Law of behavior of LAIGLE

Summary:

The rheological model of Laigle makes it possible to analyze the rock mechanics behavior. The development of this model of behavior was initiated following the difficulty in correctly apprehending the answer of the solid mass during the excavation of an underground cavity, with an aim:

- to define the need and the nature of possible supportings to implement;
- to determine the extent of the ground around a work influenced by the digging.

The implementation of this elastoplastic model was mainly focused on the simulation of the behavior post-peak of the rock. It is supposed, accordingly, that there is no work hardening of the rock prior to the rupture of this one. That results in a linear elastic behavior to the peak of resistance (there can nevertheless be damage of the rock whereas the material is not yet in rupture). The definite criterion of plasticity is of type generalized Hoek and Brown and gives an account of the influence of the level of constraint on the shear strength. The rise in temperature of material is associated with a progressive reduction in the properties of cohesion and angle of friction accompanied by a change of volume. It is controlled by the plastic deformation déviatoire cumulated considered as only variable of work hardening.

To facilitate the integration of this model in *Code_Aster*, the law initially developed in the formalism of the principal constraints was rewritten with invariants of constraints on a basis of the model Cambou - Jafari-Sidoroff (CJS). The digital formulation is implicit compared to the criterion and explicit compared to the direction of flow.

The convention of sign used for the formulation of the equations, within the framework of this note, is that of the mechanics of the continuous mediums.

Contents

1 Notations....4
1.1 General information....4
1.2 Parameters of the model....5
2 Introduction....6
2.1 Phenomenology of the behavior of the grounds....6
2.2 Context of study and simplifying assumptions of the model....7
3 The continuous model....8
3.1 Elastic behavior....8
3.2 Criterion of plasticity....8
3.2.1 Surface of load....8
3.2.1.1 Expression of the criterion of Laigle in major and minor constraints....8
3.2.1.2 General expression....8
3.2.1.3 Pace of the thresholds....9
3.2.2 Work hardening....9
3.2.3 Law of dilatancy....10
3.2.3.1 Generalized writing....10
3.2.3.2 Determination of the intersection of the intermediate criterion and the ultimate criterion....11
3.2.4 Plastic flow....12
4 Calculation of the derivative....13
4.1 Derived from the criterion....13
4.1.1 Derived compared to the constraints....13
4.1.1.1 Derived intermediate compared to the diverter....13
4.1.1.2 Derived intermediate compared to the constraints....14
4.1.1.3 Final expression of the derivative of the criterion compared to the constraints....14
4.1.2 Derived compared to the variable from work hardening....14
4.2 Total derivative of the criterion compared to the plastic multiplier....15
4.3 Derived from the parameters compared to the variable of work hardening....15
5 Tangent operator of speed....17
6 Digital processing adapted to the nonregular models....18
6.1 Projection at the top of the cone....18
6.1.1 Definition of the jetting angle....18
6.1.2 Existence of projection....19
6.1.3 Rules of projection....23
6.1.3.1 Case where the parameter of dilatancy is negative....23
6.1.3.2 Case where the parameter of dilatancy is positive....23
6.1.3.3 Graphic interpretation....23
6.1.3.4 Equations of flow....24

6.2 Local Recutting of the step of time....24
7 Internal variables....25
7.1 V1: the plastic deformation déviatoire cumulated....25
7.2 V2: cumulated plastic voluminal deformation....25
7.3 V3: fields of behavior of the rock....25
7.4 V4: the state of plasticization....26
8 Detailed presentation of the algorithm....26
8.1 Calculation of the elastic solution....26
8.2 Calculation of the elastic criterion....26
8.3 Algorithm....27
9 Alternative on the expression of the criterion of plasticity....30
9.1 General formulation....30
9.2 Pace of the thresholds....30
10 Features and checking....32
11 Bibliography....32
12 Description of the versions of the document....32
Annexe 1 Retiming of the criterion on the triaxial one in compression....33
Annexe 2 Standardisation of Q....34
Annexe 3 Framing of the jetting angle....35

1 Notations

1.1 General information

$\boldsymbol{\sigma}$ indicate the tensor of the effective constraints in small disturbances, noted in the shape of the following vector:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sqrt{2}\sigma_{12} \\ \sqrt{2}\sigma_{13} \\ \sqrt{2}\sigma_{23} \end{pmatrix}$$

One notes:

$I_1=\mathrm{tr}(\sigma)$	first invariant of the constraints
$\boldsymbol{s}=\boldsymbol{\sigma}-\frac{I_1}{3}\boldsymbol{I}$	tensor of the constraints déviatoires
$s_{11}=\sqrt{\boldsymbol{s}.\boldsymbol{s}}$	second invariant of the tensor of the constraints déviatoires
σ_1	major principal constraint
σ_3	minor principal constraint
$\boldsymbol{e}=\boldsymbol{\varepsilon}-\frac{Tr(\varepsilon)}{3}\boldsymbol{I}$	diverter of the deformations
$\varepsilon_v=\mathrm{Tr}(\boldsymbol{\varepsilon})$	voluminal deformation
$\cos(3\theta)=2^{1/2}3^{3/2}\frac{det(\boldsymbol{s})}{s_{II}^3}$	θ being the angle of Lode
$\gamma^p=\sqrt{\frac{2}{3}e_{ij}^p e_{ij}^p}$	cumulated plastic deviatoric deformations
$\mathbf{n}$	normal of the hypersurface of deformation
$\mathbf{G}$	function controlling the evolution of the plastic deformations and describing the direction of flow
$\tilde{\boldsymbol{G}}=\boldsymbol{G}-\frac{\mathrm{Tr}(\boldsymbol{G})}{3}\boldsymbol{I}$	diverter of $\mathbf{G}$
$G=Tr(\boldsymbol{G})$	trace of $\mathbf{G}$
$\tilde{G}_{II}=\sqrt{\tilde{\boldsymbol{G}}.\tilde{\boldsymbol{G}}}$	normalizes $\tilde{\mathbf{G}}$
ψ	angle of dilatancy
φ	angle of friction
f	surface of load

1.2 Parameters of the model

Notation	Description
m	Slope of the criterion in the plan (p',q) for the very strong constraints (function of the mineralogical nature of the rock)
s	Cohesion of the medium. Representative of the damage of the rock.
a	Characterization of the concavity of the criterion, function of the level of deterioration of the rock. It defines the influence of the component of dilatancy in the behavior in the great deformations.
γ_{ult}	Plastic deformation déviatoire corresponding to the ultimate criterion
γ_e	Plastic deformation déviatoire corresponding to disappearance supplements cohesion
m_{ult}	Value of m ultimate criterion reached in γ_{ult}
m_e	Value of m intermediate criterion reached in γ_e
a_e	Value of a intermediate criterion reached in γ_e
m_{pic}	Value of m criterion of peak reached with the peak of constraint
a_{pic}	Value of a criterion of peak reached with the peak of constraint
η	Exhibitor controlling work hardening
σ_c	Compressive strength simple
γ	First parameter regulating dilatancy
ζ	Second parameter regulating dilatancy
γ_{cjs}	Parameter of form of the criterion of plasticity in the déviatoire plan
E	Young modulus
ν	Poisson's ratio
σ_{p1}	Intersection of the intermediate criterion and the criterion of peak
σ_{p2}	Intersection of the intermediate criterion and the ultimate criterion
PA	Atmospheric pressure

2 Introduction

The object of this note is to present the rheological model to analyze the rock mechanics behavior, adapted to the simulation of the underground works, introduced into *Code_Aster* and developed by the CIH [bib1]. The finality of this model is of being able to be implemented, in a fast and industrial way in order to answer the principal interrogations that the engineer during the analysis and of the design of an underground cavity is posed. The rheological law must for that remain relatively simple, as well during the identification of the parameters as in its implementation and during interpretation of the results.

2.1 Phenomenology of the behavior of the grounds

One of the characteristics of a rock, compared to a ground, that its mechanical behavior is, on a beach of important constraint, is controlled by cohesion. This cohesion is associated with a cementing of the medium, is induced during the geological history of the solid mass, and is primarily of epitaxic nature. On the contrary, the resistance of a ground is more particularly governed by the term of friction and/or dilatancy. Cohesion, of primarily capillary origin, then affects only for very weak states of stresses of containment.

This distinction between a ground and a rock is important because it directs the choice and the basic assumptions of the model of behavior.

The principal rheological phenomena associated in this context are the following:

- In the field of the small deformations, the answer of a rock, in particular under weak states of containment, can be comparable to a linear elastic behavior, slightly depend on the state of the constraints. Non-linearities of the behavior are likely to appear prior to peak of resistance, in the case of the tender rocks, for a level of constraint of about 70 to 80% of the maximum value. This threshold decreases with the increase in the average pressure for almost cancelling itself when the constraint of surconsolidation is reached (course-model). Under very low constraints of containment representative of those reigning near the underground works, these non-linearities are generally low, more especially as cementing is important, and thus the high level of surconsolidation of the rock.
- Dilatancy (increase in volume) is initiated when non-linearities appear on stress-strain curve. This dilatancy increases until there is localization within the sample. At this time, the rate of dilatancy (or the angle of dilatancy ψ) is maximum, for then gradually decreasing and cancelling themselves with the very great deformations.
- The peak of resistance is reached for constraints describing a criterion of rupture, generally curved in the plan of Mohr or the plan of the principal constraints major and minor. The assumption of a linear criterion of Mohr-Coulomb is thus only one simplifying assumption, tending, for low constraints of containment, to raise the cohesion of the medium.
- Once maximum resistance reached, the resistance of the rock decreases. This rise in temperature post-peak is all the more fast and important (in intensity) that the constraint of containment is low. This decrease is related to a damage more or less localised of the rock, according to the level of containment. Whatever this constraint, beyond the peak, **the rock cannot be regarded as continuous any more**. Its behavior is then controlled by the conditions of deformation and strength to the level of the zone of localization of the deformations.
- The appearance of one or more discontinuities kinematics within the rock is associated with a loss of cohesion. The behavior post-peak is then governed by the conditions of friction and dilatancy along the plans of discontinuity or within a band of localization of the deformations. It comes out from this reasoning that for very great deformations, the behavior of the comparable rock to a “structure”, is only rubbing, and is characterized by an ultimate angle of friction ϕ . This angle is an intrinsic data of material, function of minerals constitutive of the rock. It thus does not depend directly on the conditions of cohesion, and it can especially be regarded as independent of dimensions of the sample.

- When the behavior only becomes rubbing, it is associated with no voluminal deformation. Dilatancy was thus cancelled, and does not exist any more with the great deformations.
- The evolution between the resistance of peak and the critical condition corresponding to the great deformations, is more or less progressive according to the state of the pressures applied. For a state of null containment (simple compression), the behavior is only controlled by cohesion, and the rupture results in an immediate and brutal loss of any resistance. Rise in temperature will be more progressive as the constraint of containment increases, to become non-existent beyond of a certain constraint of containment limiting the ductile and fragile fields of behavior.

