Relation of behavior BETON_UMLV for the creep of the concrete

Summary:

This document presents the clean model of creep UMLV (behavior BETON_UMLV) , which is a way of modelling the creep of the concrete (clean and of desiccation).

One also details there the writing and the digital processing of the model. The integration of the model (i.e. the update of the constraints) is carried out according to an incremental diagram starting from the increment of total deflections provided by the total diagram of resolution.

One adds the description of the coupling between clean creep and the model of MAZARS. The integration of the model (i.e. the update of the constraints) is carried out according to a “total” diagram starting from the total deflections cumulated since the initial state before loading.

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1 Introduction

Within the framework of the studies of the long-term behavior of structures out of concrete, a dominating share of the deformations measured on structure relates to the differed deformations which appear in the concrete during its life. They comprise the withdrawals with the young age, the withdrawal of desiccation, clean creep and the creep of desiccation.

The model presented here is dedicated to the modeling of the differed deformation associated with creep, clean and of desiccation. Clean creep is, in complement of the creep of desiccation, the share of creep of the concrete which one would observe during a test without exchange of water with outside. In experiments the concrete in creep presents a growing old viscoelastic behavior. The deformation of creep observed is proportional to the constraint of loading, depends on the temperature and the hygroscopy. The growing old aspect and the dependence at the temperature are not taken into account by this law.

Clean creep. The first model of creep of the concretes introduces previously into Code_Aster (see for example [R7.01.01] and [bib4]) was developed in optics to predict the longitudinal deflections of creep under uniaxial constraints. The generalization of this model, in order to take into account a state of multiaxial stresses, is done then via a Poisson's ratio of creep arbitrary, constant and equal, or close, elastic Poisson's ratio. However, determination *a posteriori* Poisson's ratio of effective creep shows its dependence with respect to the way of loading. In addition, the concrete of certain works of the Park EDF, the such containment systems of nuclear reactor, is subjected in a state of biaxial stresses. This report led to the clarification of the law of deformations of clean creep UMLV (University of Marne-the-Valley, partner in the development of this model) for which the Poisson's ratio of creep is a direct consequence of the calculation of the principal deformations.

Creep of desiccation. The model suggested here is that of Bazant [bib6]. It is a law purely viscous.

In *Code_Aster*, the model of creep presented here is used under the name of `BETON_UMLV`.

2 Assumptions

Assumption 1 (H.P.P.)
The law is written within the framework of the small disturbances.

Assumption 2 (partition of the deformations)
In small deformations, the tensor of the total deflections is broken up into several terms relating to the processes considered. As regards the description of the various mechanisms of deformations differed from the concretes, one admits that the total deflection is written:

$$\varepsilon = \underbrace{\varepsilon^{e}}_{\text{déformation élastique}} + \underbrace{\varepsilon^{fp}}_{\text{fluage propre}} + \underbrace{\varepsilon^{fdess}}_{\text{fluage de dessiccation}} + \underbrace{\varepsilon^{\Re}}_{\text{retrait endogène}} + \underbrace{\varepsilon^{rd}}_{\text{retrait de dessiccation}} + \underbrace{\varepsilon^{th}}_{\text{déformation thermique}} \quad \text{éq 2-1}$$

In this document, one does not describe the taking into account of the various types of withdrawals (for that, to see the documentation of *Code_Aster* [R7.01.12]), so that [éq 2-1] is reduced to:

$$\varepsilon = \varepsilon^{e} + \varepsilon^{fp} + \varepsilon^{fdess} \quad \text{éq 2-2}$$

Assumption 3 (decomposition of the components of creep)

In a general way, clean creep can be modelled by combining the elastic behavior of the solid and the viscous behavior of the fluid. For the law presented, clean creep is described like the combination of the elastic behavior of the hydrates and the aggregates and the viscous behavior of water.
In the case of the model BETON_BURGER, the assumption is carried out that clean creep can be broken up into a process uncoupling a spherical part and a deviatoric part. The tensor of the total deflections of clean creep is written then:

$$\underline{\underline{\varepsilon}}^{fp} = \underbrace{\varepsilon^{fs} \cdot \underline{\underline{1}}}_{\text{partie sphérique}} + \underbrace{\underline{\underline{\varepsilon}}^{fd}}_{\text{partie déviatorique}} \quad \text{with} \quad \varepsilon^{fs} = \frac{1}{3} \cdot tr\, \underline{\underline{\varepsilon}}^{fp} \quad \text{éq 2-3}$$

The tensor of the constraints can be developed according to a similar form:

$$\underline{\underline{\sigma}} = \underbrace{\sigma^{s} \cdot \underline{\underline{1}}}_{\text{partie sphérique}} + \underbrace{\underline{\underline{\sigma}}^{d}}_{\text{partie déviatorique}} \quad \text{éq 2-4}$$

The model BETON_BURGER suppose a total decoupling between the spherical and deviatoric components of clean creep: the deformations induced by the spherical constraints are purely spherical and the deformations induced by the deviatoric constraints are purely deviatoric. On the other hand, the cumulated viscous deformations have an effect on the viscous properties of the fluid, some is its source (spherical or deviatoric). To take account of the effect of internal moisture, the deformations are multiplied by internal relative moisture:

$$\varepsilon^{s} = h \cdot f\left(\sigma^{s}\right) \quad \text{and} \quad \underline{\underline{\varepsilon}}^{d} = h \cdot f\left(\underline{\underline{\sigma}}^{d}\right) \quad \text{éq 2-5}$$

Or h indicate internal relative moisture.
The condition [éq 2-5] makes it possible to check a posteriori that the deformations of clean creep are proportional to the relative humidity.

3 Description of the model

3.1 Clean creep

3.1.1 Description of the spherical part of clean creep

The spherical constraints are at the origin of the migration of the water adsorbed with the interfaces between the hydrates on the level of the macroporosity and absorptive within microporosity in capillary porosity. The diffusion of the inter-lamellate water of the pores of hydrates towards capillary porosity is carried out in an irreversible way. The total spherical deformation of creep is thus written as the sum of a reversible part and an irreversible part:

$$\varepsilon^{fs} = \underbrace{\varepsilon_r^{fs}}_{\substack{partie \\ réversible}} + \underbrace{\varepsilon_i^{fs}}_{\substack{partie \\ irréversible}} \quad \text{éq 3.1.1-1}$$

The process of deformation spherical of creep is controlled by the system of coupled equations according to (equations [éq 3.1.1-2] and [éq 3.1.1-3]):

$$\dot{\varepsilon}_r^{fs} = \frac{1}{\eta_r^s} \cdot \left[h \cdot \sigma^s - k_r^s \cdot \varepsilon_r^{fs} \right] - \dot{\varepsilon}_i^{fs} \quad \text{éq 3.1.1-2}$$

where k_r^s indicate rigidity connect associated with the skeleton formed by blocks with hydrates on a mesoscopic scale;

and η_r^s viscosity connects associated with the mechanism with diffusion within capillary porosity.

$$\dot{\varepsilon}_i^{fs} = \frac{1}{\eta_i^s} \left\langle \left[k_r^s \cdot \varepsilon^{fs} - \left(k_r^s + k_i^s \right) \cdot \varepsilon_i^{fs} \right] - \left[h\sigma^s - k_r^s \cdot \varepsilon_r^{fs} \right] \right\rangle^+ \quad \text{éq 3.1.1-3}$$

where k_i^s indicate rigidity connect intrinsically associated with the hydrates on a microscopic scale;

and η_i^s viscosity connects associated with the interfoliaceous mechanism of diffusion.

