Law CJS in géomechanics

Summary:

The law here is presented CJS who applies to the soil mechanics. One specifies:

- the description of the model,
- the integration of the law in *Code_Aster*,
- the description of the introduced routines.

Contents

1 Notations

The notations used here are the usual notations of the soil mechanics, to which the notations suitable are added for the writing of the parameters of law CJS.

One also gives the correspondence, if it takes place, between the parameters of the law and their notations in Aster.

A	parameter of the model	A_CJS
b	parameter of the model	B_CJS
c	parameter of the model	C_CJS
n	parameter of the model	N_CJS
K	modulus of voluminal deformation elastic	
K_o^e	parameter of the model	
K_o^p	parameter of the model	KP
G	elastic modulus of rigidity	
G_o^e	parameter of the model	
$\mathbf{G}^d$	function controlling the evolution of the plastic deformations déviatoires	
$\mathbf{s}$	diverter of the tensor of the constraints	
I_1	first invariant of the constraints	
p_{co}	pressure of initial criticism	PCO
P_a	pressure of reference of the model	Pa
$f^i,\ f^d$	thresholds of the plastic mechanisms isotropic and déviatoire	
Q_{iso}	variable interns model corresponding to the acceptable limit of the déviatoire plan	
$\mathbf{q}, \mathbf{Q}$	tensors of the model	
$R, \mathbf{X}$	internal variables of the model corresponding to the average radius and the center of the surface of load in the déviatoire plan	
R_m	parameter of the model	RM
R_c	parameter of the model	RC
λ^i, λ^d	plastic multipliers of the mechanisms isotropic and déviatoire	
$\varepsilon, \varepsilon^e, \varepsilon^{ip}, \varepsilon^{dp}$	tensors of the deformations respectively total, elastic, plastic isotropic and plastic déviatoires	
ε_v	voluminal deformations	
β	parameter of the model	BETA_CJS
γ	parameter of the model	GAMMA_CJS
θ	angle of Lode	
φ	function limiting the evolution of $\mathbf{X}$	
μ	parameter of the model	MU_CJS
Q_{init}	parameter of the model	Q_INIT

Notice :

Foreword: Contrary for the use of géomechanics, the convention of sign retained is that of the mechanics of the continuous mediums, i.e tractions are counted positively.

2 Introduction

The model `CJS` is an elastoplastic law of behavior adapted to the modeling of granular materials. It was developed at the Central School of Lyon ([bib1], [bib2], [bib3]).

The version `CJS` established in *Code_Aster* is a model arranged hierarchically including several levels of complexity. In its most complete expression, the model has two surfaces of load: one is activated by the isotropic requests, the other by the requests déviatoires. The first undergoes an isotropic work hardening and the second a mixed work hardening (isotropic and kinematic). The elastic law is of hypoelastic type nonlinear.

3 Description of the law `CJS`

3.1 Partition of the deformations

The increment of total deformation breaks up into three parts, relative to each concerned mechanism:

$$\dot{\varepsilon}_{ij}=\dot{\varepsilon}^{e}_{ij}+\dot{\varepsilon}^{ip}_{ij}+\dot{\varepsilon}^{dp}_{ij}$$

where $\dot{\varepsilon}^{e}_{ij}$, $\dot{\varepsilon}^{ip}_{ij}$ and $\dot{\varepsilon}^{dp}_{ij}$ are respectively the increments of elastic strain, isotropic plastic deformation and plastic deformation déviatoire.

3.2 Elastic mechanism

The elastic part of the law is of hypoelastic type, whose general expression is:

$$\dot{\varepsilon}^{e}_{ij}=\frac{\dot{s}_{ij}}{2\,G}+\frac{\dot{I}_1}{9\,K}\delta_{ij}$$

where I_1 is the first invariant of the constraints: $I_1=tr(\sigma)$, $\mathbf{s}$ tensor of the constraints is the déviatoire part, and where K and G are respectively the voluminal modulus of deformation and the modulus of rigidity rubber bands. Those depend on the state of stresses according to:

$$K=K^{e}_{o}\left(\frac{I_1+Q_{init}}{3\,P_a}\right)^{n},\quad G=G^{e}_{o}\left(\frac{I_1+Q_{init}}{3\,P_a}\right)^{n}$$

K^{e}_{o}, G_o, P_a and n are parameters of the model. P_a is a pressure of reference equal to -100 kPa.

3.3 Isotropic plastic mechanism

The surface of corresponding load f^{i} is, within the space of principal constraints, a plan perpendicular to the hydrostatic axis, that is to say:

$$f^{i}(\sigma,\,Q_{iso})=-\frac{(I_1+Q_{init})}{3}+Q_{iso}$$

where Q_{iso} is the thermodynamic force which depends on the internal variable q according to:

$$\dot{Q}_{iso}=K^{p}\,\dot{q}=K^{p}_{o}\left(\frac{Q_{iso}}{P_a}\right)^{n}\dot{q}$$

K^{p}_{o}, P_a and n are the parameters of the plastic mechanism déviatoire (P_a and n are identical to those of the elastic mechanism). The rule of normality makes it possible to express the evolution of the plastic deformation and the variable of work hardening according to the evolution of the plastic multiplier λ^i :

$$\dot{\varepsilon}_{ij}^{ip}=\dot{\lambda}^i \frac{\partial f^i}{\partial \sigma_{ij}}=-\frac{1}{3}\dot{\lambda}^i \delta_{ij} \text{ and } \dot{q}=-\dot{\lambda}^i \frac{\partial f^i}{\partial Q_{iso}}=-\dot{\lambda}^i$$

Taking into account the second equation, the law of work hardening can be also put in the form:

$$\dot{Q}_{iso}=-\dot{\lambda}^i K_o^p\left(\frac{Q_{iso}}{P_a}\right)^n$$

3.4 Plastic mechanism déviatoire

The surface of load of this second plastic mechanism is a convex surface with ternary symmetry defined by the equation:

$$f^d(\sigma,\ R, \boldsymbol{X})=q_{II}\ h(\theta_q)+R(\ I_1+Q_{init})$$

with $q_{ij}=s_{ij}-I_1\ X_{ij}$

$$q_{II}=\sqrt{q_{ij}\ q_{ij}}$$

$$h(\theta_q)=\left(1+\gamma\cos(3\theta_q)\right)^{1/6}=\left(1+\gamma\quad\sqrt{54}\ \frac{\det(\boldsymbol{q})}{q_{II}^3}\right)^{1/6}.$$

The scalar R and the tensor $\boldsymbol{X}$ the average radius and the center of the surface of load in the déviatoire plan represent respectively.

$\boldsymbol{s}$, $\boldsymbol{q}$ and $\boldsymbol{X}$ are tensors déviatoires. γ is a parameter which translates the dissymmetrical behavior of the grounds into compression and extension. θ is the angle of Lode.

This surface of load evolves according to two types of work hardening: isotropic work hardening and kinematic work hardening.

Notice :

The expression of the angle of Lode is found in the following way:
In a reference mark $(H, \boldsymbol{i}, \boldsymbol{j})$ *déviatoire plan the vector* $\mathbf{HM}$ *can be given starting from the distance* $HM = \rho$ *and of the angle of Lode* θ_s *(cf [Figure 3.4-a]). Coordinates of* $\mathbf{HM}$ *are:*
$\mathbf{HM} = (\rho \sin\theta_s, \rho\cos\theta_s)$

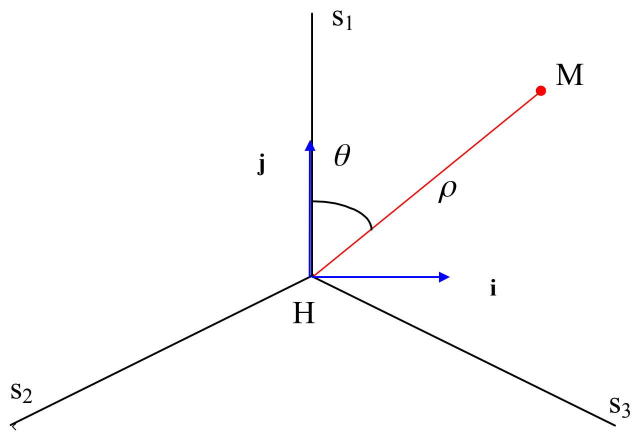


Figure 3.4-a: Angle of Lode in the déviatoire plan

The principal components of the diverter are thus:

$$s_1 = \rho\cos\theta_s \ , \ s_2 = \rho\cos\left(\frac{4\pi}{3} - \theta_s\right) \text{ and } s_3 = \rho\cos\left(\frac{2\pi}{3} - \theta_s\right)$$

Consequently, one a: $s_{II} = \sqrt{\frac{3}{2}}\rho$ *and*

$$\det(\boldsymbol{s}) = \frac{1}{4}\rho^3\cos\theta_s\left(\cos^2\theta_s - 3\sin^2\theta_s\right) = \frac{1}{4}\rho^3\cos(3\theta_s)$$

one from of deduced the relation then:

$$\cos(3\theta_s) = 2^{1/2}\,3^{3/2}\,\frac{\det(s)}{s_{II}^3}$$

The angle θ_q *is calculated in the same way.*

3.4.1 Écrouissage isotropic

The isotropic law of work hardening is written as follows:

$$\dot{R} = \frac{A\,R_m^2\,\dot{r}}{(R_m + A\,r)^2}$$

The thermodynamic force R is function of r whose evolution is given by:

$$\dot{r} = -\dot{\lambda}^d\,\frac{\partial f^d}{\partial R}\left(\frac{I_1 + Q_{init}}{3\,P_a}\right)^{-1.5} = -\dot{\lambda}^d\,(I_1 + Q_{init})\left(\frac{I_1 + Q_{init}}{3\,P_a}\right)^{-1.5}$$

By direct integration of the law of work hardening, it comes:

$$R=\frac{A\ R_m\ r}{R_m+A\ r}\text{ , that is to say too }r=\frac{R\ R_m}{A\left(R_m-R\right)}$$

The law of work hardening can thus be also expressed by:

$$\dot{R}=-\dot{\lambda}^d\ A\left(1-\frac{R}{R_m}\right)^2\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{-1.5}=\dot{\lambda}^d G^R\left(\sigma\ ,\ R\right)$$

with $G^R\left(\sigma\ ,\ R\right)=-A\left(1-\frac{R}{R_m}\right)^2\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{-1.5}$

and where R_m (which is the average radius of the elastic range in rupture) and A are parameters of the model.

3.4.2 Écrouissage kinematic

The kinematic law of work hardening is given by:

$$\dot{X}_{ij}=\frac{1}{b}\ \dot{\alpha}_{ij}$$

The thermodynamic force $\boldsymbol{X}$ is function of the variable α whose nonlinear evolution is given by:

$$\dot{\alpha}_{ij}=-\dot{\lambda}^d\left[dev\left(\frac{\partial f^d}{\partial X_{ij}}\right)-\left(I_1+Q_{init}\right)\varphi\ X_{ij}\right]\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{-1.5}$$

The term $-\left(I_1+Q_{init}\right)\varphi\,X$ allows to obtain nonlinear kinematic work hardening, representing the limitation of the evolution of the surface of load.

By taking account of $\frac{\partial f^d}{\partial X_{ij}}=\frac{\partial f^d}{\partial q_{kl}}\ \frac{\partial q_{kl}}{\partial X_{ij}}=-\left(I+Q_{init_1}\right)\frac{\partial f^d}{\partial q_{ij}}$, and while posing: $Q_{ij}=dev\left(\frac{\partial f^d}{\partial q_{ij}}\right)$, it comes finally for the law from work hardening:

$$\dot{X}_{ij}=\dot{\lambda}^d\ \frac{1}{b}\left(Q_{ij}+\varphi\ X_{ij}\right)\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{-1.5}=\dot{\lambda}^d\ G^X_{ij}\left(\sigma\ ,\boldsymbol{X}\right)$$

with $G^X_{ij}\left(\sigma\ ,\boldsymbol{X}\right)=\frac{1}{b}\left(Q_{ij}+\varphi\ X_{ij}\right)\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{-1.5}$.

where φ a function which limits the evolution of $\boldsymbol{X}$ and is a parameter of the model.

The tensor $\boldsymbol{Q}$ is calculated according to the formula:

$$Q_{ij}=\frac{1}{h\left(\theta\right)^5}\left[\left(1+\frac{\gamma}{2}\ \cos\left(3\ \theta\right)\right)\frac{q_{ij}}{q_{II}}+\frac{\gamma\sqrt{54}}{6q^2_{II}}dev\left(\frac{\partial\det\left(\boldsymbol{q}\right)}{\partial q_{ij}}\right)\right]$$

The preceding expression is obtained in the following way. One a:

$$\frac{\partial f^d}{\partial q_{ij}}=h(\theta_q)\,\frac{\partial q_{II}}{\partial q_{ij}}+q_{II}\,\frac{\partial h(\theta_q)}{\partial q_{ij}}$$

where $\frac{\partial q_{II}}{\partial q_{ij}}$ and $\frac{\partial h(\theta_q)}{\partial q_{ij}}$ are respectively given by:

$$\frac{\partial q_{II}}{\partial q_{ij}}=\frac{q_{ij}}{q_{II}}$$

$$\frac{\partial h(\theta_q)}{\partial q_{ij}}=\frac{1}{6\,h(\theta_q)^5}\,\frac{\partial}{\partial q_{ij}}\left(1+\gamma\,\sqrt{54}\,\frac{\det(\boldsymbol{q})}{q_{II}^3}\right)=\frac{-\gamma\cos(3\,\theta_q)}{2\mathrm{h}(\theta_q)^5}\,\frac{q_{ij}}{q_{II}^2}+\frac{\gamma\sqrt{54}}{6\mathrm{h}(\theta_q)^5\,q_{II}^3}\,\frac{\partial\det(\boldsymbol{q})}{\partial q_{ij}}$$

from where

$$\frac{\partial f^d}{\partial q_{ij}}=\frac{1}{h(\theta_q)^5}\left[\left(1+\frac{\gamma}{2}\,\cos(3\theta_q)\right)\frac{q_{ij}}{q_{II}}+\frac{\gamma\sqrt{54}}{6\mathrm{q}_{II}^2}\left(\frac{\partial\det(\boldsymbol{q})}{\partial q_{ij}}\right)\right]$$

The function φ , as for it is given by:

$$\varphi=\varphi_o\,h(\theta_s)\,Q_{II}$$

where $Q_{II}=\sqrt{Q_{ij}\,Q_{ij}}$ and $h(\theta_s)=\left(1+\gamma\,\cos(3\theta_s)\right)^{1/6}=\left(1+\gamma\,\sqrt{54}\,\frac{\det(\boldsymbol{s})}{s_{II}^3}\right)^{1/6}$. The term φ_o express yourself according to characteristic with the rupture of material.

3.4.3 Law of evolution of the plastic mechanism déviatoire

In granular materials, a variation of volume can occur for a loading déviatoire purely. This variation of volume is related on the discontinuous aspect of material and the conditions kinematics which result during the loading. This particular phenomenon does not make it possible to define the plastic deformations déviatoires starting from the only rule of normality. This is why the plastic mechanism déviatoire is nonassociated. There thus exists a potential function controlling the evolution of the deformations:

$$\dot{\varepsilon}_{ij}^{dp}=\dot{\lambda}^d\,G_{ij}^d$$

The potential function is defined starting from the following kinematic condition:

$$\dot{\varepsilon}_v^{dp}=-\beta\,\left(\frac{s_{II}}{s_{II}^c}-1\right)\frac{|s_{ij}\,\dot{e}_{ij}^{dp}|}{s_{II}}$$

where β is a parameter of the model and s_{II}^c represent the characteristic state of stress. A surface, from form identical to the surface of load within the space of constraints, separates the contracting states from the dilating states. This surface, known as characteristic, has as an equation:

$$f^c=s_{II}^c\,h(\theta_s)+R_c\,(I_1+Q_{init})$$

where R_c is a parameter corresponding to the average radius of this characteristic surface. The kinematic condition can be also put in the form:

$$\dot{\varepsilon}_v^{dp}+\beta\left(\frac{s_{II}}{s_{II}^c}-1\right)\frac{\left|s_{ij}\,\dot{e}_{ij}^{dp}\right|}{s_{II}}=\dot{\varepsilon}_v^{dp}+\beta\left(\frac{s_{II}}{s_{II}^c}-1\right)\frac{\left|s_{ij}\,\dot{e}_{ij}^{dp}\right|}{s_{ij}\,\dot{e}_{ij}^{dp}}\frac{s_{ij}\,\dot{e}_{ij}^{dp}}{s_{II}}$$
$$=\dot{\varepsilon}_v^{dp}+\frac{\beta'}{s_{II}}\,s_{ij}\,\dot{e}_{ij}^{dp}$$
$$=\dot{\varepsilon}_v^{dp}+\frac{\beta'}{s_{II}}\,s_{ij}\,\dot{\varepsilon}_{ij}^{dp}=0$$

where $\beta'=\beta\left(\frac{s_{II}}{s_{II}^c}-1\right)signe\left(s_{ij}\ \dot{\varepsilon}_{ij}^{dp}\right)$.

