Law of behavior ENDO_ISOT_BETON

Summary:

This documentation presents the theoretical writing and the digital integration of the law of behavior ENDO_ISOT_BETON who describes an asymmetrical local damage mechanism of the concretes, with effect of restoration of rigidity.

Contents

1 Introduction – Scope of application

The law of behavior ENDO_ISOT_BETON aim at modelling most simply possible fragile elastic concrete behavior. It can be seen like an extension of the law ENDO_FRAGILE [R5.03.18] (with which it keeps a proximity of unquestionable formulation) for applications of Génie Civil.

As for the law ENDO_FRAGILE, the material is isotropic. Rigidity can decrease, the loss of rigidity measured by an evolving scalar of 0 (healthy material) to 1 (completely damaged material).
On the other hand, contrary to ENDO_FRAGILE, the loss of rigidity distinguishes traction from compression, to privilege the damage in traction. Moreover this loss of rigidity can disappear by return in compression, it acts of the phenomenon of restoration of rigidity to refermeture. It as should be noted as this law of damage aims at describing the rupture of the concrete in traction; it is not thus adapted at all to the description of the nonlinear behavior of the concrete in compression. It thus supposes that the concrete remains in a moderate compactness.

The law ENDO_ISOT_BETON present softening, which generally involves a loss of ellipticity of the equations of the problem and consequently a localization of the deformations, from where a pathological dependence to the grid.

Lastly, the softening character of the behavior also involves the appearance of instabilities, physics or parasites, which result in snap - backs on the total answer and return the piloting of the essential loading in statics. The piloting of the type PRED_ELAS [R5.03.80] then seems the mode of control of the loading more adapted.

2 Law of behavior

2.1 Theoretical writing

If one seeks to take account of the effect of refermeture, it is necessary to pay a great attention to the continuity of the constraints according to the deformations (what is an essential condition for a law of behavior in a computation software by finite elements), confer [bib1]. Indeed, if one models this effect in a too simplistic way, the law of behavior is likely great to present a discontinuous answer.

To take account of the refermeture (i.e the transition between traction and compression), it is necessary to start by finely describing what one calls traction and compression, knowing that in traction (resp. compression) the crack will be considered “open” (resp. “closed”). A natural solution is to place itself in a clean reference mark of deformation. In such a reference mark, the elastic free energy is written (λ and μ indicating the coefficients of Lamé):

$$\Phi(\varepsilon)\frac{\lambda}{2}(tr\,\varepsilon)^2+\mu\sum_i \varepsilon_i^2$$ **éq 2.1-1**

One can then define:

- a traction or voluminal compression, according to the sign of $tr\,\varepsilon$,
- a traction or compression in each clean direction, according to the sign of ε_i .

According to the rather reasonable principle according to - in a case of traction ("open crack"), one corrects the elastic energy of a factor of damage; in a case of compression ("closed crack"), one keeps the expression of elastic energy -, the free energy endommageable is written:

$$\Phi(\varepsilon, d)=\frac{\lambda}{2}(tr\,\varepsilon)^2\left(H(-tr\,\varepsilon)+\frac{1-d}{1+\gamma d}H(tr\,\varepsilon)\right)+\mu\sum_i \varepsilon_i^2\left(H(-\varepsilon_i)+\frac{1-d}{1+\gamma d}H(\varepsilon_i)\right) \quad \textbf{éq 2.1-2}$$

It is noticed that the free energy is continuous with each regime change. It is even continuously derivable compared to the deformations, since it is nap of derivable functions (the function $x^2 H(x)$ is derivable) and the continuity of the derivative partial at the points $tr\,\varepsilon=0$ and $\varepsilon_i=0$ is immediate. The constraints then are clarified (by knowing that they will be everywhere continuous functions of the deformations). As in elasticity, the clean reference mark of the constraints coincides with the clean reference mark of the deformations, result shown in appendix.

One writes the constraints in the clean reference mark:

$$\sigma_{ii}=\lambda(tr\,\varepsilon)\left(H(-tr\,\varepsilon)+\frac{1-d}{1+\gamma d}H(tr\,\varepsilon)\right)+2\mu\,\varepsilon_{ii}\left(H(-\varepsilon_{ii})+\frac{1-d}{1+\gamma d}H(\varepsilon_{ii})\right) \quad \textbf{éq 2.1-3}$$

In this form, the continuity of the constraints with respect to the deformations is clear. The figure opposite watch the constraint σ_{11} in the plan $(\varepsilon_{1,}\varepsilon_2)$ with constant damage (case 2D, plane deformation). The effect of refermeture as well as the continuity of the constraints are quite visible.

