Law of behavior CAM_CLAY

Summary:

The Camwood-Clay model one of the elastoplastic models known and the most are the most used in soil mechanics. It is especially adapted to argillaceous materials. There are several types of models Camwood-Clay, that presented here is most current and is called modified Camwood-Clay. This model is characterized by hammer-hardenable surfaces of load in the shape of ellipses in the diagram of the first two invariants of the constraints. Inside these surfaces of reversibility, the material is elastic nonlinear. There exists moreover, in a point of each ellipse, a critical condition characterized by a worthless variation of volume. The whole of these points constitutes a line separating the zones from dilatancy and contractance of material like zones of negative and positive work hardening. Work hardening is governed by only one scalar variable and the normal rule of flow is adopted.

Contents

1 Notations

σ indicate the tensor of the effective constraints in small disturbances defined as being the difference between the total constraints and the pressure of water in the case of the water-logged soils, noted in the shape of the following vector:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sqrt{2}\,\sigma_{12} \\ \sqrt{2}\,\sigma_{23} \\ \sqrt{2}\,\sigma_{31} \end{pmatrix}$$

One notes:

$\underline{\underline{P=-\frac{1}{3}tr(\sigma)}}$	**constraint of containment**
$s=\sigma+PI$	diverter of the constraints
$I_2=\frac{1}{2}tr(s.s)$	second invariant of the constraints
$Q=\sigma_{eq}=\sqrt{3I_2}$	equivalent constraint
$\varepsilon=\frac{1}{2}(\nabla u+\nabla^T u)$	total deflection
$\varepsilon=\varepsilon_e+\varepsilon_p+\varepsilon_{th}$	partition of the deformations (elastic, plastic, thermal)
$\underline{\underline{\varepsilon_v=-tr(\varepsilon)+3\alpha(T-T_0)}}$	**voluminal total deflection**
$\varepsilon_V^p=-tr(\varepsilon^p)$	voluminal plastic deformation
$\tilde{\varepsilon}=\varepsilon+\frac{1}{3}\varepsilon_v I$	diverter of the deformations
$\tilde{\varepsilon}^e=\tilde{\varepsilon}-\tilde{\varepsilon}^p$	diverter of the elastic strain
$\tilde{\varepsilon}^p=\varepsilon^p+\frac{1}{3}\varepsilon_v^p I$	deviatoric plastic deformation
$\varepsilon_{eq}^e=\sqrt{\frac{2}{3}tr(\tilde{\varepsilon}^e.\tilde{\varepsilon}^e)}$	equivalent elastic strain
$\varepsilon_{eq}^p=\sqrt{\frac{2}{3}tr(\tilde{\varepsilon}^p.\tilde{\varepsilon}^p)}$	equivalent plastic deformation

e index of the vacuums of the material (report of the volume of the pores on the volume of the solid matter constituents)

e_0 initial index of the vacuums

ϕ porosity (report of the volume of the pores on total volume)

κ coefficient of swelling (elastic slope in a hydrostatic test of compression)

M slope of the right-hand side of critical condition

$$k_0=\frac{(1+e_0)}{\kappa}$$

P_{cr} variable interns model, critical pressure equal to half of the pressure of consolidation P_{cons}

λ coefficient of compressibility (plastic slope in a hydrostatic test of compression)

$$k=\frac{(1+e_0)}{(\lambda-\kappa)}$$

μ elastic coefficient of shearing (coefficient of Lamé)

f surface of load

Λ plastic multiplier

I^d tensor unit of order 2 whose term running is δ_{ij}

I_4^d tensor unit of order 4 whose term running is δ_{ijkl}

2 Introduction

The model describes here is the model known as of modified Camwood-Clay. The initial Camwood-Clay model was developed by the school of soil mechanics of Cambridge in the Sixties. He predicted too important deviatoric deformations under weak loading deviatoric, and was modified by Burland and Roscoe in 1968 [bib1].

2.1 Phenomenology of the behavior of the grounds

The materials poroplastic such as certain clays are characterized by the following behaviors:

- the strong porosity of these materials causes unrecoverable deformations under hydrostatic loading corresponding to an important reduction of porosity. This mechanism purely contractor is sometimes called “collapse”,
- under loading deviatoric, these materials show a contracting phase followed by a phase where the material becomes deformed with constant plastic volume or dilates.

For the two types of loading, the energy blocked in material evolves according to the number of contact between the grains. For a hydrostatic loading, the number of contact increases, as well as blocked energy, one thus has positive work hardening. For a loading deviatoric, the material can become deformed without variation of volume to many intergranular contacts constant. Moreover, one can observe in the tests of the localizations of deformations accompanied by a strong dilatancy. In these zones, the number of grains in decreasing contact, there is reduction in blocked energy and thus softening.

These behaviors are highlighted primarily by triaxial compression tests of revolution. These observations bring to apply that there exists a plastic threshold whose evolution is controlled by two mechanisms: one purely contractor associated with the hydrostatic constraint, and a mechanism deviatoric controlled by internal friction being held with constant volume and possibly dilating with the approach of the localization.
All the interest of the Camwood Clay model lies in its faculty to describe these phenomena with a minimum of ingredients and in particular only one surfaces of load and a work hardening associated with only one scalar variable.

2.2 Behaviour under hydrostatic compression

During a hydrostatic test of compression, the grounds present an index of the vacuums which decrease logarithmiquement with the exerted hydrostatic pressure (cf [Figure 2.2-a]). e_0 being the initial index of the vacuums under initial loading. Until a pressure P^0_{cons} called pressure of consolidation, the behavior is reversible, the slope κ diagram $(e, Ln\ P)$ elastic coefficient of swelling is called. P^0_{cons} corresponds to the maximum pressure which the material during its history underwent. Beyond this preconsolidation, the diagram presents a new slope λ (coefficient of compressibility) more marked and appearance of unrecoverable deformations. P^0_{cons} thus corresponds to an evolutionary elastoplastic threshold.

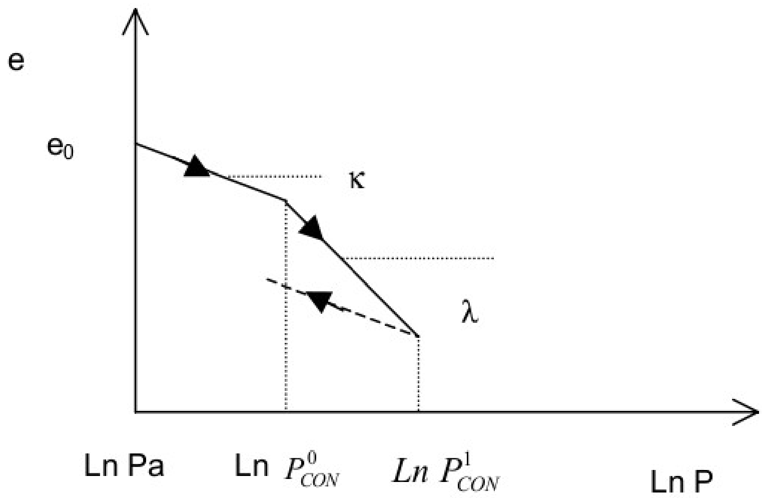


Figure 2.2-a: Hydrostatic test of loading and unloading

Note:

The diagram above corresponds to a set of measurements where the effective constraint is stabilized. Indeed, in the process of consolidation of the grounds, it is the water contained in the pores which takes again initially the hydrostatic pressure with very little deformation, before running out and letting the skeleton become deformed. After consolidation of material and stabilization of the pressure of water, the effective constraint (forced total minus pressure of water) is stabilized and deferred on the graph. The relations of behavior in the saturated porous environments are generally expressed with the effective constraints according to the assumption of Terzaghi.

2.3 Behavior under loading deviatoric

The triaxial compression tests of revolution make it possible to control at the same time the deviatoric component Q and the spherical component P loading. According to the report of these two components, one observes a plastic behavior purely dilating ($\frac{Q}{P-P_{trac}}>M$) or contracting ($\frac{Q}{P-P_{trac}}<M$), line $Q=M\left(P_{cr}-P_{trac}\right)$ representing the whole of the critical points on surfaces of load where the mechanical state evolves without plastic change of volume. The basic Camwood Clay model makes the assumption that the rates of plastic deformations are normal on the surface of load f ($\dot{\varepsilon}_v^p=\dot{\Lambda}\frac{\partial f}{\partial P}, \dot{\tilde{\varepsilon}}^p=\dot{\Lambda}\frac{\partial f}{\partial Q}$) . Moreover, plastic work in an unspecified point of the surface of load is considered equal to plastic work with the critical condition.

3 Camwood Clay law modified

3.1 Assumptions of modeling

The model is written in small disturbances.
The coefficients of the model do not depend on the temperature.

3.2 Surface of load

The expression of the surface of load is written in the following way:

$$f(P,Q,P_{cr})=Q^2+M^2\left(P-P_{trac}\right)^2-2\mathrm{M}^2\left(P-P_{trac}\right)P_{cr} \qquad \leq 0 \qquad \textbf{éq 3.2-1}$$

In the plan (P,Q) , the expression represents a family of ellipses, centered on P_{cr} who is related to pressure of consolidation: $P_{cons}=2P_{cr}-P_{trac}$ (cf [Figure 3.2-a). P_{cr} will be the parameter of work hardening of the model.

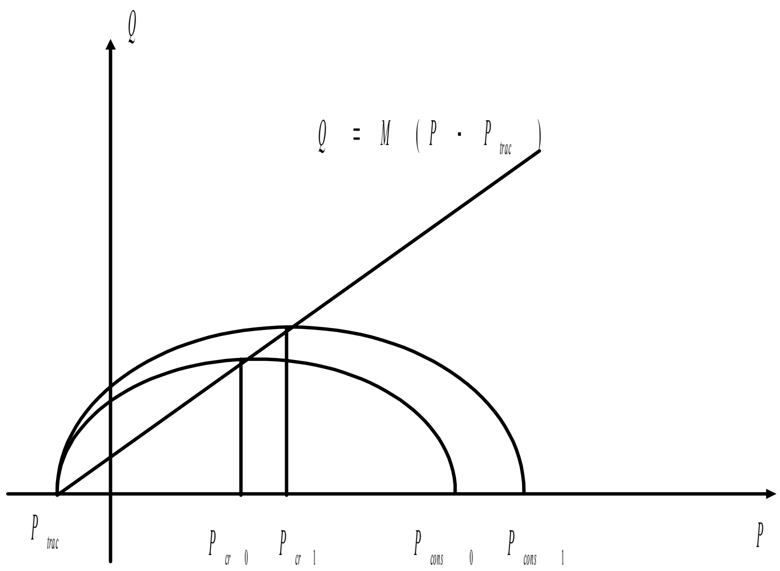


Figure 3.2-a: Family of hammer-hardenable surfaces of load

When $f=0$ and $P-P_{trac}<P_{cr}$ the material is dilating ($\dot{\varepsilon}_v^p<0$) and P_{cr} is decreasing (softening).
When $f=0$ and $P-P_{trac}>P_{cr}$ the material is contacting ($\dot{\varepsilon}_v^p>0$) and P_{cr} is increasing (hardening).

3.3 Elastic law and law of work hardening

The assumption of the decoupling of the partly hydrostatic and deviatoric elastic law and the additional assumption are made that the modulus of rigidity is constant.

One thus considers an isotropic elastic law, with a linear deviatoric part and a non-linear voluminal part:

Déviatoire part :

$$\tilde{\varepsilon}^e = \frac{s}{2\mu} \qquad \textbf{éq 3.3-1}$$

Voluminal part :

$$\dot{\varepsilon}_v^e = -\frac{\dot{e}}{1+e_0} \text{ ou } e = e_0 - \kappa\, Ln\left(\frac{P}{K_{cam}}\right) \text{ si } P < \text{Pconsolidation} \qquad \textbf{éq 3.3-2}$$

The law [éq 3.3-2] is in fact derived from a test oedometric where one measures the variation of the index of the vacuums according to the loading [Figure 2.2-a]. Let us recall that a homogeneous test oedometric consists in increasing the axial effective constraint all while maintaining the deformation radial worthless on a cylindrical test-tube.