It is then possible to seek to express this kinematic condition starting from a tensor n in the form:

$$\dot{\varepsilon}_{ij}^{dp}\,n_{ij}=0$$

i.e., after decomposition of each term in déviatoire parts and hydrostatic:

$$\dot{\varepsilon}_{ij}^{dp}\,n_{ij}=\left(\dot{e}_{ij}^{dp}+\frac{1}{3}\,\dot{\varepsilon}_v^{dp}\,\delta_{ij}\right)\left(n_1\,s_{ij}+n_2\,\delta_{ij}\right)=n_1\,s_{ij}\,\dot{e}_{ij}^{dp}+n_2\,dt\,\varepsilon_v^{dp}=0$$

One from of deduced the relation $\frac{n_1}{n_2}=\frac{\beta'}{s_{II}}$, which added to the condition of standardisation $\boldsymbol{n}:\boldsymbol{n}=1$, led to the expressions:

$$n_1=\frac{\frac{\beta'}{s_{II}}}{\sqrt{\beta'^2+3}}\quad\text{and}\quad n_2=\frac{1}{\sqrt{\beta'^2+3}}\text{, that is to say}\quad n_{ij}=\frac{\beta'\,\frac{s_{ij}}{s_{II}}+\delta_{ij}}{\sqrt{\beta'^2+3}}$$

The law of evolution of $\dot{\varepsilon}_{ij}^{dp}$ must be such as the kinematic condition is satisfied. It is thus proposed to take the projection of $\dot{\varepsilon}_{ij}^{dp}$ on the hypersurface of deformation of normal $\boldsymbol{n}$, that is to say:

$$\dot{\varepsilon}_{ij}^{dp}=\dot{\lambda}^d\left(\frac{\partial f^d}{\partial\sigma_{ij}}-\left(\frac{\partial f^d}{\partial\sigma_{kl}}\,n_{kl}\right)n_{ij}\right)=\dot{\lambda}^d\,G_{ij}^d$$

with $G_{ij}^d=\frac{\partial f^d}{\partial\sigma_{ij}}-\left(\frac{\partial f^d}{\partial\sigma_{kl}}\,n_{kl}\right)n_{ij}$.

In addition, for the calculation of the potential, one can note that:

$$\frac{\partial f^d}{\partial\sigma_{ij}}=\frac{\partial f^d}{\partial q_{kl}}\,\frac{\partial q_{kl}}{\partial\sigma_{ij}}+R\,\delta_{ij}$$
$$=\left[dev\left(\frac{\partial f^d}{\partial q_{kl}}\right)+\frac{1}{3}\,\frac{\partial f^d}{\partial q_{mm}}\,\delta_{kl}\right]\left[\delta_{ik}\,\delta_{jl}-\delta_{ij}\left(\frac{1}{3}\,\delta_{kl}+X_{kl}\right)\right]+R\,\delta_{ij}$$
$$=Q_{kl}\,\delta_{ik}\,\delta_{jl}-\delta_{ij}\left(\frac{1}{3}\,Q_{kl}\,\delta_{kl}+Q_{kl}\,X_{kl}\right)+\frac{1}{3}\,\frac{\partial f^d}{\partial q_{mm}}\left[\delta_{ik}\,\delta_{jl}\,\delta_{kl}-\delta_{ij}\left(\frac{1}{3}\,\delta_{kl}\,\delta_{kl}+\delta_{kl}\,X_{kl}\right)\right]+R\,\delta_{ij}$$
$$=Q_{ij}-\left(Q_{kl}\,X_{kl}-R\right)\delta_{ij}$$

3.4.4 Rough surface

The state of rupture results from the nonlinear nature of the laws of work hardening and the existence of limiting values associated with the variables of work hardening R and $\mathbf{X}$. Limit of R, noted R_m, is reached when r tends towards the infinite one. Limit of X_{ij} is reached when $\dot{X}_{ij}$ becomes null.
Under these conditions:

$$Q_{ij}=\varphi\ X_{ij} \text{ and } Q_{II}=\varphi\ X_{II\,\mathbf{lim}} \Rightarrow X_{II\,\mathbf{lim}}=\frac{1}{\varphi_o\ h(\theta_s)}$$

WITH the state of rupture one thus has [Figure 3.4.4-a]:

$$q_{II}=\frac{s_{II}+I_1\ X_{II\,\mathbf{lim}}\ \cos\alpha}{\cos(\theta_s-\theta_q)}$$

By replacing this expression and the value of R in rupture, in the equation of the surface of breaking load, one obtains the equation of a limiting envelope for surfaces of load:

$$f^r=s_{II}\ h(\theta_s)+R_r\ (I_1+Q_{init})=0$$

with $R_r=\frac{\cos\alpha}{\varphi_o}+\frac{h(\theta_s)}{h(\theta_q)}\ R_m\ \cos(\theta_s-\theta_q)$, average radius of the envelope, which is determined starting from the mechanical characteristics with the rupture of material. The value of φ_o can then be deduced about it:

$$\varphi_o=\frac{\cos\alpha}{R_r-\frac{h(\theta_s)}{h(\theta_q)}\ R_m\ \cos(\theta_s-\theta_q)}$$

with $\cos\alpha=\frac{q_{II}^2-s_{II}^2-(I_1\ X_{II})^2}{2\ s_{II}\ I_1\ X_{II}}$

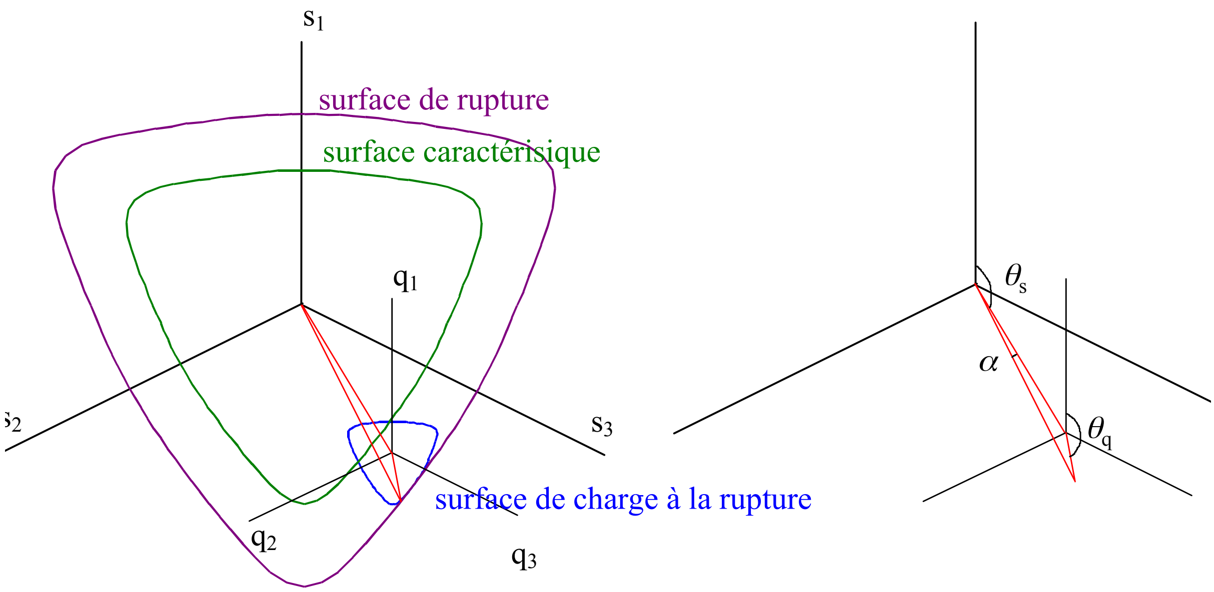


Figure 3.4.4-a: Representation of the characteristic, rough surfaces and of load in the déviatoire plan

In addition, R_r is related to the maximum angle of friction and depends on the average constraint and the relative density. To take into account the dependence of the maximum angle of friction according to the average constraint and of the relative density, the relation is considered:

$$R_r = R_c + \mu \ \ln \left(\frac{3 \ p_c}{I_1 + Q_{init}} \right)$$

where R_c and μ are parameters of the model. p_c is the average constraint criticizes, i.e. the minimal average constraint (it is negative with our convention of sign) known by material during its history. It depends on the initial relative density according to the classical concept critical line in the plan $(e, \ \ln|p|)$:

$$p_c = p_{co} \ \exp(-c \ \varepsilon_v)$$

where p_{co} is the initial critical pressure and $1/c$ is the slope of the right-hand side of critical condition in the plan $(|\varepsilon_v|, \ \ln|p|)$.

3.5 Hierarchisation of the model

3.5.1 Description summary of three levels CJS

Starting from the complete description of the model given above, one deduces three levels from increasing complexity whose characteristics are summarized in the following table:

	Elastic mechanism	Isotropic plastic mechanism	Plastic mechanism déviatoire
CJS1	linear	not activated	activated, perfect plasticity
CJS2	nonlinear	activated	activated, isotropic work hardening
CJS3	nonlinear	activated	activated, kinematic work hardening

Table 3.5.1-1: Various mechanisms used by the various levels of model CJS

3.5.2 Assessment of parameters CJS

In addition, one can also summarize the correspondence between the various levels of the model and the parameters associated with each one of them:

	n	K_o^e	G_o^e	K^p	γ	β	R_c	A	b	R_m	μ	p_{co}	c	p_a
CJS1		×	×		×	×				×				×
CJS2	×	×	×	×	×	×	×	×		×				×
CJS3	×	×	×	×	×	×	×		×	×	×	×	×	×

Table 3.5.2-1: Assessment of the various parameters according to levels CJS

In *Code_Aster*, elastic parameters of the model `CJS` (K_o^e and G_o) are directly taken into account in the elastic characteristics of material, i.e. through the Young modulus E and the Poisson's ratio NU .

In *Code_Aster*, the user does not indicate the level explicitly CJS that it selected. In fact indeed the choice of the various parameters determines the corresponding level. We have to summarize the following logical tests which are integrated in the code:

- if $n=0$ then level CJS1,
- if ($n\neq 0$ and $A\neq 0$) then level CJS2,
- if ($n\neq 0$ and $A=0$) then level CJS3.

Note:

The user must fix the value of P_a equalize with $-100\,kPa$ according to the selected units. Moreover, for CJS3, the value of p_{co} must be negative.

3.5.3 Correspondence with the cohesion and the angle of friction

The mechanics of the grounds have the habit to use the concepts of cohesion Cohesion c, of angle of friction φ and of angle of dilatancy: ψ. These parameters are used in the law of Mohr Coulomb. Level 1 of law CJS makes it possible to find a behavior very close by making the following choice to parameters:

$$\left(\frac{1-\gamma}{1+\gamma}\right)^{1/6}=\frac{3-\sin(\varphi)}{3+\sin(\varphi)}$$

$$R_m=\frac{2\sqrt{\frac{2}{3}}\sin(\varphi)(1-\gamma)^{1/6}}{3-\sin(\varphi)}$$

$$Q_{init}=-3\,c\,\text{cotan}(\varphi)$$

$$\beta=\frac{-2\sqrt{6}\sin\psi}{3-\sin\psi}$$

4 Integration of the law CJS

We detail the integration of the law below CJS according to or activated mechanisms:

- nonlinear rubber band,
- nonlinear rubber band and isotropic plastic
- nonlinear rubber band and plastic déviatoire
- nonlinear rubber band, isotropic plastic and plastic déviatoire.

In each case, the goal is to calculate, starting from the fields known with the state less ε^-, σ^- and of the increment of deformation $\Delta\varepsilon$, the new state of stress σ^+.

In the sequence of calculations, one starts by making the assumption that only the nonlinear elastic mechanism intervenes. An elastic prediction is thus carried out. This prediction is then used to calculate the functions of load f^i and f^d, one seeks to know if one goes then beyond the thresholds:

- if $f^i\leq 0$ and $f^d\leq 0$, the elastic prediction is regarded as new state of stress,

- if $f^i>0$ and $f^d\leq 0$, one makes the isotropic integration of the mechanisms elastic nonlinear and plastic,
- if $f^i\leq 0$ and $f^d>0$, one makes the integration of the mechanisms elastic nonlinear and plastic déviatoire,
- if $f^i>0$ and $f^d>0$, one makes the integration of the mechanisms elastic nonlinear, plastic isotropic and plastic déviatoire.

At exit of elastoplastic calculation, when only one plastic threshold was initially exceeded, one recomputes each function of load. Indeed, it is possible that while seeking to bring back itself on one of the thresholds, one then exceeds the other threshold not activated initially by the elastic prediction. In this case, one solves then by integrating all the mechanisms.

4.1 Choice of the internal variables

Variables q , r and α are equivalent to the associated thermodynamic forces Q_{iso} , R and $\mathbf{X}$. For this reason and since their geometrical significance is more obvious, we will retain like internal variables for the integration of law CJS, the sizes Q_{iso} , R and $\mathbf{X}$.
In addition, we add to the number of the internal variables:

- the sign of the product $s_{ij}\varepsilon_{ij}^{dp}$
- the elastic or elastoplastic state of material, while noting:
  - 0: elastic state
  - 1: elastoplastic state, isotropic plastic mechanism
  - 2: elastoplastic state, plastic mechanism déviatoire
  - 3: elastoplastic state, plastic mechanisms isotropic and déviatoire

Finally, the internal variables are stored in a vector `VI` in the following order:

Internal index of variable		CJS1	CJS2	CJS3
3D	2D	CJS1	CJS2	CJS3
1	1	$Q_{iso}=\infty$	Q_{iso}	Q_{iso}
2	2	$R=R_m$	R	$R=R_m$
3	3	0	0	X_{11}
4	4	0	0	X_{22}
5	5	0	0	X_{33}
6	6	0	0	$\sqrt{2}\,X_{12}$
7	–	0	0	$\sqrt{2}\,X_{13}$
8	–	0	0	$\sqrt{2}\,X_{23}$
9	7	$\frac{q_{II}\,h(\theta_q)}{\lvert R_m(I_1+Q_{init})\rvert}$	$\frac{q_{II}\,h(\theta_q)}{\lvert R(I_1+Q_{init})\rvert}$	$\frac{q_{II}\,h(\theta_q)}{\lvert R_m(I_1+Q_{init})\rvert}$
10	8		$\frac{R}{R_m}$	$\frac{X_{II}}{X_{II}^{\lim}}$
11	9		$\lvert\frac{3\mathrm{Q}}{I_1+Q_{init}}\rvert$	$\lvert\frac{3\mathrm{Q}}{I_1+Q_{init}}\rvert$
12	10	Iteration count internal	Iteration count internal	Iteration count internal
13	11	local test reached	local test reached	local test reached
14	12	no. of recutting	no. of recutting	no. of recutting
15	13	$signe(s_{ij}\,\varepsilon_{ij}^{dp})$	$signe(s_{ij}\,\varepsilon_{ij}^{dp})$	$signe(s_{ij}\,\varepsilon_{ij}^{dp})$
16	14	0,1,2,3 state of material	0,1,2,3 state of material	0,1,2,3 state of material

4.2 Integration of the nonlinear elastic mechanism

In the elastic case, the new state of stress σ^+ , checks simply:

$$\sigma_{ij}^{+}=\sigma_{ij}^{-}+D_{ijkl}(\sigma^{+})\,\Delta\,\varepsilon_{kl}$$

The dependence of the nonlinear tensor of elasticity according to the state of stresses is summarized in fact with:

$$D_{ijkl}(\sigma^{+})=D_{ijkl}^{lineaire}\left(\frac{I_1^{+}+Q_{init}}{3\;P_a}\right)^{n}$$

where $D_{ijkl}^{lineaire}$ is the tensor of isotropic linear elasticity classical, obtained from K_o^e and G_o or by equivalence from `E` and `Naked`.

Of this relation, it is deduced in particular that the first invariant of the constraints satisfied:

$$I_1^{+}-I_1^{-}-3\;K_o^{e}\left(\frac{I_1^{+}+Q_{init}}{3\;P_a}\right)^{n}\;tr(\Delta\,\varepsilon)=0$$

This nonlinear equation is solved by a method of the secant for CJS2 and CJS3, by differentiating the cases following the sign from $tr(\Delta\varepsilon)$. With regard to the model CJS1, for which the parameter n is null, the explicit resolution is immediate, since one has then

$$I_1^+ = I_1^- + 3\,K_o^e\,tr(\Delta\varepsilon)$$

In the case general, the knowledge of I_1^+ and thus of the term $\left(\frac{I_1^+ + Q_{init}}{3\,P_a}\right)^n$ allows to define the nonlinear operator of elasticity $D_{ijkl}(\sigma^+)$. Obtaining the new state of stress is then direct.

4.3 Isotropic integration of the mechanisms elastic nonlinear and plastic

In this case, the new state of stress σ^+ , checks:

$$\sigma_{ij}^+ = \sigma_{ij}^- + D_{ijkl}(\sigma^+)\left(\Delta\varepsilon_{kl} - \Delta\varepsilon_{kl}^{ip}\right)$$

Being given the simple form, plastic deformations of the isotropic plastic mechanism:

$$\Delta\varepsilon_{ij}^{ip} = -\frac{1}{3}\Delta\lambda^i\delta_{ij}$$

the nonlinear system to solve is composed of:

- LE_{ij} : the elastic law state: $\sigma_{ij}^+ - \sigma_{ij}^- - D_{ijkl}(\sigma^+)\left(\Delta\varepsilon_{kl} + \frac{1}{3}\Delta\lambda^i\delta_{kl}\right) = 0$
- LQ : the law of work hardening of the internal variable Q_{iso} : $Q_{iso}^+ - Q_{iso}^- - \Delta\lambda^i\,G^{Q_{iso}}(Q_{iso}^+) = 0$
- FI : the equation of the surface of isotropic load: $-\frac{I_1^+ + Q_{init}}{3} + Q_{iso}^+ = 0$

Schematically, one thus seeks to solve the system $R(Y)=0$, where the unknown factor Y is given by $Y=(\sigma_{ij}^+, Q_{iso}^+, \Delta\lambda^i)$ and where $R=(LE_{ij}, LQ, FI)$. The resolution of $R(Y)=0$ is done by the method of Newton:

- initialization and calculation of a solution of test
- iterations of Newton: resolution of $\frac{DR}{DY}(Y^p)\,DY^{p+1} = -R(Y^p)$
- test of convergence: if convergence $Y = Y^p$; if not $Y^{p+1} = Y^p + DY^{p+1}$ and $p=p+1$

We detail these three stages below.