2.2 Context of study and simplifying assumptions of the model

The will to develop a model easy to implement is necessarily accompanied by simplifications, resulting from a compromise between the expected objectives, the conditions of use of the model (quality of the data input, times and cost available…) and the means put in work to ensure these developments. These compromises are primarily the following:

- **A linear elastic behavior** to the peak of resistance. This amounts supposing that there is no work hardening of the rock prior to the rupture of this one.
- **Only a criterion of rupture in shearing is retained**. This means that if the rock is crushed in an isotropic way, the behavior remains elastic, and that there are not damage and work hardening of material under this kind of way. During the phases of excavation of an underground work with implementation of a light supporting, the average pressure in the solid mass located in the vicinity can only decrease (or to remain constant in the ideal case of a circular cavity subjected to an isotropic request, for a linear elastic behavior). Plasticization under isotropic constraint, that one can find on a Cape-Model or on a law of the Camwood-Clay type did not seem essential to us taking into account the searched objectives, and in the case of an isothermal and short-term request.

During the development of this model, we voluntarily focused ourselves on the study and the simulation of the behavior post-peak of the rock. In this field of behavior, the resistance of material is supposed to be controlled, according to the state of the constraints and the level of damage of the rock, by cohesion, dilatancy or friction.

Cohesion defines the resistance of material as long as this one remains continuous. It is active to the peak of resistance, and has only little influence on the behavior softening, unless cohesion is representative of a ductile “adhesive” (case of the grounds injected by silicate freezing,…).

As cohesion worsens by damage, dilatancy increases, to reach its maximum value at the time of the loss of continuity of the medium. At this time, under the effect of the shearing of induced discontinuity, this dilatancy is degraded gradually and slowly. The rheology of the rock evolves then to a behavior purely rubbing.

3 The continuous model

3.1 Elastic behavior

The elastic behavior is controlled by a linear law, with a constant module independent of the state of stresses. The 2 parameters characterizing this behavior are the modulus of elasticity E and the Poisson's ratio ν.

$$\dot{s}=2\mu\left(\dot{e}-\dot{e^p}\right) \quad \text{éq 3.1-1}$$

$$\dot{I}_1=3K\left(\dot{\varepsilon}_v-\dot{\varepsilon}_v^p\right) \quad \text{éq 3.1-2}$$

3.2 Criterion of plasticity

The adopted formulation is that of [bib2].

3.2.1 Surface of load

3.2.1.1 Expression of the criterion of Laigle in major and minor constraints

$$f=\left(\frac{\sqrt{\frac{2}{3}}}{\sigma_c}\right)^{\frac{1}{a(\gamma^p)}}\left[\left(|\sigma_1-\sigma_3|\right)\frac{1}{a\left(\gamma^p\right)}-(\sigma_c)\frac{1}{a\left(\gamma^p\right)}\left(\frac{m\left(\gamma^p\right)}{\sigma_c}(-\sigma_3)+s(\gamma^p)\right)\right] \quad \text{éq 3.2.1.1 - 1}$$

3.2.1.2 General expression

One transforms the preceding expression according to the first invariant and of the diverter of the constraints, by a retiming of the criterion on triaxial in compression, to obtain:

$$f=\left(\frac{g\left(\boldsymbol{s}\right)}{\sigma_c h_c^0}\right)^{\frac{1}{a\left(\gamma^p\right)}}-u\left(\boldsymbol{\sigma},\gamma^p\right)\leq 0 \quad \text{éq 3.2.1.2 - 1}$$

with:

$$h\left(\theta\right)=\left(1+\gamma_{\text{cjs}}\cos(3\theta)\right)^{1/6}=\left(1+\gamma_{\text{cjs}}\sqrt{54}\frac{det(\boldsymbol{s})}{s_{II}^3}\right)^{1/6} \quad \text{éq 3.2.1.2 - 2}$$

$$\left|\begin{array}{l} h_c^0=h\left(\theta=\frac{\pi}{3}\right)=\left(1-\gamma_{\text{cjs}}\right)^{1/6} \\ h_t^0=\left(1+\gamma_{\text{cjs}}\right)^{1/6}\end{array}\right.$$

$$g(\boldsymbol{s})=s_{II}h(\theta) \quad \text{éq 3.2.1.2 - 3}$$

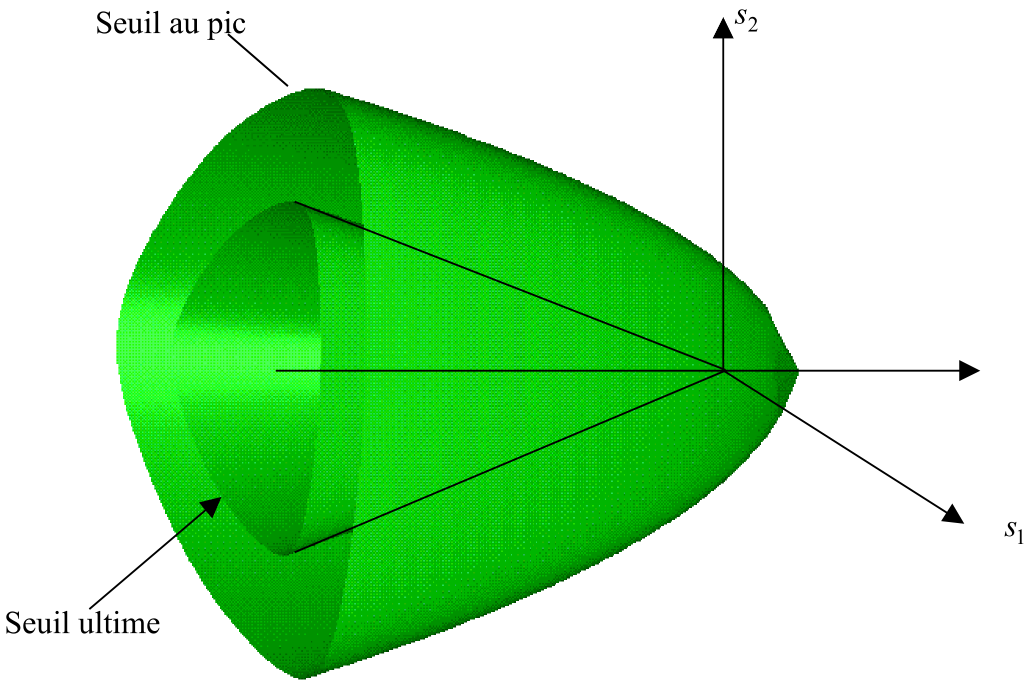
$$u(\boldsymbol{\sigma},\gamma^p)=-\frac{m(\gamma^p)k(\gamma^p)}{\sqrt{6}\sigma_c}\frac{g(\boldsymbol{s})}{h_c^0}-\frac{m(\gamma^p)k(\gamma^p)}{3\sigma_c}I_1+s(\gamma^p).k(\gamma^p) \quad \text{éq 3.2.1.2 - 4}$$

Note:

- *One shows [Appendix 1] the equivalence of the two expressions*
- *It is shown that a second formulation of the criterion with a retiming on triaxial in compression and extension is possible but we do not choose it. It however is presented to the chapter [§9].*

3.2.1.3 Pace of the thresholds

One traces the pace of the thresholds to the criterion of peak and the ultimate criterion.



3.2.2 Work hardening

To translate rise in temperature post-peak of the rock one defines laws of variations of the parameters m, S and has criterion according to the internal variable of work hardening γ^p (it is the deformation déviatoire plastic cumulated, proportional to the second invariant of the tensor of the deformations déviatoires, corresponding to the plastic distortion).

$$s(\gamma^p)=\left(1-\frac{\gamma^p}{\gamma^e}\right) \quad \text{si}\, \gamma^p<\gamma_e$$
$$s(\gamma^p)=0 \quad \text{si}\, \gamma^p\geq\gamma_e \qquad \text{éq 3.2.2-1}$$

si $\underline{\gamma^p>\gamma_{\text{ult}}}$ - $\qquad a=1$

$m=m_{\text{ult}}$

If not

$$\Omega\left(\gamma^p\right)=\left(\frac{\gamma^p}{\gamma^e}\right)^{\eta}\frac{a_e-a_{\text{pic}}}{1-a_e}\frac{\gamma_{\text{ult}}-\gamma_e}{\gamma_{\text{ult}}-\gamma^p}$$ éq 3.2.2-2

$$a\left(\gamma^p\right)=\frac{a_{\text{pic}}+\Omega\left(\gamma^p\right)}{1+\Omega\left(\gamma^p\right)}$$ éq 3.2.2-3

$$m\left(\gamma^p\right)=\frac{\sigma_c}{\sigma_{\text{p1}}}\left[\left(m^{\text{pic}}\frac{\sigma_{\text{p1}}}{\sigma_c}+1\right)^{\frac{a_{\text{pic}}}{a\left(\gamma^p\right)}}-s\left(\gamma^p\right)\right]\quad \text{si}\,\gamma^p<\gamma_e$$

$$m\left(\gamma^p\right)=\frac{\sigma_c}{\sigma_{\text{p2}}}\left[\left(m_e\frac{\sigma_{\text{p2}}}{\sigma_c}\right)^{\frac{a_{\text{pic}}}{a\left(\gamma^p\right)}}\right]\quad \text{si}\,\gamma^p\geq\gamma_e$$ éq 3.2.2-4

$$k\left(\gamma^p\right)=\left(\frac{2}{3}\right)^{\frac{1}{2\text{a}\left(\gamma^p\right)}}.$$ éq 3.2.2-5

These laws of evolutions for each of the 3 parameters are dependent from/to each other and observe the conditions of intersection of the criteria during the phase of work hardening [bib1].

Note:

The condition of coherence to respect door on the continuity of the parameter m in γ_e :

$$\lim_{\gamma^p\to\gamma_e} m\left(\gamma^p\right)=\frac{\sigma_c}{\sigma_{\text{p1}}}\left[\left(m_{\text{pic}}\frac{\sigma_{\text{p1}}}{\sigma_c}+1\right)^{\frac{a_{\text{pic}}}{a\left(\gamma_p\right)}}-s\left(\gamma^p\right)\right]$$

that is to say:

$$m_e=\frac{\sigma_c}{\sigma^{\text{p1}}}\left(m_{\text{pic}}\frac{\sigma_{\text{p1}}}{\sigma_c}+1\right)^{\frac{a_{\text{pic}}}{a_e}}$$ *éq 3.2.2-6*

3.2.3 Law of dilatancy

3.2.3.1 Generalized writing

The law of dilatancy (it is admitted that the value of dilatancy is inversely proportional to that of cohesion) can be generalized while writing:

$$\sin\psi=\sin\left(\psi\left(\alpha'\right)\right)=\gamma\frac{\alpha'-m_{\text{ult}}-1}{\zeta\alpha'+m_{\text{ult}}+1}$$ éq 3.2.3.1 - 1

with:

$$\alpha'=\alpha'\left(I_{1,}g(\boldsymbol{s}),\sigma_{t0}\right)=\frac{\tilde{\sigma}_1-\sigma_{t0}}{\tilde{\sigma}_3-\sigma_{t0}} \quad \text{éq 3.2.3.1 - 2}$$

$s_3=\sqrt{\frac{2}{3}}s_{II}\cos(\theta);\quad s_1=\sqrt{\frac{2}{3}}s_{II}\cos(\theta+\frac{2\pi}{3});\quad s_2=\sqrt{\frac{2}{3}}s_{II}\cos(\theta-\frac{2\pi}{3});$ where θ is the angle of Lode

