Modeling of the shocks and friction in transitory analysis by modal recombination

Summary:

This document describes the physical laws of contact with friction between structures and the modeling which is made by it in the transitory algorithm of analysis by modal recombination DYNA_VIBRA (TYPE_CALCUL=' TRAN' and BASE_CALCUL=' GENE') with COMPORTEMENT=' DIS_CHOC'. For the various linear connections not - of contact usable, one details the calculation of the sizes defining the conditions of contact.

The diagrams of use used are described in [R5.06.04].

Contents

Contents

1 Introduction

The problems of shock with friction which interest EDF relate to for example the modeling of the tubular vibrations of structures maintained by supports with games, or separated by games weak and thus being able to make contact. The tubes of the steam generators, the pencils of the control rods, the assemblies of fuel are examples of structures which one wishes to model the vibrations.

The major consequence of the vibrations in the presence of game is to cause shocks as well as friction between the structure and its supports or the structures from where risks of wear. This document describes the type of non-linearities introduced by the presence of these games, as well as modeling used to take them into account in the algorithm of modal recombination.

2 Relations of contact between two structures

Two relations govern the contact between two structures:

- Lrelation of unilateral contact has which expresses the non-interpenetrability between the solid bodies,
- the relation of friction which governs the variation of the tangential stresses in the contact. One will retain for these developments a simple relation: the law of friction of Coulomb.

2.1 Relation of unilateral contact

Are two structures Ω_1 and Ω_2 . One notes $d_N^{1/2}$ the normal distance enters the structures and $F_N^{1/2}$ the force of normal reaction of Ω_1 on Ω_2 (see figure 2-1).

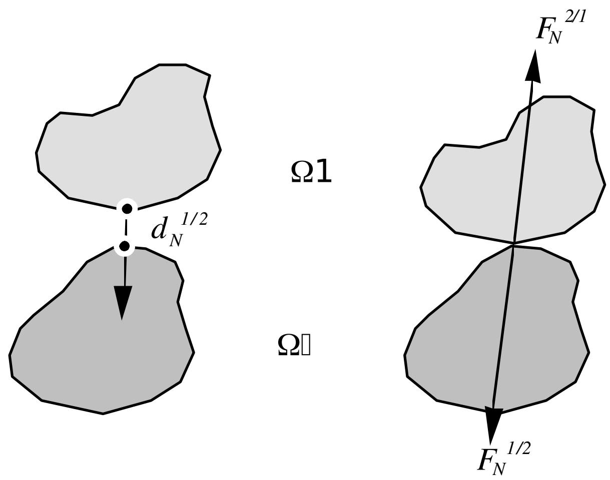

Figure 2-1: Normal distance and normal reaction

The law of the action and the reaction imposes:

$$F_N^{1/2} = -F_N^{2/1} \quad (1)$$

Conditions of unilateral contact, still called conditions of Signorini (see [1]), express themselves in the following way:

$$\begin{cases} d_N^{1/2} \geq 0 \\ F_N^{1/2} \geq 0 \\ d_N^{1/2} F_N^{1/2} = 0 \end{cases} \quad (2)$$

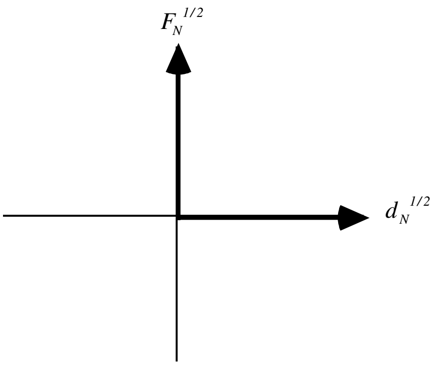

Figure 2-2: Graph of the relation of unilateral contact

The chart of the law of unilateral contact on the figure 2-2 translated a relation force-displacement which is not differentiable. It is thus not usable in a simple way in a dynamic calculation algorithm.
If one restricts the study with the case of a tubular structure in the presence of an indeformable support, one notes $d_n \left(d_n = d_N^{1/2} \right)$ the normal distance to the support, and F_n reaction of this last ($F_n = F_N^{2/1} = -F_N^{1/2}$ to see figure 2-3).

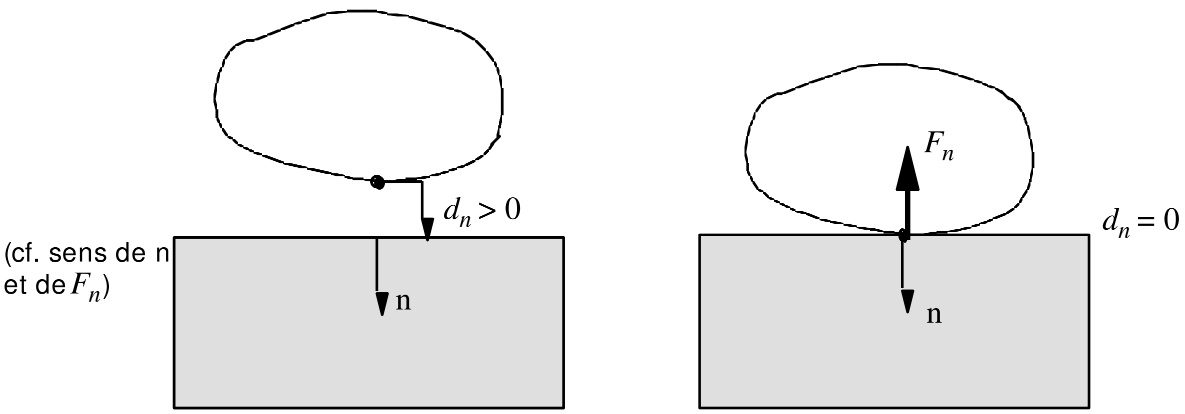

Figure 2-3: Normal distance and normal reaction enter a structure and a support

The expression of the conditions of normal contact, expressing the limitation of displacements due to the support is worth:

$$\begin{cases} d_n \geq 0 \\ F_n \geq 0 \\ d_n F_n = 0 \end{cases} \quad (3)$$

2.2 Law of friction of Coulomb

The law of Coulomb expresses a tangential limitation of effort $\boldsymbol{F}_T^{1/2}$ of tangential reaction of Ω_1 on Ω_2. That is to say $\dot{\boldsymbol{u}}_T^{1/2}$ the relative speed of Ω_1 compared to Ω_2 in a point of contact and is μ the coefficient of friction of Coulomb, one has (see [1]):

$$\begin{cases} s=\|\boldsymbol{F}_T^{1/2}\|-\mu F_N^{1/2}\le 0 \\ \exists\lambda \text{ tel que } \dot{\boldsymbol{u}}_T^{1/2}=\lambda \boldsymbol{F}_T^{1/2} \\ \lambda\le 0 \\ \lambda . s=0 \end{cases} \quad (4)$$

and the law of the action and the reaction:

$$\boldsymbol{F}_T^{1/2}=-\boldsymbol{F}_T^{2/1} \quad (5)$$

The chart of the law of Coulomb on the figure 2-4 translated it too character not-differentiable law and is thus not simple to use in a dynamic algorithm.

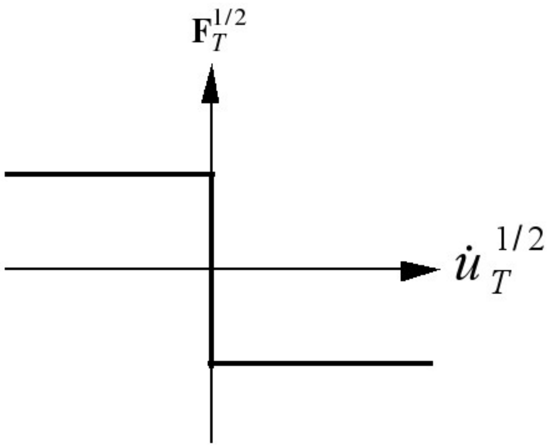

Figure 2-4: Graph of the law of friction of Coulomb

If one restricts the study with the case of a tubular structure in the presence of an indeformable support, only tangential stress $\boldsymbol{F}_T^{2/1}=\boldsymbol{F}_T$ is used, the law of friction expresses itself in the following way:

$$\begin{cases} s=\|\boldsymbol{F}_T\|-\mu F_n\le 0 \\ \exists\lambda \text{ tel que } \dot{\boldsymbol{u}}_T=\lambda \boldsymbol{F}_T \\ \lambda\le 0 \\ \lambda s=0 \end{cases} \quad (6)$$

A current extension of the law of Coulomb, resulting from the experiment, consists in having two coefficients of friction: one for adherence, noted μ_s , the other for the slip, noted μ_d , with $\mu_s>\mu_d$. One has then in phase of adherence $\|\boldsymbol{F}_T\|\le\mu_s F_n$ and in phase of slip $\|\boldsymbol{F}_T\|=\mu_d F_n$.

3 Modeling contactfriction by penalization

3.1 Model of normal force of contact

The principle of the penalization applied to the graph of the figure 2-2 consist in introducing a univocal relation $F_N^{1/2}=f_\epsilon\left(d_N^{1/2}\right)$ by means of a parameter ϵ. The graph of f_ϵ must tend towards the graph of Signorini when ϵ tends towards zero (see [2]). One of the possibilities consists in proposing a linear relation enters $d_N^{1/2}$ and $F_N^{1/2}$:

$$\begin{cases} F_N^{1/2}=-\frac{1}{\epsilon}d_N^{1/2} \text{ si } d_N^{1/2}\leq 0 \\ F_N^{1/2}=0 \text{ sinon} \end{cases} \quad (7)$$

If one notes $K_N=\frac{1}{\epsilon}$ (called commonly **stiffness of shock**) one finds the classical relation, modelling an elastic shock:

$$F_N^{1/2}=-K_N d_N^{1/2} \quad (8)$$

The approximate graph of the law of contact with penalization is on the figure 3-1.

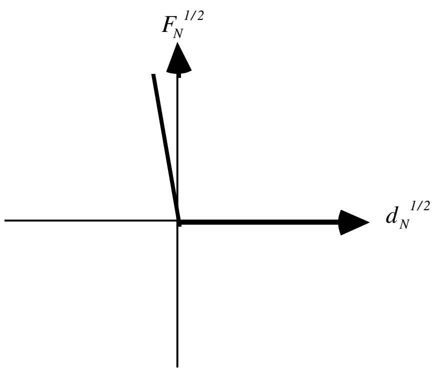

Figure 3-1: Graph of the relation of unilateral contact approached by penalization

To take account of a possible loss of energy in the shock, a damping of shock is introduced C_N. The expression of the normal force of contact is expressed then by:

$$F_N^{1/2}=-K_N d_N^{1/2}-C_N \dot{u}_N^{1/2} \quad (9)$$

Où $\dot{u}_N^{1/2}$ is the relative normal speed of Ω_1 compared to Ω_2. To respect the relation of Signorini (not blocking), one must on the other hand check a posteriori that $F_N^{1/2}$ is positive or worthless. Only the positive part will thus be taken $\langle \cdot \rangle^+$ expression (9):

$$\begin{cases} \langle x\rangle^+=x \text{ si } x\geq 0 \\ \langle x\rangle^+=0 \text{ si } x<0 \end{cases} \quad (10)$$