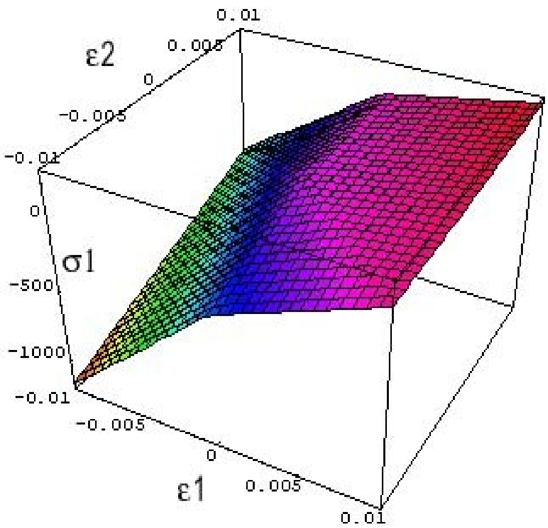

Figure 2-a: illustration of continuity.

The thermodynamic force F^d associated with the internal variable of damage is written:

$$F^d=-\frac{\partial\Phi}{\partial d}=\frac{1+\gamma}{(1+\gamma d)^2}\left(\frac{\lambda}{2}(tr\,\varepsilon)^2 H(tr\,\varepsilon)+\mu\sum_i \varepsilon_i^2 H(\varepsilon_i)\right)$$ **éq 2.1-4**

It remains to define the evolution of the damage. The diagram selected is that of the generalized standard models. A criterion should be defined, that one takes in the form:

$$f(F^d)=F^d(\varepsilon,d)-k$$ **éq 2.1-5**

where K defines the threshold of damage. In order to take into account, on the level of the evolution of the damage, the effect of containment, the threshold k depends on the state of deformation, in the form:

$$k=k_0-k_1(tr\,\varepsilon)H(-tr\,\varepsilon)$$ **éq 2.1-6**

One compels oneself to remain in the field:

$$f(F^d)\le 0$$ **éq 2.1-7**

The evolution of the variable of damage is then determined by the conditions of Kuhn-Tucker:

$$\begin{cases}\dot{d}=0 \; pour \; f<0\\ \dot{d}\ge 0 \; pour \; f=0\end{cases}$$ **éq 2.1-8**

For a uniaxial request, the resulting curve is shown in the figure 2-b, for two values of containment (see 2.3.2.2 paragraph). In compression, the behavior remains roughly linear here, which represents the behavior of material only up to 3-4 times the value of resistance (uniaxial) to traction, SYT. It is clear that the use of the law is then indicated as much as one remains in this terminal. In the figure 2-b, the test-tube is initially charged in traction and sudden thus a damage. Then, it is discharged and charged after in compression in the same direction. The resumption of rigidity in compression (refermeture of crack) will thus be noticed.

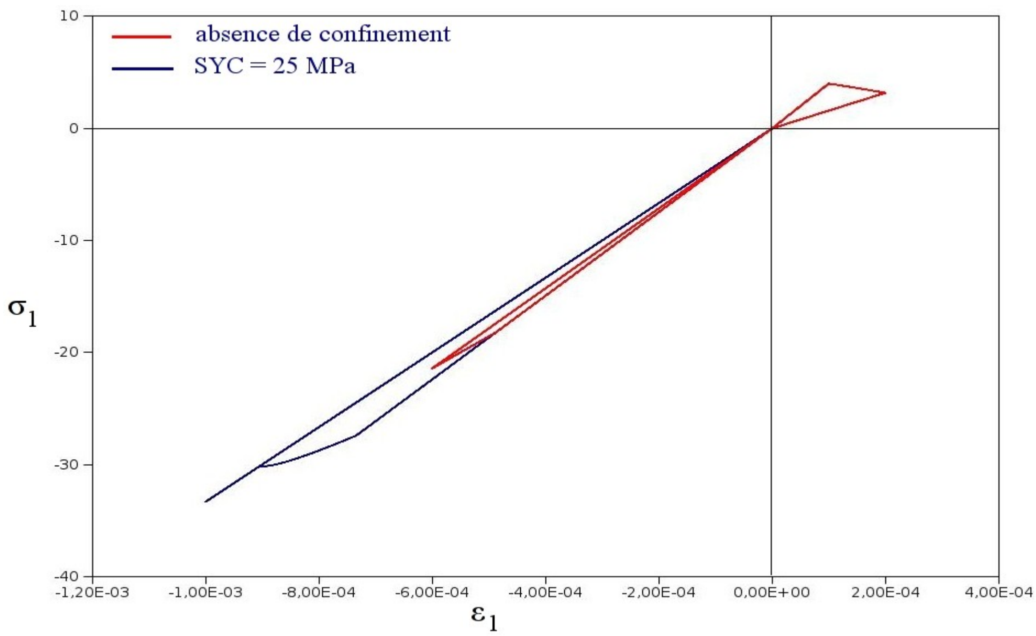

Figure 2-b: answer to a uniaxial request.