$\sigma_1=\frac{I_1}{3}+s_1;\quad \sigma_2=\frac{I_1}{3}+s_2;\quad \sigma_3=\frac{I_1}{3}+s_3;$

$$\begin{cases}\tilde{\sigma}_1=\sigma_i\,\text{avec}\,i\,\text{tel que}|\sigma_i|=\max\left(|\sigma_j|,\,j=1,2,3\right)\\ \tilde{\sigma}_3=\sigma_i\,\text{avec}\,i\,\text{tel que}|\sigma_i|=\max\left(|\sigma_j|,\,j=1,2,3\right)\end{cases}$$

Note:

A condition to respect is that the report $\frac{\gamma}{\zeta}$ *Reste lower than 1. In the case of very resistant hard stones, subjected to relatively low constraints of containment, the law of dilatancy can thus tend towards this report. If the two parameters are unit one finds the expression of the law of Rowe describing the law of dilatancy for non-cohesive soils. This approach amounts preserving the same expression as for a strongly damaged rock, by comparing the effect of cohesion to that of an additional containment of value* σ_{t0} *.*

Characterization of σ_{t0} according to the parameters (has, m, S) characterizing the rock

- Case where $s(\gamma^p)=0$
  Disappearance of cohesion, one poses $\sigma_{t0}=0$

- Case where $s(\gamma^p)\neq 0$

$$\sigma_{t0}=\sigma_{t0}\left(\phi_{0,}C_0\right)=2C_0\sqrt{\frac{1-\sin\phi_0}{1+\sin\phi_0}} \quad \text{éq 3.2.3.1 - 3}$$

with:

$$\begin{cases}\phi_0=\phi_0(m,s,a)=2\cdot\arctan\left(\sqrt{1+\text{ams}^{a-1}}\right)-\frac{\pi}{2}\\ C_0=C_0(m,s,a)=\frac{\sigma_c s^a}{\sqrt{1+\text{ams}^{a-1}}}\end{cases}$$

3.2.3.2 Determination of the intersection of the intermediate criterion and the ultimate criterion

By writing the continuity of m in γ_{ult} the following relation is obtained:

$$m(\gamma_{\text{ult}})=\frac{\sigma_c}{\sigma_{\text{p2}}}\left[\left(m_e\frac{\sigma_{\text{p2}}}{\sigma_c}\right)^{\frac{a_e}{a(\gamma_{\text{ult}})}}\right]$$

$$m_{\text{ult}}=\frac{\sigma_c}{\sigma_{\text{p2}}}\left(m_e\frac{\sigma_{\text{p2}}}{\sigma_c}\right)^{\frac{a_e}{a_{\text{ult}}}}$$

$$m_{\text{ult}}=m_e^{a_e}\left(\frac{\sigma_{\text{p2}}}{\sigma_c}\right)^{a_e-1}$$

$$\sigma_{\text{p2}}=\sigma_c\left(\frac{m_{\text{ult}}}{m_e^{a_e}}\right)^{\frac{1}{a_e-1}}$$ éq 3.2.3.2 - 1

3.2.4 Plastic flow

The adopted formalism is rewritten on the basis of model CJS [R7.01.13]. When the constraints reach the edge of the field of reversibility, of the plastic deformations develop. To calculate them, there exists a function potential controlling the evolution of the deformations and defined by the relation $\dot{\boldsymbol{\varepsilon}}^p=\dot{\lambda}\boldsymbol{G}$ where $\dot{\lambda}$ is the plastic multiplier and

$$\boldsymbol{G}=\frac{\partial f}{\partial\boldsymbol{\sigma}}-\left(\frac{\partial f}{\partial\boldsymbol{\sigma}}\boldsymbol{n}\right)\boldsymbol{n}\ .$$ éq 3.2.4-1

The potential function is obtained starting from the following kinematic condition:

$$\dot{\varepsilon}_v^p=-\beta'\frac{\boldsymbol{s}.\dot{\boldsymbol{\varepsilon}}^p}{s_{II}}$$ éq 3.2.4-2

The parameter of dilatancy β' is calculated starting from the angle of dilatancy ψ (defined by [éq 3.2.3.1 - 1]) by the formula:

$$\beta'=\beta'(\psi)=-\frac{2\sqrt{6}\sin(\psi)}{3-\sin(\psi)}$$
$$\beta'=0\ \text{si}\ \gamma^p>\gamma_{\text{ult}}(1-10^{-3})$$ **éq 3.2.4-3**

Note:

β' is positive when $\gamma^p=0$ and in compression, then it becomes negative when plasticity develops. It is always negative in traction

It is then possible to seek to express the kinematic condition [éq 3.2.4-2] starting from a tensor $\mathbf{n}$ in the form:

$$\boldsymbol{n}.\dot{\boldsymbol{\varepsilon}}^p=0$$ éq 3.2.4-4

After decomposition of each term in déviatoire parts and hydrostatic, one finds the expression:

$$(n_I s_{ij}+n_2\delta_{ij}).\left(\dot{e}_{ij}^p+\frac{1}{3}\dot{\varepsilon}_v^p\delta_{ij}\right)=n_1 s_{ij}\dot{e}_{ij}^p+n_2\dot{\varepsilon}_v^p=0$$

One from of deduced the relation $\frac{n_1}{n_2}=\frac{\beta'}{s_{\mathrm{II}}}$ who, added to the condition of standardisation of the tensor $\mathbf{n}$, led to the expression:

$$\boldsymbol{n}=\frac{\beta_{'}\frac{\boldsymbol{s}}{s_{\mathrm{II}}}+\boldsymbol{I}}{\sqrt{\beta'^2+3}}$$ éq 3.2.4-5

The law of evolution of $\dot{\boldsymbol{\varepsilon}}^p$ must be such as the kinematic condition is satisfied. It is thus proposed to take the projection of $\dot{\boldsymbol{\varepsilon}}^p$ on $\mathbf{n}$ (normal of the hypersurface of deformation), that is to say:

$$\dot{\boldsymbol{\varepsilon}}^{\boldsymbol{p}}=\dot{\lambda}\boldsymbol{G}=\dot{\lambda}\left(\frac{\partial f}{\partial\boldsymbol{\sigma}}-\left(\frac{\partial f}{\partial\boldsymbol{\sigma}}\boldsymbol{n}\right)\boldsymbol{n}\right)$$

One from of also deduced the condition relating to the plastic voluminal deformation:

$$\dot{\varepsilon}_v^p=\dot{\lambda}G$$ éq 3.2.4-6

4 Calculation of the derivative

4.1 Derived from the criterion

4.1.1 Derived compared to the constraints

4.1.1.1 Derived intermediate compared to the diverter

One leaves: $\frac{\partial g}{\partial s_{\mathrm{ij}}}=h(\theta)\frac{\partial s_{\mathrm{II}}}{\partial s_{\mathrm{ij}}}+s_{\mathrm{II}}\frac{\partial h(\theta)}{\partial s_{\mathrm{ij}}}$

where $\frac{\partial s_{\mathrm{II}}}{\partial s_{\mathrm{ij}}}$ and $\frac{\partial h(\theta)}{s_{\mathrm{ij}}}$ are respectively given by:

$$\frac{s_{\mathrm{II}}}{s_{\mathrm{ij}}}=\frac{s_{\mathrm{ij}}}{s_{\mathrm{II}}}$$

$$\begin{aligned}\frac{\partial h(\theta)}{\partial s_{\mathrm{ij}}}&=\frac{1}{6\mathrm{h}(\theta)_5}\frac{\partial}{\partial s_{\mathrm{ij}}}\left(1+\gamma_{\mathrm{cjs}}\sqrt{54}\frac{det(\underline{\mathbf{s}})}{s_{\mathrm{II}}^3}\right)\\&=\frac{-\gamma_{\mathrm{cjs}}\cos(3\theta)}{2\mathrm{h}(\theta)^5 s_{\mathrm{II}}^2}s_{\mathrm{ij}}+\frac{\gamma_{\mathrm{cjs}}\sqrt{54}}{6\mathrm{h}(\theta)^5 s_{\mathrm{II}}^3}\left(\frac{\partial det(\underline{\mathbf{s}})}{\partial s_{\mathrm{ij}}}\right)\end{aligned}$$

Finally:

$$\frac{\partial g}{\partial s_{\mathrm{ij}}}=\frac{1}{h(\theta)^5}\left[\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos(3\theta)\right)\frac{s_{\mathrm{ij}}}{s_{\mathrm{II}}}+\frac{\gamma_{\mathrm{cjs}}\sqrt{54}}{6\mathrm{s}_{\mathrm{II}}^2}\left(\frac{\partial det(\underline{\mathbf{s}})}{\partial s_{\mathrm{ij}}}\right)\right]$$

And consequently:

$$\frac{\partial g}{\partial s_{ij}}=\frac{1}{h(\theta)^5}\left[\left(1+\frac{\gamma_{cjs}}{2}\cos(3\theta)\right)\frac{\mathbf{s}}{s_{II}}+\frac{\gamma_{cjs}\sqrt{54}}{6s_{II}^2}\left(\frac{\partial\, det(\underline{\mathbf{s}})}{\partial s_{ij}}\right)\right] \quad \text{éq 4.1.1.1 - 1}$$

4.1.1.2 Derived intermediate compared to the constraints

One poses by definition: $Q_{ij}=dev\left(\frac{\partial g}{\partial s_{ij}}\right)$

$$\frac{\partial g}{\partial \sigma_{ij}}=\frac{\partial g}{\partial s_{kl}}\frac{\partial s_{kl}}{\partial \sigma_{ij}}=\left[dev\left(\frac{\partial g}{\partial s_{kl}}\right)+\frac{1}{3}\frac{\partial g}{\partial s_{mm}}\delta_{kl}\right]\left[\delta_{ik}\delta_{jl}-\frac{1}{3}\delta_{ij}\delta_{kl}\right]$$

$$\frac{\partial g}{\partial \sigma_{ij}}=Q_{kl}\delta_{ik}\delta_{jl}-\frac{1}{3}\delta_{ij}Q_{kl}\delta_{kl}+\frac{1}{3}\frac{\partial g}{\partial q_{mm}}\left[\delta_{ik}\delta_{jl}\delta_{kl}-\frac{1}{3}\delta_{ij}\delta_{kl}\delta_{kl}\right]$$

$$\frac{\partial g}{\partial \sigma_{ij}}=Q_{ij}$$

It is then enough to take the deviatoric part of $\frac{\partial g}{\partial s_{ij}}$ to obtain:

$$\frac{\partial g}{\partial \sigma_{ij}}=Q_{ij}=dev\left(\frac{\partial g}{\partial s_{ij}}\right)=\frac{1}{h(\theta)^5}\left[\left(1+\frac{\gamma_{cjs}}{2}\cos(3\theta)\right)\frac{s_{ij}}{s_{II}}+\frac{\gamma_{cjs}\sqrt{54}}{6s_{II}^2}dev\left(\frac{\partial\, det(\underline{\mathbf{s}})}{\partial s_{ij}}\right)\right]$$

And consequently:

$$\boldsymbol{Q}=\frac{\partial g}{\partial \boldsymbol{\sigma}}=\frac{1}{h(\theta)^5}\left[\left(1+\frac{\gamma_{cjs}}{2}\cos(3\theta)\right)\frac{\mathbf{s}}{s_{II}}+\frac{\gamma_{cjs}\sqrt{54}}{6s_{II}^2}dev\left(\frac{\partial\, det(\underline{\mathbf{s}})}{\partial s}\right)\right] \quad \text{éq 4.1.1.2 - 1}$$