Note:

Pressures P correspond to tests drained or not. Nevertheless, in a modeling with Code_Aster the constraints handled in the laws of behavior are effective i.e. that one does not take into account the hydrostatic pressure of the fluid which can circulate in the pores, this one being calculated in modelings THM.

Tests of voluminal loading (cf. [Figure 2.2-a]) we bring to the following elastic law:

$$k_0 P + K_{cam} = \left(k_0 P_0 + K_{cam}\right) \exp\left[k_0 \left(\varepsilon_v^e - \varepsilon_{v0}^e\right)\right] \text{ avec } k_0 = \frac{\left(1+e_0\right)}{\kappa} \qquad \textbf{éq 3.3-3}$$

In the same way, the growth of the surface of load in phase of contractance, its decrease in dilatancy, and the experimental results suggest writing:

$$P_{cr} = P_{cr}^0 \exp\left[k \left(\varepsilon_v^p - \varepsilon_{v0}^p\right)\right] \text{ , avec } k = \frac{\left(1+e_0\right)}{\left(\lambda-\kappa\right)} \qquad \textbf{éq 3.3-4}$$

ε_{v0}^p and e_0 correspond to the voluminal deformation and the index of the initial vacuums, determined by extrapolation of the oedometric curve of the test to the pressure K_{cam} (cf [Figure 2.2-a]).

3.4 Plastic law of flow

The two plastic variables are the voluminal plastic deformation ε_v^p and the tensor deviatoric of the plastic deformations $\tilde{\varepsilon}^p$. The internal variable is also ε_v^p but associated by the strength of work hardening P_{cr} . The material standard is not generalized. The rule of flow is written:

$$\dot{\varepsilon}^p = \dot{\Lambda}\frac{\partial f}{\partial \sigma}, \ , \ \dot{\varepsilon}_v^p = -\dot{\Lambda}\frac{\partial F}{\partial P_{cr}} , \qquad \textbf{éq 3.4-1}$$

Λ being the plastic multiplier.

By breaking up the first term, one obtains:

$$\dot{\varepsilon}_v^p = \dot{\Lambda}\frac{\partial f}{\partial P} \qquad \dot{\tilde{\varepsilon}}^p = \dot{\Lambda}\frac{\partial f}{\partial s} \qquad \dot{\varepsilon}_v^p = -\dot{\Lambda}\frac{\partial F}{\partial P_{cr}} \qquad \textbf{éq 3.4-2}$$

knowing that:

$$P = -\frac{1}{3}tr(\sigma) \quad \text{et} \quad \varepsilon_v = -tr(\varepsilon) + 3\alpha(T - T_0) \qquad \textbf{éq 3.4-3}$$

F is the plastic potential associated with the phenomenon of work hardening. Let us note that the third part of [éq 3.4-2] is only formal. Indeed, one knows $\dot{\varepsilon}_v^p$ by the first relation thus one knows the evolution of P_{cr} .

3.5 Energy writing and plastic module of work hardening

One is thus within the not generalized "standard" material framework (one uses three potentials then: the surface of load f , plastic potential F , and free energy ψ . Even in this configuration less favorable than the traditional framework of not generalized standard materials, one is ensured to satisfy the second principle with thermodynamics [bib4]. Using the condition of consistency (expressing that the point representative of the loading "follows" the surface of load) which is written in the following way:

$$df = \frac{\partial f}{\partial P}dP + \frac{\partial f}{\partial Q}dQ + \frac{\partial f}{\partial P_{cr}}dP_{cr} = 0 \ , \qquad \textbf{éq 3.5-1}$$

the expression of the plastic multiplier is determined [bib4]:

$$\Lambda = \frac{1}{H_p}\frac{\partial f}{\partial \sigma}d\sigma = -\frac{1}{H_p}\frac{\partial f}{\partial P_{cr}}dP_{cr} \qquad \textbf{éq 3.5-2}$$

with [bib4]:

$$H_p = \frac{\partial f}{\partial P_{cr}}\frac{\partial^2 \psi}{\partial {\varepsilon_v^p}^2}\frac{\partial F}{\partial P_{cr}} \ , \ \text{où } H_p \text{ est le module d'écrouissage} \qquad \textbf{éq 3.5-3}$$

The identification of the first and third part of [éq 3.4-2] makes it possible to calculate F who is written:

$$F=-\int\frac{\partial f}{\partial P}dP_{cr}=M^2P_{cr}\left(P_{cr}-2\mathrm{P}+2\mathrm{P}_{trac}\right) \quad \textbf{éq 3.5-4}$$

Concept of work hardening being associated with that of blocked energy:

$$P_{cr}=\frac{\partial\psi}{\partial\varepsilon_v^p} \quad \text{donc} \quad dP_{cr}=\frac{\partial^2\psi}{\partial^2\varepsilon_v^p}d\varepsilon_v^p \quad \textbf{éq 3.5-5}$$

where ψ is the density of free energy:

$$\psi=\frac{3}{2}\mu\left(\varepsilon_{eq}^e\right)^2+\frac{P_0}{k_0}\exp\left(k_0\varepsilon_v^e\right)+\frac{P_{cr}^0}{k}\exp\left(k\left(\varepsilon_v^p-\varepsilon_{v0}^p\right)\right) \quad \textbf{éq 3.5-6}$$

By using them [éq 3.4-2], [éq 3.5-4] and [éq 3.5-6], one can draw according to [éq 3.5-3] the expression from the plastic module of work hardening:

$$H_p=\frac{\partial f}{\partial P_{cr}}\frac{\partial^2\psi}{\partial{\varepsilon_v^p}^2}\frac{\partial F}{\partial P_{cr}}=4kM^4\left(P-P_{trac}\right)P_{cr}\left(P-P_{trac}-P_{cr}\right) \quad \textbf{éq 3.5-7}$$

The module of work hardening is positive in phase of contractance $\left(P-P_{trac}>P_{cr}\right)$ and negative in phase of dilatancy $\left(P-P_{trac}<P_{cr}\right)$. For $P-P_{trac}=P_{cr}$, the behavior is plastic perfect and proceeds with constant plastic volume.

3.6 Incremental relations

The equation [éq 3.4-3] and the condition of consistency give the relations of flow:

$$d\varepsilon_v^p=\frac{1}{k}\left[\left(\frac{1}{P_{cr}}-\frac{1}{\left(P-P_{trac}\right)}\right)dP+\frac{Q}{M^2\left(P-P_{trac}\right)P_{cr}}dQ\right] \quad \textbf{éq 3.6-1}$$

$$d\varepsilon_{eq}^p=\frac{1}{k}\left[\frac{Q}{M^2\left(P-P_{trac}\right)P_{cr}}dP+\frac{Q^2}{M^4\left(P-P_{trac}\right)P_{cr}\left(P-P_{trac}-P_{cr}\right)}dQ\right] \quad \textbf{éq 3.6-2}$$

$$d\tilde{\varepsilon}^p=d\varepsilon_{eq}^p\frac{3}{2}\frac{s}{Q} \quad \textbf{éq 3.6-3}$$

The rearrangement of [éq 3.6-1] and [éq 3.6-2] conduit with:

$$\frac{d\varepsilon_{eq}^p}{d\varepsilon_v^p}=\frac{Q}{M^2\left(P-P_{trac}-P_{cr}\right)} \quad \textbf{éq 3.6-4}$$

i.e. with the equation [éq 3.6-3],

$$\frac{d\tilde{\varepsilon}^p}{d\varepsilon_v^p}=\frac{3}{2}\frac{s}{M^2\left(P-P_{trac}-P_{cr}\right)} \quad \textbf{éq 3.6-5}$$

Typical case of the critical point:

For $f=0$ et $P-P_{trac}=P_{cr}$: $\dot{P}_{cr}=0$, $\dot{\varepsilon}_v^p=0$. One from of deduced, by considering the elastic law: $\dot{P}=k_0 P\dot{\varepsilon}_v$. The condition of consistency gives us $\dot{Q}=0$.

3.7 Summary of the relations of behavior

Elasticity

$$s=2\mu\tilde{\varepsilon}^e \quad \text{éq 3.7-1}$$

$$P=P_0\exp(k_0\Delta\varepsilon_v^e)+\frac{K_{cam}}{k_0}\left(\exp\left(k_0\Delta\varepsilon_v^e\right)-1\right) \quad \text{éq 3.7-2}$$

Plasticity

The criterion: $f(\sigma, P_{cr})=Q^2+M^2(P-P_{trac})^2-2\mathrm{M}^2(P-P_{trac})P_{cr}=0$ with $(Q=\sigma_{eq})$

$$\frac{\partial f}{\partial\sigma}=\underbrace{\left(-\frac{1}{3}\frac{\partial f}{\partial P}I^d+\frac{3}{2}\frac{\partial f}{\partial Q}\frac{s}{Q}\right)} \quad \text{éq 3.7-3}$$

thus:

$$\dot{\tilde{\varepsilon}}^p=3\dot{\Lambda}s \quad \text{éq 3.7-4}$$

$$\dot{\varepsilon}_v^p=\dot{\Lambda}2\mathrm{M}^2(P-P_{trac}-P_{cr}) \quad \text{éq 3.7-5}$$

Work hardening

$$P_{cr}(\varepsilon_v^p)=P_{cr0}\exp\left(k\left(\varepsilon_v^p-\varepsilon_v^{p_0}\right)\right) \quad \text{éq 3.7-6}$$

Elastic behavior: If $f<0$ then:

$$\dot{P}_{cr}=0 \quad \text{éq 3.7-7}$$

$$\dot{\tilde{\varepsilon}}_{eq}^p=0, \dot{\varepsilon}_v^p=0 \quad \text{éq 3.7-8}$$

$$\dot{s}=2\mu\dot{\tilde{\varepsilon}} \quad \text{éq 3.7-9}$$

$$\dot{P}=(k_0 P+K_{cam})\dot{\varepsilon}_v \quad \text{éq 3.7-10}$$

Elastoplastic behavior: If $f=0$ and $\dot{f}=0$ then:

$$\dot{P}_{cr}\neq 0 \quad ; \quad \dot{P}_{cr}=k\dot{\varepsilon}_v^p P_{cr} \quad \text{éq 3.7-11}$$

$$\dot{\tilde{\varepsilon}}^p=3\dot{\Lambda}s \quad \text{si} \quad P-P_{trac}\neq P_{cr} \quad \text{éq 3.7-12}$$

$$\dot{\varepsilon}_v^p=\dot{\Lambda}2\mathrm{M}^2(P-P_{trac}-P_{cr}) \quad \text{si} \quad P-P_{trac}\neq P_{cr} \quad \text{éq 3.7-13}$$

$$\dot{s}=2\mu\dot{\tilde{\varepsilon}} \quad \text{éq 3.7-14}$$

$$\dot{P}=(k_0 P+K_{cam})\dot{\varepsilon}_v \quad \text{éq 3.7-15}$$

Note:

- *From the only unknown factor* $\dot{\varepsilon}_v^p$ *, one can deduce the other unknown factors* $\tilde{\dot{\varepsilon}}^p$ *and* $\dot{P}_{cr}$ *.*
- *If* $P-P_{trac}=P_{cr}$ *:* $\dot{\varepsilon}_v^p=0$ *,* $\dot{Q}=\dot{P}_{cr}=0,\ \dot{P}=k_0 P\ \dot{\varepsilon}_v$ *.*

4 Digital integration of the relations of behavior

4.1 Recall of the problem

For an increment of loading given and a set of variables given (initial field of displacement, constraint and variable interns), one solves the discretized total system (2.2.2.2 - 1 of [bib3]) which seeks to satisfy the equilibrium equations.

The resolution of this system gives us Δu , therefore $\Delta\varepsilon$. One thus seeks locally, in each point of Gauss, the increment of constraint and of variable correspondent interns with $\Delta\varepsilon$ and which satisfies the law with behavior.

The following notations are employed: $A^{-}, A, \Delta A$ for the quantity evaluated at the known moment T, the moment $t+\Delta t$ and its increment, respectively. The equations are discretized in an implicit way, expressed according to the unknown variables at the moment $t+\Delta t$.