One notes that in uniaxial pure load endommageante ($\dot{d}\geqslant 0$), the thermodynamic force is expressed: $F^d(d)=-E\dfrac{\varepsilon^2.\xi_{,d}(d)}{2}$, having noted $\xi(d)=\dfrac{1-d}{1+\gamma d}$. Lcondition of coherence has is written then:

$$\dot{f}=-E\,\varepsilon\,\dot{\varepsilon}\,\xi_{,d}-E\frac{\varepsilon^2\dot{d}\,\xi_{,dd}}{2}=0 \qquad \textbf{éq 2.1-9}$$

From where: $\dot{d}\;=\;-\dfrac{2\dot{\varepsilon}\,\xi_{,d}}{\varepsilon\,\xi_{,dd}}$ and thus the law of damaging uniaxial pure load membrane is:

$$\dot{\sigma}\;=\;E\left(\xi_{,d}\dot{d}\,\varepsilon+\xi_{\dot{\varepsilon}}\right)\;=\;E\,\dot{\varepsilon}\left(\xi-2\frac{\xi_{,d}^2}{\xi_{,dd}}\right)=\frac{-E}{\gamma}\dot{\varepsilon} \qquad \textbf{éq 2.1-10}$$

what makes it possible to interpret the role of the parameter γ : the slope being constant, which is the justification of the algebraic form of the function ξ .

Note:

From a formal point of view, the generalized standard materials are characterized by a potential of dissipation function positively homogeneous of degree 1, transformed of Legendre-Fenchel of the indicating function of the field of reversibility, which is thus worth here:

$$\Delta(\dot{d})=\sup_{F^d/f(F^d)\leq 0}F^d\dot{d}=k\,\dot{d}+I_{IR^+}(\dot{d}) \qquad \textbf{éq 2.1-11}$$

One will note the presence of an indicating function relating to $\dot{d}$, which ensures that the damage is increasing.

It still remains to take into account the fact that the damage is raised by 1. From an intuitive point of view, that seems easy. To keep a writing completely compatible with the formalism generalized standard, it is enough to introduce an indicating function of the acceptable field into the expression of the free energy:

$$\begin{aligned}\Phi(\varepsilon,d)=\;&\frac{\lambda}{2}(tr\,\varepsilon)^2\left(H(-tr\,\varepsilon)+\frac{1-d}{1+\gamma d}H(tr\,\varepsilon)\right)+\\&\mu\sum_i\varepsilon_i^2\left(H(-\varepsilon_i)+\frac{1-d}{1+\gamma d}H(\varepsilon_i)\right)+I_{]-\infty;1]}(d)\end{aligned} \qquad \textbf{éq 2.1-12}$$

The introduction of this indicating function prevents the damage from exceeding 1, indeed, for $d=1$, $F^d=\dfrac{-\partial f}{\partial d}=-\infty$, and the damage does not evolve any more.

2.2 Taking into account of the withdrawal and the temperature

The law of behavior takes into account a possible withdrawal of desiccation, a possible endogenous withdrawal and a possible thermal deformation. Deformation ε it is question in this document being then the “elastic strain” $\tilde{\varepsilon}=\varepsilon-\varepsilon^{th}-\varepsilon^{rd}-\varepsilon^{\mathrm{re}}$.
On the other hand, the parameters materials in question in the next paragraph are regarded as constants (in particular, they cannot depend on the temperature, in the current level of development)

2.3 Identification of the parameters

The parameters of the law of behavior are 4 or 5 (see following paragraphs). They are classically provided in the operator DEFI_MATERIAU.

2.3.1 Elastic parameters

They are simplest: it is the two classical parameters, Young modulus and Poisson's ratio, provided under the keyword ELAS or ELAS_FO of DEFI_MATERIAU.

2.3.2 Parameters of damage

According to whether the user wants to use the dependence of the threshold with containment or not, it is necessary to provide 2 or 3 parameters to control the law of damage.