4.3.1 Initialization and solution of test

We take simply for $Y^0 = (\sigma_{ij}^0, Q_{iso}^0, \Delta\lambda^{i^0})$, following values:

$\sigma_{ij}^0 = \sigma_{ij}^{elas}$: constraints given by the elastic prediction,

$Q_{iso}^0 = Q_{iso}^-$: variable interns with T

$\Delta\lambda^{i^0} = 0$: plastic multiplier no one

Contrary to the other elastoplastic mechanisms, here a solution of test is not calculated.

4.3.2 Iterations of Newton

The resolution of $\frac{DR}{DY}\left(Y^p\right)\ DY^{p+1}=-R\left(Y^p\right)$ naturally require the calculation of the derivative of LE_{ij} , LQ and FI compared to each component of Y . One a:

$$\frac{DR}{DY}=\begin{bmatrix}\frac{\partial LE_{ij}}{\partial \sigma_{kl}} & \frac{\partial LE_{ij}}{\partial Q_{iso}} & \frac{\partial LE_{ij}}{\partial \Delta \lambda^i}\\ \frac{\partial LQ}{\partial \sigma_{kl}} & \frac{\partial LQ}{\partial Q_{iso}} & \frac{\partial LQ}{\partial \Delta \lambda^i}\\ \frac{\partial FI}{\partial \sigma_{kl}} & \frac{\partial FI}{\partial Q_{iso}} & \frac{\partial FI}{\partial \Delta \lambda^i}\end{bmatrix}$$

with:

$$\frac{\partial LE_{ij}}{\partial \sigma_{kl}\delta}=\delta_{ik}\delta_{jl}-\frac{\partial D_{ijmn}}{\partial \sigma_{kl}}\left(\Delta \epsilon_{mn}+\frac{1}{3}\Delta \lambda^i \delta_{mn}\right)=\delta_{ik}\delta_{jl}-D_{ijmn}^{lineaire}\left(\Delta \epsilon_{mn}+\frac{1}{3}\Delta \lambda^i \delta_{mn}\right)\frac{n}{3\ P_a}\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{n-\delta}\delta_{kl}$$

$$\frac{\partial LE_{ij}}{\partial Q_{iso}}=0$$

$$\frac{\partial LE_{ij}}{\partial \Delta \lambda^i}=-\frac{1}{3}\ D_{ijmn}\delta_{mn}$$

$$\frac{\partial LQ}{\partial \sigma_{kl}}=0$$

$$\frac{\partial LQ}{\partial Q_{iso}}=1-\Delta \lambda^i\frac{\partial G^{Q_{iso}}}{\partial Q_{iso}}=1+\Delta \lambda^i\frac{n\ K_o^p}{P_a}\left(\frac{Q_{iso}}{P_a}\right)^{n-1}$$

$$\frac{\partial LQ}{\partial \Delta \lambda^i}=-\ G^{Q_{iso}}$$

$$\frac{\partial FI}{\partial \sigma_{kl}}=-\frac{1}{3}\delta_{kl}$$

$$\frac{\partial FI}{\partial Q_{iso}}=1$$

$$\frac{\partial FI}{\partial \Delta \lambda^i}=0$$

4.3.3 Test of convergence

The iterations of Newton are continued as much as the relative error $\frac{\|DY^{p+1}\|}{\|Y^{p+1}-Y^0\|}$ remain higher than the tolerance allowed by the user and defined by the keyword RESI_INTE_RELA. The standard used here is the vectorial standard: $\|x\|=\sqrt{\sum_i x_i^2}$.

4.4 Integration of the mechanisms elastic nonlinear and plastic déviatoire

In this case, the new state of stress σ^+ , checks:

$$\sigma_{ij}^+=\sigma_{ij}^-+D_{ijkl}(\sigma^+)\left(\Delta\epsilon_{kl}-\Delta\epsilon_{kl}^{dp}\right)$$

The plastic deformations of the plastic mechanism déviatoire are given by the potential G^d :

$$\Delta\epsilon_{ij}^{dp}=\Delta\lambda^d G_{ij}^d$$

One from of deduced that the nonlinear system to solve is composed of:

- LE_{ij} : the elastic law state: $\sigma_{ij}^+-\sigma_{ij}^--D_{ijkl}(\sigma^+)\left(\Delta\epsilon_{kl}-\Delta\lambda^d G_{kl}^d(\sigma^+,R^+,\boldsymbol{X}^+)\right)=0$
- LR : the law of work hardening of the variable R : $R^+-R^--\Delta^d G^R(\sigma^+,R^+)=0$
- LX_{ij} : the law of work hardening of the variable X_{ij} : $X_{ij}^+-X_{ij}^--\Delta\lambda^d G^X(\sigma^+,\boldsymbol{X}^+)=0$
- FD : the equation of the surface of load déviatoire: $q_{II}^+ h(\theta_q^+)+R^+(I_1^++Q_{init})=0$

As in the preceding paragraph one solves by the method of Newton the system $R(Y)=0$, where the unknown factor Y is given by $Y=(\sigma_{ij}^+, R^+, X_{ij}^+, \Delta\lambda^d)$ and where $R=(LE_{ij}, LR, LX_{ij}, FD)$.

4.4.1 Initialization and solution of test

Starting from the state at the moment T $(\sigma_{ij}^-, R^-, X_{ij}^-)$, we seek a solution of test which brings us closer to the final solution. For that we solve the following equation:

$$f^d\left(\sigma_{ij}^-+D_{ijkl}^-\left(\Delta\epsilon_{kl}-\Delta\lambda^d G_{kl}^{d-}\right), R^-+\Delta\lambda^d G^{R-}, X_{ij}^-+\Delta\lambda^d G_{ij}^{X-}\right)=0$$

with $D_{ijkl}^-=D_{ijkl}(\sigma^-)$, $G_{kl}^{d-}=G_{kl}^d(\sigma^-, R^-,\boldsymbol{X}^-)$, $G^{R-}=G^R(\sigma^-, R^-)$, $G_{ij}^{X-}=G_{ij}^X(\sigma^-,\boldsymbol{X}^-)$ and where the unknown factor is the plastic multiplier $\Delta\lambda^d$, by only one iteration of Newton, i.e. finally of we let us have:

$$\frac{\partial f^d}{\partial\Delta\lambda^d}\Big|_{\Delta\lambda^d=0}\Delta\lambda^d=-f^d{}_{|\Delta\lambda^d=0} \quad \text{that is to say still} \quad \Delta\lambda^d=-\frac{f^d_{|\Delta\lambda^d=0}}{\frac{\partial f^d}{\partial\Delta\lambda^d}\Big|_{\Delta\lambda^d=0}}$$

with:

$$\frac{\partial f^d}{\partial\Delta\lambda^d}=h(\theta_q)\frac{\partial q_{II}}{\partial\Delta\lambda^d}+q_{II}\frac{\partial h(\theta_q)}{\partial\Delta\lambda^d}+(I_1+Q_{init})\frac{\partial R}{\partial\Delta\lambda^d}+R\frac{\partial I_1}{\partial\Delta\lambda^d}$$

Moreover,

one a: $I_1=I_1^-+3K^-\left(tr(\Delta\epsilon)-\Delta\lambda^d tr(\boldsymbol{G}^{d-})\right)$ then: $\frac{\partial I_1}{\partial\Delta\lambda^d}=-3K^- tr(\boldsymbol{G}^{d-})$

one a: $R=R^-+\Delta\lambda^d G^{R-}$ then: $\frac{\partial R}{\partial\Delta\lambda^d}=G^{R-}$

one a:

$$q_{ij}=\sigma_{ij}^{-}+D_{ijkl}^{-}\left(\Delta\ \varepsilon_{kl}-\Delta\ \lambda^{d}\ G_{kl}^{d-}\right)-\left[I_1^{-}+3\ K^{-}\left(tr(\Delta\ \varepsilon)-\Delta\ \lambda^{d}\ tr\left(G^{d-}\right)\right)\right]\left[\frac{1}{3}\ \delta_{ij}+X_{ij}^{-}+\Delta\ \lambda^{d}\ G_{ij}^{X-}\right]$$

then: $$\frac{\partial q_{ij}}{\partial\Delta\ \lambda^{d}}_{|\Delta\ \lambda^{d}=0}=-D_{ijkl}^{-}\ G_{kl}^{d-}+3\ K^{-}\ tr\left(G^{d-}\right)\left(\frac{1}{3}\ \delta_{ij}+X_{ij}^{-}\right)-G_{ij}^{X-}\left(I_1^{-}+3\ K^{-}\ tr(\Delta\boldsymbol{\varepsilon})\right)$$

one a: $$\frac{\partial q_{II}}{\partial\Delta\ \lambda^{d}}=\frac{\partial q_{II}}{\partial q_{ij}}\ \frac{\partial q_{ij}}{\partial\Delta\ \lambda^{d}}=\frac{q_{ij}}{q_{II}}\ \frac{\partial q_{ij}}{\partial\Delta\ \lambda^{d}}$$ and $$\frac{\partial h(\theta_q)}{\partial\Delta\ \lambda^{d}}=\frac{\partial h(\theta_q)}{\partial q_{ij}}\ \frac{\partial q_{ij}}{\partial\Delta\ \lambda^{d}}$$

Ultimately, we take for the solution of test: $Y^{0}=\left(\sigma_{ij}^{0},\ R^{0},\ X_{ij}^{0},\ \Delta\ \lambda^{d0}\right)$, with the following values:

$\Delta\ \lambda^{d0}$: the value found according to the preceding formulation.

$$\sigma_{ij}^{0}=\sigma_{ij}^{-}+D_{ijkl}^{-}\left(\Delta\ \varepsilon_{kl}-\Delta\ \lambda^{d0}\ G_{kl}^{d-}\right)$$
$$R^{0}=R^{-}+\Delta\ \lambda^{d0}\ G^{R-}$$
$$X_{ij}^{0}=X_{ij}^{-}+\Delta\ \lambda^{d0}\ G_{ij}^{X-}$$

4.4.2 Iterations of Newton

$\frac{DR}{DY}$ is given here by:

$$\frac{DR}{DY}=\begin{bmatrix}\frac{\partial LE_{ij}}{\partial\sigma_{kl}} & \frac{\partial LE_{ij}}{\partial R} & \frac{\partial LE_{ij}}{\partial X_{ij}} & \frac{\partial LE_{ij}}{\partial\Delta\ \lambda^{d}}\\ \frac{\partial LR}{\partial\sigma_{kl}} & \frac{\partial LR}{\partial R} & \frac{\partial LR}{\partial X_{ij}} & \frac{\partial LR}{\partial\Delta\ \lambda^{d}}\\ \frac{\partial LX_{ij}}{\partial\sigma_{kl}} & \frac{\partial LX_{ij}}{\partial R} & \frac{\partial LX_{ij}}{\partial X_{ij}} & \frac{\partial LX_{ij}}{\partial\Delta\ \lambda^{d}}\\ \frac{\partial FD}{\partial\sigma_{kl}} & \frac{\partial FD}{\partial R} & \frac{\partial FD}{\partial X_{ij}} & \frac{\partial FD}{\partial\Delta\ \lambda^{d}}\end{bmatrix}$$

with:

$$\frac{\partial LE_{\mathbf{ij}}}{\partial\sigma_{kl}}=\delta_{ik}\ \delta_{jl}-D_{ijmn}^{lineaire}\left(\Delta\ \varepsilon_{mn}-\Delta\ \lambda^{d}\ G_{mn}^{d}\right)\frac{n}{3\ P_a}\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{n-1}\ \delta_{kl}+D_{ijmn}\ \Delta\ \lambda^{d}\ \frac{\partial G_{mn}^{d}}{\partial\sigma_{kl}}$$

$$\frac{\partial LE_{\mathbf{ij}}}{\partial R}=D_{ijmn}\ \Delta\ \lambda^{d}\ \frac{\partial G_{mn}^{d}}{\partial R}$$

$$\frac{\partial LE_{\mathbf{ij}}}{\partial X_{kl}}=D_{ijmn}\ \Delta\ \lambda^{d}\ \frac{\partial G_{mn}^{d}}{\partial X_{kl}}$$

$$\frac{\partial LE_{\mathbf{ij}}}{\partial\Delta\ \lambda^{d}}=D_{ijmn}\ G_{mn}^{d}$$

$$\frac{\partial LR}{\partial\sigma_{kl}}=-\Delta\ \lambda^{d}\ \frac{\partial G^{R}}{\partial\sigma_{kl}}=-\Delta\ \lambda^{d}\ \frac{A}{2}\left(1-\frac{R}{R_m}\right)^{2}\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{-1,5}\ \delta_{kl}$$

$$\frac{\partial' LR}{\partial' R}=1-\Delta\ \lambda^d\ \frac{\partial' G^R}{\partial' R}=1-\Delta\ \lambda^d\ \frac{2\ A}{R_m}\left(1-\frac{R}{R_m}\right)\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{-1,5}$$

$$\frac{\partial LR}{\partial X_{kl}}=0$$

$$\frac{\partial LR}{\partial \Delta\ \lambda^d}=-G^R$$

$$\frac{\partial LX_{\boldsymbol{ij}}}{\partial \sigma_{kl}}=-\Delta\ \lambda^d\ \frac{\partial G_{ij}^X}{\partial \sigma_{kl}}$$

$$\frac{\partial LX_{\boldsymbol{ij}}}{\partial R}=0$$

$$\frac{\partial LX_{\boldsymbol{ij}}}{\partial X_{kl}}=\delta_{ik}\ \delta_{jl}-\Delta\ \lambda^d\ \frac{\partial G_{ij}^X}{\partial X_{kl}}$$

$$\frac{\partial LX_{\boldsymbol{ij}}}{\partial \Delta\ \lambda^d}=-G_{ij}^X$$

$$\frac{\partial FD}{\partial \sigma_{kl}}=\frac{\partial f^d}{\partial \sigma_{kl}}=Q_{kl}-\left(Q_{mn}\ X_{mn}-R\right)\ \delta_{kl}$$

$$\frac{\partial FD}{\partial R}=I_1$$

$$\frac{\partial FD}{\partial X_{kl}}=\frac{\partial f^d}{\partial X_{kl}}$$

$$\frac{\partial FD}{\partial \Delta\ \lambda^d}=0$$

In addition, the calculation of the terms $\frac{\partial G_{mn}^d}{\partial \sigma_{kl}}$, $\frac{\partial G_{mn}^d}{\partial R}$, $\frac{\partial G_{mn}^d}{\partial X_{kl}}$, $\frac{\partial G_{ij}^X}{\partial \sigma_{kl}}$, $\frac{\partial G_{ij}^X}{\partial X_{kl}}$ and $\frac{\partial f^d}{\partial X_{kl}}$ Ci - is detailed after, as well as the calculation of useful intermediate terms:

- calculation of $\frac{\partial f^d}{\partial X_{kl}}$:

$$\begin{aligned}\frac{\partial f^d}{\partial X_{kl}}&=q_{II}\ \frac{\partial h\left(\theta_q\right)}{\partial X_{kl}}+h\left(\theta_q\right)\ \frac{\partial q_{II}}{\partial X_{kl}}\\&=q_{II}\ \frac{\partial h\left(\theta_q\right)}{\partial q_{mn}}\ \frac{\partial q_{mn}}{\partial X_{kl}}+h\left(\theta_q\right)\ \frac{\partial q_{II}}{\partial q_{mn}}\ \frac{\partial q_{mn}}{\partial X_{kl}}\\&=q_{II}\ \frac{\partial h\left(\theta_q\right)}{\partial q_{mn}}\ \frac{\partial q_{mn}}{\partial X_{kl}}+h\left(\theta_q\right)\ \frac{q_{mn}}{q_{II}}\ \frac{\partial q_{mn}}{\partial X_{kl}}\\&=\left(q_{II}\ \frac{\partial h\left(\theta_q\right)}{\partial q_{mn}}+h\left(\theta_q\right)\ \frac{q_{mn}}{q_{II}}\right)\frac{\partial q_{mn}}{\partial X_{kl}}\\&=-I_1\left(q_{II}\ \frac{\partial h\left(\theta_q\right)}{\partial q_{mn}}+h\left(\theta_q\right)\ \frac{q_{mn}}{q_{II}}\right)\delta_{mk}\ \delta_{nl}\end{aligned}$$

$$=-I_1\left(\frac{\partial f^d}{\partial q_{kl}}\right)$$

One will notice for the continuation that:

$$dev\left(\frac{\partial f^d}{\partial X_{kl}}\right)=-I_1\ Q_{kl}$$

- calculation of $\frac{\partial\left(\frac{\partial f^d}{\partial\sigma_{ij}}\right)}{\partial\sigma_{kl}}$:

$$\frac{\partial\left(\frac{\partial f^d}{\partial\sigma_{ij}}\right)}{\partial\sigma_{kl}}=\frac{\partial\left(\frac{\partial f^d}{\partial\sigma_{ij}}\right)}{\partial q_{mn}}\ \frac{\partial q_{mn}}{\partial\sigma_{kl}}$$

$$=\left(\frac{\partial\left(Q_{ij}-\left(Q_{rs}\ X_{rs}-R\right)\ \delta_{ij}\right)}{\partial q_{mn}}\right)\frac{\partial q_{mn}}{\partial\sigma_{kl}}$$

$$=\left(\frac{\partial Q_{ij}}{\partial q_{mn}}-\left(\frac{\partial Q_{rs}}{\partial q_{mn}}\ X_{rs}\right)\ \delta_{ij}\right)\frac{\partial q_{mn}}{\partial\sigma_{kl}}$$

$$=\left(\frac{\partial Q_{ij}}{\partial q_{mn}}-\left(\frac{\partial Q_{rs}}{\partial q_{mn}}\ X_{rs}\right)\delta_{ij}\right)\left(\delta_{mk}\ \delta_{nl}-\delta_{kl}\left(\frac{\delta_{mn}}{3}+X_{mn}\right)\right)$$