4.1.1.3 Final expression of the derivative of the criterion compared to the constraints

The derivative of the criterion compared to the constraints is then:

$$\frac{\partial f}{\partial \sigma}=\frac{1}{a(\gamma^p)}\frac{1}{\sigma_c h_c^0}^{\frac{1}{a(\gamma^p)}}(g)\, r^{\frac{1-a(\gamma^p)}{a(\gamma^p)}}\boldsymbol{Q}-\frac{\partial u}{\partial \boldsymbol{\sigma}} \quad \text{éq 4.1.1.3 - 1}$$

with

$$\frac{\partial u}{\partial \boldsymbol{\sigma}}=-\frac{m(\gamma^p)k(\gamma^p)}{\sigma_c}\left(\frac{1}{\sqrt{6}h_c^0}\boldsymbol{Q}+\frac{1}{3}\boldsymbol{I}\right) \quad \text{éq 4.1.1.3 - 2}$$

4.1.2 Derived compared to the variable from work hardening

$$\frac{\partial f}{\partial \gamma^p}=-\left(\frac{1}{a(\gamma^p)}\right)^2\left(\frac{g(\mathbf{s})}{\sigma_c h_c^0}\right)^{\frac{1}{a(\gamma^p)}}\log\left(\frac{g(\mathbf{s})}{\sigma_c h_c^0}\right).\frac{\partial a}{\partial \gamma^p}-\frac{\partial u}{\partial \gamma^p} \quad \text{éq 4.1.2-1}$$

with

$$\frac{\partial u}{\partial \gamma^p}=-\frac{1}{\sqrt{6}\sigma_c}\frac{\partial(km)}{\partial \gamma^p}(\gamma^p)\frac{g}{h_c^0}-\frac{1}{3\sigma_c}\frac{\partial(km)}{\partial \gamma^p}(\gamma^p)I_1+\frac{\partial(ks)}{\partial \gamma^p}(\gamma^p) \quad \text{éq 4.1.2-2}$$

4.2 Total derivative of the criterion compared to the plastic multiplier

Let us consider the function:

$$f^{*}(\Delta\lambda)=f\left(\boldsymbol{s}^{e}-2\mu\Delta\lambda\tilde{\boldsymbol{G}},\boldsymbol{I}_1^{e}-3\mathrm{K}\Delta\lambda G,\gamma^{p}+\Delta\lambda\sqrt{\frac{2}{3}}\tilde{G}_{\mathrm{II}}\right) \qquad \text{éq 4.2-1}$$

Where $\mathbf{G}$ is a fixed tensor independent of. $\Delta\lambda$ It is of this function of which we seek the zero to find the state of stress:

$$\frac{\partial f^{*}}{\partial\Delta\lambda}=-\frac{\partial f}{\partial\boldsymbol{\sigma}}.\left(2\mu\tilde{\boldsymbol{G}}+KG\boldsymbol{I}\right)+\frac{\partial f}{\partial\gamma^{p}}\sqrt{\frac{2}{3}}\tilde{G}_{\mathrm{II}} \qquad \text{éq 4.2-2}$$

4.3 Derived from the parameters compared to the variable of work hardening

$$\begin{cases}\dfrac{\partial s}{\partial\gamma^{p}}=-\dfrac{1}{\gamma_e} & si\,\gamma^{p}<\gamma_e\\[2mm] \dfrac{\partial s}{\partial\gamma^{p}}=0 & si\,\gamma^{p}\geq\gamma_e\end{cases} \qquad \text{éq 4.3-1}$$

$$\begin{cases}\dfrac{\partial m}{\partial s}=-\dfrac{\sigma_c}{\sigma_{\mathrm{p1}}} & si\,\gamma^{p}<\gamma_e\\[2mm] \dfrac{\partial m}{\partial s}=0 & si\,\gamma^{p}\geq\gamma_e\end{cases} \qquad \text{éq 4.3-2}$$

$$\begin{cases}\dfrac{\partial m}{\partial a}=-\dfrac{\sigma_c}{\sigma_{\mathrm{p1}}}\log\left(m_{\mathrm{pic}}\dfrac{\sigma_{\mathrm{p1}}}{\sigma_c}+1\right)\dfrac{a_{\mathrm{pic}}}{a^{2}}\left(m_{\mathrm{pic}}\dfrac{\sigma_{\mathrm{p1}}}{\sigma_c}+1\right)^{\frac{a_{\mathrm{pic}}}{a}} & \mathrm{si}\,\gamma^{p}<\gamma_e\\[2mm] \dfrac{\partial m}{\partial a}=-\dfrac{\sigma_c}{\sigma_{\mathrm{p2}}}\log\left(m_e\dfrac{\sigma_{\mathrm{p2}}}{\sigma_c}\right)\dfrac{a_e}{a^{2}}\left(m_e\dfrac{\sigma_{\mathrm{p2}}}{\sigma_c}\right)^{\frac{a_{\mathrm{pic}}}{a}} & \mathrm{si}\,\gamma^{p}<\gamma_e\end{cases} \qquad \text{éq 4.3-3}$$

$$\frac{\partial\Omega}{\partial\gamma^{p}}=\frac{\left(\gamma_{\mathrm{ult}}-\gamma_e\right)}{\left(\gamma_e\right)^{\eta}}\frac{a_e-a_{\mathrm{pic}}}{1-a_e}\left(\frac{\eta}{\gamma_{\mathrm{ult}}-\gamma^{p}}\left(\gamma^{p}\right)^{\eta-1}+\left(\gamma^{p}\right)^{\eta}\frac{1}{\left(\gamma_{\mathrm{ult}}-\gamma^{p}\right)^{2}}\right) \qquad \text{éq 4.3-4}$$

$$\frac{\partial a}{\partial\boldsymbol{\Omega}}=\frac{1-a_{\mathrm{pic}}}{\left(1+\Omega\right)^{2}} \qquad \text{éq 4.3-5}$$

$$\frac{\partial m}{\partial \gamma^p}=\frac{\partial m}{\partial a}\frac{\partial a}{\partial \gamma^p}+\frac{\partial m}{\partial s}\frac{\partial s}{\partial \gamma^p} \quad \text{si}\ \gamma^p<\gamma_e$$

$$\frac{\partial m}{\partial \gamma^p}=\frac{\partial m}{\partial a}\frac{\partial a}{\partial \gamma^p} \quad \text{si}\ \gamma_{\text{ult}}\left(1-10^{-3}\right)>\gamma^p\geq\gamma_e$$ éq 4.3-6

$$\frac{\partial m}{\partial \gamma^p}=0 \quad \text{si}\ \gamma_{\text{ult}}\left(1-10^{-3}\right)<\gamma^p$$

$$\frac{\partial k}{\partial \gamma^p}=-\left(\frac{2}{3}\right)^{\frac{1}{2a}}\log\left(\frac{2}{3}\right)\frac{1}{2a^2}\frac{\partial a}{\partial \gamma^p} \quad \gamma_{\text{ult}}\left(1-10^{-3}\right)>\gamma^p$$

$$\frac{\partial k}{\partial \gamma^p}=0 \quad \text{sinon}$$ éq 4.3-7

5 Tangent operator of speed

The condition

$$\dot{f}=0 \qquad \text{éq 5-1}$$

is written:

$$\dot{f}=\frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij}+\frac{\partial f}{\partial \gamma^p}\dot{\gamma}^p=0$$

From the expression of the cumulated plastic deviatoric deformation and $\gamma^p=\sqrt{\frac{2}{3}e^p_{ij}e^p_{ij}}$ relation $\dot{e}^p=\dot{\lambda}\tilde{G}$, the condition then is found:

$$\dot{f}=\frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij}+\frac{\partial f}{\partial \gamma^p}\sqrt{\frac{2}{3}}\dot{\lambda}\tilde{G}_{II}=0$$

What gives us for the plastic multiplier:

$$\dot{\lambda}=\frac{-\frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij}}{\sqrt{\frac{2}{3}}\frac{\partial f}{\partial \gamma^p}\tilde{G}_{II}}$$

By then considering the relation forced/deformations:

$$\frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij}=\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}\dot{\varepsilon}_{kl}=\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}=\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}\dot{\varepsilon}_{kl}-\dot{\lambda}\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}G_{kl}$$

and by deferring it in the expression of $\dot{\lambda}$ one can write:

$$\dot{\lambda}=-\frac{\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}\dot{\varepsilon}_{kl}-\dot{\lambda}\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}G_{kl}}{\sqrt{\frac{2}{3}}\frac{\partial f}{\partial \gamma^p}\tilde{G}_{II}}$$

That is to say:

$$\dot{\lambda}=-\frac{\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}\dot{\varepsilon}_{kl}}{\sqrt{\frac{2}{3}}\frac{\partial f}{\partial \gamma^p}\tilde{G}_{II}-\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}G_{kl}} \qquad \text{éq 5-2}$$

By deferring this result in the expression of $\dot{\sigma}_{ij}$ one finds:

$$\dot{\sigma}_{ab}=D_{abcd}\left(\dot{\varepsilon}_{cd}+\frac{\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}\dot{\varepsilon}_{kl}}{\sqrt{\frac{2}{3}}\frac{\partial f}{\partial \gamma^p}\tilde{G}_{II}-\frac{\partial f}{\partial \sigma_{ij}}D_{ijkl}G_{kl}}G_{cd}\right) \qquad \text{éq 5-3}$$

6 Digital processing adapted to the nonregular models

The law of evolution of the plastic mechanism, defined in the chapter [§3], must satisfy the kinematic condition [éq 3.2.4-2]. The projection suggested on the normal of the hypersurface of deformation can lead to a “not-solution” which results in a failure of the digital processing (see the graphic interpretation of the chapter [§ 6.1.3.3]). One proposes in this chapter to define rules of projection allowing to manage the models known as “not-regular” in their imposing projection called “to the top of the cone”.

Moreover, as for other relations of behavior, one adds the possibility of locally cutting out (at the points of Gauss) the step of time to facilitate digital integration.