The complete relation giving the normal force of contact which is retained for the algorithm is the following one:

$$\begin{cases} \text{Si } d_N^{1/2}\leq 0 \text{ alors } F_N^{1/2}=\langle -K_N d_N^{1/2}-C_N \dot{u}_N^{1/2}\rangle^+ \text{ et } F_N^{1/2}=-F_N^{2/1} \\ \text{Sinon } F_N^{1/2}=F_N^{2/1}=0 \end{cases} \quad (11)$$

3.2 Model of tangential force of contact

The graph describing the tangential force with law of Coulomb is not-differentiable for the phase of adherence $\dot{\boldsymbol{u}}_T^{1/2}=0$. One thus introduces a univocal relation binding relative tangential displacement $\boldsymbol{d}_T^{1/2}$ and the tangential force $\boldsymbol{F}_T^{1/2}=f_\xi\left(\boldsymbol{d}_T^{1/2}\right)$ by means of a parameter ξ . The graph of f_ξ must tend towards the graph of Coulomb when ξ tends towards zero (see [2]). One of the possibilities consists in writing a linear relation enters $\boldsymbol{d}_T^{1/2}$ and $\boldsymbol{F}_T^{1/2}$ for an incremental writing:

$$\boldsymbol{F}_T^{1/2}-\boldsymbol{F}_{T,0}^{1/2}=-\frac{1}{\xi}\left(\boldsymbol{d}_T^{1/2}-\boldsymbol{d}_{T,0}^{1/2}\right) \quad (12)$$

Withvec $\left(\cdot\right)_{T,0}$ quantities with the step of previous time. If a tangential stiffness is introduced $K_T=\frac{1}{\xi}$, the relation is obtained:

$$\boldsymbol{F}_T^{1/2}=\boldsymbol{F}_{T,0}^{1/2}-K_T\left(\boldsymbol{d}_T^{1/2}-\boldsymbol{d}_{T,0}^{1/2}\right) \quad (13)$$

LE approximate graph of the law of friction of Coulomb modelled by penalization is on the figure 3-2. For digital reasons, related to the dissipation of parasitic vibrations (see [3]) in phase of adherence, one is brought to add a tangential damping C_T in the expression of the tangential force. Its final expression is:

$$\boldsymbol{F}_T^{1/2}=\boldsymbol{F}_{T,0}^{1/2}-K_T\left(\boldsymbol{d}_T^{1/2}-\boldsymbol{d}_{T,0}^{1/2}\right)-C_T\dot{\boldsymbol{u}}_T^{1/2} \text{ with } \boldsymbol{F}_T^{1/2}=-\boldsymbol{F}_T^{2/1} \quad (14)$$

It is necessary moreover than this force checks the criterion of Coulomb, that is to say:

$$\|\boldsymbol{F}_T^{1/2}\|\leq\mu F_N^{1/2} \quad (15)$$

If it is not the case, one corrects the force of friction by the following formula:

$$\boldsymbol{F}_T^{1/2}=-\mu F_N^{1/2}\frac{\dot{\boldsymbol{u}}_T^{1/2}}{\|\dot{\boldsymbol{u}}_T^{1/2}\|} \text{ and } \boldsymbol{F}_T^{1/2}=-\boldsymbol{F}_T^{2/1} \quad (16)$$

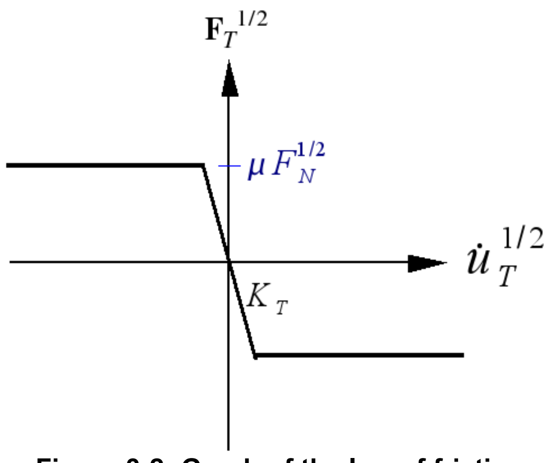

Figure 3-2: Graph of the law of friction approached by penalization

The case of the extension of the law of Coulomb with the distinction the adhesion coefficient enters μ_s and the coefficient of friction μ_d , the approximate graph of the law is modified (see figure 3-3).

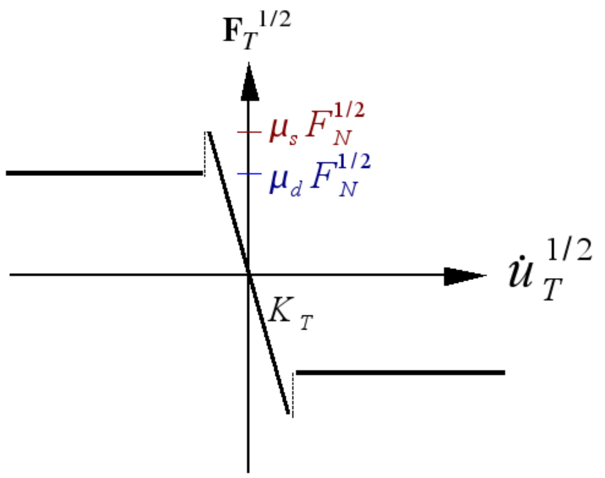

Figure 3-3: Graph of the alternative of the law of friction approached by penalization

4 Types of modelled connections of contact

S developments presented here relate to the implementation of non-linear connections with unilateral contact and friction enters one node and an obstacle or enters two nodes given.
The nodes in contact are supposed to belong to two slim structures of standard beam or to a beam and an indeformable obstacle. The nodes on which will carry the condition of contact are supposed to be carried by the average line of the beams.