- calculation of $\frac{\partial Q_{ij}}{\partial q_{mn}}$:

As a preliminary, one definite the tensor t and its déviatoire part t^d while posing:

$$t_{ij}=\frac{\partial\det\left(q\right)}{\partial q_{ij}}\ \text{ and }\ t_{ij}^d=dev\left(\frac{\partial\det\left(q\right)}{\partial q_{ij}}\right)$$

One has as follows:

$$t=\begin{bmatrix}t_{11}\\t_{22}\\t_{33}\\t_{12}\\t_{13}\\t_{23}\end{bmatrix}=\begin{bmatrix}q_{22}\ q_{33}-q_{23}\ q_{23}\\q_{11}\ q_{33}-q_{13}\ q_{13}\\q_{11}\ q_{22}-q_{12}\ q_{12}\\q_{13}\ q_{23}-q_{12}\ q_{33}\\q_{12}\ q_{23}-q_{13}\ q_{22}\\q_{12}\ q_{13}-q_{23}\ q_{11}\end{bmatrix}$$

$$\frac{\partial Q_{ij}}{\partial q_{mn}}=\frac{-5}{h(\theta_q)^6}\left[\left(1+\frac{\gamma}{2}\cos(3\theta_q)\right)\frac{q_{ij}}{q_{II}}+\frac{\gamma\sqrt{54}}{6\ q_{II}^2}dev(t_{ij})\right]\frac{\partial(h(\theta_q))}{\partial q_{mn}}$$

$$+\frac{1}{h(\theta_q)^5}\left(1+\frac{\gamma}{2}\cos(3\theta_q)\right)\frac{\partial\left(\frac{q_{ij}}{q_{II}}\right)}{\partial q_{mn}}+\frac{1}{h(\theta_q)^5}\frac{\gamma}{2}\frac{q_{ij}}{q_{II}}\frac{\partial\cos(3\theta_q)}{\partial q_{mn}}+\frac{1}{h(\theta_q)^5}\frac{\sqrt{54}\ \gamma}{6}\frac{\partial\left(\frac{t_{ij}^d}{q_{II}^2}\right)}{\partial q_{mn}}$$

$$=\frac{-5}{h(\theta_q)^6}\left[\left(1+\frac{\gamma}{2}\cos(3\theta_q)\right)\frac{q_{ij}}{q_{II}}+\frac{\gamma\sqrt{54}}{6\ q_{II}^2}dev(t_{ij})\right]\frac{\partial(h(\theta_q))}{\partial q_{mn}}+\frac{1}{h(\theta_q)^5}\left(1+\frac{\gamma}{2}\cos(3\theta_q)\right)\left(\frac{\delta_{im}\delta_{jn}}{q_{II}}-\frac{q_{ij}q_{mn}}{q_{II}^3}\right)$$

$$+\frac{1}{h(\theta_q)^5}\frac{\gamma}{2}\frac{q_{ij}\sqrt{54}}{q_{II}^4}\left(t_{mn}-3\frac{\det q}{q_{II}^2}q_{mn}\right)+\frac{1}{h(\theta_q)^5}\frac{\gamma}{6}\frac{\sqrt{54}}{q_{II}^2}\left(\frac{\partial t_{ij}^d}{\partial q_{mn}}-2t_{ij}^d\frac{q_{mn}}{q_{II}^2}\right)$$

The expression of $\frac{\partial t_{ij}^d}{\partial q_{mn}}$ clarify yourself as follows:

$$\frac{\partial t^d}{\partial q_{11}}=\begin{bmatrix}-\frac{1}{3}(q_{22}+q_{33})\\ \frac{1}{3}(-q_{22}+2\ q_{33})\\ \frac{1}{3}(2\ q_{22}-q_{33})\\ 0\\ 0\\ -q_{23}\end{bmatrix},\ \frac{\partial t^d}{\partial q_{22}}=\begin{bmatrix}\frac{1}{3}(-q_{11}+2\ q_{33})\\ -\frac{1}{3}(q_{11}+q_{33})\\ \frac{1}{3}(2\ q_{11}-q_{33})\\ 0\\ -q_{13}\\ 0\end{bmatrix},\ \frac{\partial t^d}{\partial q_{33}}=\begin{bmatrix}\frac{1}{3}(-q_{11}+2\ q_{22})\\ \frac{1}{3}(2\ q_{11}-q_{22})\\ -\frac{1}{3}(q_{11}+q_{22})\\ -q_{12}\\ 0\\ 0\end{bmatrix},$$

$$\frac{\partial t^d}{\partial q_{12}}=\begin{bmatrix}\frac{2}{3}\ q_{12}\\ \frac{2}{3}\ q_{12}\\ -\frac{4}{3}\ q_{12}\\ -q_{33}\\ q_{23}\\ q_{13}\end{bmatrix},\ \frac{\partial t^d}{\partial q_{13}}=\begin{bmatrix}\frac{2}{3}\ q_{13}\\ -\frac{4}{3}\ q_{13}\\ \frac{2}{3}\ q_{13}\\ q_{23}\\ -q_{22}\\ q_{12}\end{bmatrix},\ \frac{\partial t^d}{\partial q_{23}}=\begin{bmatrix}-\frac{4}{3}\ q_{23}\\ \frac{2}{3}\ q_{23}\\ \frac{2}{3}\ q_{23}\\ q_{13}\\ q_{12}\\ -q_{11}\end{bmatrix}$$

- calculation of $\frac{\partial G_{mn}^d}{\partial\sigma_{kl}}$:

One a:

$$\frac{\partial G_{mn}^d}{\partial\sigma_{kl}}=\frac{\partial\left(\frac{\partial f^d}{\partial\sigma_{mn}}\right)}{\partial\sigma_{kl}}-\left(\frac{\partial f^d}{\partial\sigma_{rs}}n_{rs}\right)\frac{\partial n_{mn}}{\partial\sigma_{kl}}-\left(\frac{\partial\left(\frac{\partial f^d}{\partial\sigma_{rs}}\right)}{\partial\sigma_{kl}}n_{rs}+\frac{\partial f^d}{\partial\sigma_{rs}}\frac{\partial n_{rs}}{\partial\sigma_{kl}}\right)n_{mn}$$

One definite the tensor $\tilde{\boldsymbol{n}}$ by $\tilde{n}_{ij}=\beta'\frac{s_{ij}}{s_{II}}+\delta_{ij}$

i.e. that $\boldsymbol{n}$ is then given by $n_{ij}=\frac{\tilde{n}_{ij}}{\tilde{n}_{II}}$ with $\tilde{n}_{II}=\sqrt{\beta'^2+3}$

In practice, for the calculation of β', one uses $\Delta\varepsilon_{ij}$ instead of $\Delta\varepsilon_{ij}^{dp}$, i.e. one a:

$$\beta'=\beta\left(\frac{s_{II}}{s_{II}^c}-1\right) signe\left(s_{ij}\Delta\varepsilon_{ij}\right)$$

One has then for $\frac{\partial G_{mn}^d}{\partial\sigma_{kl}}$:

$$\frac{\partial G_{mn}^d}{\partial\sigma_{kl}}=\frac{\partial\left(\frac{\partial f^d}{\partial\sigma_{mn}}\right)}{\partial\sigma_{kl}}-\left(\frac{\partial\left(\frac{\partial f^d}{\partial\sigma_{rs}}\right)}{\partial\sigma_{kl}}n_{rs}\right)n_{mn}-\left(\frac{\partial f^d}{\partial\sigma_{rs}}\frac{\partial\tilde{n}_{rs}}{\partial\sigma_{kl}}\right)\frac{\tilde{n}_{mn}}{\tilde{n}_{II}^2}-\left(\frac{\partial f^d}{\partial\sigma_{rs}}\tilde{n}_{rs}\right)\frac{\partial\tilde{n}_{mn}}{\partial\sigma_{kl}}\frac{1}{\tilde{n}_{II}^2}-\left(\frac{\partial f^d}{\partial\sigma_{rs}}\tilde{n}_{rs}\right)\tilde{n}_{mn}\frac{\partial\left(\frac{1}{\tilde{n}_{II}^2}\right)}{\partial\sigma_{kl}}$$

with:

$$\frac{\partial\left(\frac{1}{\tilde{n}_{II}^2}\right)}{\partial\sigma_{kl}}=\frac{\partial\left(\frac{1}{\left(\beta'^2+3\right)}\right)}{\partial\sigma_{kl}}=-\frac{1}{\left(\beta'^2+3\right)^2}\frac{\partial\left(\beta'^2\right)}{\partial\sigma_{kl}}=-\frac{2\beta^2\left(\frac{s_{II}}{s_{II}^c}-1\right)}{\left(\beta'^2+3\right)^2}\frac{\partial\left(\frac{s_{II}}{s_{II}^c}\right)}{\partial\sigma_{kl}}$$

- calculation of $\frac{\partial\left(\frac{s_{II}}{s_{II}^c}\right)}{\partial\sigma_{kl}}$:

$$\frac{\partial\left(\frac{s_{II}}{s_{II}^c}\right)}{\partial\sigma_{kl}}=\frac{1}{s_{II}^c}\frac{\partial\left(s_{II}\right)}{\partial\sigma_{kl}}-\frac{s_{II}}{s_{II}^{c^2}}\frac{\partial\left(s_{II}^c\right)}{\partial\sigma_{kl}}$$

$$=\frac{1}{s_{II}^c}\frac{\partial\left(s_{II}\right)}{\partial s_{mn}}\frac{\partial s_{mn}}{\partial\sigma_{kl}}-\frac{s_{II}}{s_{II}^{c^2}}\frac{\partial\left(-\frac{R_c\left(I_1+Q_{init}\right)}{h\left(\theta_s\right)}\right)}{\partial\sigma_{kl}}$$

$$=\frac{1}{s_{II}^c}\frac{s_{mn}}{s_{II}}\left(\delta_{mk}\delta_{nl}-\frac{1}{3}\delta_{mn}\delta_{kl}\right)-\frac{s_{II}}{s_{II}^{c^2}}\left(-\frac{R_c}{h\left(\theta_s\right)}\frac{\partial I_1}{\partial\sigma_{kl}}+\frac{R_c\left(I_1+Q_{init}\right)}{h\left(\theta_s\right)^2}\frac{\partial h\left(\theta_s\right)}{\partial\sigma_{kl}}\right)$$

$$=\frac{1}{s_{II}^c}\frac{s_{mn}}{s_{II}}\left(\delta_{mk}\delta_{nl}-\frac{1}{3}\delta_{mn}\delta_{kl}\right)-\frac{s_{II}}{s_{II}^{c^2}}\left(-\frac{R_c}{h\left(\theta_s\right)}\delta_{kl}+\frac{R_c\left(I_1+Q_{init}\right)}{h\left(\theta_s\right)^2}\frac{\partial h\left(\theta_s\right)}{\partial s_{rs}}\frac{\partial s_{rs}}{\partial\sigma_{kl}}\right)$$

- calculation of $\frac{\partial \tilde{n}_{mn}}{\partial \sigma_{kl}}$:

$$\frac{\partial \tilde{n}_{mn}}{\partial \sigma_{kl}}=\beta\left(\frac{1}{s_{II}^c}-\frac{1}{s_{II}}\right) signe\left(s_{ij}\ \Delta\varepsilon_{ij}\right)\frac{\partial s_{mn}}{\partial \sigma_{kl}}+\beta\ signe\left(s_{ij}\ \Delta\varepsilon_{ij}\right) s_{mn}\left(\frac{\partial\left(\frac{1}{s_{II}^c}\right)}{\partial \sigma_{kl}}-\frac{\partial\left(\frac{1}{s_{II}}\right)}{\partial \sigma_{kl}}\right)$$

$$=\beta\left(\frac{1}{s_{II}^c}-\frac{1}{s_{II}}\right) signe\left(s_{ij}\ \Delta\varepsilon_{ij}\right)\left(\delta_{mk}\ \delta_{nl}-\frac{1}{3}\ \delta_{mn}\ \delta_{kl}\right)+\beta\ signe\left(s_{ij}\ \Delta\varepsilon_{ij}\right) s_{mn}\left(\frac{1}{s_{II}^2}\frac{\partial\left(s_{II}\right)}{\partial \sigma_{kl}}-\frac{1}{s_{II}^{c^2}}\frac{\partial s_{II}^c}{\partial \sigma_{kl}}\right)$$

- calculation of $\frac{\partial G_{mn}^d}{\partial R}$:

$$\frac{\partial G_{mn}^d}{\partial R}=\frac{\partial\left(\frac{\partial f^d}{\partial \sigma_{mn}}\right)}{\partial R}-\left(\frac{\partial f^d}{\partial \sigma_{rs}}\ n_{rs}\right)\frac{\partial n_{mn}}{\partial R}-\left(\frac{\partial\left(\frac{\partial f^d}{\partial \sigma_{rs}}\right)}{\partial R}\ n_{rs}+\frac{\partial f^d}{\partial \sigma_{rs}}\ \frac{\partial n_{rs}}{\partial R}\right) n_{mn}$$

$$=\frac{\partial\left(\frac{\partial f^d}{\partial \sigma_{mn}}\right)}{\partial R}-\left(\frac{\partial\left(\frac{\partial f^d}{\partial \sigma_{rs}}\right)}{\partial R}\ n_{rs}\right) n_{mn}$$

$$=\delta_{mn}-\left(\delta_{rs}\ n_{rs}\right) n_{mn}$$

$$=\frac{\beta'^2\ \delta_{mn}-3\ \beta'\frac{s_{mn}}{s_{II}}}{\beta'^2+3}$$

- calculation of $\frac{\partial G_{mn}^d}{\partial X_{kl}}$:

$$\frac{\partial G_{mn}^d}{\partial X_{kl}}=\frac{\partial\left(\frac{\partial f^d}{\partial \sigma_{mn}}\right)}{\partial X_{kl}}-\left(\frac{\partial f^d}{\partial \sigma_{rs}}\ n_{rs}\right)\frac{\partial n_{mn}}{\partial X_{kl}}-\left(\frac{\partial\left(\frac{\partial f^d}{\partial \sigma_{rs}}\right)}{\partial X_{kl}}\ n_{rs}+\frac{\partial f^d}{\partial \sigma_{rs}}\ \frac{\partial n_{rs}}{\partial X_{kl}}\right) n_{mn}$$

$$=\frac{\partial\left(\frac{\partial f^d}{\partial \sigma_{mn}}\right)}{\partial X_{kl}}-\left(\frac{\partial\left(\frac{\partial f^d}{\partial \sigma_{rs}}\right)}{\partial X_{kl}}\ n_{rs}\right) n_{mn}$$

- calculation of $\frac{\partial\left(\frac{\partial f^d}{\partial \sigma_{mn}}\right)}{\partial X_{kl}}$:

$$\frac{\partial\left(\frac{\partial f^d}{\partial \sigma_{mn}}\right)}{\partial X_{kl}}=\frac{\partial Q_{mn}}{\partial X_{kl}}-\left(\frac{\partial Q_{rs}}{\partial X_{kl}}\ X_{rs}+Q_{rs}\ \frac{\partial X_{rs}}{\partial X_{kl}}\right)\delta_{mn}$$

$$=\frac{\partial Q_{mn}}{\partial q_{ij}}\frac{\partial q_{ij}}{\partial X_{kl}}-\left(\left(\frac{\partial Q_{rs}}{\partial q_{ij}}\frac{\partial q_{ij}}{\partial X_{kl}}\right)X_{rs}+Q_{rs}\,\delta_{kr}\,\delta_{ls}\right)\delta_{mn}$$

$$=-I_1\frac{\partial Q_{mn}}{\partial q_{ij}}\delta_{ik}\,\delta_{jl}-\left(\left(-I_1\frac{\partial Q_{rs}}{\partial q_{ij}}\delta_{ik}\,\delta_{jl}\right)X_{rs}+Q_{rs}\,\delta_{kr}\,\delta_{ls}\right)\delta_{mn}$$