6.1 Projection at the top of the cone

6.1.1 Definition of the jetting angle

One places oneself in this chapter within the framework of finished increase. The equations translating the elastic behavior are written:

$$\boldsymbol{s}=\boldsymbol{s}^{-}+2\mu\left(\Delta\boldsymbol{e}-\Delta\boldsymbol{r}^{p}\right)=\boldsymbol{s}^{e}-2\mu\,\Delta\boldsymbol{e}^{p} \qquad \text{éq 6.1.1-1}$$

$$I_1=I_1^{-}+3K\left(\Delta\varepsilon_v-\Delta\varepsilon_v^{p}\right)=I_1^{e}-\Delta\varepsilon_v^{p} \qquad \text{éq 6.1.1-2}$$

One can also express the kinematic condition starting from the tensor $\mathbf{n}$ (cf paragraph [§3.2.4]):

$$\boldsymbol{n}.\Delta\boldsymbol{\varepsilon}^{p}=0 \qquad \text{éq 6.1.1-3}$$

By deferring the two equations translating the elastic behavior in the preceding expression one finds:

$$\Delta\boldsymbol{e}^{p}=\frac{1}{2\mu}\left(\boldsymbol{s}^{e}-\boldsymbol{s}\right) \qquad \text{éq 6.1.1-4}$$

$$\Delta\varepsilon_v^{p}=\frac{1}{3K}\left(I_1^{e}-I_1\right) \qquad \text{éq 6.1.1-5}$$

One then expresses the kinematic condition by the following relation:

$$\boldsymbol{n}.\left(\frac{1}{2\mu}\left(\boldsymbol{s}^{e}-\boldsymbol{s}\right)+\frac{1}{3K}\left(\frac{1}{3K}\left(I_1^{e}-I_1\right)\boldsymbol{I}\right)\right)=0 \text{ with } \boldsymbol{n}=\frac{\beta'\dfrac{\boldsymbol{s}}{s_{II}}+\boldsymbol{I}}{\sqrt{\beta'^2+3}}$$

Maybe by combining the two preceding relations where $\mathbf{n}$ indicate the normal of the hypersurface of deformation:

$$\frac{1}{2\mu}\frac{\beta'\dfrac{\boldsymbol{s}}{s_{II}}+\boldsymbol{I}}{\sqrt{\beta'^2+3}}\left(\boldsymbol{s}^{e}-\boldsymbol{s}\right)+\frac{1}{9K}\left(I_1^{e}-I_1\right).\mathrm{Tr}\left(\boldsymbol{n}\right)=0$$

$$\frac{1}{2\mu}\beta'\frac{\boldsymbol{s}.\left(\boldsymbol{s}^{e}-\boldsymbol{s}\right)}{s_{II}}+\frac{1}{3K}\left(I_1^{e}-I_1\right)$$

This last equation defines the point $\left(I_1, \mathbf{s}\right)$ like a projection of the point $\left(I_1^e, \mathbf{s}^e\right)$ on the criterion. The point $\left(I_1, s_{II}\right)$ will be the oblique projection of the point $\left(I_1^e, s_{II}^e\right)$, projection whose direction varies with θ . One can give the chart of it of the chapter [§ 6.1.3.3].

The preceding relation can then be rewritten as follows:

$$I_1^e - I_1 = -\beta' \frac{3K}{2\mu} \frac{\boldsymbol{s}.\left(\boldsymbol{s}^e - \boldsymbol{s}\right)}{s_{II}} \quad \text{éq 6.1.1-6}$$

The jetting angle then is defined φ_s by the relation:

$$\cos\varphi_s = \frac{\boldsymbol{s}.\left(\boldsymbol{s}^e - \boldsymbol{s}\right)}{s_{II}\sqrt{\left(\boldsymbol{s}^e - \boldsymbol{s}\right)\left(\boldsymbol{s}^e - \boldsymbol{s}\right)}} \quad \text{éq 6.1.1-7}$$

By deferring the definition of the angle φ_s in the relation of projection one finds the relation:

$$\frac{I_1^e - I_1}{\sqrt{\left(\boldsymbol{s}^e - \boldsymbol{s}\right)\left(\boldsymbol{s}^e - \boldsymbol{s}\right)}} = -\beta' \frac{3K}{2\mu} \cos\varphi_s \quad \text{éq 6.1.1-8}$$

6.1.2 Existence of projection

The principle of this paragraph is to discuss on the question the existence the angle φ_s such as projection point $\left(I_1^e, \boldsymbol{s}^e\right)$ always belongs to the surface of load. These problems appear essential for projections around the top of the surface of load, in other words when $\boldsymbol{s} \rightarrow \boldsymbol{0}$. There is by definition the relation:

$$\cos\varphi_s = \frac{\boldsymbol{s}.\left(\boldsymbol{s}^e - \boldsymbol{s}\right)}{s_{II}\sqrt{\left(\boldsymbol{s}^e - \boldsymbol{s}\right)\left(\boldsymbol{s}^e - \boldsymbol{s}\right)}} = \frac{\boldsymbol{s}.\left(\boldsymbol{s}^e - \boldsymbol{s}\right)}{s_{II}\left\|\boldsymbol{s}^e - \boldsymbol{s}\right\|} \quad \text{éq 6.1.2-1}$$

By combining this equation with the expression: $\boldsymbol{s} = \boldsymbol{s}^e - 2\mu\,\Delta\,\boldsymbol{e}^p = \boldsymbol{s}^e - 2\mu\,\Delta\lambda\,\tilde{\boldsymbol{G}}$

One obtains:

$$\cos\varphi_s = \frac{\boldsymbol{s}.\tilde{\boldsymbol{G}}}{s_{II}\tilde{G}_{II}} \quad \text{éq 6.1.2-2}$$

One seeks an estimate of $\cos\varphi_s$.

Stage 1: estimate of $\dfrac{\mathbf{s}.\tilde{\mathbf{G}}}{s_{II}}$

One places oneself in this paragraph under the conditions: $\boldsymbol{s} \rightarrow \boldsymbol{0}$ and $f = 0$.

By definition of $\tilde{\mathbf{G}}$ and of $\mathbf{G}$ one a: $\tilde{\boldsymbol{G}}.\boldsymbol{s} = \left(\boldsymbol{G} - \frac{\operatorname{Tr}\left(\boldsymbol{G}\right)}{3}\boldsymbol{I}\right).\boldsymbol{s} = \boldsymbol{G}.\boldsymbol{s} = \left(\frac{\partial f}{\partial\boldsymbol{\sigma}} - \left(\frac{\partial f}{\partial\boldsymbol{\sigma}}\boldsymbol{n}\right)\boldsymbol{n}\right).\boldsymbol{s}$

For preoccupations with a simplification of calculation one brings back the resolution of f with the resolution of the equation:

$$f=\left(\frac{g(\boldsymbol{s})}{\sigma_c h_c^0}\right)^{\frac{1}{a(\gamma^p)}}-u(\boldsymbol{\sigma},\gamma^p)=0\Rightarrow f_2=\left(\frac{g(\boldsymbol{s})}{\sigma_c h_c^0}\right)-u(\boldsymbol{\sigma},\gamma^p)^{a(\gamma^p)}=0 \qquad \text{éq 6.1.2.3}$$

By derivation of this new function one finds the relation:

$$\frac{\partial f_2}{\partial\boldsymbol{\sigma}}=\left(\frac{1}{\sigma_c h_c^0}\right)\frac{\partial g}{\partial\boldsymbol{\sigma}}-a(\gamma^p)u(\boldsymbol{\sigma},\gamma^p)^{a(\gamma^p)-1}\frac{\partial u}{\partial\boldsymbol{\sigma}}=\left(\frac{1}{\sigma_c h_c^0}\right)\boldsymbol{Q}-a(\gamma^p)u(\boldsymbol{\sigma},\gamma^p)^{a(\gamma^p)-1}\frac{\partial u}{\partial\boldsymbol{\sigma}}$$

with: $\frac{\partial u}{\partial\boldsymbol{\sigma}}=-\frac{m(\gamma^p)k(\gamma^p)}{\sigma_c}\left(\frac{1}{\sqrt{6}h_c^0}\boldsymbol{Q}+\frac{1}{3}\boldsymbol{I}\right)$

Who gives after simplification:

$$\frac{\partial f_2}{\partial\boldsymbol{\sigma}}=A\boldsymbol{Q}+B\boldsymbol{I} \qquad \text{éq 6.1.2.4}$$

Where:

$$\begin{cases}A=\frac{1}{\sigma_c h_c^0}\left(1+\frac{a(\gamma^p)m(\gamma^p)k(\gamma^p)}{\sqrt{6}}u(\sigma,\gamma^p)^{a(\gamma^p)-1}\right)\\ B=\frac{a(\gamma^p)m(\gamma^p)k(\gamma^p)}{3\sigma_c h_c^0}u(\boldsymbol{\sigma},\gamma^p)^{a(\gamma^p)-1}\end{cases} \qquad \text{éq 6.1.2.5}$$

One has as follows: $\frac{\partial f_2}{\partial\boldsymbol{\sigma}}.\boldsymbol{n}=(A\boldsymbol{Q}+b\boldsymbol{I})\frac{\beta'\frac{\boldsymbol{s}}{s_{II}}+\boldsymbol{I}}{\sqrt{\beta'^2+3}}=\frac{\beta'}{\sqrt{\beta'^2+3}}\frac{A}{s_{II}}\boldsymbol{Q}.\boldsymbol{s}+\frac{3B}{\sqrt{\beta'^2+3}}$

And consequently:

$$\begin{aligned}\tilde{\boldsymbol{G}}.\boldsymbol{s}&=\left(\frac{\partial f}{\partial\boldsymbol{\sigma}}-\left(\frac{\partial f}{\partial\boldsymbol{\sigma}}\boldsymbol{n}\right)\boldsymbol{n}\right).\boldsymbol{s}\\ &=\left(A\boldsymbol{Q}+B\boldsymbol{I}-\left(\frac{\beta'}{\sqrt{\beta'^2+3}}\frac{A}{s_{II}}\boldsymbol{Q}.\boldsymbol{s}+\frac{3B}{\sqrt{\beta'^2+3}}\right)\frac{\beta'\frac{\boldsymbol{s}}{s_{II}}+\boldsymbol{I}}{\sqrt{\beta'^2+3}}\right)\\ &=\frac{3A}{\beta'^2+3}\boldsymbol{Q}.\boldsymbol{s}-\frac{3B\beta'}{\beta'^2+3}s_{II}\end{aligned}$$

From where it is deduced that:

$$\frac{\tilde{\boldsymbol{G}}.\boldsymbol{s}}{s_{II}}=\frac{3A}{\beta'^2+3}\frac{\boldsymbol{Q}.\boldsymbol{s}}{s_{II}}-\frac{3B\beta'}{\beta'^2+3} \qquad \text{éq 6.1.2.6}$$

By definition of $\mathbf{Q}$ one a:

$$\boldsymbol{Q}\,.\,\boldsymbol{s}=\mathrm{dev}\left(\frac{\partial g}{\partial \boldsymbol{s}}\right).\,\boldsymbol{s}=\frac{1}{h(\theta)^5}\left[\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos(3\theta)\right)\frac{\boldsymbol{s}}{s_{\mathrm{II}}}+\frac{\gamma_{\mathrm{cjs}}\sqrt{54}}{6\,s_{\mathrm{II}}^2}\mathrm{dev}()\right].\,\boldsymbol{s}$$

$$=\frac{1}{h(\theta)^5}\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos(3\theta)\right)s_{\mathrm{II}}$$

$$=h(\theta)\,s_{\mathrm{II}}$$

One expresses finally:

$$\frac{\tilde{\boldsymbol{G}}\,.\,\boldsymbol{s}}{s_{\mathrm{II}}}=\frac{3\mathrm{A}}{\beta'^2+3}h(\theta)-\frac{3\beta'}{\beta'^2+3} \quad \text{éq 6.1.2.7}$$

When $\boldsymbol{s}\rightarrow\boldsymbol{0}$ then $u(\boldsymbol{\sigma},\gamma^p)\rightarrow 0 \text{ et } A\rightarrow\frac{1}{\sigma_c h_c^0}, B\rightarrow 0$

And thus:

When $\boldsymbol{s}\rightarrow\boldsymbol{0}$ then $\frac{\tilde{\boldsymbol{G}}\,.\,\boldsymbol{s}}{s_{\mathrm{II}}}\underset{s\rightarrow\boldsymbol{0}}{\rightarrow}\frac{3\mathrm{h}(\theta)}{\sigma_c h_c^0(\beta'^2+3)}$ éq 6.1.2.8

Stage 2: estimate of $\tilde{G}_{II}$

One places oneself in this paragraph under the conditions: $\boldsymbol{s}\rightarrow\boldsymbol{0}, A\rightarrow\frac{1}{\sigma_c h_c^0}, b\rightarrow 0$

$$\tilde{\boldsymbol{G}}=\left(\frac{\partial f}{\partial\boldsymbol{\sigma}}-\left(\frac{\partial f}{\partial\boldsymbol{\sigma}}\boldsymbol{n}\right)\boldsymbol{n}\right)$$

$$\left(\frac{1}{\sigma_c h_c^0}\boldsymbol{Q}+B\,\boldsymbol{I}-\left(\frac{\beta'}{\sqrt{\beta'^2+3}}\frac{1}{\sigma_c h_c^0 s_{\mathrm{II}}}\boldsymbol{Q}.\,\boldsymbol{s}+\frac{3\mathrm{B}}{\sqrt{\beta'^2+3}}\right)\frac{\beta'\frac{\boldsymbol{s}}{s_{\mathrm{II}}}+\boldsymbol{I}}{\sqrt{\beta'^2+3}}\right)$$

$$\frac{1}{\sigma_c h_c^0}\boldsymbol{Q}-\frac{\beta'^2 h(\theta)}{(\beta'^2+3)\sigma_c h_c^0 s_{\mathrm{II}}}\boldsymbol{s}$$

$$\tilde{G}_{\mathrm{II}}^2\,\tilde{\boldsymbol{G}}.\,\tilde{\boldsymbol{G}}=\frac{Q_{\mathrm{II}}^2}{(\sigma_c h_c^0)^2}+\frac{\beta'^4 h^2(\theta)s_{\mathrm{II}}^2}{(\beta'^2+3)^2(\sigma_c h_c^0)_2 s_{\mathrm{II}}^2}-2\frac{\beta'^2 h^2(\theta)}{(\beta'^2+3)(\sigma_c h_c^0)^2}$$

$$\frac{1}{\sigma_c h_c^{02}}\left(Q_{\mathrm{II}}^2-\frac{\beta'^2(\beta'^2+6)h^2(\theta)}{(\beta'^2+3)^2}\right)$$

It is shown [Appendix 2] that:

$$Q_{\mathrm{II}}^2=\frac{1}{h(\theta)^{10}}\left[\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos(3\theta)\right)_2+\frac{\gamma_{\mathrm{cjs}}^2}{4}+\gamma_{\mathrm{cjs}}\cos(3\theta)\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos(3\theta)\right)\right] \quad \text{éq 6.1.2.9}$$

and thus like $h(\theta)=\left(1+\cos(3\theta)\right)^{1/6}$:

$$\tilde{G}_{\mathrm{II}}^{2}=\frac{1}{\sigma_c h_c^{22}}\left(\frac{1}{h(\theta)^{10}}\left[\left(\frac{1}{2}+\frac{h(\theta)^6}{2}\right)^2+\frac{{\gamma_{\mathrm{cjs}}}^2}{4}+\left(h(\theta)^6-1\right)\left(\frac{1}{2}+\frac{h(\theta)^6}{2}\right)\right]-\frac{\beta'^2\left(\beta'^2+6\right)h^2(\theta)}{\left(\beta'^2+3\right)^2}\right)$$

$$\tilde{G}_{2}^{\mathrm{II}}=\frac{1}{\left(\sigma_c h_c^0\right)^2}\left(\frac{3\mathrm{h}(\theta)^2}{4}+\frac{1}{2h(\theta)^4}+\frac{\gamma_{\mathrm{cjs}}^2-1}{4\mathrm{h}(\theta)^{10}}-\frac{\beta'^2\left(\beta'^2+6\right)h^2(\theta)}{\beta'^2+3^2}\right)$$

$$\tilde{G}_{\mathrm{II}}^{2}=\left(\frac{h(\theta)}{\sigma_c h_c^0}\right)^2\left[\frac{1}{2\mathrm{h}(\theta)^6}+\frac{\gamma_{\mathrm{cjs}}^2-1}{4h(\theta)^{12}}+\left(\frac{3}{\beta'^2+3}\right)^2-\frac{1}{4}\right]$$

And consequently:

$$\tilde{G}_{\mathrm{II}}=\left(\frac{h(\theta)}{\sigma_c h_c^0}\right)\sqrt{\left(\frac{3}{\beta'^2+3}\right)^2-\frac{1}{4}+\frac{1}{2\mathrm{h}(\theta)^6}+\frac{\gamma_{\mathrm{cjs}}^2-1}{4\mathrm{h}(\theta)^{12}}} \qquad \text{éq 6.1.2.10}$$

Stage 3: estimate of $\cos(\varphi_s)$

One deduces from the two paragraphs precedent the expression of the following jetting angle:
When $s\rightarrow 0$ then:

$$\cos\varphi_s tward \frac{3}{\left(\beta'^2+3\right)\sqrt{\frac{3}{\beta'^2+3}^2-\frac{1}{4}+\frac{1}{2\left(1+\gamma_{\mathrm{cjs}}\cos(3\theta)\right)}+\frac{\gamma_{\mathrm{cjs}}^2-1}{4\left(1+\gamma_{\mathrm{cjs}}\cos(3\theta)\right)^2}}} \qquad \text{éq 6.1.2.11}$$

It is noticed that φ_s depends on the angle of Lode θ , and that consequently limit of the jetting angle when $s\rightarrow 0$ do not exist. However a framing of $\cos\varphi_s$ we allows to determine a zone of projection at the top a priori (demonstration of the framing in [Appendix 3]):

$$\cos\varphi_s^{\min}\leq\cos\varphi_s\leq\cos\varphi_s^{\max}$$

with:

$$\begin{cases}\cos\varphi_s^{\min}=\dfrac{3}{\left(\beta'^2+3\right)\sqrt{\left(\dfrac{3}{\beta'^2+3}\right)^2+\dfrac{{\gamma_{\mathrm{cjs}}}^2}{4\left(1-\gamma_{\mathrm{cjs}}^2\right)}}}\\ \cos\varphi_s^{\max}=1\end{cases} \qquad \textbf{éq 6.1.2.12}$$

6.1.3 Rules of projection

One calls I_1^0 the intersection of the field of reversibility with the hydrostatic axis. One obtains:

$$I_1^0=\frac{3\sigma_c . s(\gamma_p)}{m(\gamma_p)} \quad \text{éq 6.1.3-1}$$

While deferring I_1^0 and the framing of $\cos\varphi_s$, when $\boldsymbol{s}\to\boldsymbol{0}$, in the relation $\frac{I_1^e-I_1}{\sqrt{(\boldsymbol{s}^e-\boldsymbol{s})(\boldsymbol{s}^e-\boldsymbol{s})}}=-\beta'\frac{3\mathrm{K}}{2\mu}\cos\varphi_s$, one from of deduced them following rules of projection according to the sign of the parameter of dilatancy β' , and for values of I_1^e and of s_{II}^e data.

6.1.3.1 Case where the parameter of dilatancy is negative

If $\frac{I_1^e-I_1^0}{s_{\mathrm{II}}^e}<-\beta'\frac{3\mathrm{K}}{2\mu}\cos\varphi_s^{\min}$ then projection will be regular;

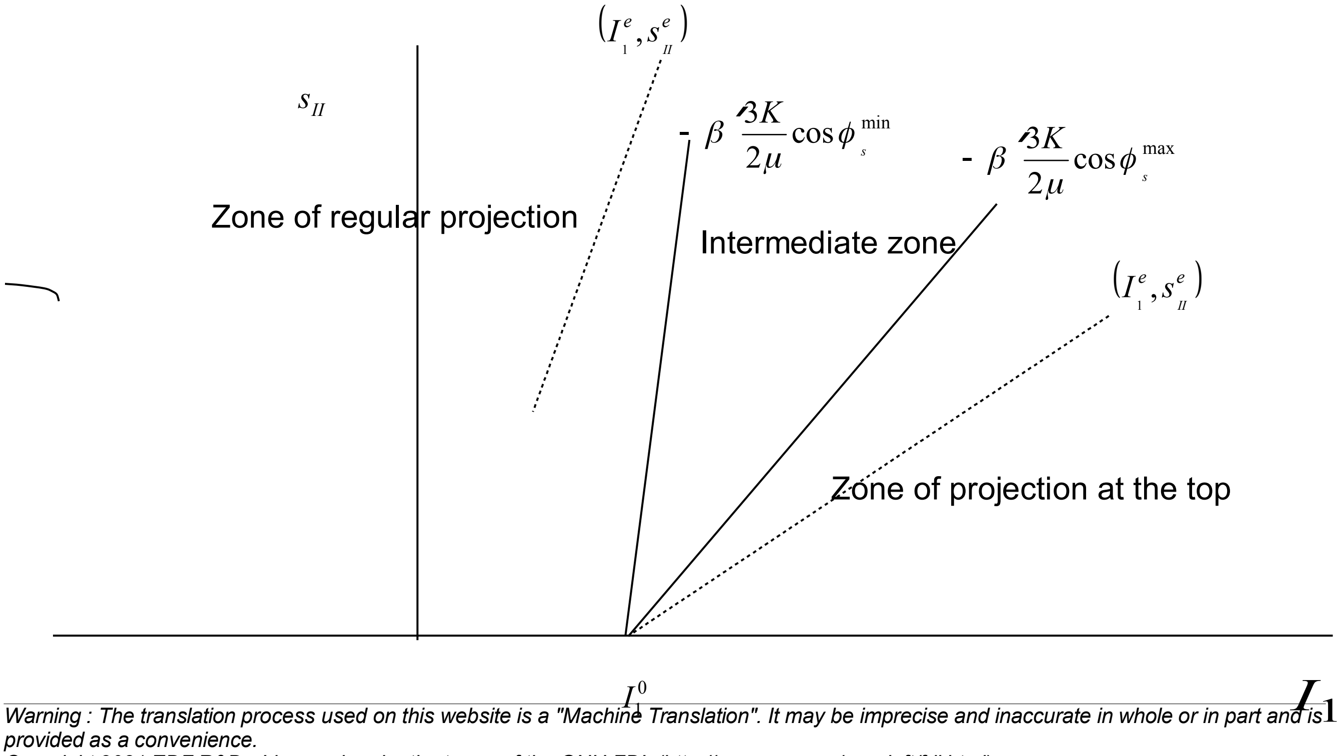
If $\frac{I_1^e-I_1^0}{s_{\mathrm{II}}^e}>-\beta'\frac{3\mathrm{K}}{2\mu}\cos\varphi_s^{\max}$ then projection will be at the top.

6.1.3.2 Case where the parameter of dilatancy is positive

If $\frac{I_1^e-I_1^0}{s_{\mathrm{II}}^e}<-\beta'\frac{3\mathrm{K}}{2\mu}\cos\varphi_s^{\max}$ then projection will be regular;

If $\frac{I_1^e-I_1^0}{s_{\mathrm{II}}^e}>-\beta'\frac{3\mathrm{K}}{2\mu}\cos\varphi_s^{\min}$ then projection will be at the top.