2.3.2.1 Use without dependence with containment

In this case, it is considered that the parameter k_1 is null. It should be noted that the compactness of the concrete must remain moderate so that the law remains valid (compressive stress about some times the constraint with the peak of traction, in absolute value).
The user must inform, under the keyword BETON_ECRO_LINE of DEFI_MATERIAU, values of:

- SYT : limit of simple tensile stress,
- D_SIGM_EPSI : slope of the curved post-peak in traction.

The value of SYC is then calculated automatically to have k1=0 and take its minimal value, that is to say:

$$SYC = SYT\sqrt{\frac{1+\nu-2\nu^2}{2\nu^2}}$$

2.3.2.2 Use with dependence with containment

In this case, the dependence with containment makes it possible the concrete to keep a realistic behaviour in compression until the order of magnitude of appearance of nonthe linearity in compression, given by SYC, Cf below (classically, a stress en compressive of about ten times the constraint with the peak of traction, in absolute value).

The user must inform, under the keyword BETON_ECRO_LINE of DEFI_MATERIAU, values of:

- SYT : limit of simple tensile stress,
- SYC : limit of the simple compressive stress,
- D_SIGM_EPSI : slope of the curved post-peak in traction.

The figure 2-b watch the behavior with two values of containment. The increase in parameter SYC has like effect to prolong the linear behavior of material.

2.3.2.3 Passage des values “user” with the values “models”

For information, one obtains the values of γ , k_0 and possibly k_1 (if the user informed SYC) by the following formulas, cf [éq 2.1-10]:

$$\gamma = \frac{-E}{\mathrm{D\_SIGM\_EPSI}}$$

$$k_0=(SYT)^2\left(\frac{1+\gamma}{2E}\right)\left(\frac{1+\nu-2\nu^2}{1+\nu}\right)$$
$$k_1=SYC\frac{(1+\gamma)\nu^2}{(1+\nu)(1-2\nu)}-k_0\frac{E}{(1-2\nu)(SYC)}$$

2.4 Digital integration

Two points are to be regulated before establishing the model: the first relates to the evaluation of the damage; the second consists in calculating the tangent matrix, calculation made a little more delicate than usually by the passage in a clean reference mark of deformation.
One places oneself here within the framework of the implicit integration of the laws of behavior. The dependence of the criterion according to containment [éq 2.1-6] is taken into account in explicit form, i.e the threshold k is entirely determined by the state of deformation of the preceding step, this to simplify the integration of the model.

2.4.1 Evaluation of the damage

As one will see it, a simple scalar equation makes it possible to obtain the damage, which makes it possible to avoid a recourse to the iterative methods.
One notes d^- the damage with the preceding step and d^+ the evaluation of the damage with the step running to the current iteration which will be the damage with the current step when convergence is reached. Simplest to evaluate the damage of the current iteration is to suppose that one reaches the criterion at the current moment, which results in:

$$f(F^d)=0 \Rightarrow \frac{1+\nu}{(1+\gamma d)^2}\left(\frac{\lambda}{2}(tr\,\varepsilon)^2 H(tr\,\varepsilon)+\mu\sum_i \varepsilon_i^2 H(\varepsilon_i)\right)=k \qquad \textbf{éq 2.4.1-1}$$

what gives:

$$d^{test}=\frac{1}{\gamma}\left(\sqrt{\frac{1+\gamma}{k}\left(\frac{\lambda}{2}(tr\,\varepsilon)^2 H(tr\,\varepsilon)+\mu\sum_i \varepsilon_i^2 H(\varepsilon_i)\right)}-1\right) \qquad \textbf{éq 2.4.1-2}$$

3 cases arise:

- $d^{test}\le d^-$: that wants to say that at the moment running, the criterion is not reached, one concludes from it that $d^+=d^-$,
- $d^-\le d^{test}\le 1$: the criterion is thus reached, the condition of coherence implies $d^+=d^{test}$,
- $d^{test}\ge 1$: the material is then ruined in this point, from where $d^+=1$.

2.4.2 Calculation of the tangent matrix

The tangent matrix is the sum of two terms, the first expressing the forced relation/deformation with constant damage, the second resulting from the condition $f=0$. Indeed, one can write:

$$\frac{\partial\sigma_{ij}}{\partial\varepsilon_{kl}}=\left\{\frac{\partial\sigma_{ij}}{\partial\varepsilon_{kl}}\right\}_{d=c^{te}}+\frac{\partial\sigma_{ij}}{\partial d}\left\{\frac{\partial d}{\partial\varepsilon_{kl}}\right\}_{f=0}$$ **éq 2.4.2-1**

If the user asks for calculation with tangent matrix (cf documentation of STAT_NON_LINE, [U4.51.03]), the law of behavior provides the expression given by [éq 2.4.2-1]. On the other hand, if the user asks for calculation with the matrix of discharge, the law of behavior provides the secant matrix, i.e. the first term of the member of right-hand side of [éq 2.4.2-1].