4.2 Calculation of the constraints and internal variables

The elastic prediction of the deviatoric constraint is written:

$$s^e = s^- + 2\mu\Delta\tilde{\varepsilon} \quad \textbf{éq 4.2-1}$$

however one can always write s at the moment + as being:

$$s = s^- + 2\mu\Delta\tilde{\varepsilon}^e \quad \textbf{éq 4.2-2}$$

These two equations enable us to deduce s according to s^e :

$$s = s^e - 2\mu\Delta\tilde{\varepsilon} + 2\mu\Delta\tilde{\varepsilon}^e \quad \textbf{éq 4.2-3}$$

$$\text{ou} \quad s = s^e - 2\mu\Delta\tilde{\varepsilon}^p \quad \textbf{éq 4.2-4}$$

While replacing $\Delta\tilde{\varepsilon}^p$ by its expression according to $\Delta\varepsilon_v^p$, one obtains:

$$s = \frac{s^e}{1 + \dfrac{3\mu\Delta\varepsilon_v^p}{M^2\left(P - P_{trac} - P_{cr}\right)}} \quad \textbf{éq 4.2-5}$$

from where,

$$Q = \frac{Q^e}{1 + \dfrac{3\mu\Delta\varepsilon_v^p}{M^2\left(P - P_{trac} - P_{cr}\right)}} \quad \textbf{éq 4.2-6}$$

By supposing that k_0 is independent of the temperature, the incremental writing of P is written:

$$P=P^{-}\exp\left[k_0\varepsilon_v^e-k_0\varepsilon_v^{e^-}\right]+\frac{K_{cam}}{k_0}\left(\exp\left[k_0\varepsilon_v^e-k_0\varepsilon_v^{e^-}\right]-1\right)$$ **éq 4.2-8**

$$P=P^{-}\exp\left[k_0\Delta\varepsilon_v^e\right]+\frac{K_{cam}}{k_0}\left(\exp\left[k_0\Delta\varepsilon_v^e\right]-1\right)$$ **éq 4.2-9**

$$\Delta P=P^{-}\left(\exp\left[k_0\Delta\varepsilon_v^e\right]-1\right)+\frac{K_{cam}}{k_0}\left(\exp\left[k_0\Delta\varepsilon_v^e\right]-1\right)$$ **éq 4.2.10**

In the same way one can write the expression of P^e according to P^- :

$$P^e=P^{-}\exp\left[k_0\Delta\varepsilon_v\right]+\frac{K_{cam}}{k_0}\left(\exp\left[k_0\Delta\varepsilon_v\right]-1\right)$$ **éq 4.2-11**

from where the expression of P at the moment + is:

$$P=P^e\exp\left[-k_0\Delta\varepsilon_v^p\right]+\frac{K_{cam}}{k_0}\left(\exp\left[-k_0\Delta\varepsilon_v^p\right]-1\right)$$ **éq 4.2-12**

In the incremental writing of P_{cr} , the coefficient k does not depend on the temperature, one thus finds the expression following:

$$P_{cr}=P_{cr0}\exp\left[k\left(\varepsilon_v^p-\varepsilon_v^{p_0}\right)\right]$$ **éq 4.2-13**

$$P_{cr}=P_{cr}^{-}\exp\left[k\Delta\varepsilon_v^p\right]$$ **éq 4.2-14**

$$\Delta P_{cr}=P_{cr}^{-}\left[\exp\left(k\Delta\varepsilon_v^p\right)-1\right]$$ **éq 4.2-15**

Summary:

$f\left(s^e, P^e, P_{cr}^{-}\right)\leq 0$ in this case $\Delta P_{cr}=0$ that is to say $s=s^{-}+\Delta s=s^e$
$P=P^e$

$f\left(s^e, P^e, P_{cr}^{-}\right)>0$ in this case $\Delta P_{cr}>0$, $\Delta\tilde{\varepsilon}^p\neq 0$ and $\Delta\varepsilon_v^p\neq 0$
that is to say $s=s^e-2\mu\Delta\tilde{\varepsilon}^p$

$$P=P^e\exp\left[-k_0\Delta\varepsilon_v^p\right]+\frac{K_{cam}}{k_0}\left(\exp\left[-k_0\Delta\varepsilon_v^p\right]-1\right)$$

$$P_{cr}=P_{cr}^{-}\exp\left[k\Delta\varepsilon_v^p\right]$$

Note:

The principal unknown factor is $\Delta\varepsilon_v^p$.

4.3 Calculation of the unknown factor $\Delta\varepsilon_v^p$

By deferring in the criterion the expressions of P and Q according to P^e and of Q^e and by using the equation [éq 4.2-6]:

$$Q_e^2=-\left[1+\frac{3\mu\Delta\varepsilon_v^p}{M^2(P-P_{trac}-P_{cr})}\right]^2 M^2\left(P-P_{trac}\right)\left(P-P_{trac}-2P_{cr}\right)$$ **éq 4.3-1**

$$Q_e^2=-M^2\left[1+\frac{3\mu\Delta\varepsilon_v^p}{M^2\left(P_e\exp\left[-k_0\Delta\varepsilon_v^p\right]+\frac{K_{cam}}{k_0}\left(\exp\left[-k_0\Delta\varepsilon_v^p\right]-1\right)-P_{trac}-P_{cr}^-\exp\left[k\Delta\varepsilon_v^p\right]\right)}\right]^2$$
$$\left(P_e\exp\left[-k_0\Delta\varepsilon_v^p\right]+\frac{K_{cam}}{k_0}\left(\exp\left[-k_0\Delta\varepsilon_v^p\right]-1\right)-P_{trac}\right)$$
$$\left(P_e\exp\left[-k_0\Delta\varepsilon_v^p\right]+\frac{K_{cam}}{k_0}\left(\exp\left[-k_0\Delta\varepsilon_v^p\right]-1\right)-P_{trac}-2P_{cr}^-\exp\left[k\Delta\varepsilon_v^p\right]\right)$$ **éq 4.3-2**

In under following paragraph one determines limits with this function which facilitate the resolution of the equation [éq 4.3-2] with for example the method of the cords or by the method of Newton.

4.4 Determination of the terminals of the function

One poses $\Delta\varepsilon_v^p=x$ the unknown factor of the problem.
One thus has:

$$P(x)=P^e\exp(-k_0x)+\frac{K_{cam}}{k_0}(\exp(-k_0x)-1)$$ **éq 4.4-1**

$$P_{cr}(x)=P_{cr}^-\exp(kx)$$ **éq 4.4-2**

$$\Lambda(x)=\frac{x}{2M^2\left(P(x)-P_{trac}-P_{cr}(x)\right)}$$ **éq 4.4-3**

$$Q(x)=\frac{Q^e}{1+6\ \mu\ \Lambda(x)}$$ **éq 4.4-4**

$$f(x)=Q^2(x)+M^2\left(P(x)-P_{trac}\right)^2-2M^2\left(P(x)-P_{trac}\right)P_{cr}(x)=0$$ **éq 4.4-5**

At the point $x=0\ ;\ P(0)=P^e\ ;\ P_{cr}(0)=P_{cr}^-\ ;\ \lambda(0)=0\ ;\ Q(0)=Q^e$ éq 4.4-6

$$f(0)=Q^{e^2}+M^2\left(P^e-P_{trac}\right)\left(P^e-P_{trac}-2P_{cr}^-\right) \quad \textbf{éq 4.4-7}$$

$$f(0)>0$$

At the point:

$$P-P_{trac}=P_{cr} \;;\; \Lambda(x_b)=\infty \;;\; Q(x_b)=0 \text{ et } f(x_b)=-M^2\left(P-P_{trac}\right)^2 \quad \textbf{éq 4.4-8}$$

$$f(x_b)<0$$

In $x=0$; $f(0)>0$ and in $x=x_b$; $f(x_b)<0$

One seeks X between 0 and x_b ; to determine it, one writes:

$$P(x_b)-P_{trac}=P_{cr}(x_b)$$

$$\Leftrightarrow P^e\exp(-k_0x_b)+\frac{K_{cam}}{k_0}\exp(-k_0x_b)-P_{cr}^-\exp(kx_b)=\frac{K_{cam}}{k_0}+P_{trac} \quad \textbf{éq 4.4-9}$$

It is a nonlinear equation in x_b , one makes a development limited of order 1 to deduce the expression from x_b :

If $P^e - P_{cr}^- - P_{trac} = 0$; $x_b = 0$ and $\Delta\varepsilon_v^p = 0$

If $k_0P^e + K_{cam} + kP_{cr}^- \neq 0$; $x_b=\left(\dfrac{P^e-P_{cr}^--P_{trac}}{k_0P^e+K_{cam}+kP_{cr}^-}\right)$

If not one makes a limited development of order 2 and one finds;

$$\left(P^e-P_{cr}^--P_{trac}\right)-\left(k_0P^e+K_{cam}+kP_{cr}^-\right)x_b+\frac{1}{2}\left(k_0P^e+K_{cam}-kP_{cr}^-\right)x_b^2=0$$

Like $k_0P^e + K_{cam} + kP_{cr}^- = 0$ then $k_0P^e + K_{cam} - kP_{cr}^- \neq 0$

And one solves

$$\left(P^e-P_{cr}^--P_{trac}\right)+\frac{1}{2}\left(k_0P^e+K_{cam}-kP_{cr}^-\right)x_b^2=0$$

If $P^e - P_{cr}^- - P_{trac} = 0$; $x_b = 0$ and $\Delta\varepsilon_v^p = 0$

If not $x_b=\pm\sqrt{\dfrac{-2\left(P^e-P_{cr}^--P_{trac}\right)}{\left(k_0P^e+K_{cam}-kP_{cr}^-\right)}}$

If σ <0 one chooses a value for x_b approached either $x_b=\dfrac{1}{k_0+k}Log\left(\dfrac{\left|P^e-P_{trac}\right|}{P_{cr}^-}\right)$

If not one has the choice between two values of x_b ;

The following test is made:

If $\left(P^e - P_{trac} > P_{cr}^-\right)$ then $x_b = \sqrt{\dfrac{-2\left(P^e - P_{cr}^- - P_{trac}\right)}{\left(k_0 P^e + K_{cam} - kP_{cr}^-\right)}}$; the solution would be positive; x>0

If $\left(P^e - P_{trac} < P_{cr}^-\right)$ then $x_b = -\sqrt{\dfrac{-2\left(P^e - P_{cr}^- - P_{trac}\right)}{\left(k_0 P^e + K_{cam} - kP_{cr}^-\right)}}$; the solution would be negative; x<0

4.5 Typical case of the critical point

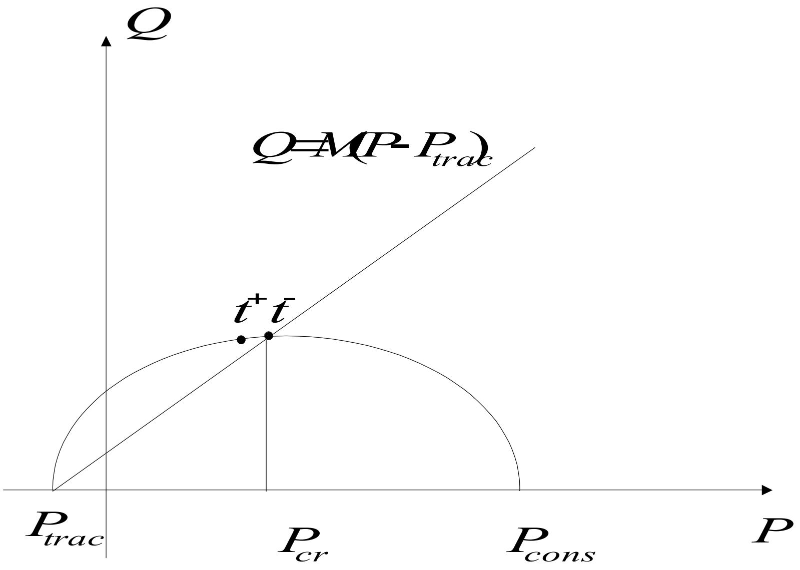


Figure 4.5-a: Mechanical state around the critical point

So at the moment t^- one reaches the critical condition, then $P_{cr}^+ = P_{cr}^-$, $\Delta\varepsilon_v^p = 0$ and $Q^- = MP^-$. If $f = 0,\ \dot{f} = 0$, then the point (P, Q) at the moment t^+ moves on the initial ellipse (cf [Figure 4.5-a]). One deduces immediately from the elastic law and the condition $\Delta\varepsilon_v^p = 0$:

$$\Delta P = k_0\ \Delta\ \varepsilon_v P^- \quad \textbf{éq 4.5-1}$$

The criterion being checked at the moment t^+ , one has while using [éq 4.5-1]:

$$Q^{+2}=M^2 P^+(2P_{cr}^- - P^+)=M^2(P^- + \Delta P)(P^- - \Delta P)=M^2 P^{-(2)}(1-k_0^2 \Delta\varepsilon_v^2)=Q^{-(2)}(1-k_0^2 \Delta\varepsilon_v^2)$$ **éq 4.5-2**

In addition the diverter of the constraints can be written:

$$s=s^e-2\mu\ \Delta\tilde{\varepsilon}^p=s^e-2\mu\ \Lambda\frac{\partial f}{\partial s}=s^e-6\mu\ \Lambda\ s$$ **éq 4.5-3**

One from of deduced:

$$1+6\ \mu\ \Lambda=\frac{Q^e}{Q}\ ,$$ **éq 4.5-4**

and:

$$s=\frac{Q^-\sqrt{(1-k_0^2\Delta\varepsilon_v^2}}{Q^e}\ s^e$$ **éq 4.5-5**

4.6 Summary

The discretization of the equations and the law of implicit behavior of manner leads to the resolution of the equation [éq 4.3-2].