6.1.3.3 Graphic interpretation



6.1.3.4 Equations of flow

In the intermediate zone one solves the equations corresponding to a regular projection. If this resolution does not give a solution one then solves the equations of flow of projection at the top.

In the case of projection at the top there are the relations:

$$\mathbf{s}=\mathbf{0}$$ éq 6.1.3.4 - 1

$$I_1^0=\frac{3\sigma_c . s\left(\gamma^p\right)}{m\left(\gamma^p\right)}$$ éq 6.1.3.4 - 2

$$\Delta\gamma^p=\frac{1}{2\mu}\sqrt{\frac{2}{3}}s_{\mathrm{II}}^e$$ éq 6.1.3.4 - 3

6.2 Local Recutting of the step of time

As for other relations of behavior (the model CJS for example) one added the possibility for the model of LAIGLE of redécouper locally (at the points of Gauss) the step of time in order to facilitate digital integration. This possibility is managed by the operand ITER_INTE_PAS keyword CONVERGENCE of the operator STAT_NON_LINE. If the value of ITER_INTE_PAS (itepas) is worth 0.1 or – 1 it has no recutting there (note: 0 are the value by default). If itepas is positive recutting is systematic, if it is negative recutting is taken into account only in the event of nondigital convergence.

Recutting consists in carrying out the integration of the plastic mechanism with an increment of deformation whose components correspond to the components of the initial increment of deformation divided by the absolute value of itepas (cf Doc. STAT_NON_LINE [U4.51.03]).

7 Internal variables

For implementation the data-processing we retained the 4 following internal variables:

7.1 V1: the plastic deformation déviatoire cumulated

The variable of work hardening γ^p is proportional to the second invariant of the tensor of the deformations déviatoires.

$$\gamma^p = \sqrt{\frac{2}{3} e^p_{ij} e^p_{ij}}$$

with $$e^p_{ij} = \varepsilon^p_{ij} - \frac{\mathrm{tr}\left(\varepsilon^p_{ij}\right)}{3} \delta_{ij}$$

7.2 V2: cumulated plastic voluminal deformation

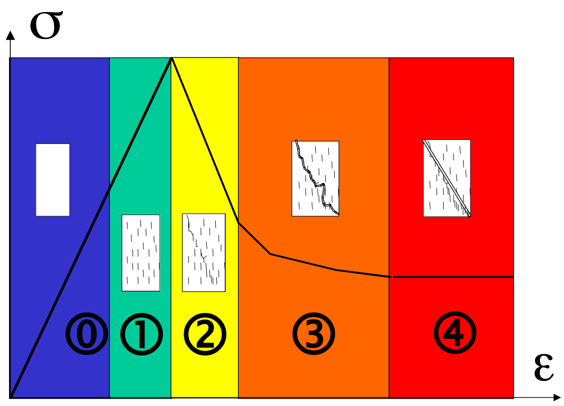
The plastic voluminal deformation is defined by the relation presented to the paragraph [§3.2.4] on the law of evolution of the plastic mechanism: $\dot{\varepsilon}^p_v = \dot{\lambda} G$

7.3 V3: fields of behavior of the rock

Five fields of behavior, numbered from 0 to 4 (cf appears), are identified to make it possible to have a relatively simple representation of the state of damage of the rock, since the intact rock to the rock in a residual state. These fields are function of the cumulated plastic deformation déviatoire γ^p and of the state of stress. Each increment of number of field defines the passage in a field of higher damage.

- If the diverter is lower than 70% of the diverter of peak, then the material is in field 0;
- If not:
  - If $\gamma^p = 0$ then the material is in field 1;
    1) If $0 < \gamma^p < \gamma^e$ then the material is in field 2;
  - If $\gamma_e < \gamma^p < \gamma_{\mathrm{ult}}$ then the material is in field 3;
  - If $\gamma^p > \gamma_{\mathrm{ult}}$ then the material is in field 4.

Domaine	Etat de la roche
0	Intacte
1	Endommagement pré-pic
2	Endommagement post-pic
3	Fissurée
4	Fracturée



7.4 V4: the state of plasticization

It is an internal indicator with *Code_Aster*. It is worth 0 if the point of gauss is in elastic load or discharge, and is worth 1 if the point of gauss is in plastic load.

8 Detailed presentation of the algorithm

One retains a formulation implicit compared to the criterion and explicit compared to the direction of flow: the criterion will have to be checked at the end of the step, whereas the direction of flow is that calculated at the beginning of the step (and thus the value of dilatancy will be also that calculated at the beginning of the step of time).

One places oneself in a material point, and one considers that are given:

- The tensor of increase in deformations $\Delta \varepsilon$ from where one deduces Δe and; $\Delta \varepsilon_v$
- Constraints at the beginning of the step D $\boldsymbol{\sigma}^-$ 'where L' one deduces $\boldsymbol{s}^-$ and I_1^- ;
- Values of the internal variables at the beginning of the step of time (only cumulated plastic deformation γ^{p^-} is necessary).

It is a question of calculating:

- Constraints at the end of the step of time $\boldsymbol{\sigma}$;
- Internal variables in the end of the step of time (γ^p , ε_v^p , fields of behavior);
- The tangent behavior at the end of the step: $\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}}$

8.1 Calculation of the elastic solution

$$\Delta \varepsilon^e = \Delta \varepsilon^- - \alpha \Delta T$$

$$\boldsymbol{s}^e = \boldsymbol{s}^- + 2\mu \Delta \boldsymbol{e}$$

$$I_{¿}^e = I_1^- + 3K \Delta \varepsilon_v$$

8.2 Calculation of the elastic criterion

Calculation of $g^e = s_{II}^e h(\theta^e)$

Calculation of, $m^- = m(\gamma^{p^-})$ $s^- = s(\gamma^{p^-})$, $a^- = a(\gamma^{p^-})$ and $k^- = k(a^-)$

Calculation of $u^e = -\frac{m^- k^-}{\sqrt{6}\sigma_c} \frac{g^e}{h_c^0} - \frac{m^- k^-}{3\sigma_c} I_1^e + s^- . k^-$

Calculation of $f^e = \left(\frac{g^e}{\sigma_c h_c^0}\right)^{\frac{1}{a^-}} - u^e$

8.3 Algorithm

If $f^e > 0$

Calculation of:

$$I_1^{0-} = \frac{3\sigma_c . s^-}{m^-}; \quad g^- = g(\boldsymbol{s}^-)$$

$$\phi_0^- = \phi_0(m^-, s^-, a^-); \quad C_0^- = C_0(m^-, s^-, a^-); \quad \sigma_0^- = \sigma_{t0}(\phi_0^-, C_0^-)$$

$$\alpha'^- = \alpha'(I_1^-, g^-, \sigma_{t0}^-); \quad \psi^- = \psi(\alpha'^-); \quad \beta'^- = \beta'(\psi^-)$$

Calculation a priori of projection at the top

$\boldsymbol{s} = \boldsymbol{0}$; Calculation of $\gamma^p = \gamma^{p^-} + \frac{1}{2\mu}\sqrt{\frac{2}{3}} s_{II}^e = \gamma^{p^{\text{sommet}}}$ and of $I_1 = \frac{3\sigma_c . s(\gamma^p)}{m(\gamma^p)} = I_1^{\text{sommet}}$.

If $\begin{cases} (I_1^e - I_1^{\text{sommet}}) < -\frac{3K}{2\mu}\beta' s_n^e \cos\varphi_s^{\max}; \text{si } \beta'^- < 0 \\ (I_1^e - I_1^{\text{sommet}}) < -\frac{3K}{2\mu}\beta' s_n^e \cos\varphi_s^{\min}; \text{si } \beta'^- \geq 0 \end{cases}$

Projection at the top is not retained a priori. The regular solution is calculated.

$$\boldsymbol{Q}^- = \begin{cases} \boldsymbol{Q}(\boldsymbol{\sigma}^-) \text{si } \boldsymbol{\sigma}^- \neq 0 \\ \boldsymbol{Q}(\boldsymbol{\sigma}^e) \text{si } \boldsymbol{\sigma}^- = 0 \end{cases} \quad \boldsymbol{n}^f = \begin{cases} \boldsymbol{n}(\beta'^-, \boldsymbol{\sigma}^-) \text{si } \boldsymbol{\sigma}^- \neq 0 \\ \boldsymbol{n}(\beta'^e, \boldsymbol{\sigma}^e) \text{si } \boldsymbol{\sigma}^- = 0 \end{cases} \quad \boldsymbol{G}^f = \begin{cases} \boldsymbol{G}(\beta'^-, \boldsymbol{\sigma}^-) \text{si } \boldsymbol{\sigma}^- \neq 0 \\ \boldsymbol{G}(\beta'^e, \boldsymbol{\sigma}^e) \text{si } \boldsymbol{\sigma}^- = 0 \end{cases}$$

If $\gamma^{p^-} = 0$

Initialization $\Delta\lambda^0 = 0; \gamma^{p^0} = \gamma^{p^-}; \boldsymbol{s}^0 = \boldsymbol{s}^e; I_1^0 = I_1^e; f^0 = f^e$

And $\begin{cases} \Delta\gamma^{p^1} = \frac{1}{10}\max|\Delta\varepsilon_{ij}^e| \\ \delta\lambda^{p^1} = \frac{\Delta\gamma^{p^1}}{\tilde{G}_{II}^{f_b}}\sqrt{\frac{2}{3}} \end{cases}$

If not

Calculation of the increase in the plastic multiplier $\Delta\lambda$ by Newton:

Initialization $\Delta\lambda^0 = 0; \gamma^{p^0} = \gamma^{p^-}; \boldsymbol{s}^0 = \boldsymbol{s}^e; I_1^0 = I_1^e; f^0 = f^e$

$$\frac{\partial u^0}{\partial\boldsymbol{\sigma}} = \frac{\partial u^-}{\partial\boldsymbol{\sigma}} = -\frac{m^-}{\sqrt{6}\sigma_c}\frac{k^-}{h_c^0}\boldsymbol{Q}^- - k^-\frac{m^-}{3\sigma_c}\boldsymbol{I}$$

$$\frac{\partial u^0}{\partial\gamma^p} = -\frac{1}{\sqrt{6}\sigma_c}\frac{\partial(km)}{\gamma^p}(\gamma^{p^-})\frac{g^e}{h_c^0} - \frac{1}{3\sigma_c}\frac{\partial(km)}{\partial\gamma^p}(\gamma^{p^-}) I_1^e + \frac{\partial(ks)}{\partial\gamma^p}(\gamma^{p^-})$$

$$\frac{\partial f^0}{\partial\boldsymbol{\sigma}} = \frac{1}{a^-}\left(\frac{1}{\sigma_c h_c^0}\right)^{\frac{1}{a^-}}(g^e)\frac{1-a^-}{a^-}\boldsymbol{Q}^- - \frac{\partial u^0}{\partial\boldsymbol{\sigma}} \neq \frac{\partial f^-}{\partial\boldsymbol{\sigma}}$$

$$\frac{\partial f^0}{\partial\gamma^p} = -\left(\frac{1}{a^-}\right)^2\left(\frac{g^e}{\sigma_c h_c^0}\right)^{\frac{1}{a^-}}\log\left(\frac{g^e}{\sigma_c h_c^0}\right)\frac{\partial a}{\partial\gamma^p}(\gamma^{p^-}) - \frac{\partial u^0}{\partial\gamma^p} \neq \frac{\partial f^-}{\partial\gamma^p}$$

$$\frac{\partial f^{0^*}}{\partial \Delta \lambda} = -\frac{\partial f^0}{\partial \boldsymbol{\sigma}} . \left(2 \mu \tilde{\boldsymbol{G}}^f + K G^f \boldsymbol{I} \right) + \frac{\partial f^0}{\partial \gamma^p} \sqrt{\frac{2}{3}} \tilde{G}^f_{\mathrm{II}}$$