2.4.2.1 Tangent matrix with constant damage

As we underlined previously, the calculation of the tangent matrix is a little delicate because of writing of the model in the clean reference mark of deformation. Thus, one easily knows the tangent matrix with constant damage in the clean reference mark of deformation, but what one seeks is this same tangent matrix in the total reference mark.

If the damage does not evolve, in the clean reference mark of deformation, the required matrix expresses a simple relation of degraded elasticity:

$$\left\{\frac{\delta\tilde{\sigma}_i}{\delta\tilde{\varepsilon}_j}\right\}_{d=C^{te}}=\lambda\left(H(-tr\,\varepsilon)+\frac{1-d}{1+\gamma d}H(tr\,\varepsilon)\right)+2\mu\delta_{ij}\left(H(-\varepsilon_j)+\frac{1-d}{1+\gamma d}H(\varepsilon_j)\right)$$ **éq 2.4.2.1 - 1**

It is now necessary to express the passage of the total reference mark to the clean reference mark of deformations, at least if the eigenvalues of deformation are different. The tangent matrix being necessary only to the algorithms of resolution **digital** (diagram of Newton), one will allow oneself, during the calculation of the tangent matrix (and **only** in this case) to disturb possible identical eigenvalues numerically (in order to make them distinct). One will notice in particular that allows, null damage, to find the matrix of elastic rigidity.

One notes with a tilde the tensors in the clean reference mark of deformation (which, one points out it, is also the clean reference mark of constraints). By definition, while noting U_i the clean vector associated with the i-ème eigenvalue, the matrix basic change $Q=(U_1 U_2 U_3)$, one a:

$$\sigma=Q\tilde{\sigma}Q^T\;\Rightarrow\;\delta\sigma_{ij}=Q_{\mathrm{im}}Q_{\mathrm{jm}}\delta\tilde{\sigma_m}+\delta Q_{\mathrm{im}}Q_{\mathrm{jm}}\tilde{\sigma_m}+Q_{\mathrm{im}}\delta Q_{\mathrm{jm}}\tilde{\sigma_m}$$

If the eigenvalues of deformation are distinct, the evolution of the clean vectors and eigenvalues is given by (cf previously [§2]):

$$\dot{U}_j.U_k=\frac{\dot{\tilde{\varepsilon}}_{jk}}{\tilde{\varepsilon}_j-\tilde{\varepsilon}_k}\text{ for }j\neq k$$ **éq 2.4.2.1 - 2**

$$\dot{\tilde{\varepsilon}}_i=\dot{\tilde{\varepsilon}}_{ii}\text{ for }j\neq k$$ **éq 2.4.2.1 - 3**

One from of deduced easily δQ :

$$\delta Q_{ij}=\sum_{k\neq j}\frac{\delta\tilde{\varepsilon}_{jk}}{\tilde{\varepsilon}_j-\tilde{\varepsilon}_k}(U_k)_i=\sum_{k\neq j}\frac{\delta\tilde{\varepsilon}_{jk}}{\tilde{\varepsilon}_j-\tilde{\varepsilon}_k}Q_{ik}$$ **éq 2.4.2.1 - 4**

While using then (the last expression being used only to obtain a clearly symmetrical matrix):

$$\delta \tilde{\varepsilon}_{ij} = Q_{ki} Q_{lj} \varepsilon_{kl} = \frac{1}{2} \left(Q_{ki} Q_{lj} + Q_{li} Q_{kj} \right) \varepsilon_{kl}$$

One thus obtains:

$$\begin{aligned}\delta\sigma_{ij} &= \sum_{m,n} Q_{im} Q_{jm} \left(\frac{\partial\tilde{\sigma}_m}{\partial\tilde{\varepsilon}_n} \right) \delta\tilde{\varepsilon}_n + \sum_m \delta Q_{im} Q_{jm} \tilde{\sigma}_m + Q_{im} \delta Q_{jm} \tilde{\sigma}_m \\ &= \sum_{m,n,k,l} Q_{im} Q_{jm} Q_{kn} Q_{ln} \left(\frac{\partial\tilde{\sigma}_m}{\partial\tilde{\varepsilon}_n} \right) \delta\varepsilon_{kl} + \frac{1}{2} \sum_{\substack{k,l \\ n\neq m}} \frac{Q_{in} Q_{km} Q_{ln} Q_{jm}}{\tilde{\varepsilon}_m - \tilde{\varepsilon}_n} \tilde{\sigma}_m \delta\varepsilon_{kl} \\ &+ \frac{1}{2} \sum_{\substack{k,l \\ n\neq m}} \frac{Q_{im} Q_{jn} Q_{km} Q_{ln}}{\tilde{\varepsilon}_m - \tilde{\varepsilon}_n} \tilde{\sigma}_m \delta\varepsilon_{kl} + \frac{1}{2} \sum_{\substack{k,l \\ n\neq m}} \frac{Q_{in} Q_{lm} Q_{kn} Q_{jm}}{\tilde{\varepsilon}_m - \tilde{\varepsilon}_n} \tilde{\sigma}_m \delta\varepsilon_{kl} + \frac{1}{2} \sum_{\substack{k,l \\ n\neq m}} \frac{Q_{im} Q_{jn} Q_{lm} Q_{kn}}{\tilde{\varepsilon}_m - \tilde{\varepsilon}_n} \tilde{\sigma}_m \delta\varepsilon_{kl}\end{aligned}$$

éq 2.4.2.1 - 5

The tangent matrix with constant damage is thus written:

$$A_{ijkl} = \frac{\partial \sigma_{ij}}{\partial \varepsilon_{kl}} = \sum_{m,n} Q_{\text{im}} Q_{\text{jm}} Q_{\text{kn}} Q_{\text{ln}} \left\{ \frac{\delta \tilde{\sigma}_m}{\delta \tilde{\varepsilon}_n} \right\}_{d=C^{te}} + \frac{1}{2} \sum_{m,n:n\neq m} \left(\frac{(Q_{\text{km}} Q_{\text{ln}} + Q_{\text{lm}} Q_{\text{kn}})(Q_{\text{in}} Q_{\text{jm}} Q_{\text{jn}} Q_{\text{im}})}{\tilde{\varepsilon}_n - \tilde{\varepsilon}_m} \right) \tilde{\sigma}_m$$

éq 2.4.2.1 - 6

2.4.2.2 Term of the tangent matrix due to the evolution of the damage

The expression to be evaluated is written:

$$\frac{\partial \sigma_{ij}}{\partial d} \left\{ \frac{\partial d}{\partial \varepsilon_{kl}} \right\}_{f=0}$$

éq 2.4.2.2 - 1

One writes the equation [éq 2.4.1-1] in the form:

$$\frac{1+\gamma}{(1+\gamma d)^2} [W(\varepsilon)] = k$$

éq 2.4.2.2 - 2

with: $W(\varepsilon) = \frac{\lambda}{2} (tr\,\varepsilon)^2 H(tr\,\varepsilon) + \mu \sum_i \varepsilon_i^2 H(\varepsilon_i)$.

While differentiating this expression, it comes:

$$-\frac{2\gamma(1+\gamma)}{(1+\gamma d)^3} W(\varepsilon) \delta d + \frac{1+\gamma}{(1+\gamma d)^2} \sigma^{el} \cdot \delta \varepsilon = 0$$

éq 2.4.2.2 - 3

with: $\sigma^{el} = \frac{\partial W}{\partial \varepsilon}$

One uses then the following equality:

$$\frac{\partial \sigma}{\partial d} = -\frac{1+\gamma}{(1+\gamma d)^2}\sigma^{el}$$ **éq 2.4.2.2 - 4**

One concludes:

$$\left\{\frac{\partial \sigma_{ij}}{\partial \varepsilon_{kl}}\right\}_{f=0} = -\frac{1+\gamma}{2\gamma(1+\gamma d)W(\varepsilon)}\sigma_{ij}^{el}\sigma_{kl}^{el}$$ **éq 2.4.2.2 - 5**

2.4.3 Case of completely damaged material

In the case of the completely damaged material, $d=1$, the rigidity of the material point can be cancelled. That poses problem for the constraint by no means; on the other hand, that can actuate worthless pivots in the matrix of rigidity. To mitigate this difficulty, one allows oneself to define a minimal rigidity, for the tangent matrix or the matrix of discharge. This minimal rigidity does not affect the value of the damage (which can reach 1) or the constraint (which can reach 0).

To preserve a reasonable conditioning of the matrix of rigidity, minimal rigidity is taken with 10^{-5} rigidity initale. An indicator χ specify the behavior during the step of current time:

1) $\chi=0$: no evolution of the damage during the step,
2) $\chi=1$: evolution of the damage during the step,
3) $\chi=2$: saturated damage $d=1$.