4.1 Connections between a node and an indeformable obstacle

4.1.1 Connections of contact node on obstacle plan

One considers a slim structure represented by elements of type beam. Its displacement is limited in a point by the presence of an obstacle made up of two infinite half-planes in the direction Y (see figure 4-1).

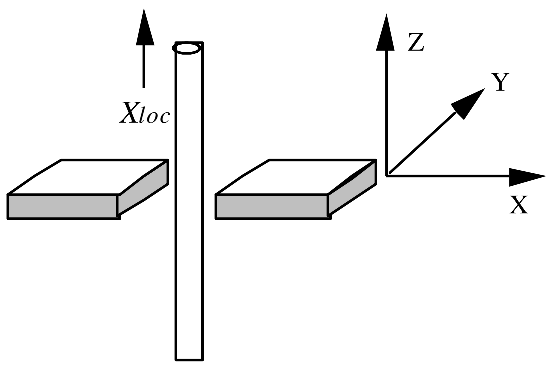

Figure 4-1: Slim structures with contact node on plan

To analyze the conditions of contact, one places oneself in the reference mark perpendicular to the axis $Xloc$, direction of neutral fibre or a generator of the beam. That is to say $NO1$, the node of the connection considered on the beam, geometry of the connection contact node on plan (called `PLAN_Y`) is described on the figure (4-2).

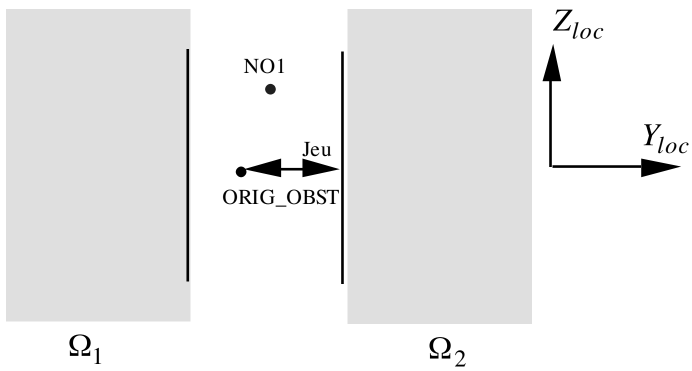

Figure 4-2: Geometry of the connection node on obstacle plan

Are $\begin{Bmatrix} Y_{loc} \\ Z_{loc} \end{Bmatrix}$ coordinates of the node $NO1$ in the reference mark (Y_{loc}, Z_{loc}) , the origin of this reference mark is the point `ORIG_OBST`. The normal distance d_N in this case, by neglecting rotations of the sections expresses itself then by:

$$d_N = -|Y_{loc}| + jeu \qquad (17)$$

The contact in this connection is judicious to take place whatever the shift in Z_{loc} between the two structures. The normal vector $\boldsymbol{n}$ in the reference mark (Y_{loc}, Z_{loc}) has for components:

$$\boldsymbol{n}=\begin{Bmatrix} sign(Y_{loc}) \\ 0 \end{Bmatrix} \quad (18)$$

Other quantities $\dot{u}_N$, F_N, $\dot{\boldsymbol{u}}_T$ and $\boldsymbol{F}_T$ are calculated in a general way as specified to [the §3].

4.1.2 Connections of contact node on concave circular obstacle

One considers a hurled structure, represented by elements of type beam. Its displacement is limited in a point by the presence of an obstacle made up of an infinite plan bored of a circular hole (see figure 4-3).

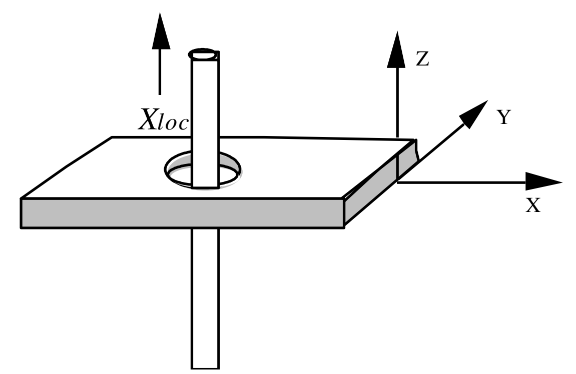

Figure 4-3: Slim structures with contact node on circular obstacle

To analyze the conditions of contact, one places oneself in the reference mark perpendicular to the axis X_{loc}, direction of neutral fibre or a generator of the beam. Are $NO1$, the node of the connection considered, geometry of the connection of contact node on circle (called `CIRCLE`) is described on the figure (4-4).

