Relations of behavior BETON_GTO ARRANGE and BETON_GRANGER_V for the clean creep of the concrete

Summary:

This document presents the clean model of creep of "Granger", which is a way of modelling the clean creep of the concrete. One also details there the writing and the digital processing of the model.

In Code_Aster, the model is declined in two laws of behavior: full version BETON_GRANGER_V and a simplified version, named BETON_GRANGER, without the effects of L '' hygroscopy and of ageing.

Contents

1 Introduction

Within the framework of the studies of the long-term behavior of the concrete structures, a dominating share of the deformations measured on structure relates to the differed deformations which appear in the concrete during its life. They comprise the withdrawals with the young age, the withdrawal of desiccation, clean creep and the creep of desiccation.

The model presented here is dedicated to the modeling of the differed deformation associated with clean creep. Clean creep is, in complement of the creep of desiccation, the share of creep of the concrete which one would observe during a test without exchange of water with outside. In experiments the concrete in clean creep presents a growing old viscous behavior. The deformation of creep observed is proportional to the constraint of loading, depends on the temperature and the hygroscopy. The longitudinal deflection is accompanied as in elasticity by a transverse deformation by opposite sign.

The selected model takes as a starting point that proposed by L. Granger [bib1]. It is model of a viscoelastic type which takes into account the effect of ageing and the hygroscopy. The effect of the temperature is not modelled.

One initially carries out a short recall on the linear viscoelastic models and one presents then the model itself like his digital integration in C*ode_hasster*.

In C*ode_hasster*, two versions are available: `BETON_GRANGER_V` the complete model, and `BETON_GRANGER`, which does not take into account ageing.

2 Recall on behaviour in creep of a viscoelastic material

The experimental curve of creep represents the evolution according to the time of the deformation of a material subjected to a constant unidimensional constraint σ. Deformation of creep ε^{fl} is, in opposition to the instantaneous strain, the share of deformation which evolves with time.

If a material has a linear viscoelastic behavior, deformation of creep of a test-tube subjected to a load σ constant is, by definition, proportional to σ. Deformation of creep (1D) as from the time of loading t_c can be written:

$$\varepsilon^{fl}(t)=\sigma J(t,t_c) \qquad (1)$$

The function $J(t,t_c)$ is an increasing function of $\left(t-t_c\right)$ and worthless for $\left(t-t_c\right)$ negative.

If the material is nongrowing old, the function of creep at a certain moment depends only on time $\left(t-t_c\right)$ run out since the moment of loading [bib3]:

$$J(t,t_c)=f(t-t_c) \qquad (2)$$

The deformation of creep is written:

$$\varepsilon^{fl}(t)=\sigma f(t-t_c) \qquad (3)$$

For a growing old viscoelastic material, the experimental curve of creep varies for two different times of loading. From the point of view of modeling, the function of creep $J(t,t_c)$ do not vary only any more according to the past time of the moment of loading; it depends in a way independent of time t and of the moment of loading t_c.

2.1 Principle of superposition of Boltzmann

The relation (1) is valid only for one constant loading. For a history of nonconstant loading one can apply the principle of superposition of Boltzmann, under the terms of the linear dependence with the constraint. If history of loading $\sigma(t)$ is broken up into n increments of load one a:

$$\sigma(t)=\sum_{i=0}^{n} H(t-t_{ci})\Delta\sigma_i \qquad (4)$$

Où H is the function of HeavIside. One can then write itexpression of the deformation of creep:

$$\varepsilon^{fl}(t)=\sum_{i=0}^{n} J(t,t_{ci})\Delta\sigma_i \qquad (5)$$

While passing in extreme cases, for an increment of loading infinitely small :

$$\varepsilon^{fl}(t)=\int_{t_c=0}^{t} J(t,t_c)\frac{\partial\sigma}{\partial t_c}\,d\,t_c \qquad (6)$$

For a linear viscoelastic material not-growing old one a:

$$\varepsilon^{fl}(t)=\int_{t_c=0}^{t} f(t-t_c)\frac{\partial\sigma}{\partial t_c}\,dt_c=f*\frac{\partial\sigma}{\partial t} \quad (7)$$

where $*$ represent the product of convolution.