- calculation of $\frac{\partial G_{ij}^X}{\partial \sigma_{kl}}$:

$$\frac{\partial G_{ij}^X}{\partial\sigma_{kl}}=-\frac{1}{2\,b}\left(Q_{ij}+\varphi\;X_{ij}\right)\left(\frac{I_1+Q_{init}}{3\;P_a}\right)^{-1,5}\frac{\partial I_1}{\partial\sigma_{kl}}+\frac{1}{b}\left(\frac{\partial Q_{ij}}{\partial\sigma_{kl}}+\frac{\partial\varphi}{\partial\sigma_{kl}}X_{ij}\right)\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\;P_a}\right)^{-1,5}$$

$$=-\frac{1}{2\,b}\left(Q_{ij}+\varphi\;X_{ij}\right)\left(\frac{I_1+Q_{init}}{3\;P_a}\right)^{-1,5}\delta_{kl}+\frac{1}{b}\frac{\partial Q_{ij}}{\partial q_{mn}}\frac{\partial q_{mn}}{\partial\sigma_{kl}}\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\;P_a}\right)^{-1,5}$$

$$+\frac{1}{b}\left(h\left(\theta_s\right)Q_{II}\frac{\partial\varphi_o}{\partial\sigma_{kl}}+\varphi_o\,Q_{II}\frac{\partial h\left(\theta_s\right)}{\partial\sigma_{kl}}+\varphi_o\,h\left(\theta_s\right)\frac{\partial Q_{II}}{\partial\sigma_{kl}}\right)X_{ij}\left(I_1+Q_{init}\right)\left(\frac{I_1+Q_{init}}{3\;P_a}\right)^{-1,5}$$

- calculation of $\frac{\partial h\left(\theta_s\right)}{\partial \sigma_{kl}}$:

$$\frac{\partial h\left(\theta_s\right)}{\partial\sigma_{kl}}=\frac{\partial h\left(\theta_s\right)}{\partial s_{mn}}\frac{\partial s_{mn}}{\partial\sigma_{kl}}$$

$$=\left(\frac{\gamma\;\sqrt{54}}{6\;h\left(\theta_s\right)^5 q_{II}^3}t_{mn}-\frac{\gamma\;\cos(3\;\theta_q)}{2\mathrm{h}\left(\theta_s\right)^5 q_{II}^2}s_{mn}\right)\left(\delta_{mk}\,\delta_{nl}-\frac{1}{3}\delta_{mn}\,\delta_{kl}\right)$$

- calculation of $\frac{\partial Q_{II}}{\partial \sigma_{kl}}$:

$$\frac{\partial Q_{II}}{\partial\sigma_{kl}}=\left(\frac{\partial Q_{II}}{\partial Q_{rs}}\frac{\partial Q_{rs}}{\partial q_{mn}}\right)\frac{\partial q_{mn}}{\partial\sigma_{kl}}$$

$$=\left(\frac{Q_{rs}}{Q_{II}}\frac{\partial Q_{rs}}{\partial q_{mn}}\right)\left(\delta_{mk}\,\delta_{nl}-\delta_{mn}\left(\frac{1}{3}\delta_{kl}+X_{kl}\right)\right)$$

- calculation of $\frac{\partial \varphi_o}{\partial \sigma_{kl}}$:

$$\frac{\partial\varphi_o}{\partial\sigma_{kl}}=\frac{1}{R_r-\frac{h\left(\theta_s\right)}{h\left(\theta_q\right)}R_m\cos\left(\theta_s-\theta_q\right)}\frac{\partial\cos\alpha}{\partial\sigma_{kl}}$$

$$-\cos\alpha\frac{\left[\frac{\partial R_r}{\partial\sigma_{kl}}-\frac{1}{h\left(\theta_q\right)}R_m\cos\left(\theta_s-\theta_q\right)\frac{\partial h\left(\theta_s\right)}{\partial\sigma_{kl}}+\frac{h\left(\theta_s\right)}{h\left(\theta_q\right)^2}R_m\cos\left(\theta_s-\theta_q\right)\frac{\partial h\left(\theta_s\right)}{\partial\sigma_{kl}}-\frac{h\left(\theta_s\right)}{h\left(\theta_q\right)}R_m\frac{\partial\cos\left(\theta_s-\theta_q\right)}{\partial\sigma_{kl}}\right]}{\left[R_r-\frac{h\left(\theta_s\right)}{h\left(\theta_q\right)}R_m\cos\left(\theta_s-\theta_q\right)\right]^2}$$

with:

$$\frac{\partial \cos\alpha}{\partial \sigma_{kl}} = \frac{1}{2\, s_{II}\, I_1\, X_{II}} \left(2\, q_{II} \frac{\partial q_{II}}{\partial \sigma_{kl}} - 2\, I_1\, X_{II}^2 \frac{\partial I_1}{\partial \sigma_{kl}} - 2\, s_{II} \frac{\partial s_{II}}{\partial \sigma_{kl}} \right)$$

$$- \frac{q_{II}^2 - (I_1\, X_{II})^2 - s_{II}^2}{s_{II}\, I_1\, X_{II}} \left(s_{II}\, X_{II} \frac{\partial I_1}{\partial \sigma_{kl}} + I_1\, X_{II} \frac{\partial s_{II}}{\partial \sigma_{kl}} \right)$$

$$= \frac{1}{s_{II}\, I_1\, X_{II}} \left[\left(q_{kl} - I_1\, X_{II}^2\, \delta_{kl} - s_{kl} \right) - \left(q_{II}^2 - (I_1\, X_{II})^2 - s_{II}^2 \right) \left(s_{II}\, X_{II}\, \delta_{kl} + I_1\, X_{II} \frac{s_{kl}}{s_{II}} \right) \right]$$

$$\frac{\partial R_r}{\partial \sigma_{kl}} = - \frac{\mu}{I_1 + Q_{init}}\, \delta_{kl}$$

$$\frac{\partial \cos(\theta_s - \theta_q)}{\partial \sigma_{kl}} = - \sin(\theta_s - \theta_q) \left(\frac{\partial \theta_s}{\partial \sigma_{kl}} - \frac{\partial \theta_q}{\partial \sigma_{kl}} \right)$$

- calculation of $\frac{\partial G_{ij}^X}{\partial X_{kl}}$:

$$\frac{\partial G_{ij}^X}{\partial X_{kl}} = \frac{1}{b} \left(\frac{\partial Q_{ij}}{\partial X_{kl}} + \varphi \frac{\partial X_{ij}}{\partial X_{kl}} \right) (I_1 + Q_{init}) \left(\frac{I_1 + Q_{init}}{3\, P_a} \right)^{-1,5}$$

$$= \frac{1}{b} \left(\frac{\partial Q_{ij}}{\partial q_{mn}} \frac{\partial q_{mn}}{\partial X_{kl}} + \varphi\, \delta_{ik}\, \delta_{jl} \right) (I_1 + Q_{init}) \left(\frac{I_1 + Q_{init}}{3\, P_a} \right)^{-1,5}$$

$$= \frac{1}{b} \left(-I_1 \frac{\partial Q_{ij}}{\partial q_{mn}} \delta_{mk}\, \delta_{nl} + \varphi\, \delta_{ik}\, \delta_{jl} \right) (I_1 + Q_{init}) \left(\frac{I_1 + Q_{init}}{3\, P_a} \right)^{-1,5}$$

4.4.3 Test of convergence

The convergence criteria remain $\frac{\| DY^{p+1} \|}{\| Y^{p+1} - Y^0 \|}$ □ `RESI_INTE_RELA`.

4.5 Integration of the mechanisms elastic nonlinear, plastic isotropic and plastic déviatoire

In this case, the new state of stress σ^+, checks:

$$\sigma_{ij}^+ = \sigma_{ij}^- + D_{ijkl}(\sigma^+) \left(\Delta\, \varepsilon_{kl} - \Delta\, \varepsilon_{kl}^{ip} - \Delta\, \varepsilon_{kl}^{dp} \right)$$

Taking into account what precedes, one from of deduced that the nonlinear system to solve is composed of:

- LE_{ij} : the elastic law state: $\sigma_{ij}^+ - \sigma_{ij}^- - D_{ijkl}(\sigma^+) \left(\Delta\, \varepsilon_{kl} + \frac{1}{3} \Delta\, \lambda^i\, \delta_{kl} - \Delta\, \lambda^d\, G_{kl}^d(\sigma^+, R^+, \boldsymbol{X}^+) \right) = 0$
- LQ : the law of work hardening of the internal variable Q_{iso} : $Q_{iso}^+ - Q_{iso}^- - D\, \lambda^i\, G^{Q_{iso}}(Q_{iso}^+) = 0$
- LR : the law of work hardening of the variable R : $R^+ - R^- - \Delta\, \lambda^d\, G^R(\sigma^+, R^+) = 0$
- LX_{ij} : the law of work hardening of the variable X_{ij} : $X_{ij}^+ - X_{ij}^- - \Delta\, \lambda^d\, G_{ij}^X(\sigma^+, \boldsymbol{X}^+) = 0$

- FI : the equation of the isotropic surface of load: $-\frac{I_1^+ + Q_{init}}{3} + Q_{iso}^+ = 0$
- FD : the equation of the surface of load déviatoire: $q_{II}^+ \; h(\theta_q^+) + R^+ (I_1^+ + Q_{init}) = 0$

As in the preceding paragraphs one solves by the method of Newton the system $R(Y)=0$, where the unknown factor Y is given by $Y=(\sigma_{ij}^+, Q_{iso}^+, R^+, X_{ij}^+, \Delta\lambda^i, \Delta\lambda^d)$ and where $R=(LE_{ij}, LQ, LR, LX_{ij}, FI, FD)$.

4.5.1 Initialization and solution of test

Starting from the state at the moment t $(\sigma_{ij}^-, Q_{iso}^-, R^-, X_{ij}^-)$, we seek a solution of test which brings us closer to the final solution. For that we solve the system of equations according to:

$$\begin{cases} f^i\left(s_{ij}^- + D_{ijkl}^+\left(De_{kl} + \frac{1}{3} D\lambda^i d_{kl} - D\lambda^d G_{kl}^d\right), Q_{iso}^- + D\lambda^i G^{Q_{iso}-}\right) = 0 \\ f^d\left(s_{ij}^- + D_{ijkl}^+\left(De_{kl} + \frac{1}{3} D\lambda^i d_{kl} - D\lambda^d G_{kl}^d\right), R^- + D\lambda^d G^{R-}, X_{ij}^- + D\lambda^d G_{ij}^{X-}\right) = 0 \end{cases}$$

with:

$D_{ijkl}^- = D_{ijkl}(\sigma^-)$, $G_{kl}^{d-} = G_{kl}^d(\sigma^-, R^-, \boldsymbol{X}^-)$, $G^{Q_{iso}-} = G^{Q_{iso}}(Q_{iso}^-)$ $G^{R-} = G^R(\sigma^-, R^-)$, $G_{ij}^{X-} = G_{ij}^X(\sigma^-, \boldsymbol{X}^-)$ and where the unknown factors are the plastic multipliers $\Delta\lambda^i$ and $\Delta\lambda^d$, by only one iteration of Newton, i.e. finally that we have:

$$\begin{aligned} \frac{\partial f^i}{\partial \Delta\lambda^i}\Big|_{\Delta\lambda^i=0,\, \Delta\lambda^d=0} \Delta\lambda^i + \frac{\partial f^i}{\partial \Delta\lambda^d}\Big|_{\Delta\lambda^i=0,\, \Delta\lambda^d=0} \Delta\lambda^d &= -f^i{}_{|\Delta\lambda^i=0,\, \Delta\lambda^d=0} \\ \frac{\partial f^d}{\partial \Delta\lambda^i}\Big|_{\Delta\lambda^i=0,\, \Delta\lambda^d=0} \Delta\lambda^i + \frac{\partial f^d}{\partial \Delta\lambda^d}\Big|_{\Delta\lambda^i=0,\, \Delta\lambda^d=0} \Delta\lambda^d &= -f^d{}_{|\Delta\lambda^i=0,\, \Delta\lambda^d=0} \end{aligned}$$

that is to say still:

$$\Delta\lambda^i = \frac{\frac{\partial f^i}{\partial \Delta\lambda^d} f^d - \frac{\partial f^d}{\partial \Delta\lambda^d} f^i}{\frac{\partial f^i}{\partial \Delta\lambda^i} \frac{\partial f^d}{\partial \Delta\lambda^d} - \frac{\partial f^i}{\partial \Delta\lambda^d} \frac{\partial f^d}{\partial \Delta\lambda^i}} \quad \text{and} \quad \Delta\lambda^d = \frac{\frac{\partial f^d}{\partial \Delta\lambda^i} f^i - \frac{\partial f^i}{\partial \Delta\lambda^i} f^d}{\frac{\partial f^i}{\partial \Delta\lambda^i} \frac{\partial f^d}{\partial \Delta\lambda^d} - \frac{\partial f^i}{\partial \Delta\lambda^d} \frac{\partial f^d}{\partial \Delta\lambda^i}}$$

with:

$$\frac{\partial f^i}{\partial \Delta\lambda^i} = -(K^- + K^{p-})$$

$$\frac{\partial f^i}{\partial \Delta\lambda^d} = K^- \; tr(G^{d-})$$

$$\frac{\partial f^d}{\partial \Delta\lambda^i} = h(\theta_q) \frac{\partial q_{II}}{\partial \Delta\lambda^i} + q_{II} \frac{\partial h(\theta_q)}{\partial \Delta\lambda^i} + (I_1 + Q_{init}) \frac{\partial R}{\partial \Delta\lambda^i} + R \frac{\partial I_1}{\partial \Delta\lambda^i}$$

$$\frac{\partial f^d}{\partial \Delta\lambda^d} = h(\theta_q) \frac{\partial q_{II}}{\partial \Delta\lambda^d} + q_{II} \frac{\partial h(\theta_q)}{\partial \Delta\lambda^d} + (I_1 + Q_{init}) \frac{\partial R}{\partial \Delta\lambda^d} + R \frac{\partial I_1}{\partial \Delta\lambda^d}$$

It is known that $\frac{\partial f^d}{\partial \Delta \lambda^d}$ is calculated in the same way that previously when only the plastic mechanism déviatoire was activated. In addition, one has, for the calculation of $\frac{\partial f^d}{\partial \Delta \lambda^i}$ and when $\Delta \lambda^i = 0$ and $\Delta \lambda^d = 0$, following relations:

$$\frac{\partial q_{ij}}{\partial \Delta \lambda^i} = \frac{1}{3} D^-_{ijkl} \delta_{kl} - 3 K^{e-} \left(\frac{1}{3} \delta_{ij} + X^-_{ij} \right)$$

$$\frac{\partial q_{II}}{\partial \Delta \lambda^i} = \frac{\partial q_{II}}{\partial q_{ij}} \frac{\partial q_{ij}}{\partial \Delta \lambda^i} = \frac{q_{ij}}{q_{II}} \left(\frac{1}{3} D^-_{ijkl} \delta_{kl} - 3 K^{e-} \left(\frac{1}{3} \delta_{ij} + X^-_{ij} \right) \right)$$

$$\frac{\partial h(\theta_q)}{\partial \Delta \lambda^i} = \frac{\partial h(\theta_q)}{\partial q_{ij}} \frac{\partial q_{ij}}{\partial \Delta \lambda^i}$$

$$\frac{\partial R}{\partial \Delta \lambda^i} = 0$$

$$\frac{\partial I_1}{\partial \Delta \lambda^i} = 3 K^-$$

Ultimately, we take for the solution of test: $Y^0 = \left(\sigma^0_{ij}, Q^0_{iso}, R^0, X^0_{ij}, \Delta \lambda^{i0}, \Delta \lambda^{d0} \right)$, with the following values:

$\Delta \lambda^{i0}$: the value found according to the preceding formulation.