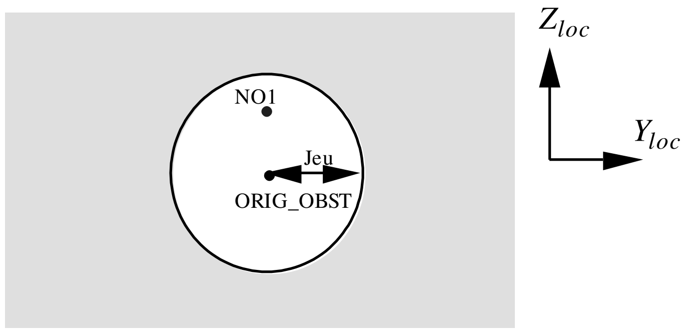

Figure 4-4: Geometry of the connection circular node obstacle

Are $\begin{Bmatrix} Y_{loc} \\ Z_{loc} \end{Bmatrix}$ coordinates of the node $NO1$ in the reference mark (Y_{loc}, Z_{loc}), the origin of this reference mark is the point `ORIG_OBST`. The normal distance d_N in this case, by neglecting rotations of the sections expresses itself then by:

$$d_N=-\sqrt{(Y_{loc}-Y_{\text{ORIG\_OBST}})^2+(Z_{loc}-Z_{\text{ORIG\_OBST}})^2}+jeu \quad (19)$$

One poses like normal vector $\boldsymbol{n}$ LE vector:

$$\boldsymbol{n}=\frac{\overline{\mathrm{ORIG\_OBST}-\mathrm{NO1}}}{\|\overline{\mathrm{ORIG\_OBST}-\mathrm{NO1}}\|} \quad (20)$$

With jeu a strictly positive distance. Other quantities $\dot{u}_N$, F_N, $\dot{\boldsymbol{u}}_T$ and $\boldsymbol{F}_T$ are calculated in a general way as specified to [the §3].

4.1.3 Connections of contact node on concave obstacle discretized by segments

One considers a hurled structure, represented by elements of type beam. Its displacement is limited in a point by the presence of an obstacle made up of a bored infinite plan of a hole of concave form unspecified which can be discretized in polar coordinates by segments (see figure 4-5).

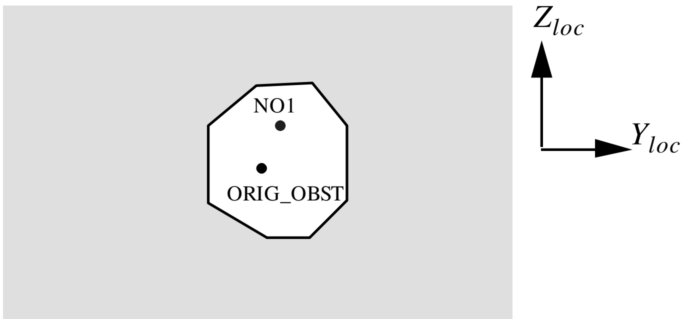

Figure 4-5: Geometry of the connection node on discretized concave obstacle

Are $\begin{Bmatrix} Y_{loc} \\ Z_{loc} \end{Bmatrix}$ coordinates of the node $NO1$ in the reference mark (Y_{loc}, Z_{loc}), the origin of this reference mark is the point `ORIG_OBST`. The facet of contact nearest to the node is searched $NO1$, the normal vector $\boldsymbol{n}$ is defined like the direct orthogonal vector with the facet on the figure 4-6.

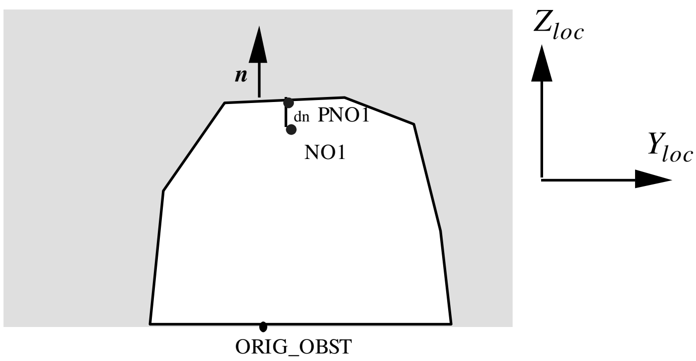

Figure 4-6: Definition of the reference mark for the case of the node on discretized concave obstacle

That is to say $PNO1$ the projection of the node $NO1$ at the facet, the normal distance d_N in this case is worth:

$$d_N=\overline{NO1-PNO1}\cdot\boldsymbol{n} \quad (21)$$

Other quantities $\dot{u}_N$, F_N, $\dot{\boldsymbol{u}}_T$ and $\boldsymbol{F}_T$ are calculated in a general way as specified to [the §3].

4.2 Connections between two nodes of two deformable structures

4.2.1 Connections of contact plan on plan

The contacts between assemblies fuel, on the level as of grids of mixture, constitute an example of contact plan on plan (see figure 4-7). One thus considers two hurled structures, being able to be modelled by beams of rectangular section on the level of the zones of contact.

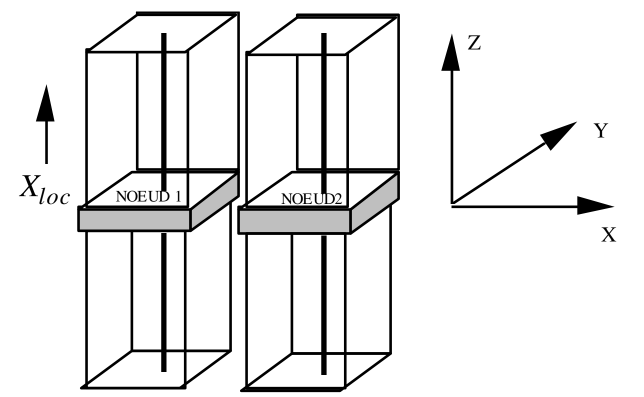

Figure 4-7: Slim structures with contact plan on plan

To analyze the conditions of contact, one places oneself in the reference mark perpendicular to the axis X_{loc}, direction of neutral fibre of the beams. Are $NO1$ and $NO2$, two nodes of the connection considered, geometry of the connection contact plan on plan (called `BI_PLAN_Y)` is described on the figure 4-8.