3 Presentation of the clean model of creep of Granger

3.1 Experimental properties of the clean creep of the concrete in uniaxial loading

The clean creep tests on test-tube reveal the following properties:

- in a range of constraint lower than approximately 50% of the breaking strength, clean creep is proportional to the constraint;
- the clean creep of a test-tube with hygroscopy h_{ext} is almost proportional to h_{ext}. The clean creep of a no-slump concrete is almost null and it is maximum for a concrete saturated with water;
- when the temperature T increase one has an acceleration of creep;
- clean creep is a strongly growing old phenomenon: the deformation does not depend only on the time passed since the setting under load but also on the moment of loading; in other words, the behavior rheological material at the moment of loading depends on its "age". In the case of the concrete, the age is the time passed since the casting.

One chooses to model the clean creep of the concrete with a linear viscoelastic model to which one will add the dependence of creep with respect to the hygroscopy as well as ageing [bib1].

The effect of the temperature, modelled in [bib1], will not be taken into account here.

3.2 Modeling of nongrowing old creep by a model of generalized Kelvin

One can show that any not-growing old linear viscoelastic body can be modelled by a series connection of chains of Kelvin, called model of generalized Kelvin. The function of creep can then be put in the form:

$$J(t, t_c) = f(t - t_c) = \sum_{s=1}^{r} J_s \left(1 - \exp\left(-\frac{t - t_c}{\tau_s}\right)\right) \quad (8)$$

Où τ_s and J_s are, respectively, the time lag and the flexibility of each chain s of Kelvin. They are plus coefficients identified on the experimental curves of creep. The choice was made in Code_Aster to limit itself to eight chains ($r=8$), which in practice is enough to reproduce in a satisfactory way the experimental curves of concrete creep.

It is very difficult to determine at the same time them J_s and τ_s as soon as the number of series of Kelvin exceeds two, because there is too much solution possible. One thus makes generally a choice a priori on τ_s and one then determines by linear regression them J_s. The time lags are selected so as to have $\tau_s = \tau_1 \cdot 10^{s-1}$.

The viscoelastic model of generalized Kelvin is enriched to take into account the effect of the hygroscopy. Ittte end, one defines an equivalent constraint.

Then, the function of creep (8) model of generalized Kevin is modified to take into account the effect of ageing.

3.3 Effect of ageing

Physically, ageing in the concrete is associated with the hydration with the young age and other phenomena like polymerization for the older concrete.

The effect of ageing is modelled by multiplying the coefficients J_s by a function of ageing $k(a(t_c))=k(t_c)$ depending on the age a material at the time of loading t_c. The function of creep becomes:

$$J(t,t_c)=k(t_c)\sum_{s=1}^{8} J_s\left(1-\exp\left(-\frac{t-t_c}{\tau_s}\right)\right) \tag{9}$$

The choice is made to use the same function of ageing for all them chains of Kelvin. From this manner one uncoupled in the function of creep (9) the contribution of the ageing (which depends on t_c) not-growing old contribution (in $t-t_c$).

In Code_hasster, the function of ageing is an user datum. A possible modeling to take into account ageing associated with the hydration is that of the CEB [bib2]:

$$\begin{cases} k(a)=\dfrac{28^{0,2}+0,1}{a^{0,2}+0,1} & \text{si } a\leqslant 28\,\text{jours} \\ k(a)=1 & \text{si } a>28\,\text{jours} \end{cases} \tag{10}$$

where a is expressed in days. The not-growing old law is found for $k(a)=constante=1$.

3.4 Effect of the hygroscopy

The equivalent constraint is defined S:

$$S=h\sigma \tag{11}$$

For a test with constant constraint one will have:

$$\varepsilon^{fl}(t)=S\,J(t,t_c)=(h\sigma)k(t_c)\sum_{s=1}^{8} J_s\left(1-\exp\left(-\frac{t-t_c}{\tau_s}\right)\right) \tag{12}$$

For a test characterized by a variable history of constraint, one will have:

$$\varepsilon^{fl}(t)=\int_{t_0}^{t} J(t,t_c)\frac{\partial S}{\partial t_c}dt_c=\sum_{s=1}^{8}\left(J_s\int_{t_0}^{t} k(t_c)\left(1-\exp\left(-\frac{t-t_c}{\tau_s}\right)\right)\frac{\partial S}{\partial t_c}dt_c\right) \tag{13}$$