$\Delta \lambda^{d0}$: the value found according to the preceding formulation.

$$\sigma^0_{ij} = \sigma^-_{ij} + D^-_{ijkl} \left(\Delta \varepsilon_{kl} + \frac{1}{3} \Delta \lambda^{i0} \delta_{kl} - \Delta \lambda^{d0} G^{d-}_{kl} \right)$$

$$Q^0_{iso} = Q^-_{iso} + \Delta \lambda^{i0} G^{Q_{iso}-}$$

$$R^0 = R^- + \Delta \lambda^{d0} G^{R-}$$

$$X^0_{ij} = X^-_{ij} + \Delta \lambda^{d0} G^{X-}_{ij}$$

4.5.2 Iterations of Newton

$\frac{DR}{DY}$ is given here by:

$$\frac{DR}{DY}=\begin{bmatrix}\frac{\partial LE_{ij}}{\partial\sigma_{kl}} & \frac{\partial LE_{ij}}{\partial Q_{iso}} & \frac{\partial LE_{ij}}{\partial R} & \frac{\partial LE_{ij}}{\partial X_{kl}} & \frac{\partial LE_{ij}}{\partial\Delta\lambda^i} & \frac{\partial LE_{ij}}{\partial\Delta\lambda^d}\\ \frac{\partial LQ}{\partial\sigma_{kl}} & \frac{\partial LQ}{\partial Q_{iso}} & \frac{\partial LQ}{\partial R} & \frac{\partial LQ}{\partial X_{kl}} & \frac{\partial LQ}{\partial\Delta\lambda^i} & \frac{\partial LQ}{\partial\Delta\lambda^d}\\ \frac{\partial LR}{\partial\sigma_{kl}} & \frac{\partial LR}{\partial Q_{iso}} & \frac{\partial LR}{\partial R} & \frac{\partial LR}{\partial X_{kl}} & \frac{\partial LR}{\partial\Delta\lambda^i} & \frac{\partial LR}{\partial\Delta\lambda^d}\\ \frac{\partial LX_{ij}}{\partial\sigma_{kl}} & \frac{\partial LX_{ij}}{\partial Q_{iso}} & \frac{\partial LX_{ij}}{\partial R} & \frac{\partial LX_{ij}}{\partial X_{kl}} & \frac{\partial LX_{ij}}{\partial\Delta\lambda^i} & \frac{\partial LX_{ij}}{\partial\Delta\lambda^d}\\ \frac{\partial FI}{\partial\sigma_{kl}} & \frac{\partial FI}{\partial Q_{iso}} & \frac{\partial FI}{\partial R} & \frac{\partial FI}{\partial X_{kl}} & \frac{\partial FI}{\partial\Delta\lambda^i} & \frac{\partial FI}{\partial\Delta\lambda^d}\\ \frac{\partial FD}{\partial\sigma_{kl}} & \frac{\partial FD}{\partial Q_{iso}} & \frac{\partial FD}{\partial R} & \frac{\partial FD}{\partial X_{kl}} & \frac{\partial FD}{\partial\Delta\lambda^i} & \frac{\partial FD}{\partial\Delta\lambda^d}\end{bmatrix}$$

where the new terms are worthless:

$$\frac{\partial LQ}{\partial R}=0\ ,\ \frac{\partial LQ}{\partial X_{kl}}=0\ ,\ \frac{\partial LQ}{\partial\Delta\lambda^d}=0\ ,\ \frac{\partial LR}{\partial Q_{iso}}=0\ ,\ \frac{\partial LR}{\partial\Delta\lambda^i}=0\ ,\ \frac{\partial LX_{\mathbf{ij}}}{\partial Q_{iso}}=0\ ,$$
$$\frac{\partial LX_{\mathbf{ij}}}{\partial\Delta\lambda^i}=0\ ,\ \frac{\partial FI}{\partial R}=0\ ,\ \frac{\partial FI}{\partial X_{kl}}=0\ ,\ \frac{\partial FI}{\partial\Delta\lambda^d}=0\ ,\ \frac{\partial FD}{\partial Q_{iso}}=0\ ,\ \frac{\partial FD}{\partial\Delta\lambda^i}=0$$

and where the already definite terms remain unchanged, except for $\frac{\partial LE_{\mathbf{ij}}}{\partial\sigma_{kl}}$ who becomes:

$$\frac{\partial LE_{\mathbf{ij}}}{\partial\sigma_{kl}}=\delta_{ik}\delta_{jl}-D^{lineaire}_{ijmn}\left(\Delta\varepsilon_{mn}+\frac{1}{3}\Delta\lambda^i\delta_{mn}-\Delta\lambda^d G^d_{mn}\right)\frac{n}{3\ P_a}\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{n-1}\delta_{kl}+D_{ijmn}\Delta\lambda^d\frac{\partial G^d_{mn}}{\partial\sigma_{kl}}$$

4.5.3 Test of convergence

The convergence criteria remain $\frac{\|DY^{p+1}\|}{\|Y^{\mathbf{p+1}}-Y^0\|}$ □ RESI_INTE_RELA

4.6 Procedure of relieving based on an estimate of the normals on the surface of load déviatoire

When the plastic mechanism déviatoire intervenes, a procedure of relieving inside the iterations of Newton is taken into account. This one makes it possible to avoid certain problems of oscillation in the calculation of the solution Y^{p+1} who lead finally to nonthe convergence of digital integration.

Thus, with the iteration $p+1$, instead of bringing up to date the unknown factor Y^{p+1} by a complete increment $\delta\ Y^{p+1}$

$$Y^{p+1}=Y^p+\delta\quad Y^{p+1}$$

one poses

$$Y_m^{p+1}=Y^p+\rho_m\ \delta\quad Y^{p+1}$$

and one seeks, by carrying out a loop on under-iterations m , to determine an optimal value of the scalar ρ_m . This value is required by considering the rotation of the normal, in the plan déviatoire, on surface f^d, during under-iterations. This normal, noted $\tilde{n}_m$, expresses itself starting from the constraints contained in the term Y_m^{p+1} by

$$\tilde{\boldsymbol{n}}_{\boldsymbol{m}}=2\,h(\theta_q)^5\,q_{II}\,\frac{\partial f^d}{\partial q_{ij}}=(2+\gamma\quad\cos(3\,\theta_q))\,q_{ij}+\frac{\sqrt{6}\,\gamma}{q_{II}}\,\frac{\partial\det(q)}{\partial q_{ij}}$$

Starting from the initial value $\rho_0=1.0$, the process set up consists of the following stages:

- calculation of the normals $\tilde{\boldsymbol{n}}_{\boldsymbol{m-1}}$ and $\tilde{\boldsymbol{n}}_{\boldsymbol{m}}$
- calculation of the swing angle φ_m between these normals: $\cos\phi_m=\frac{\tilde{\boldsymbol{n}}_{m-1}:\tilde{\boldsymbol{n}}_m}{\sqrt{\tilde{\boldsymbol{n}}_{m-1}}\sqrt{\tilde{\boldsymbol{n}}_m}}$
- test on the evolution $\cos\phi_m$:

  if $\cos\phi_m\le TOLROT$ then $\rho_{m+1}=DECREL\,\rho_m$ and $m=m+1$

  if not end of the under-iterations and $Y^{p+1}=Y_m^{p+1}$

4.7 Recutting of the step of time

As for most relations of behavior, it was introduced for the model `CJS` the possibility of redécouper locally (at the points of Gauss) the step of time in order to facilitate digital integration. This possibility is managed by the operand `ITER_INTE_PAS` keyword `CONVERGENCE` of the operator `STAT_NON_LINE`. If `itepas`, the value of `ITER_INTE_PAS`, is worth 0.1 or -1 it has no recutting there (note: 0 are the value by default). If `itepas` recutting is positive is automatic, if it is negative recutting is taken into account only in the event of nonconvergence with the step of initial time.

Recutting consists in realizing, after the phase of elastic prediction, the integration of the plastic mechanisms brought into plays with an increment of deformation whose components correspond to the components of the increment of deformation initial divided by the absolute value of `itepas`.

4.8 Various remarks

4.8.1 Calculation of the term $\cos(\theta_s-\theta_q)$

The term $\cos(\theta_s-\theta_q)$ appears in the expression of φ_o. We adopted for his calculation the same method as that used with the ECL. I.e. we determine the angles θ_s and θ_q in the manner which follows:

$$\theta_s=\frac{1}{3}\,\mathrm{Arctan}\left(\frac{\sqrt{1\text{-}\cos^2(3\theta_s)}}{\cos(3\theta_s)}\right)\quad\text{and}\quad\theta_q=\frac{1}{3}\,\mathrm{Arctan}\left(\frac{\sqrt{1\text{-}\cos^2(3\;\theta_q)}}{\cos(3\;\theta_q)}\right)$$

then we take the cosine of the difference.

These expressions of θ_s and θ_q are also useful for calculation of:

$$\frac{\partial\cos(\theta_s-\theta_q)}{\partial\sigma_{kl}}=-\sin(\theta_s-\theta_q)\left(\frac{\partial\theta_s}{\partial\sigma_{kl}}-\frac{\partial\theta_q}{\partial\sigma_{kl}}\right)$$

with $\frac{\partial\theta_s}{\partial\sigma_{kl}}=-\frac{1}{3}\sqrt{1-\cos^2(3\;\theta_s)}\frac{\sqrt{54}}{q_{II}^3}\left(t_{kl}-3\frac{\det(q)}{q_{II}^2}q_{kl}\right)$

4.8.2 Calculation of R_r

The ray of rupture introduced into model CJS3 is given by the formula

$$R_r = R_c + \mu \ \ln\left(\frac{3\ p_c}{I_1 + Q_{init}}\right)$$

In fact, when $\frac{I_1 + Q_{init}}{3} > p_c$, one must block R_r with the value of R_c . The field of dilatence disappears and one does not only admit R_r can decrease in on this side R_c . Consequently, one introduces, instead of the preceding formulation, the following expression

$$R_r = R_c + \mu \ \max\left[0,\ \ln\left(\frac{3\ p_c}{I_1 + Q_{init}}\right)\right]$$

4.8.3 Traction

Non-cohesive, the field of traction which corresponds to positive constraints is inadmissible for the grounds. From the point of view of the integration of model CJS, when the state of the constraints tends towards the top of the cone of the surface of load, the digital risk to rock in this prohibited field increases. However when that one projects oneself or when one makes a prediction in a point of this field, digital calculation leads either to an erroneous result, or with a fatal error. Indeed, traction appears numerically by a value of I_1 positive. This value poses then problem at the time to evaluate certain quantities like $\left(\frac{I_1^+ + Q_{init}}{3\ P_a}\right)^{-1.5}$; in addition it would generate from a theoretical point of view a value q_{II} negative according to the equation of the surface of load déviatoire.

Such a phenomenon was detected on several levels: in a particular way in the elastic prediction with model CJS1, and in a general way in the local iterations of Newton utilizing the mechanism déviatoire. The same answer was given in order to free itself from this pathology: it is a question of virtually projecting the constraints in the elastic range on the hydrostatic axis while posing:

$$\sigma_{11} = \sigma_{22} = \sigma_{33} = -1\ kPa$$
$$\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$$

One thus repositions the state of stresses in the field of compression while moving away little from the inadmissible initial prediction considered, and by hoping that the considerations of structures will make it possible total calculation to converge.

Moreover internal variables do not evolve and one supposes being returned in the elastic range

5 Tangent operator

The tangent operator called by the option `RIGI_MECA_TANG` corresponds to the tangent operator deduced from the problem of speed and calculated starting from the results known at the moment T.

The tangent operator called by the option `FULL_MECA` should correspond to the tangent operator with the discretized problem in an implicit way. Actually, we did not carry out this calculation. We take then, when the option `FULL_MECA` is retained, the tangent operator deduced from the problem of speed and calculated starting from the results known at the moment t+dt.

We detail below the tangent operator deduced from the problem of it speed according to or of the concerned mechanisms.

5.1 Tangent operator of the nonlinear elastic mechanism

We have simply the following nonlinear elastic relation:

$$\dot{\sigma}_{ij}=D_{ijkl}(\sigma)\,\dot{\varepsilon}_{kl}=\left(\frac{I_1+Q_{init}}{3\,P_a}\right)^n D_{ijkl}^{lineaire}\,\dot{\varepsilon}_{kl}$$

from where immediately the tangent operator:

$$H_{ijkl}^{elas.nl}=\left(\frac{I_1+Q_{init}}{3\,P_a}\right)^n D_{ijkl}^{lineaire}$$

5.2 Tangent operator of the mechanisms isotropic rubber band and plastic

In this case, we have the following relation:

$$\dot{\sigma}_{ij}=D_{ijkl}(\sigma)\left(\dot{\varepsilon}_{kl}-\dot{\varepsilon}_{kl}^{ip}\right)=D_{ijkl}(\sigma)\left(\dot{\varepsilon}_{kl}+\frac{1}{3}\dot{\lambda}^i\,\delta_{kl}\right)$$

it comes: $\dot{I}_1=3\,K\left(\dot{\varepsilon}_v+\dot{\lambda}^i\right)$

By taking account of this relation and the law of work hardening of Q_{iso}, the condition $\dot{f}^i=0$ becomes:

$$\dot{f}^i=-\frac{\dot{I}_1}{3}+\dot{Q}_{iso}=-K\left(\dot{\varepsilon}_v+\dot{\lambda}^i\right)-K^p\,\dot{\lambda}^i=0$$

that is to say: $\dot{\lambda}^i=-\frac{K}{K+K^p}\,\dot{\varepsilon}_v$

By deferring this result in the expression of $\dot{\sigma}_{ij}$, one finds:

$$\dot{\sigma}_{ij}=D_{ijkl}\left(\dot{\varepsilon}_{kl}-\frac{1}{3}\frac{K}{K+K^p}\,\dot{\varepsilon}_{mm}\,\delta_{kl}\right)=\left(D_{ijkl}-\frac{1}{3}\frac{K}{K+K^p}\,D_{ijmn}\,\delta_{mn}\,\delta_{kl}\right)\dot{\varepsilon}_{kl}$$

from where the tangent operator:

$$H_{ijkl}^{ip}=D_{ijkl}-\frac{1}{3}\frac{K}{K+K^p}\,D_{ijmn}\,\delta_{mn}\,\delta_{kl}$$

One can also write in matric form:

$$H^{ip}=\left(\frac{I_1}{3\ P_a}\right)^n\begin{bmatrix}\lambda\ -\chi\ +2\ \mu & \lambda\ -\chi & \lambda\ -\chi & 0 & 0 & 0\\ \lambda\ -\chi & \lambda\ -\chi\ +2\ \mu & \lambda\ -\chi & 0 & 0 & 0\\ \lambda\ -\chi & \lambda\ -\chi & \lambda\ -\chi\ +2\ \mu & 0 & 0 & 0\\ 0 & 0 & 0 & 2\ \mu & 0 & 0\\ 0 & 0 & 0 & 0 & 2\ \mu & 0\\ 0 & 0 & 0 & 0 & 0 & 2\ \mu\end{bmatrix}$$

where for this formula only λ and μ are the coefficient of Lamé and $\chi=\frac{K_o^{e^2}}{K_o^e+K_o^p}$.

5.3 Tangent of the mechanisms rubber band and plastic operator déviatoire

The condition $\dot{f}^d=0$ is written:

$$\dot{f}^d=\frac{\partial f^d}{\partial\sigma_{ij}}\dot{\sigma}_{ij}+\frac{\partial f^d}{\partial R}\dot{R}+\frac{\partial f^d}{\partial X_{ij}}\dot{X}_{ij}=\frac{\partial f^d}{\partial\sigma_{ij}}\dot{\sigma}_{ij}+\frac{\partial f^d}{\partial R}\dot{\lambda}^d G^R+\frac{\partial f^d}{\partial X_{ij}}\dot{\lambda}^d G_{ij}^X=0$$

The tensor G^X being purely déviatoire, the product $\frac{\partial f^d}{\partial X_{ij}}G_{ij}^X$ is reduced to:

$$\frac{\partial f^d}{\partial X_{ij}}G_{ij}^X=dev\left(\frac{\partial f^d}{\partial X_{ij}}\right)G_{ij}^X=-I_1\ Q_{ij}\ G_{ij}^X$$

The plastic multiplier can thus be put in the form:

$$\dot{\lambda}^d=\frac{1}{H^{dev}}\frac{\partial f^d}{\partial\sigma_{ij}}\dot{\sigma}_{ij}$$

while revealing the plastic module H^{dev}, given by:

$$H^{dev}=I_1^2\left(\frac{I_1+Q_{init}}{3\ P_a}\right)^{-1,5}\left[A\left(1-\frac{R}{R_m}\right)^2+\frac{1}{b}\ Q_{ij}\left(Q_{ij}+\varphi\ \ X_{ij}\right)\right]$$

The relation stress-strains then makes it possible to write:

$$\frac{\partial f^d}{\partial\sigma_{ij}}\dot{\sigma}_{ij}=\frac{\partial f^d}{\partial\sigma_{ij}}D_{ijkl}\left(\dot{\varepsilon}_{kl}-\dot{\lambda}^d G_{kl}^d\right)=\frac{\partial f^d}{\partial\sigma_{ij}}D_{ijkl}\ \dot{\varepsilon}_{kl}-\dot{\lambda}^d\frac{\partial f^d}{\partial\sigma_{ij}}D_{ijkl}\ G_{kl}^d$$

what gives finally for the plastic multiplier:

$$\dot{\lambda}^d=\frac{\frac{\partial f^d}{\partial\sigma_{ij}}D_{ijkl}\ \dot{\varepsilon}_{kl}}{H^{dev}+\frac{\partial f^d}{\partial\sigma_{ij}}D_{ijkl}\ G_{kl}^d}$$

By deferring this result in the expression of $\dot{\sigma}_{ij}$, one finds:

$$\dot{\sigma}_{ij}=D_{ijkl}\left(\dot{\varepsilon}_{kl}-\frac{\frac{\partial f^d}{\partial\sigma_{pq}}D_{pqmn}\ \dot{\varepsilon}_{mn}}{H^{dev}+\frac{\partial f^d}{\partial\sigma_{rs}}D_{rstu}\ G_{tu}^d}G_{kl}^d\right)$$

from where the tangent operator:

$$H^{dp}_{ijkl}=D_{ijkl}-D_{ijmn}\,G^d_{mn}\,\frac{\dfrac{\partial f^d}{\partial\sigma_{pq}}\,D_{pqkl}}{H^{dev}+\dfrac{\partial f^d}{\partial\sigma_{rs}}\,D_{rstu}\,G^d_{tu}}$$

The tangent operator thus obtained is not symmetrical. However for the moment law CJS is pressed on finite elements which claim a symmetrical operator. Ultimately, we retain not H^{dp}_{ijkl} but $\tilde{H}^{dp}_{ijkl}$ who is given by:

$$\tilde{H}^{dp}_{ijkl}=\frac{H^{dp}_{ijkl}+H^{dp}_{klij}}{2}\quad\text{with } ij \text{ and } kl \text{ taken in } (11,\,22,\,33,\,12,\,13,\,23)$$

5.4 Tangent operator of the mechanisms rubber band, plastics isotropic and déviatoire

One must satisfy the two following conditions: $\dot{f}^i=0$ and $\dot{f}^d=0$. Taking into account the relation stress-strains which is written:

$$\dot{\sigma}_{ij}=D_{ijkl}\left(\dot{\varepsilon}_{kl}+\frac{1}{3}\dot{\lambda}^i\,\delta_{kl}-\dot{\lambda}^d\,G^d_{kl}\right)$$

the first condition gives:

$$\dot{f}^i=-K\left(\dot{\varepsilon}_v+\dot{\lambda}^i-\dot{\lambda}^d\,G^d_v\right)-K^p\,\dot{\lambda}^i=0$$

where one posed $G^d_v=G^d_{kk}=tr\left(\boldsymbol{G}^d\right)$.