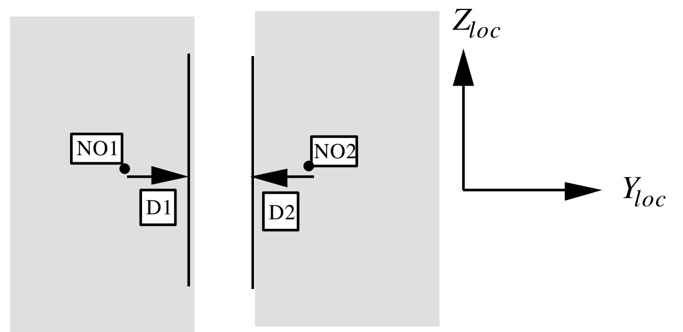

Figure 4-8: Geometry of the connection plan on plan

Are $\begin{Bmatrix} Y^i_{loc} \\ Z^i_{loc} \end{Bmatrix}$ coordinates of the node NO^i in the reference mark (Y_{loc}, Z_{loc}), the origin of this reference mark is the point `ORIG_OBST`. (`ORIG_OBST` can be provided by the user, by default `ORIG_OBST` is selected like the medium of the nodes $NO1$ and $NO2$). The normal distance d_N in this case, by neglecting rotations of the sections expresses itself then by:

$$d_N = -|Y^1_{loc} - Y^2_{loc}| - D_1 - D_2 \tag{22}$$

D_1 and D_2 are strictly positive distances. The contact in this connection is judicious to take place whatever the shift in Z_{loc} between the two structures. The normal vector $\boldsymbol{n}$ in the reference mark (Y_{loc}, Z_{loc}) has as components:

$$\boldsymbol{n} = \begin{Bmatrix} sign(Y^2_{loc} - Y^2_{loc}) \\ 0 \end{Bmatrix} \tag{23}$$

Other quantities $\dot{u}_N$, F_N, $\dot{\boldsymbol{u}}_T$ and $\boldsymbol{F}_T$ are calculated in a general way as specified to [the §3].

4.2.2 Connections of contact rings on circle

If one considers now two cylinders of circular section, modelled by elements of beam. The connection of contact between two nodes of the average lines is supposed to take place between two circles as shown in the figure 4-9.

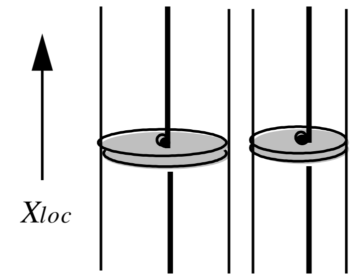

Figure 4-9: Slim structures with contact rings on circle

One places oneself in the reference mark perpendicular to the axis X_{loc} parallel with a generator of the cylinders. Are *NŒUD1* and *NŒUD2*, the two nodes of the connection considered, the geometry of the connection contact rings on circle (called `BI_CERCLE`) is described on the geometry figure 4-10.

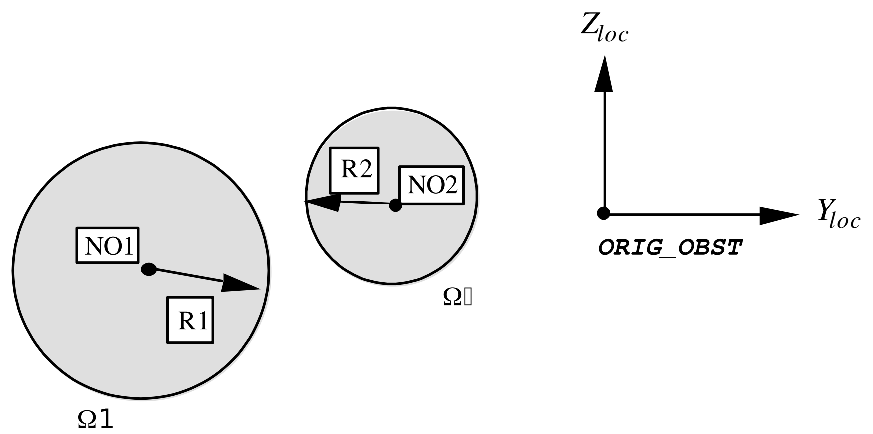

Figure 4-10: Geometry of the connection rings on circle

The normal distance d_N has as an expression:

$$d_N = -\sqrt{(Y^1_{loc} - Y^2_{loc})^2 + (Z^1_{loc} - Z^2_{loc})^2} - R_1 - R_2 \tag{24}$$

One poses like normal vector of Ω_1 towards Ω_2 LE vector:

$$\boldsymbol{n} = \frac{\overline{\text{NO2} - \text{NO1}}}{\|\overline{\text{NO2} - \text{NO1}}\|} \tag{25}$$

5 Use of the localised non-linear forces of shock and friction in modal recombination

The non-linear forces expressed above are explicit functions of the position and speed of the nodes to which the conditions of contact relate. One chooses to use the technique ofS pseudo-forces to solve the dynamic problem project. If the direct dynamic system is written:

$$M\,\ddot{\boldsymbol{X}}_t + C\,\dot{\boldsymbol{X}}_t + K\,\boldsymbol{X}_t = \boldsymbol{F}_{ext}(t) + \boldsymbol{F}_{choc}(\boldsymbol{X}_t, \dot{\boldsymbol{X}}_t) \quad (26)$$

Technique ofS pseudo-forces consists in projecting on the basis of linear system and maintaining the forces non-linear with the second member. The dynamic system project takes the shape:

$$\Phi^t M\,\Phi\,\ddot{\boldsymbol{\eta}}_t + \Phi^t C\,\Phi\,\dot{\boldsymbol{\eta}}_t + \Phi^t K\,\Phi\,\boldsymbol{\eta}_t = \Phi^t\,\Phi_{ext}(t) + \Phi^t\,\Phi_{choc}(\Phi\,\boldsymbol{\eta}_t, \Phi\,\dot{\boldsymbol{\eta}}_t) \quad (27)$$

The problem project is integrated numerically by an explicit diagram. Recommendations are given in [U2.06.04] for the choice of this base.