Notice

It is the water content C (which corresponds to the variable of order `SECH`) in not the relative humidity h that one obtains calculation Code_hasster *of drying [R7.01.12]. It is the isothermal curve of sorption-desorption which makes it possible to pass from the variable C with h. That is to say C the isothermal curve of desorption: $C=C(h)$ and $h=C^{-1}(C)$. The curve $h=C^{-1}(C)$, of empirical nature, must be well informed by the user.*

3.5 Modeling 3D

The classical assumption consists in supposing the existence of a Poisson's ratio of creep constant and equal to the elastic Poisson's ratio, that is to say $\nu_f=0{,}2$. From where for $\boldsymbol{S}=h\cdot\boldsymbol{\sigma}$ constant:

$$\boldsymbol{\varepsilon}^{fl}(t)=J(t,t_c)\cdot[(1+\nu_f)\boldsymbol{S}-\nu_f\,\mathrm{tr}(\boldsymbol{S})\boldsymbol{I}] \tag{14}$$

and thus:

$$\tilde{\boldsymbol{\varepsilon}}^{fl}(t)=J(t,t_c)\cdot(1+\nu_f)\tilde{\boldsymbol{S}} \text{ and } \mathrm{tr}(\boldsymbol{\varepsilon}^{fl}(t))=J(t,t_c)\cdot(1-2\nu_f)\mathrm{tr}(\boldsymbol{S}) \quad (15)$$

where $\tilde{\boldsymbol{\varepsilon}}$ and $\tilde{\boldsymbol{S}}$ the deviatoric parts of the tensors of strain and stress indicate.

4 Relations of behavior *Code_Aster*

One introduces into *Code_Aster* two relations of behavior associated with clean creep:

- BETON_GRANGER_V
- BETON_GRANGER

The first takes account of the whole of the effects (hygroscopy and ageing), the second does not take account of the phenomenon of ageing. They are available in modeling 2D, 3D and plane constraints.

The various parameters of the model are indicated in DEFI_MATERIAU, under the keyword BETON_GRANGER, V_BETON_GRANGER and ELAS_FO.

- Lbe 2×8 constant characteristics of the function of creep of the model of generalized Kelvin J_s , τ_s Sinformed under the keyword BETON_GRANGER, of which the use is common to both relations of behavior BETON_GRANGER and BETON_GRANGER_V.
- If the growing old relation of behavior is used BETON_GRANGER_V then one informs in more the keyword V_BETON_GRANGER the function of ageing $k(t_c)$.
- The curve of desorption, necessary to the two relations of behavior BETON_GRANGER and BETON_GRANGER_V, is well informed under the keyword ELAS_FO.

The detail of the keyword and the operands is provided in table Table 4-1.

Keyword	Explanation	Operands
BETON_GRANGER	2×8 constant characteristics of the function of creep of the model of generalized Kelvin: - J_s flexibility of chain s - τ_s time lag of chain s	J1 : J_1 TAUX_1 : τ_1 … J8 : J_8 TAUX_8 : τ_8
V_BETON_GRANGER	Function of ageing	FONC_V: $k(t_c)$
ELAS_FO	• The curve of sorption-desorption giving h according to the water content C	FONC_DESORP: $C^{-1}(C)$

Table 4-1. Information of the parameters in DEFI_MATERIAU.

5 Digital integration of the model

5.1 Discretization (1D)

Let us consider initially the nongrowing old case. Deformation of creep for only chain of Kelvin s is written in the following way (see 13):

$$\varepsilon_s^{fl}(t)=\int_{t_0}^{t} J_s\left(1-\exp\left(-\frac{t-t_c}{\tau_s}\right)\right)\frac{\partial S}{\partial t_c}dt_c \tag{16}$$