If $P^- \neq P_{cr}^-$, then one solves the equation [éq 4.3-2] whose unknown factor is $\Delta\varepsilon_v^p$.

One deduces then:

$$P_{cr}=P_{cr^-}\exp(k\Delta\varepsilon_v^p),$$
$$P=P^e\exp\left[-k_0\,\Delta\varepsilon_v^p\right]+\frac{K_{cam}}{k_0}\left(\exp\left[-k_0\,\Delta\varepsilon_v^p\right]-1\right)\ ,$$
$$\text{puis}\ \ s=\frac{s^e}{1+\dfrac{3\mu\Delta\varepsilon_v^p}{M^2\left(P-P_{trac}-P_{cr}\right)}}$$ **éq 4.6-1**

One deduces finally:

$$\Delta\tilde{\varepsilon}^p=\frac{3}{2}\frac{\Delta\varepsilon_v^p}{M^2(P-P_{trac}-P_{cr})}s$$ **éq 4.6-2**

At the critical point:

$$\Delta\varepsilon_v^p=0, P_{cr}=P_{cr}^-$$ **éq 4.6-3**

In this point, there is no evolution of work hardening, on the other hand the state of stress can continue to evolve either in contractance, or in dilatancy (the tangent with the criterion is horizontal). The new state of stresses moves on the surface of load of the preceding state.

5 Tangent operator

If the option is: RIGI_MECA_TANG , option used at the time of the prediction, the tangent operator calculated in each point of Gauss is known as of speed:

$$\dot{\sigma}_{ij}=D^{elp}_{ijkl}\dot{\varepsilon}_{kl}$$

In this case, D^{elp}_{ijkl} is calculated starting from the not discretized equations.

If the option is: FULL_MECA , option used when one reactualizes the tangent matrix with each iteration by updating the internal constraints and variables:

$$d\sigma_{ij}=A_{ijkl}\,d\varepsilon_{kl}$$

In this case, A_{ijkl} is calculated starting from the implicitly discretized equations.

5.1 Nonlinear elastic tangent operator

The elastic relation of speed is written:

$$\dot{\sigma}_{ij}=-\dot{P}\,\delta_{ij}+\dot{s}_{ij}=\left(k_0 P+K_{cam}\right)tr\,\{\,\dot{\varepsilon}\;\delta_{ij}+2\mu\;\dot{\tilde{\varepsilon}}\,\}$$ **éq 5.1-1**

$$\dot{\sigma}_{ij}=\left(k_0 P+K_{cam}-\frac{2}{3}\mu\right)tr\,\{\,\dot{\varepsilon}\;\delta_{ij}+2\mu\;\dot{\varepsilon}_{ij}\,\}$$ **éq 5.1-2**

The tangent operator in elasticity of the law noted Cam_Clay D^e is thus deduced from the following matric writing:

$$\begin{Bmatrix}\dot{\sigma}_{11}\\ \dot{\sigma}_{22}\\ \dot{\sigma}_{33}\\ \sqrt{2}\dot{\sigma}_{12}\\ \sqrt{2}\dot{\sigma}_{23}\\ \sqrt{2}\dot{\sigma}_{31}\end{Bmatrix}=\underbrace{\begin{bmatrix}k_0P+K_{cam}+\frac{4}{3}\mu & k_0P+K_{cam}-\frac{2}{3}\mu & k_0P+K_{cam}-\frac{2}{3}\mu & 0 & 0 & 0\\ k_0P+K_{cam}-\frac{2}{3}\mu & k_0P+K_{cam}+\frac{4}{3}\mu & k_0P+K_{cam}-\frac{2}{3}\mu & 0 & 0 & 0\\ k_0P+K_{cam}-\frac{2}{3}\mu & k_0P+K_{cam}-\frac{2}{3}\mu & k_0P+K_{cam}+\frac{4}{3}\mu & 0 & 0 & 0\\ 0 & 0 & 0 & 2\mu & 0 & 0\\ 0 & 0 & 0 & 0 & 2\mu & 0\\ 0 & 0 & 0 & 0 & 0 & 2\mu\end{bmatrix}}_{D^e}\begin{Bmatrix}\dot{\varepsilon}_{11}\\ \dot{\varepsilon}_{22}\\ \dot{\varepsilon}_{33}\\ \sqrt{2}\dot{\varepsilon}_{12}\\ \sqrt{2}\dot{\varepsilon}_{23}\\ \sqrt{2}\dot{\varepsilon}_{31}\end{Bmatrix}$$ **éq 5.1-3**

5.2 Plastic tangent operator of speed. Option RIGI_MECA_TANG

The total tangent operator is in this case K_{i-1} (the option RIGI_MECA_TANG called with the first iteration of a new increment of load) starting from the results known at the moment t_{i-1} [bib3].
If the tensor of the constraints with t_{i-1} is on the border of the field of elasticity, one writes the condition: $\dot{f}=0$ who must be checked jointly with the condition $f=0$. If the tensor of the constraints with t_{i-1} is inside the field, $f<0$, then the tangent operator is the operator of elasticity.

$$\dot{f}=\left(\frac{\partial f}{\partial \sigma}\right)\dot{\sigma}+\frac{\partial f}{\partial P_{cr}}\dot{P}_{cr}=0 \qquad \text{éq 5.2-1}$$

like $\dot{P}_{cr}=\frac{\partial P_{cr}}{\partial \varepsilon_v^p}\dot{\varepsilon}_v^p$, then:

$$\dot{f}=\left(\frac{\partial f}{\partial \sigma}\right)\dot{\sigma}+\frac{\partial f}{\partial P_{cr}}\frac{\partial P_{cr}}{\partial \varepsilon_v^p}\dot{\varepsilon}_v^p=0 \qquad \text{éq 5.2-2}$$

In addition $\dot{\varepsilon}^e=\dot{\varepsilon}-\dot{\varepsilon}^p$

thus:

$$D^{e^{-1}}\dot{\sigma}=\dot{\varepsilon}-\dot{\Lambda}\frac{\partial f}{\partial \sigma}\ , \qquad \text{éq 5.2-3}$$

i.e.:

$$\dot{\sigma}_{ij}=D^e_{ijkl}\dot{\varepsilon}_{kl}-\dot{\Lambda}D^e_{ijkl}\left(\frac{\partial f}{\partial \sigma}\right)_{kl} \qquad \text{éq 5.2-4}$$

The plastic module of work hardening is written according to the equation [éq 3.5-7] and by using the rule of flow:

$$H_p=\frac{\partial f}{\partial P_{cr}}\frac{\partial P_{cr}}{\partial \varepsilon_v^p}\frac{\partial F}{\partial P_{cr}}=-\frac{1}{\dot{\Lambda}}\frac{\partial f}{\partial P_{cr}}\frac{\partial P_{cr}}{\partial \varepsilon_v^p}\dot{\varepsilon}_v^p \qquad \text{éq 5.2-5}$$

The equations [éq 5.2-1] and [éq 5.2-5] give:

$$\left(\frac{\partial f}{\partial \sigma}\right)_{ij}\dot{\sigma}_{ij}-\dot{\Lambda}H_p=0 \qquad \text{éq 5.2-6}$$

Multiplication of the equation [éq 5.2-4] by $\left(\frac{\partial f}{\partial \sigma}\right)_{ij}$ give:

$$\left(\frac{\partial f}{\partial \sigma}\right)_{ij}\dot{\sigma}_{ij}=\left(\frac{\partial f}{\partial \sigma}\right)_{ij}D^e_{ijkl}\dot{\varepsilon}-\dot{\Lambda}\left(\frac{\partial f}{\partial \sigma}\right)_{ij}D^e_{ijkl}\left(\frac{\partial f}{\partial \sigma}\right)_{kl} \qquad \text{éq 5.2-7}$$

The two preceding equations make it possible to find:

$$H_p \dot{\Lambda} = \left(\frac{\partial f}{\partial \sigma}\right)_{ij} D^e_{ijkl} \dot{\varepsilon}_{kl} - \dot{\Lambda} \left(\frac{\partial f}{\partial \sigma}\right)_{ij} D^e_{ijkl} \left(\frac{\partial f}{\partial \sigma}\right)_{kl} \quad \text{éq 5.2-8}$$

from where the expression of the plastic multiplier:

$$\dot{\Lambda} = \frac{\left(\frac{\partial f}{\partial \sigma}\right)_{ij} D^e_{ijkl} \dot{\varepsilon}_{kl}}{\left(\frac{\partial f}{\partial \sigma}\right)_{ij} D^e_{ijkl} \left(\frac{\partial f}{\partial \sigma}\right)_{kl} + H_p} \quad \text{éq 5.2-9}$$

That is to say H the definite elastoplastic module like:

$$H = \left(\frac{\partial f}{\partial \sigma}\right)_{ij} D^e_{ijkl} \left(\frac{\partial f}{\partial \sigma}\right)_{kl} + H_p \quad \text{éq 5.2-10}$$

The plastic multiplier is written:

$$\dot{\Lambda} = \frac{\left(\frac{\partial f}{\partial \sigma}\right)_{ij} D^e_{ijkl} \dot{\varepsilon}_{kl}}{H} \quad \text{éq 5.2-11}$$

While replacing $\dot{\Lambda}$ by his expression in the equation [éq 5.2-4], one obtains:

$$\dot{\sigma}_{ij} = D^e_{ijkl} \dot{\varepsilon}_{kl} - \frac{1}{H} \left[\left(\frac{\partial f}{\partial \sigma}\right)_{mn} D^e_{mnop} \dot{\varepsilon}_{op} \right] . D^e_{ijkl} \left(\frac{\partial f}{\partial \sigma}\right)_{kl} \quad \text{éq 5.2-12}$$

One from of thus deduced the elastoplastic operator $D^{elp} = D^e - D^p$:

$$\dot{\sigma}_{ij} = \underbrace{\left[D^e_{ijkl} - \frac{1}{H} \left(\frac{\partial f}{\partial \sigma}\right)_{op} D^e_{ijop} \; D^e_{mnkl} \left(\frac{\partial f}{\partial \sigma}\right)_{mn} \right]}_{D^{elp}} \dot{\varepsilon}_{kl} \quad \text{éq 5.2-13}$$

with,

$$D^p_{ijkl} = \frac{1}{H} \left(\frac{\partial f}{\partial \sigma}\right)_{op} D^e_{ijop} \; D^e_{mnkl} \left(\frac{\partial f}{\partial \sigma}\right)_{mn} \quad \text{éq 5.2-14}$$