In [éq 3.1.1-3], hooks $\langle\rangle^+$ appoint the operator of Mac Cauley: $\langle x \rangle^+ = \frac{1}{2}\left(x + |x| \right)$

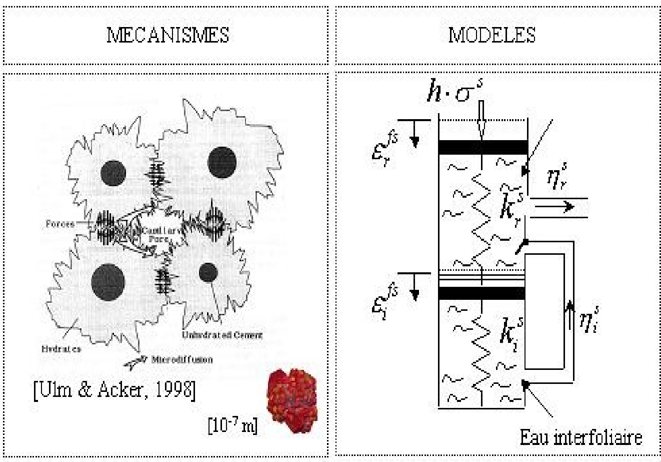

Figure 3.1.1-1 :Phenomenologic model associated with the spherical part of clean creep

3.1.2 Description of the deviatoric part

The deviatoric constraints are at the origin of a mechanism of slip (or mechanism of quasi-dislocation) of the layers of HSC in nano-porosity. Under deviatoric constraint, creep is carried out with constant volume. In addition, the law of creep UMLV supposes the deviatoric isotropy of creep. Phénoménologiquement, the mechanism of slip comprises a viscoelastic reversible contribution of water strongly adsorbed to the layers of HSC and a viscous irreversible contribution of free water:

$$\underbrace{\underline{\varepsilon}^{fd}}_{\substack{\text{déformation}\\ \text{déviatorique}\\ \text{totale}}} = \underbrace{\underline{\varepsilon}_r^{fd}}_{\substack{\text{contribution}\\ \text{eau}\\ \text{absobée}}} + \underbrace{\underline{\varepsilon}_i^{fd}}_{\substack{\text{contribution}\\ \text{eau}\\ \text{libre}}}$$ éq 3.1.2-1

$J^{ème}$ principal component of the total deviatoric deformation is governed by the equations [éq. 3.1.2-2] and [éq 3.1.2-3]:

- $\eta_r^d \dot{\varepsilon}_r^{d,j} + k_r^d \varepsilon_r^{d,j} = h \cdot \sigma^{d,j}$ éq 3.1.2-2
- $\eta_i^d \dot{\varepsilon}_i^{d,j} = h \cdot \sigma^{d,j}$ éq 3.1.2-3

where k_r^d indicate rigidity associated with the capacity with water adsorbed to transmit loads (load bearing toilets), η_r^d viscosity associated with the water adsorbed by the layers with hydrates and η_i^d indicate the viscosity of free water.

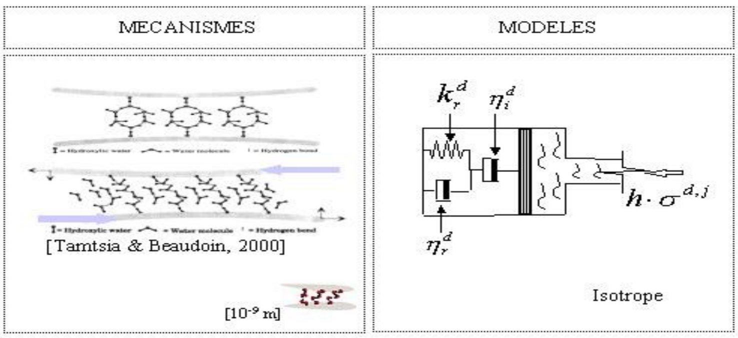

Figure 3.1.2-1: Phenomenologic model associated with the deviatoric part of clean creep.

3.2 Creep of desiccation

One supposes to be able to break up the creep of desiccation $\Delta\varepsilon^{fdess}$ in two parts called intrinsic and structural [bib4]:

$$\Delta\varepsilon^{fdess} = \Delta\varepsilon_{int}^{fdess} + \Delta\varepsilon_{struct}^{fdess}$$ éq 3.2-1

It is agreed that the structural deformation is not a component of deformation in oneself, therefore in this document the only component of the creep of desiccation relates to the intrinsic part:

$$\Delta\varepsilon^{fdess} = \Delta\varepsilon_{int}^{fdess}$$ éq 3.2-2

Bazant and al. [bib10] suggest that the drying and the application of a loading in compression simultaneously are responsible for the microphone-diffusion of the molecules between the macro-pores and the microphonepores. The microphone-diffusion of the water molecules would support the rupture of the connections between the particles of freezing inducing the deformation of creep of desiccation. It is one of the physicochemical phenomena most complicated to model resulting from a coupling between the constraint, clean creep and drying. They propose the following equation (equation of a shock absorber) to take into account the creep of desiccation (intrinsic) hasU level elementary:

$$\dot{\varepsilon}^{fdess}=\frac{|\dot{h}|\sigma}{\eta^{fd}}$$ éq 3.2-3

with:

- ε^{fdess}, deformation of the creep of desiccation,
- η^{fd} a parameter material (in $[Pa\cdot sec]$ in the S.I),
- h, the relative humidity which evolves in time, fact of the case of evolution.

4 Discretization of the equations constitutive of model

4.1 Discretization of the equations constitutive of spherical creep

One carries out a linearization with the first order of the product of the constraints and moisture:

$$\sigma(t)\cdot h(t)\approx\sigma_n\cdot h_n+\frac{t-t_n}{\Delta t_n}\left(\Delta\sigma_n\cdot h_n+\sigma_n\cdot\Delta h_n\right)$$ éq 4.1-1

After discretization of the constraints and relative humidity by functions closely connected, the spherical deformation of clean creep is discretized by the following equation:

$$\Delta\varepsilon_n^{fs}=a_n^s+b_n^s\cdot\sigma_n^s+c_n^s\cdot\sigma_{n+1}^s\Leftrightarrow\Delta\left(tr\ \underline{\underline{\varepsilon}}^f\right)=3a_n^s+b_n^s\cdot tr\ \underline{\underline{\sigma}}_n+c_n^s\cdot tr\ \underline{\underline{\sigma}}_{n+1}$$ éq 4.1-2

where σ_n^s and σ_{n+1}^s are the spherical constraints at the beginning and the step of current time.

Two cases should be distinguished according to whether the unrecoverable deformation must be taken into account or not.

1er case: the deformation of spherical creep irreversible is not taken into account, the equation [éq 4.1 - 2] can be put in the form (simple chain of Kelvin):

$$\eta_r^s\dot{\varepsilon}_r^{fs}(t)+k_r^s\varepsilon_r^{fs}(t)=h(t)\sigma_r^s(t)$$ éq 4.1-3

After discretization, the preceding equation can be put in the form:

$$\Delta\varepsilon_{r,n}^{fs}=a_{r,n}^s+b_{r,n}^s\cdot\sigma_n^s+c_{r,n}^s\cdot\sigma_{n+1}^s$$ éq 4.1-4

With:

$$\begin{cases} a_{r,n}^s=\left[\exp\left(-\dfrac{\Delta t_n}{\tau_r^s}\right)-1\right]\cdot\varepsilon_{r,n}^{fs} \\ b_{r,n}^s=\dfrac{1}{k_r^s}\left\{\left[-\left(\dfrac{2\tau_r^s}{\Delta t_n}+1\right)h_n+\dfrac{\tau_r^s}{\Delta t_n}h_{n+1}\right]\exp\left(-\dfrac{\Delta t_n}{\tau_r^s}\right)+\left[\left(\dfrac{2\tau_r^s}{\Delta t_n}-1\right)h_n-\dfrac{\tau_r^s-\Delta t_n}{\Delta t_n}h_{n+1}\right]\right\} \\ c_{r,n}^s=\dfrac{1}{k_r^s}\left[\dfrac{\tau_r^s}{\Delta t_n}\exp\left(-\dfrac{\Delta t_n}{\tau_r^s}\right)h_n-\dfrac{\tau_r^s-\Delta t_n}{\Delta t_n}h_n\right]\end{cases}$$ éq 4.1-5