The digital integration of the model is done while approximating $S(t)$ like a linear function per pieces on succession of n increments of time $\Delta t_i=t_i-t_{i-1}$ with $i=1,\cdots,n$. So $\dot{S}$ is constant on $t\in[t_{i-1},t_i]$ and equal to $\frac{\Delta S_i}{\Delta t_i}$. Deformation of chain s at time t_n is then given by the following expression:

$$\varepsilon_s^{fl}(t_n)=\sum_{i=1}^{n}\frac{\Delta S_i}{\Delta t_i}\int_{t_{i-1}}^{t_i} J_s\left(1-\exp\left(-\frac{t-t_c}{\tau_s}\right)\right)dt_c \tag{17}$$

One obtains:

$$\varepsilon_s^{fl}(t_n)=\sum_{i=1}^{n}\left(\frac{\Delta S_i}{\Delta t_i}\right)J_s\Delta t_i-\sum_{i=1}^{n}\left(\frac{\Delta S_i}{\Delta t_i}\right)J_s\tau_s\left(\exp\left(-\frac{t_n-t_i}{\tau_s}\right)-\exp\left(-\frac{t_n-t_{i-1}}{\tau_s}\right)\right) \tag{18}$$

ET thus:

$$\varepsilon_s^{fl}(t_n)=J_s\underbrace{\sum_{i=1}^{n}\Delta S_i}_{A_n^0}-\underbrace{J_s\sum_{i=1}^{n}\Delta S_i\frac{\tau_s}{\Delta t_i}\left(\exp\left(-\frac{t_n-t_i}{\tau_s}\right)\right)\left(1-\exp\left(-\frac{\Delta t_i}{\tau_s}\right)\right)}_{A_n^s}=J_sA_n^0-A_n^s \tag{19}$$

With t_{n+1} one can also write:

$$\begin{aligned}\varepsilon_s^{fl}(t_{n+1})=&J_s\sum_{i=1}^{n}\Delta S_i-J_s\sum_{i=1}^{n}\Delta S_i\frac{\tau_s}{\Delta t_i}\left(\exp\left(-\frac{t_{n+1}-t_i}{\tau_s}\right)\right)\left(1-\exp\left(-\frac{\Delta t_i}{\tau_s}\right)\right)\\&+J_s\Delta S_{n+1}-J_s\Delta S_{n+1}\frac{\tau_s}{\Delta t_{n+1}}\left(1-\exp\left(-\frac{\Delta t_{n+1}}{\tau_s}\right)\right)\end{aligned} \tag{20}$$

Shears, with $\Delta_{n+1}=t_{n+1}-t_n$:

$$\begin{aligned}\varepsilon_s^{fl}(t_{n+1})=&J_s\sum_{i=1}^{n}\Delta S_i-J_s\sum_{i=1}^{n}\Delta S_i\frac{\tau_s}{\Delta t_i}\left(\exp\left(-\frac{t_n-t_i}{\tau_s}\right)\right)\left(\exp\left(-\frac{\Delta t_{n+1}}{\tau_s}\right)\right)\left(1-\exp\left(-\frac{\Delta t_i}{\tau_s}\right)\right)\\&+J_s\Delta S_{n+1}-J_s\Delta S_{n+1}\frac{\tau_s}{\Delta t_{n+1}}\left(1-\exp\left(-\frac{\Delta t_{n+1}}{\tau_s}\right)\right)\end{aligned} \tag{21}$$

One recognizes the expression of A_n^0 and of A_n^s equation (19). In fine, one can thus write for chain s :

$$\varepsilon_s^{fl}(t_{n+1})=J_sA_n^0-A_n^s\exp\left(-\frac{\Delta t_{n+1}}{\tau_s}\right)+J_s\Delta S_{n+1}-J_s\Delta S_{n+1}\frac{\tau_s}{\Delta t_{n+1}}\left(1-\exp\left(-\frac{\Delta t_{n+1}}{\tau_S}\right)\right) \tag{22}$$

Let us consider them now eight chains of Kelvin in series. One a:

$$\varepsilon^{fl}(t_n)=\sum_{s=1}^{8}\varepsilon_s^{fl}(t_n) \tag{23}$$

Let us pose $J=\sum_{s=1}^{8}J_s$. According to (19) and (23) one has with t_n :

$$\varepsilon^{fl}(t_n)=J\,A_n^0-\sum_{s=1}^{8}A_n^s \tag{24}$$

With t_{n+1} one also has:

$$\varepsilon^{fl}(t_{n+1})=J\,A_{n+1}^0-\sum_{s=1}^{8}A_{n+1}^s \tag{25}$$

Withvec:

$$A_{n+1}^0=A_n^0+\Delta S_{n+1}$$

and

$$A_{n+1}^s=A_n^s\exp\left(-\frac{\Delta t_{n+1}}{\tau_s}\right)+J_s\Delta S_{n+1}\frac{\tau_s}{\Delta t_{n+1}}\left(1-\exp\left(-\frac{\Delta t_{n+1}}{\tau_s}\right)\right) \tag{26}$$