Calculation of H :

$$\left(\frac{\partial f}{\partial \sigma}\right)_{ij}=-\frac{2}{3}M^2\left(P-P_{trac}-P_{cr}\right)\delta_{ij}+3s_{ij} \text{ ,} \quad \textbf{éq 5.2-15}$$

who is written in vectorial notation:

$$\begin{bmatrix} -\frac{2}{3}M^2\left(P-P_{trac}-P_{cr}\right)+3s_{11} \\ -\frac{2}{3}M^2\left(P-P_{trac}-P_{cr}\right)+3s_{22} \\ -\frac{2}{3}M^2\left(P-P_{trac}-P_{cr}\right)+3s_{33} \\ 3\sqrt{2}\,s_{12} \\ 3\sqrt{2}\,s_{23} \\ 3\sqrt{2}\,s_{31} \end{bmatrix} \quad \textbf{éq 5.2-16}$$

from where the expression of:

$$D^e_{ijkl}\left(\frac{\partial f}{\partial \sigma}\right)_{kl} : \begin{bmatrix} -2k_0M^2\left(P-P_{trac}\right)\left(P-P_{trac}-P_{cr}\right)+6\mu s_{11} \\ -2k_0M^2\left(P-P_{trac}\right)\left(P-P_{trac}-P_{cr}\right)+6\mu s_{22} \\ -2k_0M^2\left(P-P_{trac}\right)\left(P-P_{trac}-P_{cr}\right)+6\mu s_{33} \\ 6\mu\sqrt{2}\,s_{12} \\ 6\mu\sqrt{2}\,s_{23} \\ 6\mu\sqrt{2}\,s_{31} \end{bmatrix} \quad \textbf{éq 5.2-17}$$

and

$$\left(\frac{\partial f}{\partial \sigma}\right)_{ij}D^e_{ijkl}\left(\frac{\partial f}{\partial \sigma}\right)_{kl}=4k_0M^4\left(P-P_{trac}\right)\left(P-P_{trac}-P_{cr}\right)^2+12\mu Q^2 \quad \textbf{where}$$

$$12\mu Q^2=18\mu\, tr\left(s\,.\,s\right) \quad \textbf{éq 5.2-18}$$

According to the equations [éq 3.5-7] and [éq 5.2-18], one can deduce the expression from H :

$$H=4M^4\left(P-P_{trac}\right)\left(P-P_{trac}-P_{cr}\right)\left(k_0\left(P-P_{trac}-P_{cr}\right)+kP_{cr}\right)+12\mu Q^2 \quad \textbf{éq 5.2-19}$$

While posing:

$$A_{ij}=-2k_0M^2\left(P-P_{trac}\right)\left(P-P_{trac}-P_{cr}\right)\delta_{ij}+6\mu s_{ij} \text{ ,} \quad \textbf{éq 5.2-20}$$

one can write the following symmetrical plastic matrix:

$$D^p=\frac{1}{H}\begin{bmatrix} A_{11}^2 & A_{11}A_{22} & A_{11}A_{33} & 6\sqrt{2}\,\mu A_{11}s_{12} & 6\sqrt{2}\,\mu A_{11}s_{23} & 6\sqrt{2}\,\mu A_{11}s_{31} \\ . & A_{22}^2 & A_{22}A_{33} & 6\sqrt{2}\,\mu A_{22}s_{12} & 6\sqrt{2}\,\mu A_{22}s_{23} & 6\sqrt{2}\,\mu A_{22}s_{31} \\ . & . & A_{33}^2 & 6\sqrt{2}\,\mu A_{33}s_{12} & 6\sqrt{2}\,\mu A_{33}s_{23} & 6\sqrt{2}\,\mu A_{33}s_{31} \\ . & . & . & 36\,\mu^2 s_{12}^2 & 36\,\mu^2 s_{12}s_{23} & 36\,\mu^2 s_{12}s_{31} \\ . & . & . & . & 36\,\mu^2 s_{23}^2 & 36\,\mu^2 s_{23}s_{31} \\ . & . & . & . & . & 36\,\mu^2 s_{31}^2 \end{bmatrix}$$ **éq 5.2-21**

5.3 Tangent operator into implicit. Option FULL_MECA

The coherent tangent operator of the option FULL_MECA is calculated like the tangent operator of speed for the current state of stresses.

Nevertheless, of the theoretical elements allowing to calculate it are given in appendix, in the paragraph 8. To note, that the equations present in the appendix suppose that the criterion passes by a state of stress worthless, P_{trac} and K_{cam} were not introduced yet there. It is necessary to think of taking them into account and with the need to reactivate these equations for the coherent tangent operator.

6 Internal parameters materials and variables

6.1 Parameters materials

Parameters E and ν obligatory under keyword ELAS are not used by law CAM_CLAY. Keyword ELAS can of this fact avoided being if the user does not need to inform α or ρ .

The data specific to the Cam_Clay model are:

- The elastic module of shearing μ ,
- The critical slope M ,
- Porosity associated with a pressure initial and related to the initial index of the vacuums: $n=\dfrac{e_0}{1+e_0}$
- Initial compressibility K_{cam} ,
- Pressure of tolerated traction P_{trac} , (must be negative)
- The elastic coefficient of swelling: κ (which leads to k_0),
- The plastic coefficient of compressibility: λ (which leads to k),
- Initial critical pressure P_{cr0} such as $P_{cr0}-P_{trac}$ that is to say equalizes with half of the pressure of consolidation,

Notice 1 :

The number of data is relatively low, which makes the model very simple. One of the most visible limitations of the model is the assumption of the alignment of the critical points on a line of slope M . This is besides the expression of the concept of internal friction. One can also interpret the size M by connecting it to the natural angle of repose of Coulomb by the relation: $\sin\,\phi=\dfrac{3M}{6+M}$. However it is known that for very cohesive materials, this angle varies when the average constraint decreases.

Besides one notes that for a chock of M on a triaxial compression test with a certain average constraint, one simulates well with this model the triaxial ones realized with a average constraint step too different but one cannot correctly consider the stages plastic for a broad range of pressure of containment (cf [bib2]). It is thus necessary to readjust M for several beaches of average constraint.

Notice 2:

The increase in constraints is connected to the voluminal increase in the deformations according to one or the other of the laws of behavior:

With Cam_Clay:

$$\Delta P=\left(k_0 P^- + K_{cam}\right)\Delta\varepsilon_v$$

$tr(\Delta\sigma)=3\left(k_0 P^- + K_{cam}\right)\Delta\varepsilon_v$ with $k_0=\frac{1+e_0}{\kappa}$ where $e_0=\frac{n}{1-n}$; n is porosity and it is a data material.

In elasticity:

$$tr(\Delta\sigma)=\frac{E}{(1-2\nu)}tr(\Delta\varepsilon)=3\mathrm{K}\Delta\varepsilon_v$$

The analogy enters the hydrostatic part of Cam_Clay and linear elasticity **at the initial state** allows to write:

$$\frac{\left(1+e_0\right)P^-}{\kappa}+K_{cam}=\frac{E}{3\left(1-2\nu\right)}$$

E and ν are not data materials but rather μ the modulus of rigidity: $\mu=\frac{E}{2\left(1+\nu\right)}$

What amounts writing the following equality while eliminating E :

$$\frac{\left(1+e_0\right)P^-}{\kappa}+K_{cam}=\frac{2\mu}{3}\frac{\left(1+\nu\right)}{\left(1-2\nu\right)} \quad \text{or} \quad \frac{\left(1+\nu\right)}{\left(1-2\nu\right)}=\frac{3\left(1+e_0\right)P^-+3\mathrm{K}_{cam}\kappa}{2\mu\kappa}$$

and one finds the expression of ν :

$$\nu=\frac{3(1+e_0)P^-+3K_{cam}\kappa-2\mu\kappa}{6(1+e_0)P^-+6K_{cam}\kappa-2\mu\kappa}$$

with the starting of calculation, P^- corresponds to the initial stress field.

one can then deduce E from ν : $E=2\mu(1+\nu)$

the following conditions are to be checked:

$$0<\nu=\frac{3\left(1+e_0\right)P^-+3\mathrm{K}_{cam}\kappa-2\mu\kappa}{6\left(1+e_0\right)P^-+6\mathrm{K}_{cam}\kappa+2\mu\kappa}\leq 0.5 \quad \text{and} \quad E>0$$

if one or the other of the two conditions is not satisfied, a message of alarm informs the user of nonthe coherence of the provided parameters.

Notice 3:

*Si P_{trac} is given null:

two possibilities for K_{cam} :

1 K_{cam} positive (of the worthless initial constraints are allowed)
2 K_{cam} no one (the constraints should absolutely be initialized)

*Si P_{trac} is given negative:
only one possibility for K_{cam} :
* K_{cam} positive as the relation should be satisfied $k_0 P_{trac} + K_{cam} > 0$
(one cannot initialize the constraints and give a zero value to K_{cam})

6.2 Internal variables

V_1 : critical pressure P_{cr}
V_2 : plastic state
V_3 : constraint of containment P
V_4 : equivalent constraint Q
V_5 : voluminal plastic deformation ε_v^p
V_6 : equivalent plastic deformation ε_{eq}^p
V_7 : index of the vacuums e

7 Implementation of a calculation with the law CAM_CLAY

7.1 Initialization of calculation

In the model CAM_CLAY, the non-linear elastic law reveals a hydrostatic constraint for a worthless voluminal deformation [éq 3.3-3].

The user adopts one of the two following choices:

- To give to the parameter material K_{cam} who represents an initial compressibility a value **positive**. Calculation can be done with a state of worthless initial stresses.

- To give to the parameter material K_{cam} who represents an initial compressibility a value **worthless**. To initialize the state of stresses according to one in the two different ways:

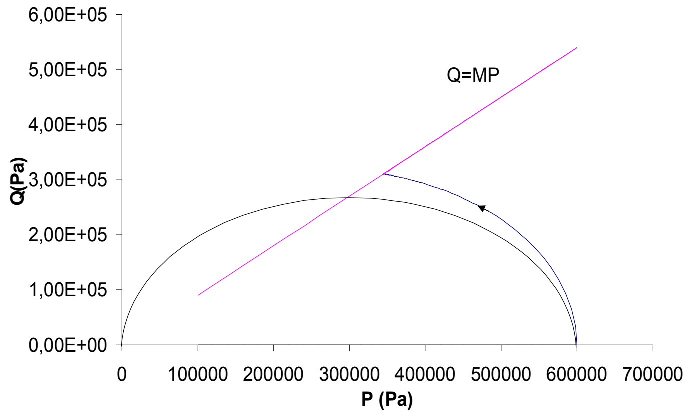
  •To carry out a linear elastic design by affecting boundary conditions such as the stress field in the structure is a uniform hydrostatic compression. One extracts from this calculation the stress field at the points of Gauss. This stress field is regarded as the initial state of the hydrostatic constraint necessary to the law CAM_CLAY in calculation STAT_NON_LINE using the model CAM_CLAY.

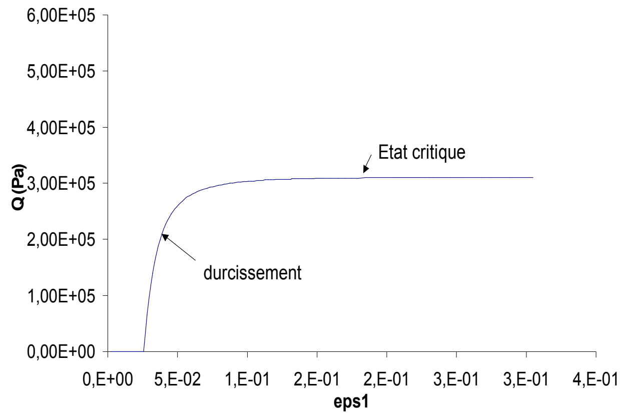
  •To use the operator CREA_CHAMP to create with the operation 'AFFE'a hydrostatic stress field at the points of Gauss, the constraint in this case is given of negative sign (convention *Aster* for compressions) and constitutes the initial state in STAT_NON_LINE according to.