The unrecoverable deformation, as for it, does not vary:

$$\Delta\varepsilon_{i,n}^{fs}=0\Rightarrow\begin{cases}a_{i,n}^s=0\\ b_{i,n}^s=0\\ c_{i,n}^s=0\end{cases}$$ éq 4.1-6

2Nd case: the deformation of spherical creep irreversible must be taken into account.
Using the linearization [éq 4.1-1],], the system of coupled equations is written:

$$\begin{cases} \dot{\varepsilon}_r^{fs}(t)+2\dot{\varepsilon}_i^{fs}(t)=\dfrac{1}{\eta_r^s}\left[\sigma_n\cdot h_n+\dfrac{t-t_n}{\Delta t_n}\left(\Delta\sigma_n\cdot h_n+\sigma_n\cdot\Delta h_n\right)-k_r^s\varepsilon_r^{fs}(t)\right] \\ \dot{\varepsilon}_i^{fs}(t)=-\dfrac{1}{\eta_i^s}\left(-2\mathrm{k}_r^s\varepsilon_r^{fs}(t)+k_i^s\varepsilon_i^{fs}(t)+\sigma_n\cdot h_n+\dfrac{t-t_n}{\Delta t_n}\left(\Delta\sigma_n\cdot h_n+\sigma_n\cdot\Delta h_n\right)\right) \end{cases} \quad \text{éq 4.1-7}$$

This system can be put in the form:

$$\underline{\dot{\varepsilon}}^{fs}(t)=\begin{bmatrix}\dot{\varepsilon}_r^{fs}(t)\\ \dot{\varepsilon}_i^{fs}(t)\end{bmatrix}=\underline{\underline{A}}:\underline{\varepsilon}_r^{fs}(t)+\underline{b}+\left(t-t_n\right)\underline{c}\Leftrightarrow\begin{cases}\dot{\varepsilon}_r^{fs}(t)=a_{rr}^s\varepsilon_r^{fs}(t)+a_{ri}^s\varepsilon_i^s(t)+b_r^s+c_r^s\left(t-t_n\right)\\ \dot{\varepsilon}_i^{fs}(t)=a_{ir}^s\varepsilon_r^{fs}(t)+a_{ii}^s\varepsilon_i^s(t)+b_i^s+c_i^s\left(t-t_n\right)\end{cases} \quad \text{éq 4.1-8}$$

$\underline{\underline{A}}$, $\underline{b}$ and $\underline{c}$ are defined as follows:

$$\underline{\underline{A}}=\begin{bmatrix} a_{rr}^s & a_{ri}^s \\ a_{ir}^s & a_{ii}^s \end{bmatrix}=\begin{bmatrix} -\dfrac{k_r^s}{\eta_r^s}-4\dfrac{k_r^s}{\eta_i^s} & 2\dfrac{k_i^s}{\eta_i^s} \\ 2\dfrac{k_r^s}{\eta_i^s} & -\dfrac{k_i^s}{\eta_i^s} \end{bmatrix}$$

$$\underline{b}=\begin{bmatrix} b_r^s \\ b_i^s \end{bmatrix}=\sigma_n\cdot h_n\begin{bmatrix} \dfrac{1}{\eta_r^s}+\dfrac{2}{\eta_i^s} \\ -\dfrac{1}{\eta_i^s} \end{bmatrix} \quad \text{éq 4.1-9}$$

$$\underline{c}=\begin{bmatrix} c_r^s \\ c_i^s \end{bmatrix}=\frac{\Delta\sigma_n\cdot h_n+\sigma_n\cdot\Delta h_n}{\Delta t_n}\begin{bmatrix} \dfrac{1}{\eta_r^s}+\dfrac{2}{\eta_i^s} \\ -\dfrac{1}{\eta_i^s} \end{bmatrix}$$

The preceding system of equations can be uncoupled and solved within the space of clean vectors. The system of equations is written indeed:

$$\dot{\varepsilon}_k^*(t)=\lambda_k\,\varepsilon_k^*(t)+b_k^*+c_k^*\left(t-t_n\right)\ \text{avec}\ \underline{\dot{\varepsilon}}^*=\begin{bmatrix}\dot{\varepsilon}_1^*\\ \dot{\varepsilon}_2^*\end{bmatrix}=\underline{\underline{P}}^{-1}\cdot\underline{\dot{\varepsilon}} \quad \text{éq 4.1-10}$$

Thus, within the space of clean vectors, the model of creep becomes equivalent to a double chain of Kelvin. It is necessary to know the solution of the homogeneous equation (without second member), as well as a particular solution in order to solve the preceding differential equation. The homogeneous solution of each of the two equations is the following one:

$$\varepsilon_k^*(t)=\mu_k\,e^{\lambda_k\cdot t} \quad \text{éq 4.1-11}$$

where μ_k is a parameter depend on the initial condition. A particular solution is obtained by the method of variation of the constant ($\mu_k=\mu_k(t)$). The following solutions then are obtained:

$$\varepsilon_k^*(t)=\mu_k\,e^{\lambda_k\cdot t}-\frac{1}{\lambda_k}\left[b_k^*+c_k^*\left(t-t_n+\frac{1}{\lambda_k}\right)\right] \quad \text{éq 4.1-12}$$

The spherical deformations of reversible and irreversible creep are then equal to:

$$\begin{cases} \varepsilon_r^{fs}(t_{n+1}) = \dfrac{\sigma_n \cdot \Delta h_n + \sigma_{n+1} \cdot h_n}{k_r^s} + \left(x_1 \mu_1 e^{\lambda_1 t_{n+1}} + \mu_2 e^{\lambda_2 t_{n+1}} \right) \\ \varepsilon_i^{fs}(t_{n+1}) = \dfrac{\sigma_n \cdot \Delta h_n + \sigma_{n+1} \cdot h_n}{k_i^s} + \left(\mu_1 e^{\lambda_1 t_{n+1}} + x_2 \mu_2 e^{\lambda_2 t_{n+1}} \right) \end{cases} \quad \text{éq 4.1-13}$$

After simplification, one then obtains the following expressions for the values of μ_k :

$$\begin{cases} \mu_1 = \dfrac{1}{(x_1 \cdot x_2 - 1) e^{\lambda_1 t_n}} \left[x_2 \left(\varepsilon_r^{fs}(t_n) - \dfrac{\sigma_n \cdot \Delta h_n + \sigma_{n+1} \cdot h_n}{k_r^s} \right) - \left(\varepsilon_i^{fs}(t_n) - \dfrac{\sigma_n \cdot \Delta h_n + \sigma_{n+1} \cdot h_n}{k_i^s} \right) \right] \\ \mu_2 = \dfrac{1}{(x_1 \cdot x_2 - 1) e^{\lambda_2 t_n}} \left[- \left(\varepsilon_r^{fs}(t_n) - \dfrac{\sigma_n \cdot \Delta h_n + \sigma_{n+1} \cdot h_n}{k_r^s} \right) + x_1 \left(\varepsilon_i^{fs}(t_n) - \dfrac{\sigma_n \cdot \Delta h_n + \sigma_{n+1} \cdot h_n}{k_i^s} \right) \right] \end{cases} \quad \text{éq 4.1-14}$$

The equation [éq 4.1-2] can thus be put in the form, after discretization:

$$\begin{cases} \Delta \varepsilon_{r,n}^{fs} = a_{r,n}^s + b_{r,n}^s \cdot \sigma_n^s + c_{i,n}^s \cdot \sigma_{n+1}^s \\ \Delta \varepsilon_{i,n}^{fs} = a_{i,n}^s + b_{i,n}^s \cdot \sigma_n^s + c_{i,n}^s \cdot \sigma_{n+1}^s \end{cases} \quad \text{éq 4.1-15}$$