More precisely, if one takes into account also the effect of ageing and by considering constant the coefficient of ageing on Δt_i , one a:

$$A_{n+1}^0=A_n^0+k(t_{n+1/2})\Delta S_{n+1}$$

and

$$A_{n+1}^s=A_n^s\exp\left(-\frac{\Delta t_{n+1}}{\tau_s}\right)+J_s k(t_{n+1/2})\Delta S_{n+1}\frac{\tau_s}{\Delta t_{n+1}}\left(1-\exp\left(-\frac{\Delta t_{n+1}}{\tau_s}\right)\right) \tag{27}$$

Note:

One noted:

- $\Delta X_i=X_i-X_{i-1}$.
- $X_{n+1/2}=\frac{X_{n+1}+X_n}{2}$

To have ε^{fl} at time t_{n+1} , one should only store A^0 and A^s step of previous time, is nine variables. In 3D them A_0 and $\boldsymbol{A}^s$ are tensors. One will then associate with the two relations of clean behaviour of creep (9x6) variable interns corresponding to the components of the tensors $\boldsymbol{A}$. They characterize the advance of creep.

The writing in increment of deformation $\Delta\varepsilon_s^{fl}(t_{n+1})=\varepsilon_s^{fl}(t_{n+1})-\varepsilon_s^{fl}(t_n)$, nearer to the programming gives as for it:

$$\Delta\varepsilon_s^{fl}(t_{n+1})=A_n^s\left(1-\exp\left(-\frac{\Delta t_{n+1}}{\tau_s}\right)\right)+J_s k(t_{n+1/2})\Delta S_{n+1}\left(1-\frac{\tau_s}{\Delta t_{n+1}}\left(1-\exp\left(-\frac{\Delta t_{n+1}}{\tau_S}\right)\right)\right) \quad (28)$$

5.2 Integration of the relation of behavior

One notes $\Delta\boldsymbol{\varepsilon}$ the increment of deformation infinitesimal such as:

$$\Delta\boldsymbol{\varepsilon}=\frac{1}{2}\left(\nabla(\Delta\boldsymbol{u})+\nabla^T(\Delta\boldsymbol{u})\right) \quad (29)$$

If account is taken, in the partition of the deformation, the thermal deformation, the deformations associated with the endogenous withdrawal and the withdrawal with desiccation, then one a:

$$\Delta\boldsymbol{\varepsilon}=\Delta\boldsymbol{\varepsilon}^e+\Delta\boldsymbol{\varepsilon}^{fl}+\Delta\boldsymbol{\varepsilon}^{th}+\Delta\boldsymbol{\varepsilon}^{ret-end}+\Delta\boldsymbol{\varepsilon}^{ret-des} \quad (30)$$

With the elastic strain such as:

$$\boldsymbol{\varepsilon}^e=\boldsymbol{H}\,\boldsymbol{\sigma} \quad (31)$$

$\boldsymbol{H}$ is the tensor of elasticity of Hooke. The thermal deformation is written:

$$\boldsymbol{\varepsilon}^{th}=\alpha(T-T_{ref})\boldsymbol{I}_d \quad (32)$$

For a temperature T data, with α the thermal dilation coefficient and T_{ref} the temperature of reference. Deformations of endogenous withdrawal and desiccation:

$$\boldsymbol{\varepsilon}^{ret-end}=-\beta\xi\boldsymbol{I}_d \text{ and } \boldsymbol{\varepsilon}^{ret-des}=\kappa(C_{ref}-C)\boldsymbol{I}_d \quad (33)$$

Withvec ξ hydration, C water concentration, C_{ref} drying of reference and (β,κ) characteristic materials.