7.2 Examples of results got on triaxial compression tests

The following figures show triaxial ways of loading with evolutions of the axial deformation according to the diverter Q . They result from digital calculations carried out with the model CAM_CLAY established in *Code_Aster*. These test were carried out by using a modeling of the type KIT_HM in not drained condition (this condition easily enables us to charge in a purely deviatoric way, the hydrostatic part of the loading being taken again by pressure of water). Shapes of the curves obtained numerically with *Code_Aster* completely comparable to the schematic curves are presented in the paper of Charlez [bib2].

In the first test, the material is normally consolidated, i.e. the hydrostatic starting pressure is equal to the pressure of consolidation (in this case 6.10^5 Pa). Work hardening (positive) starts at the beginning of the deviatoric phase, without preliminary elastic phase. Hardening continues to a stage of perfect plasticity when one reaches the point criticizes ($Q=MP$).
As for three other tests, the deviatoric phase starts for a value of the average effective constraint lower than the pressure of consolidation, the material is of this surconsolidé fact.
If P is higher than P_{cr} equalize with $3.10^5\,Pa$, the specific point of the loading cuts the surface of load before the critical line. There will be thus three specific phases: an elastic phase, a contracting plastic phase then a perfect plastic phase.
If $P=P_{cr}$, the behavior is plastic perfect right after the elastic phase.
In the case where P is lower than P_{cr} , the point representative of the loading cuts the critical line before the surface of load which it reaches during a purely elastic way. In this configuration, the behavior is lenitive and dilating and blocked energy decreases. The point representative of the loading joined then the critical condition where the material will enter in perfect plasticity.
The behavior `CAM_CLAY` a behavior continuement contractor cannot produce/dilating. The point representative of the loading is obliged to pass by the critical condition where the whole of the parameters of work hardening (plastic voluminal deformation, critical pressure, blocked energy) become stationary [bib2].


Q=MP
Q(Pa)
P (Pa)


Etat critique
durcissement
Q(Pa)
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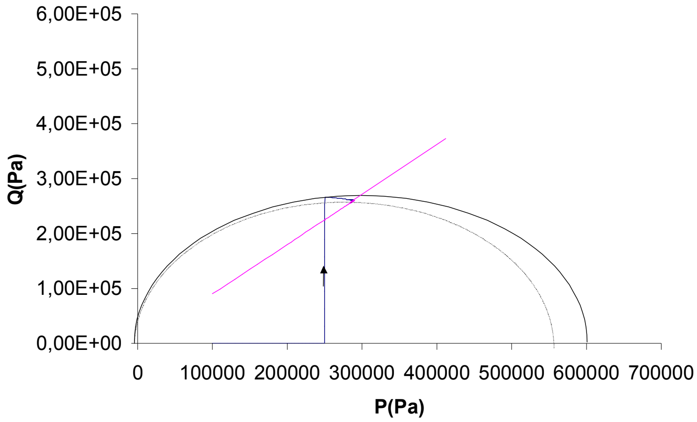
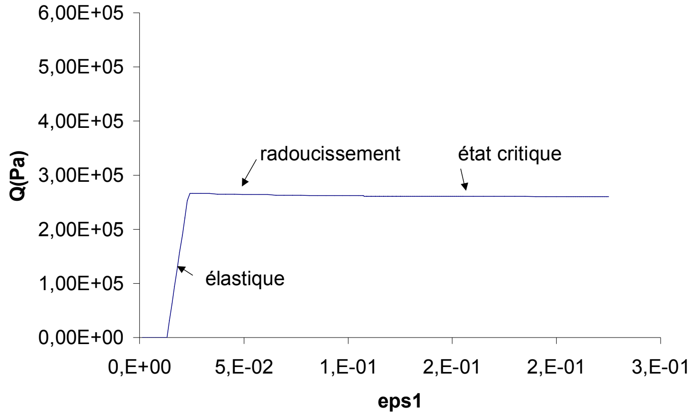

Q(Pa)
P(Pa)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Q(Pa)
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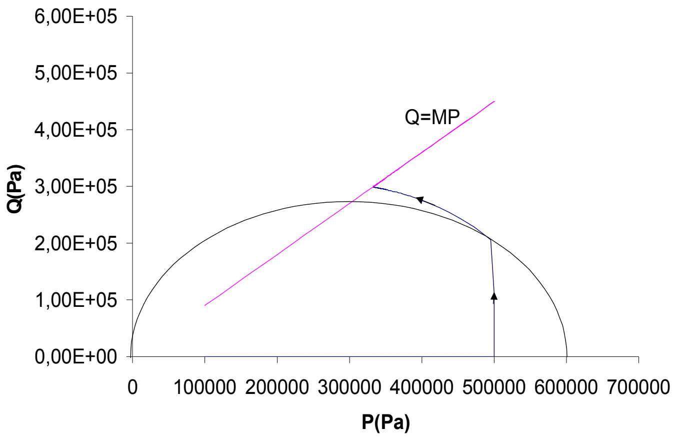
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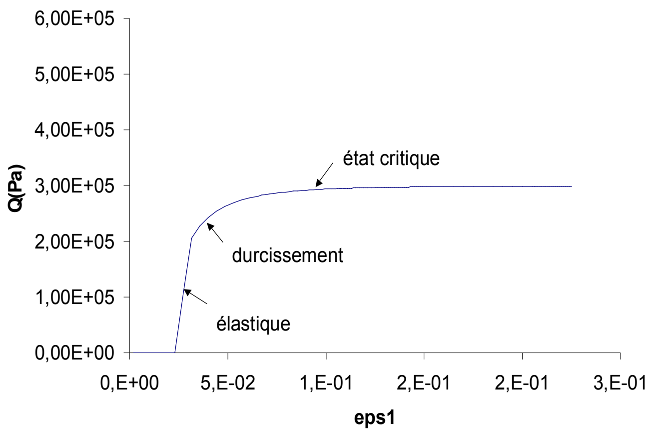
Q(Pa)
P(Pa)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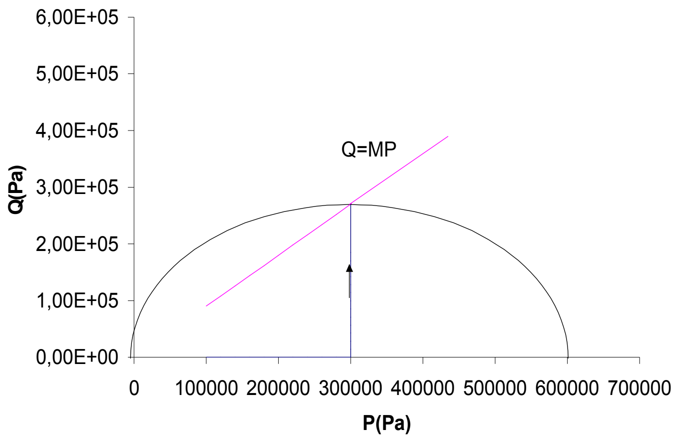
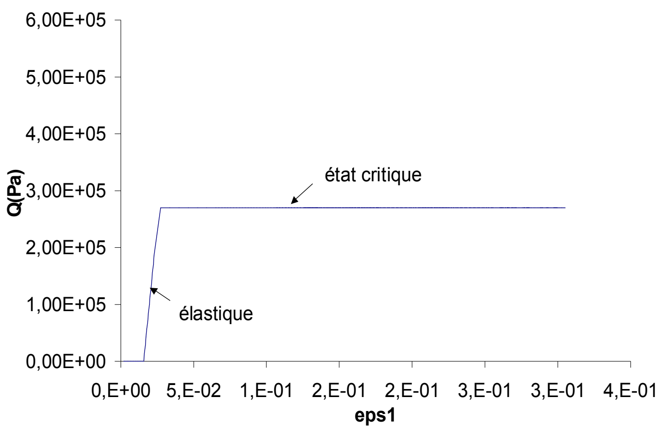

Q=MP
Q(Pa)
P(Pa)


état critique
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8 Appendix: Tangent operator into implicit. Option FULL_MECA

We present in this appendix of the elements of calculation of the coherent tangent operator.

8.1 Case general

8.1.1 Treatment of the deviatoric part

It is considered here that the variation of loading is purely deviatoric $(\delta P=0)$.
The increment of the deviatoric constraint is written in the form:

$$\Delta s_{ij}=2\mu\left(\Delta\tilde{\varepsilon}_{ij}-\Delta\tilde{\varepsilon}_{ij}^{p}\right) \quad \textbf{éq 8.1.1-1}$$

Around the point of balance $\left(\sigma^{-}+\Delta\sigma\right)$, a variation is considered δs deviatoric part of the constraint:

$$\delta s_{kl}=2\mu\left(\delta\tilde{\varepsilon}_{kl}-\delta\tilde{\varepsilon}_{kl}^{p}\right) \quad \textbf{éq 8.1.1-2}$$

Calculation of $\delta\tilde{\varepsilon}_{kl}^{p}$:

It is known that:

$$\Delta\tilde{\varepsilon}_{kl}^{p}=3\Lambda\ s_{kl} \quad \textbf{éq 8.1.1-3}$$

By deriving this equation compared to the deviatoric constraint, one obtains:

$$\delta\tilde{\varepsilon}_{kl}^{p}=3\ \delta\ \Lambda\ s_{kl}+3\ \Lambda\ \delta\ s_{kl} \quad \textbf{éq 8.1.1-4}$$

Calculation of $\delta\Lambda$:
One a:

$$\begin{aligned}\Lambda&=\frac{1}{H_p}\left[\left(\frac{\partial f}{\partial\sigma}\right)_{mn}\Delta\sigma_{mn}\right]=\frac{1}{H_p}\left[\left(\frac{\partial f}{\partial s}\right)_{mn}\Delta s_{mn}+\frac{\partial f}{\partial P}\Delta P\right]\\&=\frac{1}{H_p}\left[3s_{mn}\Delta s_{mn}+2M^2\left(P-P_{cr}\right)\Delta P\right]\end{aligned} \quad \textbf{éq 8.1.1-5}$$

If one considers only the evolution of the deviatoric part of σ $(\delta P=0)$, then:

$$\delta\ (\Lambda\ H_p)=\delta\ \Lambda H_p+\Lambda\delta H_p=\left[3\delta s_{mn}\Delta s_{mn}+3s_{mn}\delta s_{mn}\right]-2M^2\Delta P\delta P_{cr} \quad \textbf{éq 8.1.1-6}$$

However: $\delta\ P_{cr}=kP_{cr}\ \delta\ \varepsilon_v^P$.

$$\text{Comme }\ \Delta\varepsilon_v^p=2\Lambda M^2(P-P_{cr}),\ \text{ on a }\ \delta\varepsilon_V^p=2\delta\Lambda M^2(P-P_{cr})-2M^2\Lambda\delta P_{cr}\ , \quad \textbf{éq 8.1.1-7}$$

From where:

$$2\delta\ \Lambda M^2(P-P_{cr})=\left[\frac{1}{kP_{cr}}+2\Lambda M^2\right]\delta P_{cr}. \quad \textbf{éq 8.1.1-8}$$

In addition,

$$H_p = 4kM^4 PP_{cr}(P-P_{cr}) \text{ et } \delta H_p = 4kM^4 P(P-2P_{cr})\delta P_{cr}. \qquad \textbf{éq 8.1.1-9}$$

By injecting this last equation in the equation [éq 5.3.1-6], one obtains:

$$\delta \Lambda H_p + \left[4\Lambda kM^4 P(P-2P_{cr}) + 2M^2 \Delta P\right]\delta P_{cr} = \left[3\delta s_{mn}\Delta s_{mn} + 3s_{mn}\delta s_{mn}\right] \qquad \textbf{éq 8.1.1-10}$$

While using the relation [éq 5.3.1-8], it comes then:

$$\delta \Lambda = \frac{\left[3\delta s_{mn}\Delta s_{mn} + 3s_{mn}\delta s_{mn}\right]}{(H_p + A)} \qquad \textbf{éq 8.1.1-11}$$

with $A = \left[4k\Lambda M^4 P(P-2P_{cr}) + 2M^2\Delta P\right]\left[\dfrac{M^2(P-P_{cr})}{\dfrac{1}{2kP_{cr}} + \Lambda M^2}\right]$