The second condition led to:

$$\frac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,\dot{\varepsilon}_{kl}+\frac{1}{3}\dot{\lambda}^i\,\frac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,\delta_{kl}-\dot{\lambda}^d\,\frac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,G^d_{kl}-H^{dev}\,\dot{\lambda}^d=0$$

Thus, plastic multipliers $\dot{\lambda}^i$ and $\dot{\lambda}^d$ are obtained by solving the system:

$$\begin{cases}-\left(K+K^p\right)\dot{\lambda}^i+K\,G^d_v\,\dot{\lambda}^d=K\,\dot{\varepsilon}_v\\[2mm]-\dfrac{1}{3}\,\dfrac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,\delta_{kl}\,\dot{\lambda}^i+\left(\dfrac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,G^d_{kl}+H^{dev}\right)\dot{\lambda}^d=\dfrac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,\dot{\varepsilon}_{kl}\end{cases}$$

that is to say:

$$\dot{\lambda}^i=\frac{K\,G^d_v\,\dfrac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,\dot{\varepsilon}_{kl}-\left(\dfrac{\partial f^d}{\partial\sigma_{mn}}\,D_{mnpq}\,G^d_{pq}+H^{dev}\right)K\,\dot{\varepsilon}_v}{\left(K+K^p\right)\left(\dfrac{\partial f^d}{\partial\sigma_{rs}}\,D_{rstu}\,G^d_{tu}+H^{dev}\right)-\dfrac{1}{3}\,K\,G^d_v\,\dfrac{\partial f^d}{\partial\sigma_{vw}}\,D_{wvxy}\,\delta_{xy}}$$

$$\dot{\lambda}^d=\frac{\left(K+K^p\right)\dfrac{\partial f^d}{\partial\sigma_{ij}}\,D_{ijkl}\,\dot{\varepsilon}_{kl}-\dfrac{1}{3}K\,\dfrac{\partial f^d}{\partial\sigma_{mn}}\,D_{mnpq}\,\delta_{pq}\,\dot{\varepsilon}_v}{\left(K+K^p\right)\left(\dfrac{\partial f^d}{\partial\sigma_{rs}}\,D_{rstu}\,G^d_{tu}+H^{dev}\right)-\dfrac{1}{3}\,K\,G^d_v\,\dfrac{\partial f^d}{\partial\sigma_{vw}}\,D_{vwxy}\,\delta_{xy}}$$

These expressions are still written:

$$\dot{\lambda}^i = T_{1_{kl}} \dot{\varepsilon}_{kl} \quad \text{and} \quad \dot{\lambda}^d = T_{2_{kl}} \dot{\varepsilon}_{kl}$$

where tensors $\boldsymbol{T_1}$ and $\boldsymbol{T_2}$ are given by:

$$T_{1_{kl}} = \frac{K\, G_v^d \frac{\partial f^d}{\partial \sigma_{ij}} D_{ijkl} - \left(\frac{\partial f^d}{\partial \sigma_{mn}} D_{mnpq}\, G_{pq}^d + H^{dev} \right) K\, \delta_{kl}}{\left(K + K^p\right) \left(\frac{\partial f^d}{\partial \sigma_{rs}} D_{rstu}\, G_{tu}^d + H^{dev} \right) - \frac{1}{3} K\, G_v^d \frac{\partial f^d}{\partial \sigma_{vw}} D_{vwxy}\, \delta_{xy}}$$

$$T_{2_{kl}} = \frac{\left(K + K^p\right) \frac{\partial f^d}{\partial \sigma_{ij}} D_{ijkl} - \frac{1}{3} K \frac{\partial f^d}{\partial \sigma_{mn}} D_{mnpq}\, \delta_{pq}\, \delta_{kl}}{\left(K + K^p\right) \left(\frac{\partial f^d}{\partial \sigma_{rs}} D_{rstu}\, G_{tu}^d + H^{dev} \right) - \frac{1}{3} K\, G_v^d \frac{\partial f^d}{\partial \sigma_{vw}} D_{vwxy}\, \delta_{xy}}$$

By deferring the expressions $\dot{\lambda}^i$ and $\dot{\lambda}^d$ of in the formula of $\dot{\sigma}_{ij}$, one finds:

$$\dot{\sigma}_{ij} = D_{ijkl} \left(\dot{\varepsilon}_{kl} + \frac{1}{3} T_{1_{nm}} \dot{\varepsilon}_{nm}\, \delta_{kl} - T_{2_{pq}} \dot{\varepsilon}_{pq}\, G_{kl}^d \right)$$

from where the tangent operator:

$$H_{ijkl}^{idp} = D_{ijkl} + \frac{1}{3} D_{ijmn}\, \delta_{mn}\, T_{1_{kl}} - D_{ijpq}\, G_{pq}^d\, T_{2_{kl}}$$

This tangent operator not being symmetrical, we retain not H_{ijkl}^{idp} but $\tilde{H}_{ijkl}^{idp}$ who is given by:

$$\tilde{H}_{ijkl}^{idp} = \frac{H_{ijkl}^{idp} + H_{klij}^{idp}}{2} \quad \text{with } ij \text{ and } kl \text{ taken in } (11,\ 22,\ 33,\ 12,\ 13,\ 23)$$

6 Aster sources

6.1 List of the modified and added routines

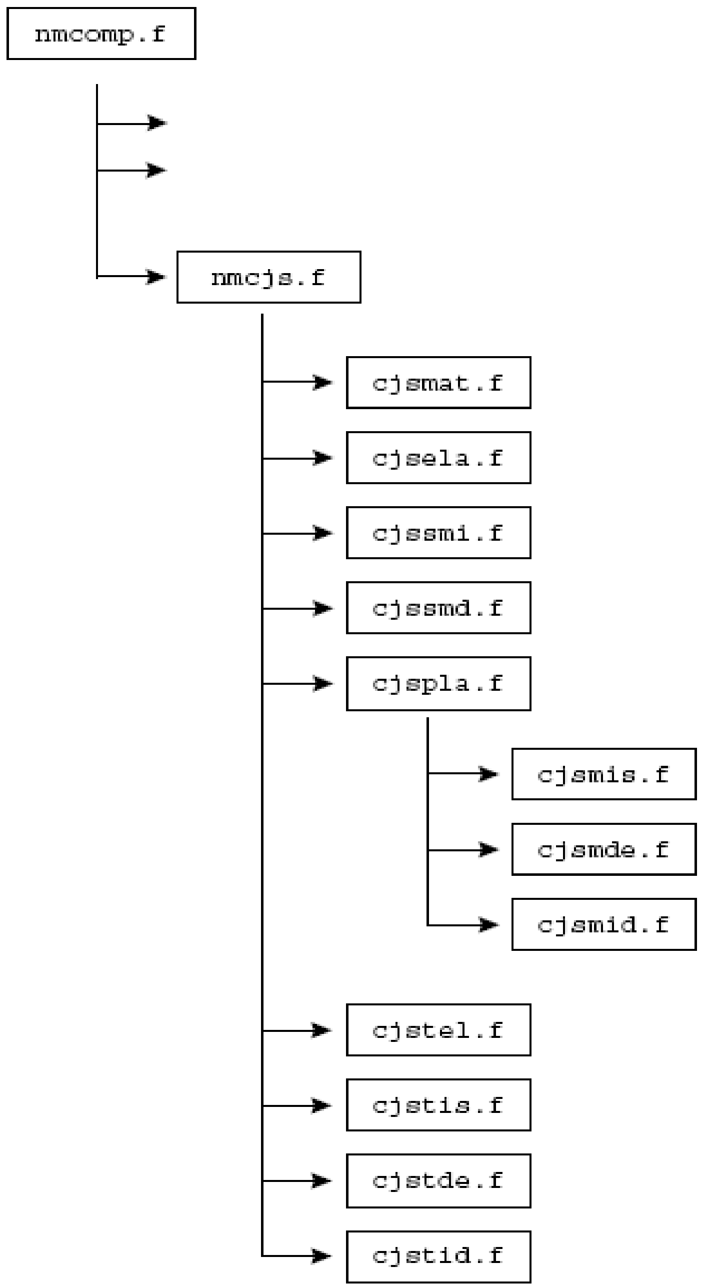
Only the routine `nmcomp.f` was modified. It makes it possible to call, when behavior CJS is chosen, the routine `nmcjs.f`, starting point of the integration of the law.

The whole of routines FORTRAN developed within the framework of the integration of law CJS in *Code_Aster* is the following:

```
cjsc3q.f,    cjsci1.f,    cjsdtd.f,    cjsela.f,    cjside.f,    cjsiid.f,
cjsjde.f,    cjsjid.f,    cjsjis.f,    cjsmat.f,    cjsmde.f,    cjsmid.f,
cjsmis.f,    cjsnor.f,    cjspla.f,    cjsqco.f,    cjsqij.f,    cjssmd.f,
cjssmi.f,    cjst.f,      cjstde.f,    cjstel.f,    cjstid.f,    cjstis.f,
lcdete.f,    nmcjs.f,     cjsinp.f,    cjsncn.f,    cjsncv.f,    cjsnvi.f,
cjsqq.f.
```

6.2 Top-level flowchart of the principal routines

Principal routines FORTRAN for the integration of law CJS are connected in the following way:



6.3 Details of the features of developed routines FORTRAN

6.3.1 Routine: CJSC3Q

Objective: calculation of $\cos(3\,\theta_q)$

Variables of entry and exit :

```
IN     SIG      :  CONSTRAINTS
       X        :  VARIABLES HAMMER-HARDENED MOVIES
        Pa       :  CLOSE ATMOSPHERIC (DATA MATERIAL)

OUT    Q        :  DEV. (SIG) - TRACE (SIG) *X
       QII      :  SQRT (QIJ*QIJ)
       COS3TQ   :  SQRT (54) *DET (Q)/(QII ** 3)
```

6.3.2 Routine: CJSCI1

Objective:

resolution of the equation $I_1^+ - I_1^- - 3\,K_o^e \left(\frac{I_1^+}{3\,P_a}\right)^n tr(\Delta\,\varepsilon) = 0$ by the method of the secant, for the nonlinear elastic behavior

Variables of entry and exit :

```
IN     CRIT  : CONVERGENCE CRITERIA
       MATER: COEFFICIENTS MATERIAL WITH T+DT
       LIFO  : INCREMENT OF DEFORMATION
       SIGD  : CONSTRAINT WITH T
OUT    I1    : TRACE OF SIG WITH T+DT
       LEAFLET: LOGICAL VARIABLE INDICATING TRACTION
```

6.3.3 Routine: CJSDTD

Objective:

calculation of the derivative of the tensor $\boldsymbol{t}^d$ compared to $\boldsymbol{q}$

Variables of entry and exit :

```
IN     MOD    :  MODELING
       Q      :  TENSOR (6 COMPONENTS)
OUT    DTDDQ  :  TENSOR RESULT (6 COMPONENTS)
```

6.3.4 Routine: CJSELA

Objective:

nonlinear elastic design of the constraints

Variables of entry and exit :

```
IN     MOD    :  MODELING
       CRIT   :  CONVERGENCE CRITERIA
       MATERF:  COEFFICIENTS MATERIAL WITH T+DT
       SIGD   :  CONSTRAINT  WITH T
       LIFO   :  INCREMENT OF DEFORMATION
```

```
OUT    SIGF    :  CONSTRAINT WITH T+DT
```

Organization of CJSELA
- calculation of the first invariant of the constraints I1 with t+dt:
  - call of CJSCI1
- calculation of the coefficients of the elastic matrix and assembly of the matrix
- calculation of the increment of the constraints and the constraints with t+dt:
  - call of LCPRMV and LCSOVE

6.3.5 Routine: CJSIDE

Objective:

for the integration of the plastic mechanism déviatoire, calculation of a solution of test in order to start the local iterations of Newton then.

Variables of entry and exit :

```
IN     MOD      :  MODELING
       MATER    :  COEFFICIENTS MATERIAL WITH T+DT
       EPSD     :  DEFORMATION WITH T+DT
       LIFO     :  INCREMENT OF DEFORMATION
       YD       :  VARIABLES WITH T = (SIGD, VIND, LAMB)
VAR    GD       :  TENSOR OF THE LAW D FLOW PLASTIC DEV.
OUT    DY       :  SOLUTION D TEST
```

Organization of CJSIDE
- calculation of the elastic operator,
- calculation of laws of work hardening G^R and $\boldsymbol{G}^X$,
- calculation of the law of flow of the plastic mechanism déviatoire $\boldsymbol{G}^d$,
- calculation of the threshold f^d , of its derivative $\dfrac{\partial f^d}{\partial \Delta\ \lambda^d}$ and of the plastic multiplier $\Delta\ \lambda^d$,
- calculation of the solution of test

6.3.6 Routine: CJSIID

Objective:

for the simultaneous integration of the plastic mechanisms isotropic and déviatoire, calculation of a solution of test in order to start the local iterations of Newton then.

Variables of entry and exit :

```
IN     MOD      :  MODELING
       MATER    :  COEFFICIENTS MATERIAL WITH T+DT
       EPSD     :  DEFORMATION WITH T+DT
       LIFO     :  INCREMENT OF DEFORMATION
       YD       :  VARIABLES WITH T = (SIGD, VIND, LAMB)
VAR    GD       :  TENSOR OF THE LAW D FLOW PLASTIC DEV.
OUT    DY       :  SOLUTION D TEST
```

Organization of CJSIID

- calculation of the elastic operator,
- calculation of laws of work hardening G^R and $\mathbf{G}^X$,
- calculation of the law of flow of the plastic mechanism déviatoire $\mathbf{G}^d$,
- calculation of the thresholds f^i and f^d , of their derivative $\frac{\partial f^i}{\partial \Delta \lambda^i}$, $\frac{\partial f^i}{\partial \Delta \lambda^d}$, $\frac{\partial f^d}{\partial \Delta \lambda^i}$ and $\frac{\partial f^d}{\partial \Delta \lambda^d}$, and of the plastic multipliers $\Delta \lambda^i$ and $\Delta \lambda^d$,
- calculation of the solution of test

6.3.7 Routine: CJSJDE

Objective:

calculation of $DRDY$ and R for the resolution of $\frac{DR}{DY}(Y^p)\ DY^{p+1}=-R(Y^p)$ (plastic mechanism déviatoire)

Variables of entry and exit :

```
IN     MOD       :  MODELING
       MATER     :  COEFFICIENTS MATERIAL WITH T+DT
       EPSD      :  DEFORMATION WITH T
       LIFO      :  INCREMENT OF DEFORMATION
       YD        :  VARIABLES WITH T = (SIGD, VIND, LAMBDD)
       YF        :  VARIABLES WITH T+DT = (SIGF, VINF, LAMBDF)
VAR    GD        :  TENSOR OF THE LAW D FLOW PLASTIC DEV.
OUT    R         :  SECOND MEMBER
       SIGN     :  SIGN OF S: DEPSDP
       DRDY      :  JACOBIEN
```

Organization of CJSJDE

- calculation of the elastic operator,
- calculation of laws of work hardening G^R and $\mathbf{G}^X$,
- calculation of the law of flow of the plastic mechanism déviatoire $\mathbf{G}^d$,
- calculation of multiple derivative intermediaries
- calculation of the terms $\frac{\partial G^R}{\partial \sigma_{ij}}$, $\frac{\partial G^R}{\partial R}$, $\frac{\partial G^X_{mn}}{\partial \sigma_{ij}}$, $\frac{\partial G^X_{mn}}{\partial X_{ij}}$, $\frac{\partial G^d_{mn}}{\partial \sigma_{ij}}$, $\frac{\partial G^d_{mn}}{\partial R}$, $\frac{\partial G^d_{mn}}{\partial X_{ij}}$
- calculation of the components of $DRDY$ and R
- assembly of $DRDY$ and R

6.3.8 Routine: CJSJID

Objective:

calculation of $DRDY$ and R for the resolution of $\frac{DR}{DY}\left(Y^p\right)\ DY^{p+1}=-R\left(Y^p\right)$ (plastic mechanisms isotropic and déviatoire)

Variables of entry and exit :

```
IN     MOD       :  MODELING
       MATER     :  COEFFICIENTS MATERIAL WITH T+DT
       EPSD      :  DEFORMATION WITH T
       LIFO      :  INCREMENT OF DEFORMATION
       YD        :  VARIABLES WITH T = (SIGD, VIND, LAMBDD)
       YF        :  VARIABLES WITH T+DT = (SIGF, VINF, LAMBDF)
VAR    GD        :  TENSOR OF THE LAW D FLOW PLASTIC DEV.
OUT    R         :  SECOND MEMBER
       SIGN     :  SIGN OF S: DEPSDP
       DRDY      :  JACOBIEN
```

Organization of CJSJID

- calculation of the elastic operator,
- calculation of laws of work hardening $G^{Q_{iso}}$, G^R and $\boldsymbol{G}^{\boldsymbol{X}}$,
- calculation of the law of flow of the plastic mechanism déviatoire $\boldsymbol{G}^{\boldsymbol{d}}$,
- calculation of multiple derivative intermediaries
- calculation of the terms $\frac{\partial G^{Q_{iso}}}{\partial Q_{iso}}$, $\frac{\partial G^R}{\partial \sigma_{ij}}$, $\frac{\partial G^R}{\partial R}$, $\frac{\partial G^X_{mn}}{\partial \sigma_{ij}}$, $\frac{\partial G^X_{mn}}{\partial X_{ij}}$, $\frac{\partial G^d_{mn}}{\partial \sigma_{ij}}$, $\frac{\partial G^d_{mn}}{\partial R}$, $\frac{\partial G^d_{mn}}{\partial X_{ij}}$
- calculation of the components of $DRDY$ and R
- assembly of $DRDY$ and R

6.3.9 Routine: CJSJIS

Objective:

calculation of $DRDY$ and R for the resolution of $\frac{DR}{DY}\left(Y^p\right)\ DY^{p+1}=-R\left(Y^p\right)$ (isotropic plastic mechanism)

Variables of entry and exit :

```
IN     MOD       :  MODELING
       MATER     :  COEFFICIENTS MATERIAL WITH T+DT
       LIFO      :  INCREMENT OF DEFORMATION
       YD        :  VARIABLES WITH T = (SIGD, VIND, LAMBDD)
       YF        :  VARIABLES WITH T+DT = (SIGF, VINF, LAMBDF)
OUT    R         :  SECOND MEMBER
       DRDY      :  JACOBIEN
```

Organization of CJSJIS

- calculation of the elastic operator,
- calculation of the components of $DRDY$ and R

- assembly of $DRDY$ and R

6.3.10 Routine: CJSMAT

Objective:

recovery of data materials, amongst components of the fields, internal variables and of selected level CJS.