Note:

In the continuation of the document, one will note $\Delta\varepsilon^A=\Delta\varepsilon^{th}+\Delta\varepsilon^{ret\ end}+\Delta\varepsilon^{ret\ des}$.

In 3D, for the deviatoric part one has thus the following constraint:

$$\tilde{\sigma}=2\mu\tilde{\varepsilon}^e=\frac{2\mu}{2\mu^-}\tilde{\sigma}^-+2\mu\Delta\tilde{\varepsilon}-2\mu\Delta\tilde{\varepsilon}^{fl} \quad (34)$$

And deformation of creep:

$$\tilde{\varepsilon}^{fl}(t_{n+1})=(1+\nu_f)\left(J\tilde{A}_{n+1}^0-\sum_{s=1}^{8}\tilde{A}_{n+1}^s\right) \quad (35)$$

To reduce the writing, one will note:

$$A_{n+1}\Rightarrow A$$
$$A_n\Rightarrow A^- \quad (36)$$

Initially, the increment of equivalent constraint is expressed (deviatoric) from (11):

$$\Delta\tilde{S}=h\tilde{\sigma}-h^-\tilde{\sigma}^- \quad (37)$$

While applying the deviatoric part of (28) (nap on all the chains) on the expression of the deviatoric constraint (34):

$$\tilde{\sigma}=\frac{2\mu}{2\mu^{-}}\tilde{\sigma}^{-}+2\mu\Delta\tilde{\varepsilon}-2\mu(1+\nu_f)\left[\sum_s\left\{\tilde{A}^{-,s}\left(1-\exp\left(-\frac{\Delta t}{\tau_s}\right)\right)\right\}+k(t_{n+1/2})\Delta\tilde{S}\sum_s\left\{J_s\left(1-\frac{\tau_s}{\Delta t}\left(1-\exp\left(-\frac{\Delta t}{\tau_S}\right)\right)\right)\right\}\right] \quad (38)$$

One injects there (37), which gives:

$$\tilde{\sigma}\left[1+2\mu(1+\nu_f)(h\,k(t_{n+1/2}))\sum_s\left(J_s 1-\frac{\tau_s}{\Delta t}\left(1-\exp\left(-\frac{\Delta t}{\tau_S}\right)\right)\right)\right]=\frac{2\mu}{2\mu^{-}}\tilde{\sigma}^{-}+2\mu\Delta\tilde{\varepsilon}-2\mu(1+\nu_f)\left[\sum_s\left\{\tilde{A}^{-,s}\left(1-\exp\left(-\frac{\Delta t}{\tau_s}\right)\right)\right\}-k(t_{n+1/2})h^{-}\tilde{\sigma}^{-}\sum_s\left\{J_s\left(1-\frac{\tau_s}{\Delta t}\left(1-\exp\left(-\frac{\Delta t}{\tau_S}\right)\right)\right)\right\}\right] \quad (39)$$

In the same way, for the spherical part one a:

$$tr(\sigma)=\frac{3K}{3K^{-}}tr(\sigma^{-})+3K\,tr(\Delta\varepsilon)-3K\,tr(\Delta\varepsilon^{fl})-3K\,tr(\Delta\varepsilon^{A}) \quad (40)$$

With the expression trace of the deformations of creep (nap on all the chains):

$$tr(\varepsilon^{fl})=(1-2\nu_f)\sum_s\left\{tr(A^{-,s})\left(1-\exp\left(-\frac{\Delta t}{\tau_s}\right)\right)\right\}+k(t_{n+1/2})tr(S)\sum_s\left\{J_s\left(1-\frac{\tau_s}{\Delta t}\left(1-\exp\left(-\frac{\Delta t}{\tau_S}\right)\right)\right)\right\} \quad (41)$$

Finally:

$$tr(\sigma)\left[1+3K(1-2\nu_f)(h\,k(t_{n+1/2}))\sum_s\left\{J_s\left(1-\frac{\tau_s}{\Delta t}\left(1-\exp\left(-\frac{\Delta t}{\tau_S}\right)\right)\right)\right\}\right]=\frac{3K}{3K^{-}}tr(\sigma^{-})+3K\,tr(\Delta\varepsilon)-3K\,tr(\Delta\varepsilon^{A})-3K(1-2\nu_f)\left[\sum_s\left\{tr(A^{-,s})\left(1-\exp\left(-\frac{\Delta t}{\tau_s}\right)\right)\right\}-h^{-}tr(\sigma^{-})k(t_{n+1/2})\sum_s\left\{J_s\left(1-\frac{\tau_s}{\Delta t}\left(1-\exp\left(-\frac{\Delta t}{\tau_S}\right)\right)\right)\right\}\right] \quad (42)$$