One then obtains immediately the variation of the deviatoric part of the plastic deformation:

$$\delta\tilde{\varepsilon}_{kl}^p = \frac{9}{(H_p + A)}(\Delta s_{mn}\delta s_{mn}s_{kl} + s_{mn}\delta s_{mn}s_{kl}) + \frac{9}{H_p}s_{mn}\Delta s_{mn}\delta s_{kl} + \frac{6}{H_p}M^2(P - P_{cr})\Delta P\delta s_{kl}$$

éq 8.1.1-12

δs_{ij} is written then:

$$\delta s_{ij} = 2\mu\delta\{\tilde{\varepsilon}_{ij} - \frac{18\mu}{(H_p + A)}\left[\left(\Delta s_{kl}s_{ij}\delta s_{kl} + s_{kl}s_{ij}\delta s_{kl}\right)\right] - \frac{18\mu}{H_p}s_{kl}\Delta s_{kl}\delta s_{ij} - \frac{12\mu}{H_p}M^2\left(P-P_{cr}\right)\Delta P\delta s_{ij}$$

éq 8.1.1-13

who becomes by separating the terms in variation from constraints and the term in variation of total deflection:

éq 8.1.1-14

or in tensorial writing:

$$\left\{I_4^d\left(1 + \frac{12\mu}{H_p}M^2\left(P-P_{cr}\right)\Delta P + \frac{18\mu}{H_p}\Delta s : s\right) + \frac{18\mu}{(H_p + A)}(s + \Delta s)\otimes s\right\}\delta s = 2\mu\delta\{\tilde{\varepsilon}\} \qquad \textbf{q8.1.1-15}$$

that one can still write by symmetrizing the tensor $(s+\Delta s)\otimes s$:

$$\left\{I_4^d\left(1 + \frac{12\mu}{H_p}M^2\left(P-P_{cr}\right)\Delta P + \frac{18\mu}{H_p}\Delta s : s\right) + \frac{18\mu}{(H_p + A)}\aleph\right\}\delta s = 2\mu\delta\{\tilde{\varepsilon}\} \qquad \textbf{éq 8.1.1-16}$$

with: $\aleph = \frac{1}{2}\left[((s+\Delta s)\otimes s) + (s\otimes(s+\Delta s))^T\right]$

Calculation of $\aleph$, while posing: $T_{ij} = s_{ij} + \Delta s_{ij}$

$$T \otimes s = \begin{bmatrix} T_{11}s_{11} & T_{11}s_{22} & T_{11}s_{33} & \sqrt{2}T_{11}s_{12} & \sqrt{2}T_{11}s_{23} & \sqrt{2}T_{11}s_{31} \\ T_{22}s_{11} & T_{22}s_{22} & T_{22}s_{33} & \sqrt{2}T_{22}s_{12} & \sqrt{2}T_{22}s_{23} & \sqrt{2}T_{22}s_{31} \\ T_{33}s_{11} & T_{33}s_{22} & T_{33}s_{33} & \sqrt{2}T_{33}s_{12} & \sqrt{2}T_{33}s_{23} & \sqrt{2}T_{33}s_{31} \\ \sqrt{2}T_{12}s_{11} & \sqrt{2}T_{12}s_{22} & \sqrt{2}T_{12}s_{33} & 2T_{12}s_{12} & 2T_{12}s_{23} & 2T_{12}s_{31} \\ \sqrt{2}T_{23}s_{11} & \sqrt{2}T_{23}s_{22} & \sqrt{2}T_{23}s_{33} & 2T_{23}s_{12} & 2T_{23}s_{23} & 2T_{23}s_{31} \\ \sqrt{2}T_{31}s_{11} & \sqrt{2}T_{31}s_{22} & \sqrt{2}T_{31}s_{33} & T_{31}s_{12} & 2T_{31}s_{23} & 2T_{31}s_{31} \end{bmatrix}$$ **éq 8.1.1-17**

$$\aleph = \frac{1}{2}\left[(T \otimes s) + (T \otimes s)^T\right]$$ **éq 8.1.1-18**

That is to say:

$$C = \left\{ I_4^d \left(\frac{1}{2\mu} + \frac{6}{H_p} M^2 \left(P - P_{cr}\right) \Delta P + \frac{9}{H_p} \Delta s : s \right) + \frac{9}{(H_p + A)} \aleph \right\}$$ **éq 8.1.1-19**

one poses:

$$c = \frac{9}{H_p} \left(\Delta s : s\right)$$ **éq 8.1.1-20**

and

$$d = \frac{6}{H_p} M^2 \left(P - P_{cr}\right) \Delta P$$ **éq 8.1.1-21**

The symmetrical matrix C dimensions (6.6) is too large to be presented whole, one breaks up it into 4 parts C_1 , C_2 , C_3 and C_4 :

$$C = \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix}$$

with

$$C_1=\begin{bmatrix} \frac{1}{2\mu}+c+d+\frac{9}{(H_p+A)}s_{11}T_{11} & \frac{9}{2(H_p+A)}(T_{11}s_{22}+T_{22}s_{11}) & \frac{9}{2(H_p+A)}(T_{11}s_{33}+T_{33}s_{11}) \\ \frac{9}{2(H_p+A)}(T_{22}s_{11}+T_{11}s_{22}) & \frac{1}{2\mu}+c+d+\frac{9}{(H_p+A)}T_{22}s_{22} & \frac{9}{2(H_p+A)}(T_{22}s_{33}+T_{33}s_{22}) \\ \frac{9}{2(H_p+A)}(T_{33}s_{11}+T_{11}s_{33}) & \frac{9}{2(H_p+A)}(T_{22}s_{33}+T_{33}s_{22}) & \frac{1}{2\mu}+c+d+\frac{9}{(H_p+A)}T_{33}s_{33} \end{bmatrix}$$

éq 8.1.1-22

$$C_2=\begin{bmatrix} \frac{9\sqrt{2}}{2(H_p+A)}(T_{11}s_{12}+s_{11}T_{12}) & \frac{9\sqrt{2}}{2(H_p+A)}(T_{11}s_{23}+s_{11}T_{23}) & \frac{9\sqrt{2}}{2(H_p+A)}(T_{11}s_{13}+s_{11}T_{13}) \\ \frac{9\sqrt{2}}{2(H_p+A)}(T_{22}s_{12}+s_{22}T_{12}) & \frac{9\sqrt{2}}{2(H_p+A)}(T_{22}s_{23}+s_{22}T_{23}) & \frac{9\sqrt{2}}{2(H_p+A)}(T_{22}s_{13}+s_{22}T_{13}) \\ \frac{9\sqrt{2}}{2(H_p+A)}(T_{33}s_{12}+s_{33}T_{12}) & \frac{9\sqrt{2}}{2(H_p+A)}(T_{33}s_{23}+s_{33}T_{23}) & \frac{9\sqrt{2}}{2(H_p+A)}(T_{33}s_{13}+s_{33}T_{13}) \end{bmatrix}$$

éq 8.1.1-23

$$C_3=C_2$$

éq 8.1.1-24

$$C_4=\begin{bmatrix} \frac{1}{2\mu}+c+d+\frac{18}{(H_p+A)}s_{12}T_{12} & \frac{9}{(H_p+A)}(T_{12}s_{23}+T_{23}s_{12}) & \frac{9}{(H_p+A)}(T_{12}s_{23}+T_{23}s_{12}) \\ \frac{9}{(H_p+A)}(T_{23}s_{12}+T_{12}s_{23}) & \frac{1}{2\mu}+c+d+\frac{18}{(H_p+A)}T_{23}s_{23} & \frac{9}{(H_p+A)}(T_{23}s_{13}+T_{13}s_{23}) \\ \frac{9}{(H_p+A)}(T_{13}s_{12}+T_{12}s_{13}) & \frac{9}{(H_p+A)}(T_{13}s_{23}+T_{23}s_{13}) & \frac{1}{2\mu}+c+d+\frac{18}{(H_p+A)}T_{13}s_{13} \end{bmatrix}$$

éq 8.1.1-25

Calculation of the rate of variation of volume:

$$\Delta\varepsilon_v^p=2\mathrm{M}^2\Lambda(P-P_{cr}),\quad \delta\varepsilon_v^p=2\mathrm{M}^2\delta\Lambda(P-P_{cr})-2\mathrm{M}^2\Lambda\delta\ P_{cr}=B\delta\ \Lambda=\frac{3\mathrm{B}}{(H_p+A)}(s+\Delta s).\delta\ s$$

éq 8.1.1-26

with: $$B=2\mathrm{M}^2(P-P_{cr})-2\mathrm{M}^2\Lambda\frac{M^2(P-P_{cr})}{\frac{1}{2kP_{cr}}+M^2\Lambda}.$$

éq 8.1.1-27

or while using [éq 5.3.1-11]

$$\delta\varepsilon_v^p=\frac{3\mathrm{B}}{(H_p+A)}(s+\Delta s).\delta\ s$$

éq 8.1.1-28

One thus has:

$$\delta\ \varepsilon_{ij}=(C_{ijkl}-\frac{B}{(H_p+A)}(s+\Delta s)_{kl}\delta_{ij})\ \delta\ s_{kl}$$

éq 8.1.1-29

8.1.2 Treatment of the hydrostatic part

It is considered now that the variation of loading is purely spherical ($\delta\ s=0$).
The increment of P is written in the form:

$$\Delta P=P^{-}\exp\left(k_0\,\Delta\varepsilon_v^e\right)-P^{-}$$ **éq 8.1.2-1**

The derivation of this equation gives:

$$\delta P=k_0\,P\left(\delta\varepsilon_v-\delta\varepsilon_v^p\right)$$ **éq 8.1.2-2**

Calculation of $\delta\varepsilon_v^p$:

It is known that:

$$\Delta\varepsilon_v^p=\Lambda 2M^2\left(P-P_{cr}\right)$$ **éq 8.1.2-3**

By differentiating this equation, one obtains:

$$\delta\varepsilon_v^p=2\mathrm{M}^2\left(\delta\ \Lambda\left(P-P_{cr}\right)+\Lambda\left(\delta P-\delta P_{cr}\right)\right)$$ **éq 8.1.2-4**

One knows the expression of Λ :

$$\Lambda=\frac{2\mathrm{M}^2\left(P-P_{cr}\right)\Delta P+3\mathrm{s}\Delta\mathrm{s}}{H_p}=\frac{b}{H_p}$$ **éq 8.1.2-5**

while posing

$$b=2\mathrm{M}^2\left(P-P_{cr}\right)\Delta P+3\mathrm{s}\Delta\mathrm{s}$$ **éq 8.1.2-6**

While differentiating $\Delta\Lambda$, it comes:

$$\delta\Lambda=\frac{2\mathrm{M}^2}{H_p}\left[\left(P-P_{cr}\right)\delta P+\left(\delta P-\delta P_{cr}\right)\Delta P\right]-\frac{4\,kM^4\,b}{H_p^2}\left[\delta\,PP_{cr}\left(2\mathrm{P}-P_{cr}\right)+\delta P_{cr}\,P\left(P-2\mathrm{P}_{cr}\right)\right]$$ **éq 8.1.2-7**

One seeks the expression of δP_{cr} according to $\delta\Lambda$:

One has

$$\delta P_{cr}=kP_{cr}\,\delta\varepsilon_v^p$$ **éq 8.1.2-8**

One can write:

$$\frac{\delta P_{cr}}{kP_{cr}}=\delta \Lambda 2M^2\left(P-P_{cr}\right)+\Lambda 2M^2\left(\delta P-\delta P_{cr}\right)$$ **éq 8.1.2-9**

$$\delta P_{cr}\left(\frac{1+\Lambda 2M^2 kP_{cr}}{kP_{cr}}\right)=\delta \Lambda 2M^2\left(P-P_{cr}\right)+\Lambda 2M^2\delta P$$ **éq 8.1.2-10**

$$\delta P_{cr}=\left(\frac{2\mathrm{M}^2\left(P-P_{cr}\right)kP_{cr}}{1+2\,kP_{cr}\Lambda M^2}\right)\delta\Lambda+\left(\frac{2\Lambda \mathrm{M}^2 kP_{cr}}{1+2\,kP_{cr}\Lambda M^2}\right)\delta P$$ **éq 8.1.2-11**