With:

$$\begin{cases} a_{r,n}^s = \left[\dfrac{x_1 \cdot x_2 \cdot e^{\lambda_1 \Delta t_n} - e^{\lambda_2 \Delta t_n}}{x_1 \cdot x_2 - 1} - 1 \right] \cdot \varepsilon_{r,n}^{fs} - x_1 \cdot \left[\dfrac{e^{\lambda_1 \Delta t_n} - e^{\lambda_2 \Delta t_n}}{x_1 \cdot x_2 - 1} \right] \cdot \max_{k \le n} \left(\varepsilon_{i,k}^{fs} \right) \\ b_{r,n}^s = \dfrac{\Delta h_n}{k_r^s} \cdot \left[\dfrac{-x_1 \cdot x_2 \cdot e^{\lambda_1 \Delta t_n} + e^{\lambda_2 \Delta t_n}}{x_1 \cdot x_2 - 1} + 1 \right] + \dfrac{\Delta h_n}{k_i^s} \cdot x_1 \cdot \left[\dfrac{e^{\lambda_1 \Delta t_n} - e^{\lambda_2 \Delta t_n}}{x_1 \cdot x_2 - 1} \right] \\ c_{r,n}^s = \dfrac{h_n}{k_r^s} \cdot \left[\dfrac{-x_1 \cdot x_2 \cdot e^{\lambda_1 \Delta t_n} + e^{\lambda_2 \Delta t_n}}{x_1 \cdot x_2 - 1} + 1 \right] + \dfrac{h_n}{k_i^s} \cdot x_1 \cdot \left[\dfrac{e^{\lambda_1 \Delta t_n} - e^{\lambda_2 \Delta t_n}}{x_1 \cdot x_2 - 1} \right] \end{cases} \quad \text{éq 4.1-16}$$

$$\begin{cases} a_{i,n}^s = x_2 \cdot \left[\dfrac{e^{\lambda_1 \Delta t_n} - e^{\lambda_2 \Delta t_n}}{x_1 \cdot x_2 - 1} \right] \cdot \varepsilon_{r,n}^{fs} + \left[\dfrac{x_1 \cdot x_2 \cdot e^{\lambda_2 \Delta t_n} - e^{\lambda_1 \Delta t_n}}{x_1 \cdot x_2 - 1} - 1 \right] \cdot \max_{k \le n} \left(\varepsilon_{i,k}^{fs} \right) \\ b_{i,n}^s = \dfrac{\Delta h_n}{k_r^s} \cdot x_2 \cdot \left[\dfrac{e^{\lambda_2 \Delta t_n} - e^{\lambda_1 \Delta t_n}}{x_1 \cdot x_2 - 1} \right] + \dfrac{\Delta h_n}{k_i^s} \cdot \left[\dfrac{-x_1 \cdot x_2 \cdot e^{\lambda_2 \Delta t_n} + e^{\lambda_1 \Delta t_n}}{x_1 \cdot x_2 - 1} + 1 \right] \\ c_{i,n}^s = \dfrac{h_n}{k_r^s} \cdot x_2 \cdot \left[\dfrac{e^{\lambda_2 \Delta t_n} - e^{\lambda_1 \Delta t_n}}{x_1 \cdot x_2 - 1} \right] + \dfrac{h_n}{k_i^s} \cdot \left[\dfrac{-x_1 \cdot x_2 \cdot e^{\lambda_2 \Delta t_n} + e^{\lambda_1 \Delta t_n}}{x_1 \cdot x_2 - 1} + 1 \right] \end{cases} \quad \text{éq 4.1-17}$$

In the equations [éq 4.1-16] and [éq 4.1-17] the parameters λ_1 , λ_2, x and x_2 are function of the intrinsic parameters of material. With each step of calculation, it is necessary to save **two internal variables** $\varepsilon_{r,n}^{fs}$, last reversible spherical deformation obtained and $\max_{k \le n} \left(\varepsilon_{i,k}^{fs} \right)$, i.e. $\varepsilon_{i,n}^{fs}$, greatest reversible spherical deformation obtained in the history of the element. The choice to retain the expressions [éq 4.1-5] and [éq 4.1-6] (not of deformation unrecoverable), or the expressions [éq 4.1 - 16] and [éq 4.1-17] (existence of unrecoverable deformations) to determine the increment of total spherical deformation is carried out *a posteriori* according to the sign of $\Delta\varepsilon_{i,n}^{fs}$ in [éq 4.1-15].

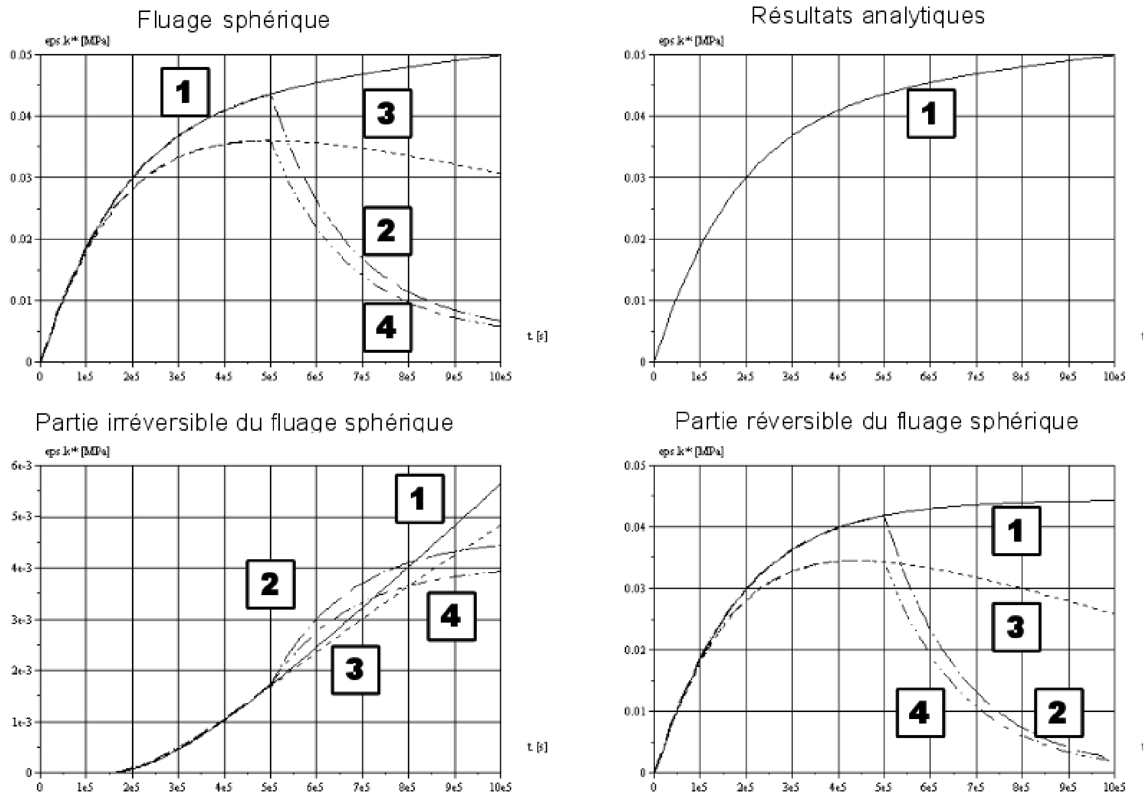

Figure 4.1-1 : Réponses digital obtained by using the discretized expressions [éq 4.1-3] with [éq 4.1-17] for four stories of loading: 1 level of unit constraint to constant moisture (100%), 2 level of unit constraint to linearly decreasing moisture of 100% to 50%,3 level of unit constraint during half of the duration of the calculation followed by a recouvrance to half of the initial constraint on the second part of calculation; moisture is supposed to be constant (100%), 4 the mechanical loading is identical to 3; moisture decrease linearly of 100% to 50%.