One from of deduced then σ since $\sigma_{ij}=\tilde{\sigma}_{ij}+\frac{1}{3}tr\,\sigma\,\delta_{ij}$

5.3 Variables of state

The variables of state of the two relations of behavior are thus:

- $\boldsymbol{\sigma}$: tensor of the constraints,
- $\boldsymbol{\varepsilon}$: tensor of the deformations,
- T : temperature,
- C : water concentration,
- ξ : hydration degree,
- $\boldsymbol{A}_s$: tensors characteristic of the advance of creep, are 6×9 variables,
- a : the age of the concrete.

Components of eight tensors $\boldsymbol{A}_s$ and a are the internal variables of the law of behavior (`BETON_GRANGER` like `BETON_GRANGER_V`). The dimension of each tensor $\boldsymbol{A}_s$ depends on modeling (four components in 2D and six in 3D). One thus has:

- in 3D, 55 internal variables:
  - VI1… VI6: components of the tensor $\boldsymbol{A}_1$
  - VI7… VI12: components of the tensor $\boldsymbol{A}_2$
  - …
  - VI43… VI48: components of the tensor $\boldsymbol{A}_8$
  - VI49… VI54: components of the tensor $\boldsymbol{A}_0$
  - VI55: a
- in 2D, 37 internal variables:
  - VI1… VI4: components of the tensor $\boldsymbol{A}_1$
  - VI5… VI8: components of the tensor $\boldsymbol{A}_2$
  - …
  - VI29… VI32: components of the tensor $\boldsymbol{A}_8$
  - VI33… VI36: components of the tensor $\boldsymbol{A}_0$
  - VI37: a

5.4 Tangent matrix

The tangent matrix is expressed by the following formula:

$$\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}}=\frac{\partial \tilde{\boldsymbol{\sigma}}}{\partial \boldsymbol{\varepsilon}}+\frac{1}{3}\frac{\partial(tr\,\boldsymbol{\sigma})}{\partial \boldsymbol{\varepsilon}}\boldsymbol{I}_d \quad (43)$$

With:

$$\frac{\partial \tilde{\boldsymbol{\sigma}}}{\partial \boldsymbol{\varepsilon}}=\frac{\partial \tilde{\boldsymbol{\sigma}}}{\partial \tilde{\boldsymbol{\varepsilon}}}\frac{\partial \tilde{\boldsymbol{\varepsilon}}}{\partial \boldsymbol{\varepsilon}} \text{ and } \frac{\partial(tr\,\boldsymbol{\sigma})}{\partial \boldsymbol{\varepsilon}}=\frac{\partial(tr\,\boldsymbol{\sigma})}{\partial(tr\,\boldsymbol{\varepsilon})}\frac{\partial(tr\,\boldsymbol{\varepsilon})}{\partial \boldsymbol{\varepsilon}} \quad (44)$$

What requires the evaluation of the following expressions:

$$\frac{\partial \tilde{\varepsilon}_{ij}}{\partial \varepsilon_{kl}}=\delta_{ik}\delta_{jl}-\frac{1}{3}\delta_{ij}\delta_{kl} \text{ and } \frac{\partial(tr\,\boldsymbol{\varepsilon})}{\partial \varepsilon_{ij}}=\delta_{ij} \text{ with } I_{ijkl}=\delta_{ik}\delta_{jl} \quad (45)$$

For an Iteration of NewtoN, one derives the expression (39):

$$\frac{\partial\tilde{\boldsymbol{\sigma}}}{\partial\boldsymbol{\varepsilon}}\left[1+2\mu(1+\nu_f)(h\,k(t_{n+1/2}))\sum_s\left(J_s 1-\frac{\tau_s}{\Delta t}\left(1-\exp\left(-\frac{\Delta t}{\tau_S}\right)\right)\right)\right]=2\mu\,\boldsymbol{I}_d \quad (46)$$