One poses

$$c=\frac{2\mathrm{M}^2 kP_{cr}\left(P-P_{cr}\right)}{\left[1+2\mathrm{M}^2 kP_{cr}\Lambda\right]}\ ,$$ **éq 8.1.2-12**

$$a=\frac{2\mathrm{M}^2 kP_{cr}\Lambda}{\left[1+2\mathrm{M}^2 kP_{cr}\Lambda\right]}$$ **éq 8.1.2-13**

One has then:

$$\delta P_{cr}=a\delta P+c\delta\Lambda$$ **éq 8.1.2-14**

By replacing the expression of δP_{cr} in $\delta\Lambda$ [éq 5.3.2-7], one finds:

$$\delta\Lambda=\left[2\mathrm{M}^2\left(P-P_{cr}\right)\delta P+2\mathrm{M}^2\left(\delta P-c\delta\Lambda-a\delta P\right)\Delta P\right].\frac{1}{H_p}$$
$$-\frac{4\,kM^4 b}{H_p^2}\left[\delta\,PP_{cr}\left(2\mathrm{P}-P_{cr}\right)+\left(c\delta\Lambda+a\delta P\right)P\left(P-2\mathrm{P}_{cr}\right)\right]$$ **éq 8.1.2-15**

By gathering the terms in $\delta\Lambda$ and those in δP , one finds:

$$\delta\Lambda=\frac{f}{e}\delta P$$ **éq 8.1.2-16**

with,

$$f=\frac{1}{H_p}\left[2\mathrm{M}^2\left(P-P_{cr}\right)+2\mathrm{M}^2\Delta P-2\,aM^2\Delta P\right]$$
$$-\frac{4\,kM^4 b}{H_p^2}\left[\left(2\mathrm{P}-P_{cr}\right)P_{cr}+aP\left(P-2\mathrm{P}_{cr}\right)\right]$$ **éq 8.1.2-17**

$$e=1+\frac{2\,cM^2\Delta P}{H_p}+\frac{4\,bckM^4}{H_p^2}P\left(P-2\mathrm{P}_{cr}\right)$$ **éq 8.1.2-18**

The expression of $\delta\varepsilon_v^p$ thus becomes:

$$\delta\varepsilon_v^p=2\mathrm{M}^2\left[\Lambda-a\Lambda-\Lambda c\frac{f}{e}+\frac{f}{e}\left(P-P_{cr}\right)\right]\delta P$$ **éq 8.1.2-19**

from where the expression of $\delta\varepsilon_v$ according to δP :

$$\delta P = \frac{k_0 P}{G} \delta \varepsilon_v$$ **éq 8.1.2-20**

$$G = 1 + 2\mathrm{M}^2 k_0 P \left(\Lambda - a\Lambda - \Lambda \frac{f}{e} c + \frac{f}{e} \left(P - P_{cr} \right) \right)$$ **éq 8.1.2-21**

Calculus of the variation of deviatoric deformation:

$$\delta\ \tilde{\varepsilon}_{ij} = \delta\ \tilde{\varepsilon}^p = 3\delta\ \Lambda\ s = 3\frac{f}{e}\ \delta\ Ps_{ij}$$ **éq 8.1.2-22**

One thus has finally:

$$\delta \varepsilon_{ij} = F_{ij} \delta P$$ **éq 8.1.2-23**

with

$$F = \frac{3\mathrm{f}}{e} s - \frac{G}{3\mathrm{k}_0 P} 1^d$$ **éq 8.1.2-24**

8.1.3 Tangent operator

The tangent operator connects the variation of total constraint to the variation of total deflection. Since the increment of the total deflection under loading deviatoric is written:

$$\delta\ \varepsilon_{ij} = (C_{ijkl} - \frac{B}{(H_p + A)} (s + \Delta s)_{kl} \delta_{ij})\ D^1_{klmn} \delta\ \sigma_{mn}\ ,$$ **éq 8.1.3-1**

with:

$$D^1 = \begin{bmatrix} 2/3 & -1/3 & -1/3 & 0 & 0 & 0 \\ -1/3 & 2/3 & -1/3 & 0 & 0 & 0 \\ -1/3 & -1/3 & 2/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$ **éq 8.1.3-2**

projection in space deviatoric,

and that under spherical loading one a:

$$\delta\ \varepsilon_{ij} = F_{ij}\ D^2_{kl}\ \delta\ \sigma_{kl}$$ **éq 8.1.3-3**

with:

$$D^2 = \begin{bmatrix} -1/3 \\ -1/3 \\ -1/3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$ **éq 8.1.3-4**

hydrostatic projection, one has then:

$$\delta \sigma_{ij} = A_{ijkl} \delta \varepsilon_{kl}$$ **éq 8.1.3-5**

with:

$$A_{ijkl}=\left[(C_{ijmn}-\frac{B}{(H_p+A)}(s+\Delta s)_{mn}\delta_{ij})\ D_{mnkl}1+F_{ij}\,D^2_{kl}\right]^{-1}$$ **éq 8.1.3-6**

the discretized tangent operator.

8.2 Tangent operator at the critical point

If the point of load is at the critical point $\left(P=P_{cr}\right)$, the general expression of the tangent operator is not valid any more. This appears in particular by divide by 0 (see the equations of [§ 5.3.1]). One details in what follows the coherent tangent operator to the critical point while passing as for the case general by the partly deviatoric and partly hydrostatic decomposition.

8.2.1 Treatment of the deviatoric part

Let us remind that the critical point, the expressions of the plastic multiplier Λ and of its derivation $\delta\Lambda$ are written in the following way:

$$\Lambda=\left(\frac{Q^e}{Q}-1\right)/6\mu \quad \text{and} \quad \delta\Lambda=\frac{\delta Q^e}{6\mu Q}-\frac{Q^e\,\delta Q}{6\mu Q^2}$$ **éq 8.2.1-1**

with,

$$\delta Q^e=\frac{3}{2}\frac{s^e\,\delta s^e}{Q^e} \quad \text{and} \quad \delta Q=\frac{3}{2}\frac{s\delta s}{Q}$$ **éq 8.2.1-2**

from where the expression of $\delta\Lambda$:

$$\delta\Lambda=\frac{1}{6\mu}\frac{3}{2}\left[\frac{s^e\,\delta s^e}{Q^e Q}-\frac{Q^e\,s\delta s}{Q^3}\right]$$ **éq 8.2.1-3**

Let us point out in the same way the expression of δs :

$$\delta s_{ij}=2\mu\left(\delta\tilde{\varepsilon}_{ij}-3\delta\Lambda s_{ij}-3\Lambda\delta s_{ij}\right)$$

While replacing Λ and $\delta\Lambda$ by their expressions, one can write:

$$\delta s_{ij}=2\,\mu\delta\ \{\ \tilde{\varepsilon}_{ij}-\frac{3}{2}\frac{s^e_{kl}\,\delta s^e_{kl}}{Q^e Q}s_{ij}+\frac{3}{2}\frac{Q^e}{Q^3}s_{kl}\,\delta s_{kl}\,s_{ij}-\left(\frac{Q^e}{Q}-1\right)\delta s_{ij}\text{¿}$$ **éq 8.2.1-4**

$$\delta s_{kl}\left[\delta_{ijkl}+\frac{Q^e}{Q}\delta_{ijkl}-\delta_{ijkl}-\frac{3}{2}\frac{Q^e}{Q^3}s_{kl}\,.\,s_{ij}\right]=2\mu\left[\delta_{ijkl}-\frac{3}{2}\frac{s^e_{kl}\,.\,s_{ij}}{Q^e Q}\right]\delta\,\tilde{\varepsilon}_{kl}$$ **éq 8.2.1-5**

or in tensorial writing:

$$\delta s\underbrace{\left[\frac{Q^e}{Q}I_4^d-\frac{3}{2}\frac{Q^e}{Q^3}s\otimes s\right]}_{G}=2\mu\underbrace{\left[I_4^d-\frac{3}{2}\frac{s^e\otimes s}{Q^e Q}\right]}_{H}\delta\,\tilde{\varepsilon}$$ **éq 8.2.1-6**

Like δs does not depend on $\delta\varepsilon_v$, one can confuse $\delta\,\tilde{\varepsilon}$ with $\delta\varepsilon$.

By using the tensor of projection within the space of deviatoric constraints D^1 [éq 5.3.3-2], one can write:

$$\delta\varepsilon=\frac{D^1.G.H^{-1}}{2\mu}.\delta\sigma \quad \textbf{éq 8.2.1-7}$$

8.2.2 Treatment of the hydrostatic part

In tensorial writing, there is the following relation:

$$I^d\,\delta P=k_0\,P\,\delta\varepsilon_v \quad . \quad \textbf{éq 8.2.2-1}$$

according to the equation [éq 5.3.2-2] with $\delta\varepsilon_v^p=0$ at the critical point.

Like δP does not depend on $\delta\tilde{\varepsilon}$ then one can confuse $\delta\tilde{\varepsilon}$ with $\delta\varepsilon$.

$$I^d\,\delta P=k_0\,P\,\delta\varepsilon \quad \textbf{éq 8.2.2-2}$$

By using the tensor of projection within the space of hydrostatic constraints D^2 [éq 5.3.3-4], one can write:

$$\delta\varepsilon=\frac{I^d}{k_0\,P}\,D^2\,\delta\sigma \quad \textbf{éq 8.2.2-3}$$

8.2.3 Tangent operator

By combining the contributions of the two parts deviatoric and hydrostatic, one finds the writing of the tangent operator who connects the variation of the total constraint to the variation of the total deflection at the critical point:

$$\delta\varepsilon=\left[\frac{D^1.G.H^{-1}}{2\mu}+\frac{I^d}{k_0\,P}\,D^2\right].\delta\sigma$$

or

$$\delta\sigma_{ij}=A_{ijkl}\,\delta\varepsilon_{kl} \quad \textbf{éq 8.2.2-4}$$

with

$$A_{ijkl}=\left[\frac{D^1.G.H^{-1}}{2\mu}+\frac{I^d}{k_0\,P}D^2\right]^{-1} \quad \textbf{éq 8.2.2-5}$$

9 Bibliography

- I.B BURLAND, K.H. ROSCOE: One the generalized stress strain behaviour of wet clay, Engineering plasticity Cambridge Heyman-Leckie, 1968.
- PH. A. CHARLEZ (Total Report): Example of model poroplastic: the Camwood model - Clay.
- M.ABBAS: Quasi-static nonlinear algorithm. Reference material Aster [R5.03.01].
- J. LEMAITRE, J.L. CHABOCHE: mechanics of solid materials, Dunod 1985.
- ROSCOE K.H., SCHOFIELD A.N., WROTH C.P., “One the yielding of soils”, Geotechnics, vol. 8, n°1, p. 22-53, 1958.
- SCHOFIELD A.N., WROTH C.P., “Critical state soil mechanics”, McGraw Hill, New York, 1968.
- BURLAND J.B., ROSCOE K.H., “One the generalized stress-strain behaviour of wet clay”, Plasticity Engineering, Heyman-Leckie, Cambridge, 1968.

10 Checking

The law of behavior `CAM_CLAY` is checked by the following tests:

SSNP136	Test of foundation slipping by with the law of `CAM_CLAY`	[V6.03.136]
SSNV160	Hydrostatic test with the law `CAM_CLAY`	[V6.04.160]
SSNV202	Test œdometric drained with the law of `CAM_CLAY`	[V6.04.202]
WTNV122	Triaxial compression test not drained with the law `CAM_CLAY`	[V7.31.122]

11 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
6.4	J.EL-GHARIB, G.DEBRUYNE EDF-R&D/AMA	Initial text
7.3	J.El-Gharib, EDF-R&D/AMA	Tangent operator for the critical point
9.4	J.El-Gharib, EDF-R&D/AMA	Modification tangent operator, addition of internal variables, cf cards REX 10585 and 10700