2.5 Description of the internal variables

The model has two internal variables:

1) $VI(1)$: damage d ,
2) $VI(2)$: indicator χ

3 Piloting by elastic prediction

The piloting of the type PRED_ELAS control intensity of the loading to satisfy a certain equation related to the value with the function threshold f^{el} during the elastic test. Consequently, only the points where the damage is not saturated will be taken into account. The algorithm which deals with this mode of piloting, cf [R5.03.80], requires the resolution of each one of these points of Gauss of the following scalar equation in which $\Delta\tau$ is a data and η the unknown factor:

$$\tilde{f}^{el}(\eta)=\Delta\tau \qquad \textbf{éq 4-1}$$

The method used to version 8 was the following one: the function $\tilde{f}^{el}$ provides the value of the function threshold during an elastic test when the field of displacement breaks up in the following way according to the scalar parameter η :

$$u=u_0+\eta u_1 \qquad \textbf{éq 4-2}$$

where u_0 and u_1 are given. Thanks to the linearity in small deformations of the operators deformation (calculation of the deformations starting from displacements) and regularized deformation, one also obtains the following decompositions:

$$\varepsilon=\varepsilon_0+\eta\,\varepsilon_1 \text{ and } \bar{\varepsilon}=\bar{\varepsilon}_0+\eta\,\bar{\varepsilon}_1 \qquad \textbf{éq 4-3}$$

The function f^{el} presenting the good property to be convex, the equation [éq 4-1] presents zero, one or two solutions, which are required as follows:

1) Determination amongst solutions by study at the boundaries $\pm\infty$ and possibly (if the value at the two boundaries is each time positive) determination if f^{el} present a negative minimum;
2) Determination of a framing of each solution starting from the preceding study
3) Determination of the solution (for a convex function knowing the framing, this research is simple and fast)

Since version 9, for more parameter, ease of use $\Delta\tau$ corresponds with the increment of damage which one seeks to obtain for at least a point of the structure.

One then does not seek any more one parameter of piloting η who makes leave the criterion a value $\Delta\tau$ with the damage resulting from the step of previous time (cf Eq 4-1), but a parameter η who brings back for us on the criterion with a damage increased by $\Delta\tau$:

$$\tilde{f}^{el}(\eta, d^-)=\Delta\tau \;\Rightarrow\; \tilde{f}^{el}(\eta, d^-+\Delta\tau)=0$$

This coefficient $\Delta\tau$ is calculated in the following way:

$$\Delta\tau=\frac{\Delta t}{\text{COEF\_MULT}}$$

where Δt corresponds to the increment of time defined in the list of moments of calculation and COEF_MULT is the coefficient specified by the keyword COEF_MULT option PILOTING in the operator STAT_NON_LINE [U4.51.03].

4 Bibliography

1 P.B. BADEL: Contributions to the digital simulation of structures out of reinforced concrete. Thesis of the University Paris VI, 2001.

5 Features and checking

This document relates to the law of behavior ENDO_ISOT_BETON (keyword BEHAVIOR of STAT_NON_LINE) and its associated material ENDO_ISOT_BETON (order DEFI_MATERIAU).

This law of behavior is checked by the cases following tests:

COMP005cd	Test of the behavior. Simulation in a material point.	Not documented
SSLA103e	Calculation of the withdrawal of desiccation and the endogenous withdrawal on a cylinder	[V3.06.103]
SSNS106	Damage of a plate planes under requests varied with the law of behavior GLRC_DM	[V6.05.106]
SSNS108	Simulation of test SAFE by the progressive push	[V6.05.108]
SSNV149	Test of ENDO_ISOT_BETON	[V6.04.149]
SSNV169	Coupling creep – damage	[V6.04.169]
WTNV121	Damping of the concrete with a law of damage	[V7.31.121]

6 Description of the versions of the document

Index document	Version Aster	Author (S) Organization (S)	Description of the modifications
B	7.4	P.Badel EDF-R&D/AMA	Initial text
C	8.5	P.Badel EDF-R&D/AMA	Correction of sign page 10: it missed a sign – with the second member of the 2.4.2.2 equation - 4 like in the following equation 2.4.2.2 - 5
D	9.4	V.Godard EDF-R&D/AMA	Modification of piloting by elastic prediction.
	10.3	F.Voldoire EDF-R&D/AMA	Addition of a passage page 5 explaining the slope in uniaxial load.

Annexe 1 Demonstration of the clean reference mark of constraint

The term traces some in energy does not pose a problem: it is invariant by any change of reference mark.