To carry out simulations of Figure 4.1-1 the following parameters were retained: $k_r^s=2{,}0\text{e}+5$ [MPa]; $\eta_r^s=4{,}0\text{e}+10$ [MPa.s]; $k_i^s=1{,}0\text{e}+4$ [MPa]; $\eta_i^s=1{,}0\text{e}+11$ [MPa.s]. Calculation comprises 200 intervals of 5000 [S].

4.2 Discretization of the equations constitutive of creep deviatoric

After discretization of the constraints and relative humidity by functions closely connected, the deviative tensor of the deformations of clean creep is discretized by the following equation:

$$\Delta\underline{\varepsilon}_n^{fd}=\underline{a}_n^d+b_n^d\cdot\underline{\sigma}_n^d+c_n^d\cdot\underline{\sigma}_{n+1}^d \qquad \text{éq 4.2-1}$$

where $\underline{\sigma}_n^d$ and $\underline{\sigma}_{n+1}^d$ are the tensors of the deviatoric constraints at the beginning and the step of current time.
The stages carried out are:

- One calculates the parameters compared to the deformation of clean creep deviatoric *reversible*, of which the model is:

$$\eta_r^d \dot{\underline{\varepsilon}}_r^{fd}(t)+k_r^d \underline{\varepsilon}_r^{fd}(t)=h(t)\underline{\sigma}^d(t)$$ éq 4.2-2

After discretization, the preceding equation can be put in the form:

$$\Delta \underline{\varepsilon}_{r,n}^{fd}=\underline{a}_{r,n}^d+b_{r,n}^d\cdot\underline{\sigma}_n^d+c_{r,n}^d\cdot\underline{\sigma}_{n+1}^d$$ éq 4.2-3

With:

$$\begin{cases} \underline{a}_{r,n}^d=\left[\exp\left(-\frac{\Delta t_n}{\tau_r^d}\right)-1\right]\cdot\underline{\varepsilon}_{r,n}^{f,d} \\ b_{r,n}^d=\frac{1}{k_r^d}\left\{\left[-\left(\frac{2\tau_r^d}{\Delta t_n}+1\right)h_n+\frac{\tau_r^d}{\Delta t_n}h_{n+1}\right]\exp\left(-\frac{\Delta t_n}{\tau_r^d}\right)+\left[\left(\frac{2\tau_r^d}{\Delta t_n}-1\right)h_n-\frac{\tau_r^d-\Delta t_n}{\Delta t_n}h_{n+1}\right]\right\} \\ c_{r,n}^d=\frac{1}{k_r^d}\left[\frac{\tau_r^d}{\Delta t_n}\exp\left(-\frac{\Delta t_n}{\tau_r^d}\right)h_n-\frac{\tau_r^d-\Delta t_n}{\Delta t_n}h_n\right] \end{cases}$$ éq 4.2-4

Note:

The equation [éq 4.2-4] (left reversible creep deviatoric) is similar to the equation [éq 4.1-5] (left reversible creep in the absence of unrecoverable deformations). They correspond to the discretization of a single chain of Kelvin.

One calculates the parameters compared to the deformation of clean creep deviatoric, whose model is:

$$\eta_i^d \dot{\underline{\varepsilon}}_i^{f,d}(t)=h(t)\underline{\sigma}^d(t)$$ éq 4.2-5

After discretization, the preceding equation can be put in the form:

$$\Delta \underline{\varepsilon}_{i,n+1}^{f,d}=\underline{a}_{i,n}^d+b_{i,n}^d\cdot\underline{\sigma}_n^d+c_{i,n}^d\cdot\underline{\sigma}_{n+1}^d$$ éq 4.2-6

With:

$$\begin{cases} \underline{a}_{i,n}^d=0 \\ b_{r,n}^d=\frac{\Delta t_n\cdot h_{n+1}}{2\eta_i^d} \\ c_{r,n}^d=\frac{\Delta t_n\cdot h_n}{2\eta_i^d} \end{cases}$$ éq 4.2-7

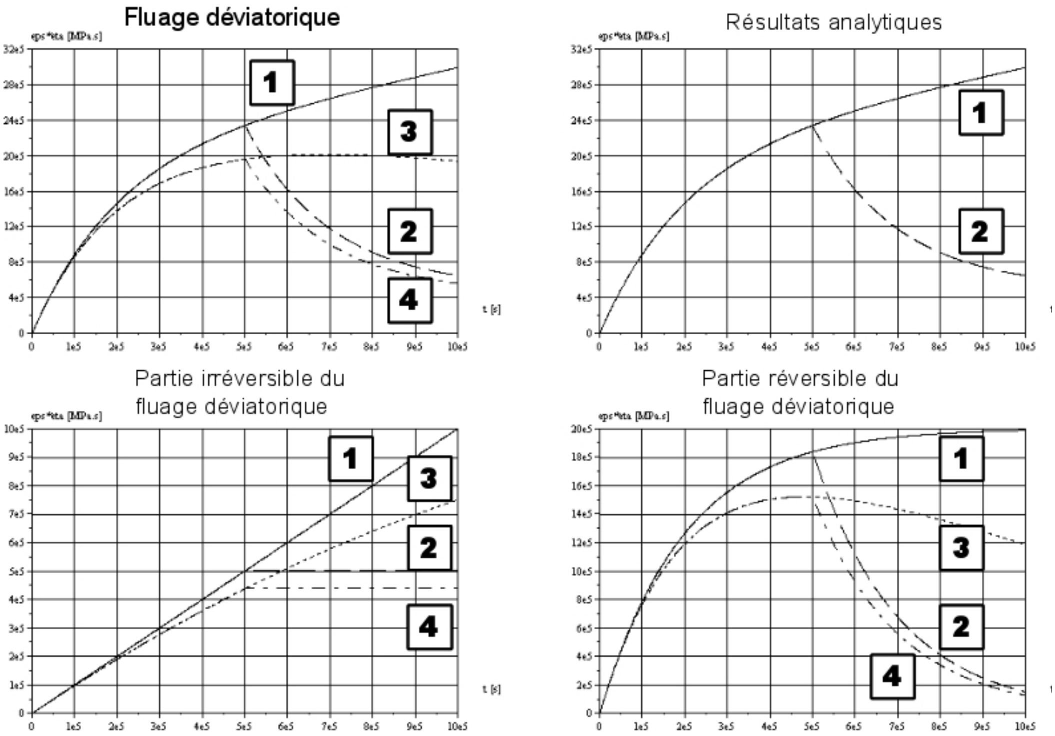

Figure 4.2-1 : Réponses digital obtained by using the discretized expressions [éq 4.2-1] with [éq 4.2-7] for four stories of loading: 1 level of unit constraint to constant moisture (100%), 2 level of unit constraint to linearly decreasing moisture of 100% to 50%,3 level of unit constraint during half of the duration of the calculation followed by a recouvrance to half of the initial constraint on the second part of calculation; moisture is supposed to be constant (100%), 4 the mechanical loading is identical to 3; moisture varies linearly decreasing from 100% to 50%.

To carry out simulations of Figure 4.2-1, the following parameters were retained: $k_r^d = 5{,}0\text{e}+4$ [MPa]; $\eta_r^d = 1{,}0\text{e}+10$ [MPa.s]; $\eta_i^d = 1{,}0\text{e}+11$ [MPa.s]. Calculation comprises 1000 intervals of 1000 [S].