Variables of entry and exit :

```
IN     IMAT   :  ADDRESS OF MATERIAL CODES
       MOD    :  TYPE OF MODELING
       TEMPF  :  TEMPERATURE WITH T+DT
OUT    MATERF:  COEFFICIENTS MATERIAL WITH T+DT
       NDT    :  TOTAL NB OF COMPONENTS TENSORS
       NDI    :  NB OF DIRECT COMPONENTS TENSORS
       NVI    :  NB OF INTERNAL VARIABLES
       NIVCJS:  LEVEL 1.2 OR 3 OF LAW CJS
```

Organization of CJSMAT

- recovery amongst components of the fields and internal variables according to modeling chosen,
- recovery of data materials,
- recognition of level CJS chosen according to the parameters given.

6.3.11 Routine: CJSMDE

Objective:

elastoplastic calculation of the constraints with the plastic mechanism deviatoire activated: resolution by the method of Newton of $R(Y)=0$

Variables of entry and exit :

```
IN     MOD      :  MODELING
       CRIT     :  CONVERGENCE CRITERIA
       MATER    :  COEFFICIENTS MATERIAL WITH T+DT
       NVI      :  NB OF INTERNAL VARIABLES
       EPSD     :  DEFORMATIONS WITH T
       LIFO     :  INCREMENT OF DEFORMATION
       SIGD     :  CONSTRAINT WITH T
       VIND     :  INTERNAL VARIABLES WITH T
       STOPNC   :  STOP IN THE EVENT OF NOT CONVERGENCE
VAR    SIGF     :  CONSTRAINT WITH T+DT
       VINF     :  INTERNAL VARIABLES WITH T+DT
       NOCONV   :  PAS DE CONVERGENCE
```

Organization of CJSMDE

- initialization of YD by the state with T
- calculation of a solution of test with CJSIDE
- buckle on the iterations of Newton
  - incrementing $YF=YD+DY$
  - calculation of $DRDY$ and R : CJSJDE
  - resolution of the system by the method of Gauss: MTGAUS
    1) actualization of the solution DY
  - test of convergence
- update of the constraints and internal variables

6.3.12 Routine: CJSMID

Objective:

elastoplastic calculation of the constraints with the plastic mechanisms isotropic and deviatoire activated: resolution by the method of Newton of $R(Y)=0$

Variables of entry and exit :

```
IN     MOD      :  MODELING
       CRIT     :  CONVERGENCE CRITERIA
       MATER    :  COEFFICIENTS MATERIAL WITH T+DT
       NVI      :  NB OF INTERNAL VARIABLES
       EPSD     :  DEFORMATIONS WITH T
       LIFO     :  INCREMENT OF DEFORMATION
       SIGD     :  CONSTRAINT WITH T
       VIND     :  INTERNAL VARIABLES WITH T
       STOPNC   :  STOP IN THE EVENT OF NOT CONVERGENCE
VAR    SIGF     :  CONSTRAINT WITH T+DT
       VINF     :  INTERNAL VARIABLES WITH T+DT
       NOCONV   :  PAS DE CONVERGENCE
```

Organization of CJSMID

- initialization of YD by the state with T
- calculation of a solution of test with CJSIID
- buckle on the iterations of Newton
  - incrementing $YF=YD+DY$
  - calculation of $DRDY$ and R : CJSJID
  - resolution of the system by the method of Gauss: MTGAUS
    1) actualization of the solution DY
  - test of convergence
- update of the constraints and internal variables

6.3.13 Routine: CJSMIS

Objective:

elastoplastic calculation of the constraints with the activated isotropic plastic mechanism: resolution by the method of Newton of $R(Y)=0$

Variables of entry and exit :

```
IN     MOD      :  MODELING
       CRIT     :  CONVERGENCE CRITERIA
       MATER    :  COEFFICIENTS MATERIAL WITH T+DT
       LIFO     :  INCREMENT OF DEFORMATION
       SIGD     :  CONSTRAINT WITH T
       VIND     :  INTERNAL VARIABLES WITH T
       STOPNC   :  STOP IN THE EVENT OF NOT CONVERGENCE
VAR    SIGF     :  CONSTRAINT WITH T+DT
       VINF     :  INTERNAL VARIABLES WITH T+DT
       NOCONV   :  PAS DE CONVERGENCE
```

Organization of CJSMIS

- initialization of YD by the elastic prediction
- buckle on the iterations of Newton
  - incrementing $YF=YD+DY$
  - calculation of $DRDY$ and R : CJSJIS
    1) resolution of the system by the method of Gauss: MTGAUS

- actualization of the solution DY
- test of convergence
- update of the constraints and internal variables
-

6.3.14 Routine: CJSNOR

Objective:

calculation of a vector parallel with $\frac{\partial\ f^d}{\partial\ q_{ij}}$

Variables of entry and exit :

```
IN     MATER: MATERIAL
       SIG   :   CONSTRAINTS
       X     :   INTERNAL VARIABLES KINEMATICS
OUT    NOR   :   ESTIMATE OF THE DITRECTION OF THE NORMAL
                 ON SURFACE DEVIATOIRE IN PLAN DEVIATOIRE
                 PERPENDICULAR WITH THE TRISECTING ONE
                 THE NOR VECTOR (1: NDT) NR IS NOT STANDARD
                 SA STANDARD IS NOR (NDT+1)
```

6.3.15 Routine: CJSPLA

Objective:

elastoplastic calculation of the constraints.

Variables of entry and exit :

```
IN     MOD    :  MODELING
       CRIT   :  CONVERGENCE CRITERIA
       MATER  :  COEFFICIENTS MATERIAL WITH T+DT
       SEUILI:  FUNCTION OF ISO LOAD. CALCULEE WITH PREDICT ELAS
       SEUILD:  FUNCTION OF LOAD DEV. CALCULEE WITH PREDICT ELAS
       NVI    :  MANY INTERNAL VARIABLES
       EPSD   :  DEFORMATIONS WITH T
       LIFO   :  INCREMENT OF DEFORMATION
       SIGD   :  CONSTRAINT WITH T
       VIND   :  INTERNAL VARIABLES WITH T
VAR    SIGF   :  CONSTRAINT WITH T+DT (IN - > ELAS, OUT - > PLASTI)
OUT    VINF   :  INTERNAL VARIABLES WITH T+DT
       MECANI:  MECHANISM (S) ACTIVATES (S)
```

Organization of CJSPLA

- assumption on the plastic mechanisms activated according to the values of the thresholds f^i and f^d calculated starting from the elastic prediction,
- treatment of the possible recutting of the step of time
- safeguard of the elastic prediction,
- elastoplastic calculation,
  - isotropic plastic mechanism: CJSMIS
  - plastic mechanism déviatoire: CJSMDE
  - plastic mechanisms isotropic and déviatoire simultaneously: CJSMID
- calculation of the thresholds starting from the constraints with t+dt
  1) call of CJSSMI and of CJSSMD
     1) if (assumption of an isotropic mechanism and f^d positive) or (assumption of a mechanism déviatoire and f^i positive): return to elastoplastic calculation with plastic mechanisms isotropic and déviatoire simultaneously,
  1) if not end of routine

6.3.16 Routine: CJSQCO

Objective:

utility routine of CJS allowing the calculation of standard sizes listed below

Variables of entry and exit :

```
IN      GAMMA    :  PARAMETER MATERIAL
        SIG      :  CONSTRAINTS
        X        :  VARIABLES HAMMER-HARDENED MOVIES
        PREF     :  CLOSE REF. FOR STANDARDISATION
        EPSSIG   :  DEVIATIVE EPSILON FOR NULLITY
        I1       :  TRACE OF THE TENSOR OF THE CONSTRAINTS
OUT     S        : DEV. (SIG)
        Software firm      : SQRT (S: S)
        SIIREL   : SII/PREF
        COS3TS   : LODE (SIG)
        HTS      : FUNCTION H (TETHA_S)
        DETS     : DETERMINANT OF S
        Q        : Q (SIG-X)
        QII      : SQRT (Q: Q)
        QIIREL   : QII/PREF
        COS3TQ
         HTQ      : FUNCTION H (TETHA_Q)
        DETQ     : DETERMINANT OF Q
```

6.3.17 Routine: CJSQIJ

Objective:

calculation of the tensor q_{ij}

Variables of entry and exit:

```
IN      NR      :  DIMENSION OF S, X, Q
         S      :  DIVERTER
        I1     :  FIRST INV.
        X      :  CENTER OF THE SURFACE OF LOAD DEVIATOIRE
OUT     Q      :  TENSOR RESULT
```

6.3.18 Routine: CJSSMD

Objective:

calculation of the threshold of the plastic mechanism déviatoire.

Variables of entry and exit :

```
IN      SIG    :  CONSTRAINT
        WINE    :  INTERNAL VARIABLES
OUT     SEUILD:  THRESHOLD ELASTICITY OF MECHANISM DEVIATOIRE
```

6.3.19 Routine: CJSSMI

Objective:

calculation of the threshold of the isotropic plastic mechanism.

Variables of entry and exit :

```
IN      SIG    :  CONSTRAINT
```

```
       WINE    :  INTERNAL VARIABLES
OUT       SEUILI:  THRESHOLD ELASTICITY OF THE ISOTROPIC MECHANISM
```

6.3.20 Routine: CJST

Objective:

calculation of $t = \dfrac{\partial \det s}{\partial s}$.

Variables of entry and exit :

```
IN     S       :  MATRIX
OUT    T       :  T (IN VECTORIAL FORM WITH RAC2)
```

6.3.21 Routine: CJSTDE

Objective:

calculation of the tangent matrix for the plastic mechanism déviatoire

Variables of entry and exit :

```
IN     MOD      :  MODELING
       MATER    :  COEFFICIENTS MATERIAL
       NVI      :  NB OF INTERNAL VARIABLES
       EPS      :  DEFORMATIONS
       SIG      :  CONSTRAINTS
       WINE      :  INTERNAL VARIABLES
OUT    DSDESY   :  TANGENT MATRIX SYMETRISEE
```

Organization of CJSTDE

- calculation of the elastic operator,
- calculation of laws of work hardening G^R and $\boldsymbol{G}^X$,
- calculation of the law of flow of the plastic mechanism déviatoire $\boldsymbol{G}^d$,
- calculation of intermediate terms
- calculation of the tangent matrix
- symmetrization of the tangent matrix

6.3.22 Routine: CJSTEL

Objective:

calculation of the tangent matrix for the elastic mechanism

Variables of entry and exit :

```
IN     MOD      :  MODELING
       MATER    :  COEFFICIENTS MATERIAL
       SIG      :  CONSTRAINTS
OUT    HOOK     :  ELASTIC OPERATOR RIGIDITY
```

Organization of CJSTEL

- calculation of the elastic operator

6.3.23 Routine: **CJSTID**

Objective:
calculation of the tangent matrix for the isotropic plastic mechanisms and déviatoire

Variables of entry and exit :
```
IN     MOD      :  MODELING
       MATER    :  COEFFICIENTS MATERIAL
       NVI      :  NB OF INTERNAL VARIABLES
       EPS      :  DEFORMATIONS
       SIG      :  CONSTRAINTS
       WINE      :  INTERNAL VARIABLES
OUT    DSDESY  :  TANGENT MATRIX SYMETRISEE
```

Organization of CJSTEL
- calculation of the elastic operator,
- calculation of laws of work hardening G^R and $\boldsymbol{G}^X$,
- calculation of the law of flow of the plastic mechanism déviatoire $\boldsymbol{G}^d$,
- calculation of intermediate terms
- calculation of the tangent matrix
- symmetrization of the tangent matrix

6.3.24 Routine: **CJSTIS**

Objective:
calculation of the tangent matrix for the isotropic plastic mechanism

Variables of entry and exit :
```
IN     MOD      :  MODELING
       MATER    :  COEFFICIENTS MATERIAL
       SIG      :  CONSTRAINTS
       WINE      :  INTERNAL VARIABLES
OUT    DSDE     :  TANGENT MATRIX
```

Organization of CJSTEL
- calculation of the tangent matrix

6.3.25 Routine: **LCDETE**

Objective:
calculation of a matrix determining 3 □ 3

Variables of entry and exit :
```
IN     With       :  MATRIX
OUT    LCDETE:  DETERMINANT
```

6.3.26 Routine: **NMCJS**

Objective:
realization of the integration of law CJS: calculation of the constraints with t+dt and/or the tangent matrix, according to the selected option of calculation.

Variables of entry and exit :
```
IN     TYPMOD  TYPE OF MODELING
```

```
       IMAT    ADDRESS OF MATERIAL CODES
       COMP    BEHAVIOR OF L ELEMENT
       CRIT    CRITERIA  BUILDINGS
       INSTAM  MOMENT T
       INSTAP  MOMENT T+DT
       TEMPM   TEMPERATURE WITH T
       TEMPF   TEMPERATURE WITH T+DT
       TREF    TEMPERATURE OF REFERENCE
       EPSD    TOTAL DEFLECTION WITH T
       LIFO    INCREMENT OF TOTAL DEFLECTION
       SIGD    CONSTRAINT WITH T
       VIND    INTERNAL VARIABLES WITH T    + INDICATING STATE T
       OPT     OPTION OF CALCULATION TO BE MADE
OUT    SIGF    CONSTRAINT WITH T+DT
       VINF    INTERNAL VARIABLES WITH T+DT + INDICATING STATE T+DT
       DSDE    MATRIX OF TANGENT BEHAVIOR WITH T+DT OR T
```

Organization of `NMCJS`

- recovery of data materials, amongst components of the fields, internal variables and of selected level CJS:
  - call of `CJSMAT`
- blocking of internal variables according to selected level CJS
- calculation of the constraints with t+dt
  - elastic prediction: `CJSELA`
  - calculation of the thresholds of the mechanisms isotropic and déviatoire: `CJSSMI` and `CJSSMD`
  - if one of the thresholds is exceeded, elastoplastic calculation: `CJSPLA`
- calculation of the tangent matrix according to the concerned mechanism
  1) rubber band: `CJSTEL`
  2) isotropic plastic: `CJSTIS`
  3) plastic déviatoire: `CJSTDE`
  4) isotropic plastic and déviatoire: `CJSTID`

7 Bibliography

[1] Mr. MALEKI, B. CAMBOU, P. DUBUJET, "hierarchical Modeling of the behavior of the grounds", French Review of Génie Civil, Volume 4 - Issue 7-8, 2000. http://dx.doi.org/10.1080/12795119.2000.9692702.

[2] B. CAMBOU, K. JAFARI, "Models behavior of the non-cohesive soils", rev. Franç. Géotech. n°44, p.p 43-55, 1988.

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8 Features and checking

This document relates to the law of behavior `CJS` (keyword `BEHAVIOR` of `STAT_NON_LINE`) and its associated material `CJS` (order `DEFI_MATERIAU`).

This law of behavior is checked by the cases following tests:

SSNV135	Triaxial compression test drained with the model `CJS` (level 1)	[V6.04.135]
SSNV136	Triaxial compression test drained with the model `CJS` (level 2)	[V6.04.136]
SSNV154	Triaxial compression test drained with the model `CJS` (level 3)	[V6.04.154]
SSNV155	Triaxial compression test drained on a turned sample of an angle of – PI/6 compared to axis X with the model `CJS` (level 2)	[V6.04.155]
WTNV100	Triaxial compression test not drained with the model `CJS` (level 1)	[V7.31.100]

9 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
6,4	*C. CHAVANT, pH. AUBERT* EDF-R&D/AMA EDF-DIS/CNEPE	Initial text