6 Precision on the use of the non- linearities of shock with friction

Non-linearities of shock between a structure and an obstacle or two structures were introduced into the algorithms of modal recombination of *Code_Aster* : an algorithm of Euler of order one and of Devogelaere of order four (see [R5.06.04]).
These algorithms are used by the operator `DYNA_VIBRATED`. The type of connection of shock between the two nodes is specified by a specific order: `DEFI_OBSTACLE`.

6.1 Definition of the type of connection of shock

The type of connection of shock is a generic concept, which does not comprise any physical information like a distance or unspecified dimension. The type of connection specifies simply the geometrical form of the connection considered.
Types of connection with shock with two nodes accepted by the order `DEFI_OBSTACLE` are described by the keywordsS following: `PLAN_Y`, `PLAN_Z`, `CIRCLE`, `BI_PLAN_Y`, `BI_PLAN_Z` or `BI_CERCLE` (see figure 6-1).

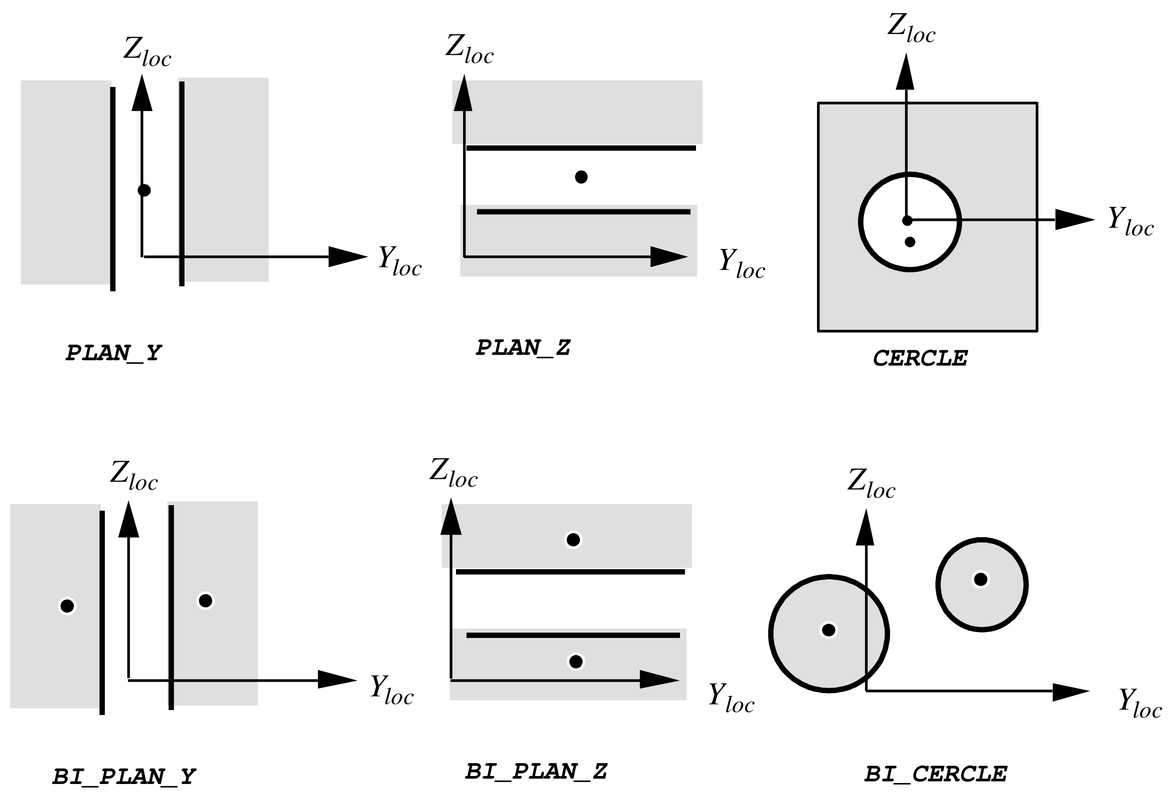

Figure 6-1: Geometries of the connections of shock

Prefix BI_ specifies that it is about a connection with two nodes.

6.2 Definition of the local reference mark for the conditions of contact

The treated structures, being regarded as cylindrical slim (circular or rectangular section), are modelled by elements of beam. The contact is treated in a plan perpendicular to the direction X_{loc} generator of the cylinders.

To define this change of reference mark completely, a local reference mark is introduced $(X_{loc}, Y_{loc}, Z_{loc})$.

The vector X_{loc} is the vector with three components provided behind the keyword `NORM_OBST`.

Using the first two nautical angles, one passes in a single way of the total reference mark (X, Y, Z) with a reference mark having X_{loc} like first basic vector (see figure 6-2). The third rotation whose angle is provided behind the keyword ANGL_VRIL give a single correspondence between the principal reference mark and the local reference mark.

Caution :

the orientation of this local reference mark is important because it is in this reference mark that the conditions of contact are analyzed, and are provided the local positions of the nodes of shock.