4.3 Discretization of the equations of the creep of desiccation

The terms related to the taking into account of the creep of desiccation break up according to the same concept as the terms of reversible clean creep [bib3]:

$$\Delta \underline{\underline{\varepsilon}}^{fdess} = \underline{\underline{a}}_n^{fdess} + b_n^{fdess} \cdot \underline{\underline{\sigma}}_n + c_n^{fdess} \cdot \underline{\underline{\sigma}}_{n+1} \qquad \text{éq. 4.3-1}$$

The expression of the various terms is the following one:

$$\begin{cases} \underline{\underline{a}}_n^{fdess} & = & 0 \\ b_n^{fdess} & = & \dfrac{\Delta h_n}{2 \cdot \eta^{fd}} \\ c_n^{fdess} & = & \dfrac{\Delta h_n}{2 \cdot \eta^{fd}} \end{cases}$$

5 Tangent matrix

By introducing the elastic modulus of rigidity μ, the diverter of the constraints at the moment $n+1$ is written according to the diverter elastic strain:

$$\underline{\sigma}_{n+1}^{d}=2\mu\,\underline{\varepsilon}_{n+1}^{ed}=\underline{\sigma}_{n}^{d}+2\mu\Delta\,\underline{\varepsilon}_{n}^{d}-2\mu\Delta\,\underline{\varepsilon}_{n}^{f,d} \quad \text{éq 5-1}$$

In substituent the deviatoric part of the clean deformation of creep by the expression [éq 4.2-1], it rises the following relation:

$$\underline{\sigma}_{n+1}^{d}\left(1+2\mu c^{d}\right)=\underline{\sigma}_{n}^{d}\left(1-2\mu b^{d}\right)+2\mu\Delta\,\underline{\varepsilon}_{n}^{d}-2\mu a^{d}\,\underline{1} \quad \text{éq 5-2}$$

Expression which induces by derivation compared to $\underline{\varepsilon}_{n+1}^{d}$:

$$\frac{\partial\underline{\sigma}_{n+1}^{d}}{\partial\underline{\varepsilon}_{n+1}^{d}}\left(1+2\mu c^{d}\right)=2\mu\,\underline{\underline{1}} \quad \text{éq 5-3}$$

By taking a similar step for the spherical part and by introducing the module of rigidity to dilation K, it follows the three following relations:

$$tr\ \underline{\sigma}_{n+1}=3K\ tr\ \underline{\varepsilon}_{n+1}^{e}=tr\ \underline{\sigma}_{n}+3K\ tr\left(\Delta\,\underline{\varepsilon}_{n}\right)-3K\ tr\left(\Delta\,\underline{\varepsilon}_{n}^{f}\right) \quad \text{éq 5-4}$$

$$tr\ \underline{\sigma}_{n+1}\left(1+3\,Kc^{s}\right)=tr\ \underline{\sigma}_{n}\left(1-3\,Kb^{s}\right)+3K\ tr\left(\Delta\,\underline{\varepsilon}_{n}\right)-Ka^{s} \quad \text{éq 5-5}$$

$$\frac{\partial\left(tr\ \underline{\sigma}_{n+1}\right)}{\partial\left(tr\ \underline{\varepsilon}_{n+1}\right)}\left(1+3\,Kc^{s}\right)=3K \quad \text{éq 5-6}$$

The tangent matrix is written finally:

$$\frac{\partial\underline{\sigma}}{\partial\underline{\varepsilon}}=\frac{\partial\underline{\sigma}^{d}}{\partial\underline{\varepsilon}}+\frac{1}{3}\frac{\partial\left(tr\ \underline{\sigma}\right)}{\partial\underline{\varepsilon}}\underline{1}=\frac{\partial\underline{\sigma}^{d}}{\partial\underline{\varepsilon}^{d}}\frac{\partial\underline{\varepsilon}^{d}}{\partial\underline{\varepsilon}}+\frac{1}{3}\frac{\partial\left(tr\ \underline{\sigma}\right)}{\partial\left(tr\ \underline{\varepsilon}\right)}\frac{\partial\left(tr\ \underline{\varepsilon}\right)}{\partial\underline{\varepsilon}}\underline{1} \quad \text{éq 5-7}$$

I.e.:

$$\frac{\partial\underline{\sigma}}{\partial\underline{\varepsilon}}=\underbrace{\frac{2\mu}{1+2\mu c^{d}}}_{\chi}\left(\underline{\underline{1}}-\frac{1}{3}\underline{1}\otimes\underline{1}\right)+\underbrace{\frac{K}{1+3\,Kc^{s}}}_{\xi}\underline{1}\otimes\underline{1} \quad \text{éq 5-8}$$

After linearization, the tangent matrix develops as follows:

$$\begin{pmatrix}\Delta\sigma_{11}\\ \Delta\sigma_{22}\\ \Delta\sigma_{33}\\ \sqrt{2}\,\Delta\sigma_{12}\\ \sqrt{2}\,\Delta\sigma_{13}\\ \sqrt{2}\,\Delta\sigma_{23}\end{pmatrix}=\begin{bmatrix}\xi+\frac{2}{3}\chi & \xi-\frac{1}{3}\chi & \xi-\frac{1}{3}\chi & 0 & 0 & 0\\ \xi-\frac{1}{3}\chi & \xi+\frac{2}{3}\chi & \xi-\frac{1}{3}\chi & 0 & 0 & 0\\ \xi-\frac{1}{3}\chi & \xi-\frac{1}{3}\chi & \xi+\frac{2}{3}\chi & 0 & 0 & 0\\ 0 & 0 & 0 & \chi & 0 & 0\\ 0 & 0 & 0 & 0 & \chi & 0\\ 0 & 0 & 0 & 0 & 0 & \chi\end{bmatrix}\cdot\begin{pmatrix}\Delta\varepsilon_{11}\\ \Delta\varepsilon_{22}\\ \Delta\varepsilon_{33}\\ \sqrt{2}\,\Delta\varepsilon_{12}\\ \sqrt{2}\,\Delta\varepsilon_{13}\\ \sqrt{2}\,\Delta\varepsilon_{23}\end{pmatrix} \quad \text{éq 5-9}$$

6 Description of the internal variables

The following table gives the correspondence between the number of the internal variables accessible by *Code_Aster* and their description:

Number of the variable	Description
1	Reversible spherical deformation
2	Irreversible spherical deformation
3	Reversible deviatoric deformation, component 11
4	Irreversible deviatoric deformation, component 11
5	Reversible deviatoric deformation, component 22
6	Irreversible deviatoric deformation, component 22
7	Reversible deviatoric deformation, component 33
8	Irreversible deviatoric deformation, component 33
9	Withdrawal of desiccation, component 11
10	Withdrawal of desiccation, component 22
11	Withdrawal of desiccation, component 33
12	Reversible deviatoric deformation, component 12
13	Irreversible deviatoric deformation, component 12
14	Reversible deviatoric deformation, component 13
15	Irreversible deviatoric deformation, component 13
16	Reversible deviatoric deformation, component 23
17	Irreversible deviatoric deformation, component 23
18	Withdrawal of desiccation, component 12
19	Withdrawal of desiccation, component 13
20	Withdrawal of desiccation, component 23

7 Coupling enters BETON_UMLV and MAZARS

The model BETON_UMLV is a model of viscoelastic behavior linear. To be able to represent the rupture of the concrete by tertiary creep, one proposes in Code_Aster to couple the model of creep with a model of damage to knowing the model of MAZARS (cf [R7.01.08]). The coupling is carried out by supposing on the one hand, that the deformations of creep are generated by the effective constraints (either, those really seen by material) and on the other hand, that only part of the deformation of creep (presumedly constant) contributes to the evolution of the damage. The diagram is carried out by maintaining the direct link between state of elastic strain and state of stresses.