And the expression (42):

$$\frac{\partial(tr\,\boldsymbol{\sigma})}{\partial(tr\,\boldsymbol{\varepsilon})}\left[1+3K(1-2\nu_f)(h\,k(t_{n+1/2}))\sum_s\left\{J_s\left(1-\frac{\tau_s}{\Delta t}\left(1-\exp\left(-\frac{\Delta t}{\tau_S}\right)\right)\right)\right\}\right]=3K\,\boldsymbol{I}_d \quad (47)$$

One considers now p doe of prediction with pas de time $[t_n, t_{n+1}]$. It is noticed as a preliminary that in 1D, the temporal derivative of the deformation of creep is expressed very simply:

$$\left(\frac{\partial\varepsilon_s^{fl}}{\partial t}\right)_{t_n}=\frac{A_s^-}{\tau_s}-J_s k(t_c)\frac{\partial S}{\partial t} \quad (48)$$

The problem is written in speed at the moment t_n:

$$\frac{\partial\tilde{\boldsymbol{\sigma}}}{\partial t}\left[1+2\mu\sum_s J_s k(t_n)h^-\right]=2\mu\frac{\partial\tilde{\boldsymbol{\varepsilon}}}{\partial t}-2\mu(1+\nu_f)\left[\sum_s\left(\frac{\tilde{A}_s^-}{\tau_s}-J_s k(t_n)\tilde{\boldsymbol{\sigma}}^-\frac{dh}{dt}\right)\right] \quad (49)$$

For the spherical part:

$$\begin{aligned}\frac{\partial(tr\,\sigma)}{\partial t}&\left[1+3\mathrm{K}(1-2\nu_f)(\sum_s J_s\cdot k(t_c)\cdot h^-)\right]=\\ &=3\mathrm{K}\frac{\partial(tr\,\varepsilon)}{\partial t}-3\mathrm{K}(1-2\nu_f)\left[\sum_s\frac{tr\,A_s^-}{\tau_s}-J_s\cdot k(t_n)\cdot(tr\,\sigma^-)\frac{dh}{dt}\right]+3\mathrm{K}(3\beta\frac{d\xi}{dt})+3\mathrm{K}(3\kappa\frac{dC}{dt})\end{aligned} \quad (50)$$

With the final one:

$$\begin{aligned}\Delta(\mathrm{tr}\,\sigma)&\left[1+3\mathrm{K}(1-2\nu_f)(\sum_s J_s\cdot k(t_n)\cdot h^-)\right] =\\ &= 3\mathrm{K}\,\Delta(\mathrm{tr}\,\varepsilon)-3\mathrm{K}(1-2\nu_f)\left[\sum_s\frac{(tr\,\boldsymbol{A}_s^-)}{\tau_s}\cdot\Delta t-J_s\cdot k(t_n)\cdot(\mathrm{tr}\,\sigma^-)\,\Delta h\right]\\ &- 3\mathrm{K}(3\alpha\,\Delta T)+3\mathrm{K}(3\beta\,\Delta\,\xi)+3\mathrm{K}(3\kappa\,\Delta C)\end{aligned} \quad (51)$$

Creep thus introduces a specific term of second member at the time of the phase of prediction which in fact is neglected, without consequence on the results.

6 Bibliography

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7 Features and checking

This document relates to the laws of behavior BETON_GRANGER, BETON_GRANGER_V (keyword BEHAVIOR of STAT_NON_LINE) and their associated materials BETON_GRANGER, V_BETON_GRANGER (order DEFI_MATERIAU).

These laws of behavior are respectively checked by the cases following tests:

BETON_GRANGER_V	SSNP116	Coupling creep/cracking - uniaxial Traction	[V6.03.116]
BETON_GRANGER	SSNV142	Clean creep test: Granger model	[V6.04.142]
BETON_GRANGER_V	SSNV105	Model BETON_GRANGER_V : creep test with taking into account of the relative humidity and ageing.	[V6.04.105]

8 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
		Initial text
7,4	S.Michel-Ponnelle EDF-R&D/AMA	
13.2	Marina Bottoni EDF-R&D/AMA	Elimination effect of the temperature