Remain the term in $\sum_i \varepsilon_i^2 \left(H(-\varepsilon_i) + \frac{1-d}{1+\gamma d} H(\varepsilon_i) \right)$.

Notation : one writes with **one** index (for example ε_i) i - ème eigenvalue of a tensor which is written (while clarifying its **two** indices) ε_{kl} .

- If the eigenvalues of the deformation are all distinct, it is shown whereas $\dot{\varepsilon}_i = \dot{\varepsilon}_{ii}$, with $\dot{\varepsilon}_{kl}$ components of $\dot{\varepsilon}$ in the fixed reference mark coinciding with the clean reference mark of deformation at the moment considered (in this reference mark one thus has $\varepsilon_{kl} = \varepsilon_k \delta_{kl}$).
  Indeed, let us write the deformations in the form:
  $$\varepsilon = \sum_i \varepsilon_i U_i \otimes U_i$$
  While differentiating this expression, it comes:
  $$\dot{\varepsilon} = \sum_i \dot{\varepsilon}_i U_i \otimes U_i + \varepsilon_i \dot{U}_i \otimes U_i + \varepsilon_i U_i \otimes \dot{U}_i$$
  By using the fact that the clean vectors are orthonormal:
  $$U_i \cdot U_j = \delta_{ij} \Rightarrow \dot{U}_i \cdot U_j + U_i \cdot \dot{U}_j = 0$$
  one obtains the variations of the eigenvalues and the clean vectors:
  $$\dot{\varepsilon}_i = \dot{\varepsilon}_{ii} \text{ and } \dot{U}_j \cdot U_k = \frac{\dot{\varepsilon}_{jk}}{\varepsilon_j - \varepsilon_k} \text{ for } j \neq k$$
  This is obviously valid only if the eigenvalues are distinct (as one can clearly see it on the expression of the variations of the clean vectors). That comes owing to the fact that the clean vectors are not continuous functions of the elements of the matrix.

- If two eigenvalues of deformations are equal (and apart from the very particular case where they are also worthless), they are either positive, or negative. Let us take the case where they are positive (the other case lends itself to a demonstration in any similar point). Energy concerning these two eigenvalues is written then: $\sum_{i=2}^{3} \varepsilon_i^2$ (the two equal eigenvalues are considered to have indices 2 and 3). By differentiating this expression, one obtains:

  $2 \sum_{i=2}^{3} \varepsilon_i d\varepsilon_i + 2\varepsilon \sum_{i=2}^{3} d\varepsilon_i$ while noting ε the common eigenvalue.

  By invariance of the trace of a matrix, here the restriction of the deformation on the clean plan considered, one obtains:

$\sum_{i=2}^{3} d\varepsilon_i = \sum_{i=2}^{3} d\varepsilon_{ii}$, whatever the evolution which underwent the clean reference mark at this time.

  For the remaining eigenvalue (distinct from both others and index 1 with the selected notations), one a: $d\varepsilon_1 = d\varepsilon_{11}$.

  By gathering these expressions, one obtains:

$$d\left(\sum_{i=1}^{3}\varepsilon_i^2 H(\varepsilon_i)\right)=d\left(\varepsilon_1^2 H(\varepsilon_1)\right)+d\left(\sum_{i=1}^{3}\varepsilon_i^2 H(\varepsilon_i)\right)=2\sum_{i=1}^{3}\varepsilon_{ii} H(\varepsilon_{ii}) d\,\varepsilon_{ii}$$

In conclusion, that the eigenvalues are distinct or not, one obtains:

$d\left(\sum_i \varepsilon_i^2 H(\varepsilon_i)\right)=2\sum_i \varepsilon_{ii} H(\varepsilon_{ii}) d\,\varepsilon_{ii}$ with the adopted notations.

This reasoning spreads easily with the case of three equal eigenvalues.

The differential of energy with constant damage is written then:

$$\begin{aligned} d\,\Phi(\varepsilon,d)\big|_{d=C^{te}} = \; & \lambda(tr\,\varepsilon) d(tr\,\varepsilon)\left(H(-tr\,\varepsilon)+\frac{1-d}{1+\gamma d}H(tr\,\varepsilon)\right)+ \\ & 2\mu\sum_i \varepsilon_{ii} d\,\varepsilon_{ii}\left(H(-\varepsilon_{ii})+\frac{1-d}{1-\gamma d}H(\varepsilon_{ii})\right) \end{aligned}$$

On this expression, it is observed well that the clean reference mark of deformation is also reference mark clean of constraint.