7.1 Implementation

7.1.1 Update of the internal variables of creep

As for the rest of the page, one limits oneself here to the description of the coupling between the damage and clean creep. One thus supposes like previously that the deformation is written according to the equation [éq 2-2]:

$$\varepsilon=\varepsilon^e+\varepsilon^f$$

where ε^e , elastic strain, contains also the contribution of the damage (micro - cracking) generated by the model of MAZARS. By considering the contributions spherical [éq 4.1-2] and deviatoric [éq 4.2-1] of clean creep ansi that of the creep of desiccation, one obtains the increment of the deformations of creep $\Delta \underline{\underline{\varepsilon}}^f$ between the moments *N* and *n+1:*

$$\Delta \underline{\underline{\varepsilon}}_n^f=\underline{\underline{a}}_n+b_n\underline{\underline{\sigma}}_n+c_n\underline{\underline{\sigma}}_{n+1} \qquad \text{éq 6.1-1}$$

with $\underline{\underline{a}}_n, b_n c_n$ coefficients of total clean creep in linear viscoelasticity.

To introduce the damage into the model, it is supposed that the deformations of creep are generated by the effective constraints, noted $\underline{\underline{\sigma}}'$ in the continuation of the document. That makes it possible to associate the deformations differed with the just part of material, as proposed in [bib1].

It is pointed out that the effective constraints can be written according to the variable of damage D or (see also documentation [R7.01.08-B]) like a law of elasticity :

$$\underline{\underline{\sigma}}'=\frac{\underline{\underline{\sigma}}}{1-D}=\underline{\underline{\underline{\underline{E}}}}\,\underline{\underline{\varepsilon}}^e \qquad \text{éq 6.1-2}$$

Consequently, the equation [éq 6.1-1] becomes:

$$\Delta \underline{\underline{\varepsilon}}_n^f=\underline{\underline{a}}_n+b_n\underline{\underline{\sigma}}'_n+c_n\underline{\underline{\sigma}}'_{n+1} \qquad \text{éq 6.1-3}$$

By using the law of elasticity [éq 6.1-2] with the equation [éq 6.1-3], one obtains the new relation for the increment of deformation of creep:

$$\Delta \underline{\underline{\varepsilon}}_n^f=\left(1+c\,\underline{\underline{\underline{\underline{E}}}}\right)^{-1}\left[\underline{\underline{a}}_n+b_n\underline{\underline{\sigma}}'_n+c_n\underline{\underline{\underline{\underline{E}}}}\left(\underline{\underline{\varepsilon}}_{n+1}-\underline{\underline{\varepsilon}}_n^f\right)\right] \qquad \text{éq 6.1-4}$$

Thus, starting from all the quantities known at the moment n , it is possible to calculate with the relation [éq 6.1-4] the deformation of creep at the moment $n+1$. Then, one easily obtains the effective constraints at the moment $n+1$ thanks to the relation [éq 6.1-2], and the internal variables of creep with the equations [éq 4.1-2] and [éq 4.2-1].

7.1.2 Evolution of the damage

One does not detail the model here of MAZARS ; the reader will be able to refer to the reference material of *Code_Aster* model of MAZARS [R7.01.08].
The basic assumption is that the damage is controlled by the elastic strain and a quota of the deformation of creep. The tensor of deformation which controls the evolution of D is given by the following relation:

$$\breve{\underline{\underline{\varepsilon}}}=\underline{\underline{\varepsilon}}^e+\chi\,\underline{\underline{\varepsilon}}^f$$ éq 6.2-1

with $0\leq\chi\leq1$ *, coefficient of coupling.*

The level of coupling increases with χ growing. There thus exist two borderline cases: $\chi=0$ and $\chi=1$.

- If, $\chi=0$ there is absence of coupling; the evolution of the damage depends only on the elastic strain and thus one cannot have tertiary creep.
- If $\chi=1$ the coupling is maximum, the damage depends on the total deflection. This case generally leads to the premature ruin of the structure.

Let us notice that the equation [éq 6.2-1] is equivalent to realise the total and elastic deflections, by using the coefficient χ like weight:

$$\breve{\underline{\underline{\varepsilon}}}=\chi\,\underline{\underline{\varepsilon}}+(1-\chi)\,\underline{\underline{\varepsilon}}^e$$ éq 6.2-2a

or with to withdraw $1-\chi$ time deformations of creep of the total deflections:

$$\breve{\underline{\underline{\varepsilon}}}=\underline{\underline{\varepsilon}}-(1-\chi)\,\underline{\underline{\varepsilon}}^f$$ éq 6.2-2b

Equivalent deformation ε_{eq} who controls the damage in the coupled law is not evaluated more starting from the elastic strain but starting from the deformations of coupling $\breve{\underline{\underline{\varepsilon}}}$:

$$\varepsilon_{eq}=\sqrt{\langle\breve{\underline{\underline{\varepsilon}}}\rangle_+:\langle\breve{\underline{\underline{\varepsilon}}}\rangle_+}$$ éq 6.2-3

$\langle\,\rangle_+$ corresponding to the positive part of the tensor.

The damage in traction D_t and in compression D_c are calculated as in the case not coupled but with the equivalent deformation defined in the equation [éq 6.2-3]:

$$D_c=1-\frac{\varepsilon_{d0}(1-A_c)}{\varepsilon_{eq}}-\frac{A_c}{\exp(B_c(\varepsilon_{eq}-\varepsilon_{d0}))}$$ éq 6.2-4a

$$D_t=1-\frac{\varepsilon_{d0}(1-A_t)}{\varepsilon_{eq}}-\frac{A_t}{\exp(B_t(\varepsilon_{eq}-\varepsilon_{d0}))}$$ éq 6.2-4b

($\varepsilon_{d0}, A_c, B_c, A_t, B_t$ *) being the parameters materials of the law of Mazars.*

As in the case not coupled, a weighted average of these two damages by the coefficient α_t finally allows to calculate the damage D_{n+1} :

$$D_{test}=\alpha_t{}^{\beta}D_t+(1-\alpha_t)^{\beta}D_c$$ éq 6.2-5a

$$D_{n+1}=\max(D_{n+1},D_{test})$$ éq 6.2-5b

Let us note that the choice which was fact is to preserve the determination of the coefficient α_t starting from the elastic strain (and not deformations of coupling):

$$\alpha_t = \frac{\sum_{i=1}^{3}\left[\left\langle \varepsilon_i^e \right\rangle_+ \varepsilon_{ti}\right]}{\left(\varepsilon_{eq}^e\right)^2}$$ éq 6.2-6

where ε_i^e are the clean deformations of the elastic tensor; components ε_{ti} tensor $\underline{\underline{\varepsilon}}_t$ are calculated starting from the eigenvalues of the elastic constraints $\sigma_i^{'}$ with the following elastic relation:

$$\varepsilon_{ti} = \frac{1+\nu}{E}\left\langle \sigma_i^{'} \right\rangle_+ - \frac{\nu}{E}\left(\left\langle \sigma_i^{'} \right\rangle_+\right)$$ éq 6.2-7

and ε_{eq}^e is the elastic deformation equivalent, calculated starting from the tensor of elastic strain $\underline{\underline{\varepsilon}}^e$:

$$\varepsilon_{eq}^e = \sqrt{\left\langle \underline{\underline{\varepsilon}}^e \right\rangle_+ : \left\langle \underline{\underline{\varepsilon}}^e \right\rangle_+}$$ éq 6.2-8

Note:

- By adopting the elastic expression [éq 6.2-8], one finds the conditions of the law of MAZARS : $\alpha_t = 1$ for pure traction and $\alpha_t = 0$ for pure compression.
- There exists a difference in the draftlies of the dependence of the Young modulus at the temperature between the law of MAZARS and the law BETON_UMLV (keyword ELAS_FO of DEFI_MATERIAU). In the first case, one uses the value of the module corresponding to the maximum temperature reached in the history of the loading; in the second case, the law user is followed. In the case of the coupling, it is the rule of the law MAZARS who is followed.