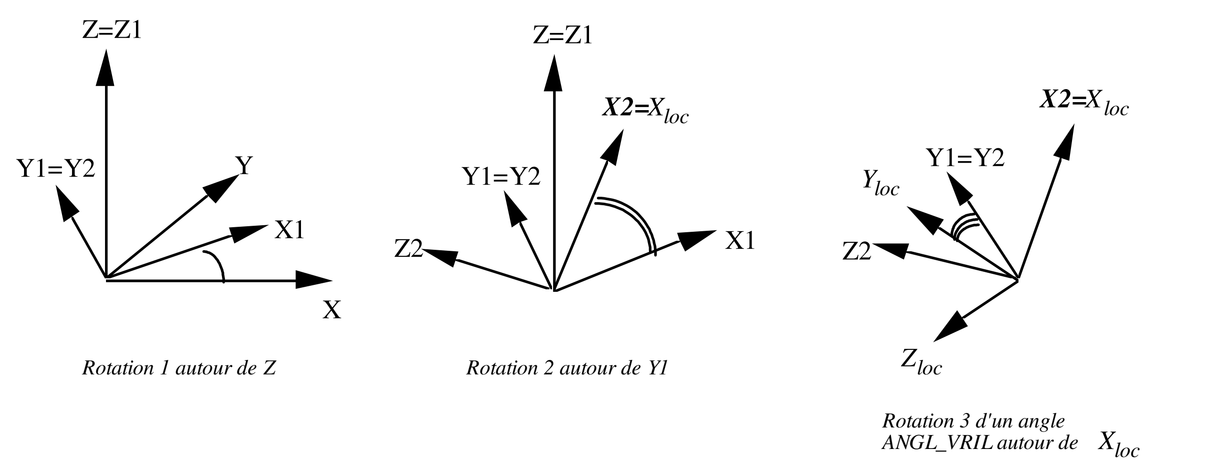

Figure 6-2: Rotations defining the local reference mark

Figure 6.2-a: Rotations defining the local reference mark

The operand ORIG_OBST allows to define the origin of the local reference mark $(Orig, X_{loc}, Y_{loc}, Z_{loc})$. This operand is optional and in theory will not be used in the case of the shocks between two nodes. The code considers whereas the origin is located in the middle of the segment connecting the two nodes.

6.3 Definition of the nodes of the connections

One specifies, behind the keyword NOEU_1 and NOEU_2, names of the two nodes of the structures on which will carry the conditions of shock. If it is about a connection between a node and an obstacle, only NOEU_1 is well informed.

6.4 Definition of dimensions characteristic of the sections

The operand GAME is used for the conditions of contact between a node and an obstacle.

Lbe operands DIST_1 and DIST_2 allow to specify dimensions characteristic of the sections of the structures surrounding the nodes of shock. In the case of the connections plan on plan, they are the thicknesses of matter surrounding the node of shock in the direction considered.

In the case of connections rings on circle, it acts of the rays of the sections surrounding the nodes of shock.

6.5 Definition of the parameters of contact

The parameters stiffnesses and damping of shock were introduced with the §3.1 and §3.2, one specifies the keywords here allowing to define them for a given connection:

- The operand RIGI_NOR is obligatory, it allows to give the value of normal stiffness of shock K_N.
- The operand AMOR_NOR is optional, it allows to give the value of normal damping of shock C_N
- The operand RIGI_TAN is optional, it allows to give the value of tangential stiffness K_T ;
- The operand AMOR_TAN is optional, it allows to give the tangential value of damping of shock C_T ;
- The operand COULOMB allows to give the value of the coefficient of Coulomb.

Foot-note:

If a stiffness K_T is defined and that the keyword AMOR_TAN is absent, the code calculates a damping optimized in order to minimize the residual oscillations in adherence (see [3]) :

$$C_T=2\cdot\sqrt{(k_i+K_T).m_i}-2\cdot x_i\cdot\sqrt{k_i.m_i} \quad ,$$

where i is the index of the dominating mode in the answer of the structure (the modal mass most important).

6.6 Use of an one-way shock

For the obstacles of the type PLAN_Y, PLAN_Z,BI_PLAN_Y or BI_PLAN_Z it is possible to activate friction in only one direction of the plan of the obstacle. The operand UNIDIRECTIONNEL allows to activate this one-way friction to cancel friction in the direction given by the operand NORM_OBST. Cette option is usable as well with the friction of the type COULOMB that with friction of type COULOMB_STAT_DYNA.
With a friction COULOMB , LE coefficient of friction is worth zero along the axis indicated in NORM_OBST and driven in the perpendicular direction.
With a friction COULOMB_STAT_DYNA, LE adhesion coefficient is worth zero along the axis indicated in NORM_OBST and driven in the perpendicular direction. The coefficient of friction is worth zero along the axis indicated in NORM_OBST and mud in the perpendicular direction.

7 Bibliography

[1] **Orsero, P.; Levesque, J. - R.; Vare, C.; Jacquart, G. and Aufaure, Mr.** (**1994**). *Support of the Dynamic course of the Structures - Code_Aster Training seminar*, EDF R & D, Report intern HP - 61/94/189.

[2] **Jean, Mr. and Moreau, J. - J.** (**1992**). *Unilaterality and dry Martini friction in the dynamics of rigid bodies collection*. In: Curnier, A. (ED.), *Proceedings of the International Mechanics Contact Symposium*, Polytechnic pp. 31-48, Presses and French Academics (Lausanne).

[3] **Oden, J. and Martins, J.** (**1992**). *Models and computational methods for dynamic friction phenomena*, Methods Computer in Applied Mechanical Engineering 52, pp. 527-634.

[4] **Jacquart, G.** (**1991**). *Methods of Ritz in non-linear dynamics - Application to systems with shock and friction localised*, EDF R & D, Report intern HP - 61/91/105.

[5] **Beaufils, B.** (**1990**). *Contribution under investigation of the vibrations and the wear of the tube bundles in transverse flow*, doctorate, University Paris VI.