study of damped structures comprising VEM. The frequency depend eigenvalues and eigenvectors are then computed ace solutions of the following problem:

$$\left([K^{*}(\lambda_r)]+\lambda_r^2[M]\right)\{\Psi_r\}=\{0\}\ , \qquad \textbf{(2.2-5)}$$

with $\lambda_{r=1,N}$ the complex eigenvalues, and $\{\Psi_{r=1,N}\}$ the associated complex modes. In this box, Eq. (2.2-2) is rewritten ace:

$$|\tilde{\omega}_r(\omega_p)-\omega_p|<\epsilon.\omega_p \qquad \forall\,\omega_p\in[\omega_1,\omega_2]\ , \qquad \textbf{(2.2-6)}$$

with $\tilde{\omega}_r=|\lambda_r|$ , the corresponding eigenfrequency in has structural damping model [6]. $\omega_p$ will Be updated by taking the been worth of the $n^{th}$ eigenfrequency for which:

$$|\tilde{\omega}_n-\omega_p|=min|\tilde{\omega}_{r=1,N}-\omega_p|\ . \qquad \textbf{(2.2-7)}$$

Computing frequency depend real gold complex modes should Be has step forward for comparison with experimental modal bases. Indeed, when structures comprising VEM are tested, to their frequency depends behaviors are physically measured. The proposed method could help to validate gold to update finite experimental element models with references.

## 2.3 Modal Improvement of the projection method

The frequency depend modes edge modal Be used to forms the basis of the projection method for answer computations of VEM. They will extend the validity domain of the basis over all the frequency arranges of interest. In combination to has static correction, either real modes will Be used for weakly damped structures, gold complex modes for highly damped structures [3]. The static correction will Be determined with the stiffness matrix realized for $\omega=0$ . Real modes will Be preferred to reduce computing times, since numerical solvers efficient are much more in this box. Goal for highly damped structures, the projection bases composed with such modes may Be insufficient to obtain accurate frequency answers. This edge Be improved using the modified Modal Strain Energy (MSE) method to reduce the errors [7].

So, frequency depends real modes, $\{\hat{\Phi}_r(\omega)\}$ , become solutions of has modified forms of Eq. (2.2-1) in order to take into account the damping matrix (imaginary share of the stiffness matrix) ace:

$$\left([K^{'}(\omega)]+\beta(\omega)[K^{''}(\omega)]-(\hat{\omega}_r(\omega))^2[M]\right)\{\hat{\Phi}_r(\omega)\}=\{0\}\ . \qquad \textbf{(2.3-1)}$$

$[K^{'}(\omega)]$ and $[K^{''}(\omega)]$ are defined by Eq. (2.1-2) and $\beta(\omega)$ is calculated by the following empirical formulated:

$$\beta(\omega)=\frac{trace[K^{''}(\omega)]}{trace[K^{'}(\omega)]}\ , \qquad \textbf{(2.3-2)}$$

where the trace for year $N\times N$ matrix is defined ace:

$$trace[A]=\sum_{j=1}^{N}A_{jj}\ . \qquad \textbf{(2.3-3)}$$

In the iterative algorithm, the uses of the modified MSE method leads to calculate $\beta(\omega_p)$ for each iteration and to search for the solutions of the resulting eigenvalue problem:

$$\left([K^{'}(\omega_p)]+\beta(\omega_p)[K^{''}(\omega_p)]-\hat{\omega}_r^2[M]\right)\{\hat{\Phi}_r\}=\{0\}\ . \qquad \textbf{(2.3-4)}$$

Another method to improve the real modes could Be the uses of residual modes whose purpose is to re-press the damping modal of VEM in the projection basis [8]. The coupling with frequency depend has modes been presented in [U2.06.04].

# 3 Implementation in *Code_Aster*

The approach to frequency depend modes described above has been implemented in *Code_Aster* via has macro command, namely `DYNA_VISCO`. This script use standard commands existing in *Code_Aster* for the conventional modal method. Python codes cuts been added for the new developments:

- of definition has frequency depend behavior,
- computation of frequency depend modes by the iterative algorithm,
- frequency answer computation with realization of the stiffness matrix At each frequency step.

In the iterations, modes are computed ace solutions of Eq. (2.1-6) via standard the `CALC_MODES` command. Modal The extraction method are described in [9]. Five eigensolutions are searched around $\omega_p$ for each iteration. This number has good compromised between securing the iterative convergence of the algorithm and limiting the total computing time. In the box where the algorithm would not converges, the number of the searched eigensolutions will Be automatically increased by the program.
The to use edge define different several VEM with behaviors and choose the frequency depends modes to Be used: real modes gold beta-modes. At this time of the implementation, it is also possible to compute frequency depends complex modes, goal they boat Be used ace has modal basis for answer computation.

# 4 References

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