7.2 Answer in traction

One proposes here an example of answer obtained with the coupled model. It is about a bar with variable section, blocked at an end and with a load of traction applied at the loose lead.

On figure 6.4-2, one represented the map of the damage at the end of calculation for $\chi = 1$; on figure 6.4-1, one shows the answer force-displacement obtained for two values of χ (1 and 0.9). One recognizes the curve of tertiary creep well, with the final rupture of the sample.

It is shown here that, in traction, the coefficient of coupling affects especially the time of the rupture.

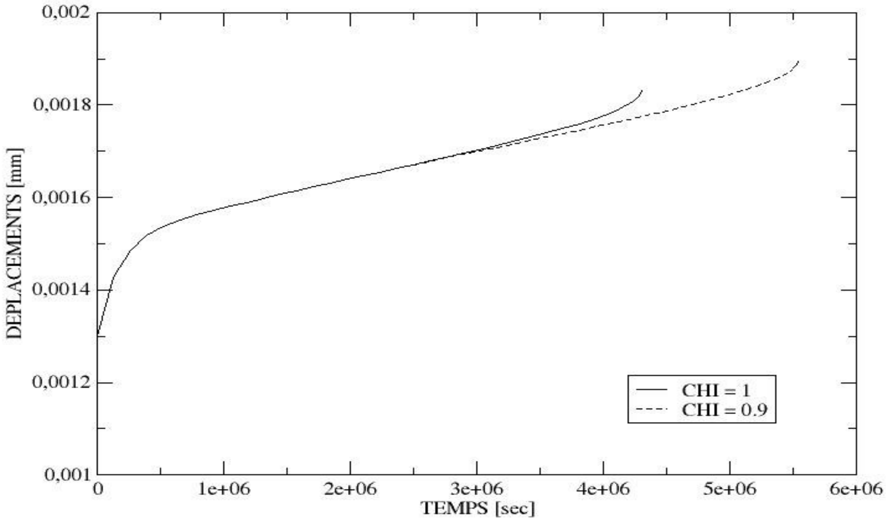

Figure 6.4-1: Answer in traction on a bar for two values of the coefficient of coupling.

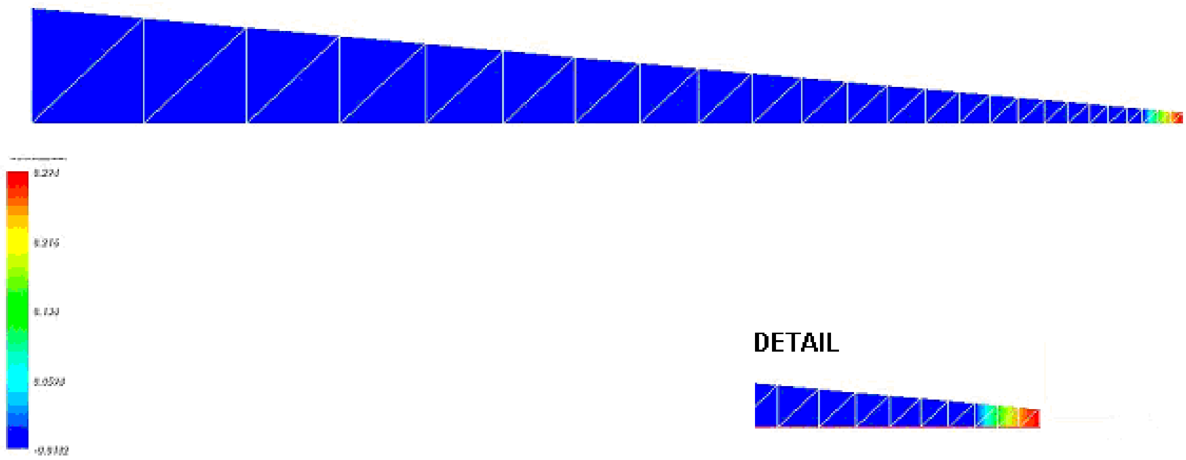

Figure 6.4-2: Damage on a bar where one applied a force of traction (χ =1).

8 Notations

$\underline{\underline{\varepsilon}}$ tensor of the total deflections

$\underline{\underline{\varepsilon}}^{f}$ tensor of the deformations of clean creep

$\underline{\underline{\varepsilon}}^{e}$ tensor of the elastic strain

$\varepsilon^{fs}\underline{\underline{1}}$ spherical part of the tensor of the deformations of clean creep

$\varepsilon_r^{fs}\underline{\underline{1}}$ reversible spherical part of the tensor of the deformations of clean creep

$\varepsilon_i^{fs}\underline{\underline{1}}$ irreversible spherical part of the tensor of the deformations of clean creep

$\underline{\underline{\varepsilon}}^{fd}$ deviatoric part of the tensor of the deformations of clean creep

$\underline{\underline{\varepsilon}}_r^{fd}$ reversible deviatoric part of the tensor of the deformations of clean creep (contribution of absorptive water)

$\underline{\underline{\varepsilon}}_i^{fd}$ irreversible deviatoric part of the tensor of the deformations of clean creep (contribution of free water)

$\underline{\underline{\sigma}}$ tensor of the total constraints

$\sigma^s \underline{\underline{1}}$ spherical part of the tensor of the constraints

$\underline{\underline{\sigma}}^d$ deviatoric part of the tensor of the constraints

h internal relative moisture

K elastic module of rigidity to dilation

k_r^s rigidity connects associated with the skeleton formed by blocks with hydrates on a mesoscopic scale

k_i^s rigidity connects intrinsically associated with the hydrates on a microscopic scale

k_r^d rigidity associated with the capacity with water adsorbed to transmit loads (*load bearing toilets*)

μ elastic modulus of rigidity

η_i^s viscosity connects associated with the inter-lamellate mechanism of diffusion

η_r^s viscosity connects associated with the mechanism with diffusion within capillary porosity

η_i^d viscosity of free water.

η_r^d viscosity associated with the water adsorbed by the layers with hydrates

η^{fd} viscosity of the creep of desiccation

χ coefficient of coupling creep-damage

$x, \underline{x}, \underline{\underline{x}}$ indicate respectively a scalar, a vector and a tensor of order 2.

$x_n, x_{n+1}, \Delta x_n$ indicate the value of the quantity respectively X at time t_n, at time t_{n+1} and variation of x during the interval $[t_n; t_{n+1}]$.

9 Bibliography

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10 Features and checking

This document relates to the law of behavior BETON_UMLV (keyword BEHAVIOR of STAT_NON_LINE) and its associated material BETON_UMLV (order DEFI_MATERIAU).

This law of behavior is checked by the cases following tests:

SSNV163	Clean calculation of creep	[V6.04.163]
SSNV174	Taking into account of the withdrawal in the model BETON_UMLV	Not documented
SSNV180	Taking into account of thermal dilation and the creep of desiccation in the model BETON_UMLV	[V6.04.180]
SSNV181	Checking of the good taking into account of shearing in the model BETON_UMLV	[V6.04.181]

And by the cases following tests in the case of coupling:

Coupled law			
ENDO_ISOT_BETON	SSLA103f	Calculation of the withdrawal of desiccation and the endogenous withdrawal on a cylinder	[V3.06.103]
ENDO_ISOT_BETON MAZARS	SSNV169	Coupling creep – damage	[V6.04.169]

11 Description of the versions of the document

Version Aster	Author (S) or contributor (S), organization	Description of the modifications
7.1	Y. the Pope EDF/R & D /MMC	Initial text
9.4	S. Michel-Ponnelle EDF/R & D /AMA Mr. Bottoni Univ. from Grenoble	Addition of coupling UMLV – MAZARS
10 4	A. Foucault EDF/R & D /AMA	Modifications equations §4 - impact card-indexes anomaly 12519
12.4	Mr. Bottoni EDF/R & D /AMA	Integration of the creep of desiccation.