Buckle iterations N

$$\frac{\partial f^n}{\partial \Delta\lambda}\delta\lambda^{n+1}=-f^n$$

$$\Delta\lambda^{n+1}=\Delta\lambda^n+\delta\lambda^{n+1}$$

$$\Delta\gamma^{p^{n+1}}=\Delta\gamma^{n+1}\sqrt{\frac{2}{3}}\tilde{G}^f_{\mathrm{II}};\quad \Delta\varepsilon^p_v=\Delta\lambda^{n+1}G^f$$

$$\boldsymbol{s}^{\boldsymbol{n+1}}=\boldsymbol{s}^e-2\mu\Delta\lambda^{p^{n+1}}\tilde{\boldsymbol{G}}^f\quad;\quad I_1^{n+1}=I_1^e-3\mathrm{K}\Delta\lambda^{p^{n+1}}G^f$$

If $\Delta\gamma^{p^{n+1}}<0$ Not convergence

Calculation $\boldsymbol{Q}^{\boldsymbol{n+1}}$

$$g^{n+1}=g\left(\boldsymbol{s}^{\boldsymbol{n+1}}\right);\ m^{n+1}=m\left(\gamma^{p^{n+1}}\right);\ s^{n+1}=s\left(\gamma^{p^{n+1}}\right);\ a^{n+1}=a\left(\gamma^{p^{n+1}}\right);\ k^{n+1}=k\left(a^{n+1}\right)$$

$$u_{n+1}=-\frac{m^{n+1}k^{n+1}}{\sqrt{6}\sigma_c}\frac{g^{n+1}}{h_c^0}-\frac{m^{n+1}k^{n+1}}{3\sigma_c}I_1^{n+1}+s^{n+1}.k^{n+1}$$

$$f^{n+1}=\left(\frac{g^{n+1}}{\sigma_c h_c^0}\right)^{\frac{1}{a^{n+1}}}-u^{n+1}$$

$$\frac{\partial u}{\partial\sigma}^{n+1}=-\frac{m}{\sqrt{6}\sigma_c}\frac{k^{n+1}}{h_c^0}\boldsymbol{Q}^{\boldsymbol{n+1}}-k^{n+1}\frac{m^{n+1}}{3\sigma_c}\boldsymbol{I}$$

$$\frac{\partial u}{\partial\gamma^p}^{n+1}=-\frac{1}{\sqrt{6}\sigma_c}\frac{\partial(km)}{\partial\gamma^p}\left(\gamma^{p^{n+1}}\right)\frac{g^{n+1}}{h_c^0}-\frac{1}{3\sigma_c}\frac{\partial(km)}{\partial\gamma^p}\left(\gamma^{p^{n+1}}\right)I_1^{n+1}+\frac{\partial(ks)}{\partial\gamma^p}\left(\gamma^{p^{n+1}}\right)$$

$$\frac{\partial f^{n+1}}{\partial\boldsymbol{\sigma}}=\frac{1}{a^{n+1}}\left(\frac{A}{\sigma_c h_c^0}\right)^{\frac{1}{a^{n+1}}}\left(g^{n+1}\right)^{\frac{A-a^{n+1}}{a^{n+1}}}\boldsymbol{Q}^{\boldsymbol{n+1}}-\frac{\partial u}{\partial\boldsymbol{\sigma}}^{n+1}$$

$$\frac{\partial f^{n+1}}{\partial\gamma^p}=-\left(\frac{1}{a^{n+1}}\right)^2\left(\frac{g^{n+1}}{\sigma_c h_c^0}\right)^{\frac{1}{a^{n+1}}}\log\left(\frac{g^{n+1}}{\sigma_c h_c^0}\right)\frac{\partial\alpha}{\partial\gamma^p}\left(\gamma^{p^{n+1}}\right)-\frac{\partial u^{n+1}}{\partial\gamma^p}$$

$$\frac{\partial f^{n+1^*}}{\partial\Delta\lambda}=-\frac{\partial f^{n+1}}{\partial\boldsymbol{\sigma}}.\left(2\mu\tilde{\boldsymbol{G}}^f+KG^f\boldsymbol{I}\right)+\frac{\partial f^{n+1}}{\partial\gamma^p}\sqrt{\frac{2}{3}}\tilde{G}^f_{\mathrm{II}}$$

If $\left|f^{n+1}/\sigma_c\right|>\varepsilon_{\mathrm{prec}}$

n=n+1

If N > no. ite internal max

If $\begin{cases}\left(I_1^e-I_1^{\mathrm{sommet}}\right)>-\dfrac{3\mathrm{K}}{2\mu}\beta' s^e_{\mathrm{II}}\cos\varphi_s^{\min};\mathrm{si}\ \beta'<0\\ \left(I_1^e-I_1^{\mathrm{sommet}}\right)>-\dfrac{3\mathrm{K}}{2\mu}\beta' s^e_{\mathrm{II}}\cos\varphi_s^{\max};\mathrm{si}\ \beta'\geq 0\end{cases}$

One retains projection at the top: $\boldsymbol{s}=\boldsymbol{0};I_1=I_1^{\mathrm{sommet}};\gamma^p=\gamma^{p^{\mathrm{sommet}}}$

If not

Not convergence

If not

Not convergence

If not

| Convergence

If FULL_MECA

Calculation of:

$$\frac{\partial \boldsymbol{\sigma}^{n+1}}{\partial \boldsymbol{\varepsilon}}=\boldsymbol{H}+\frac{\boldsymbol{H}.\boldsymbol{G}^f.\left(\frac{\partial f^{n+1}}{\partial \boldsymbol{\sigma}}\right)_T \boldsymbol{H}}{\sqrt{\frac{2}{3}}\frac{\partial f^{n+1}}{\partial \gamma^p}\tilde{G}^f_{\mathrm{II}}-\left(\frac{\partial f^{n+1}}{\partial \boldsymbol{\sigma}}\right)^T \boldsymbol{H}\,\boldsymbol{G}^f}$$

Mechanical symmetrization:

$$\frac{\partial \boldsymbol{\sigma}^{\boldsymbol{n+1}}_{\mathbf{sym}}}{\partial \boldsymbol{\varepsilon}}=\frac{1}{2}\left(\frac{\partial \boldsymbol{\sigma}^{\boldsymbol{n+1}}}{\partial \boldsymbol{\varepsilon}}+\frac{\partial \boldsymbol{\sigma}^{\boldsymbol{n+1}^T}}{\partial \boldsymbol{\varepsilon}}\right)$$

9 Alternative on the expression of the criterion of plasticity

In this alternative proposal, one expresses the criterion of plasticity according to the first invariant and the diverter of the constraints, by a retiming on triaxial in compression and extension by the following relations:

9.1 General formulation

$$f=\left(\frac{s_{\mathrm{II}}}{\sigma_c}\right)^{\frac{1}{a(\gamma^p)}}-u(\boldsymbol{\sigma},\gamma^p)\leq 0 \qquad \text{éq 9.1-1}$$

Where the expression of $u(\boldsymbol{\sigma},\gamma^p)$ is:

If $\gamma_{\mathrm{cjs}}\neq 0$

$$u(\boldsymbol{\sigma},\gamma^p)=-\frac{m(\gamma^p)k(\gamma_p)}{\sqrt{6}\sigma_c}\left(\frac{h(\theta)+h^0_t-2\mathrm{h}^0_c}{h^0_t-h^0_c}\right)-\frac{m(\gamma^p)k(\gamma^p)}{3\sigma_c}I_1+s(\gamma^p).k(\gamma^p) \qquad \text{éq 9.1-2}$$

If $\gamma_{\mathrm{cjs}}=0$

$$u(\boldsymbol{\sigma},\gamma^p)=-\frac{m(\gamma^p)k(\gamma_p)}{\sqrt{6}\sigma_c}\left(\frac{3}{2}+\frac{1}{2}\cos(3\theta)\right)-\frac{m(\gamma^p)k(\gamma^p)}{3\sigma_c}I_1+s(\gamma^p).k(\gamma^p) \qquad \text{éq 9.1-3}$$

9.2 Pace of the thresholds

One places oneself if $\gamma_{\mathrm{cjs}}=0.7\,;m=21\,;s=1\,;a=1$, then one traces the pace of the thresholds in the plan perpendicular to the hydrostatic axis (known as plan π), one standardizes compared to and one σ_c consider the two values of containments such as $I_1=0$ [Figure 9.2-a] and $I_1=-3\sigma_c$ [Figure 9.2 - B].

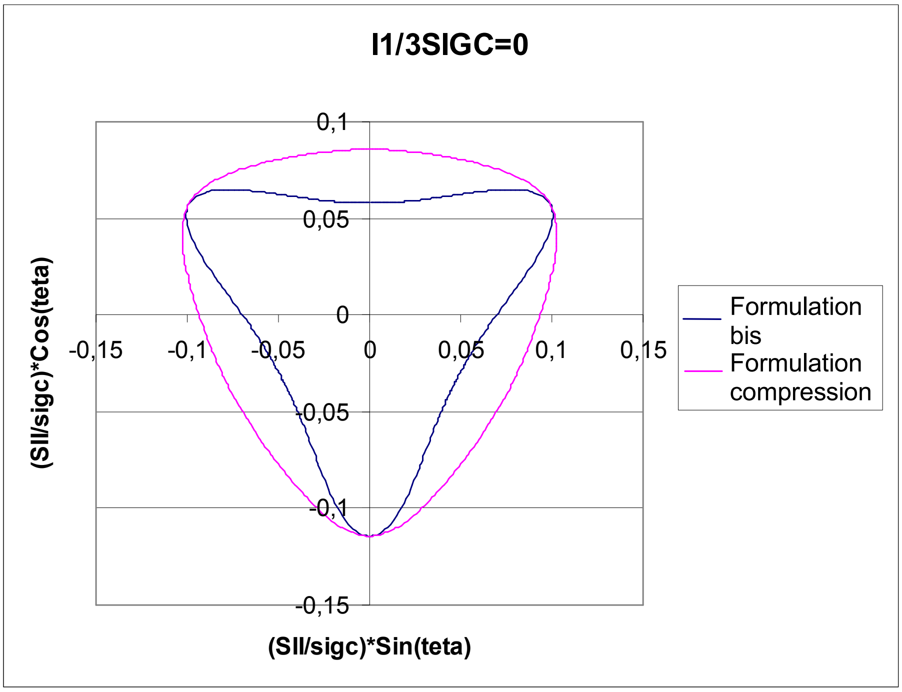


Figure 9.2-a: AllurE of the thresholds for a null containment

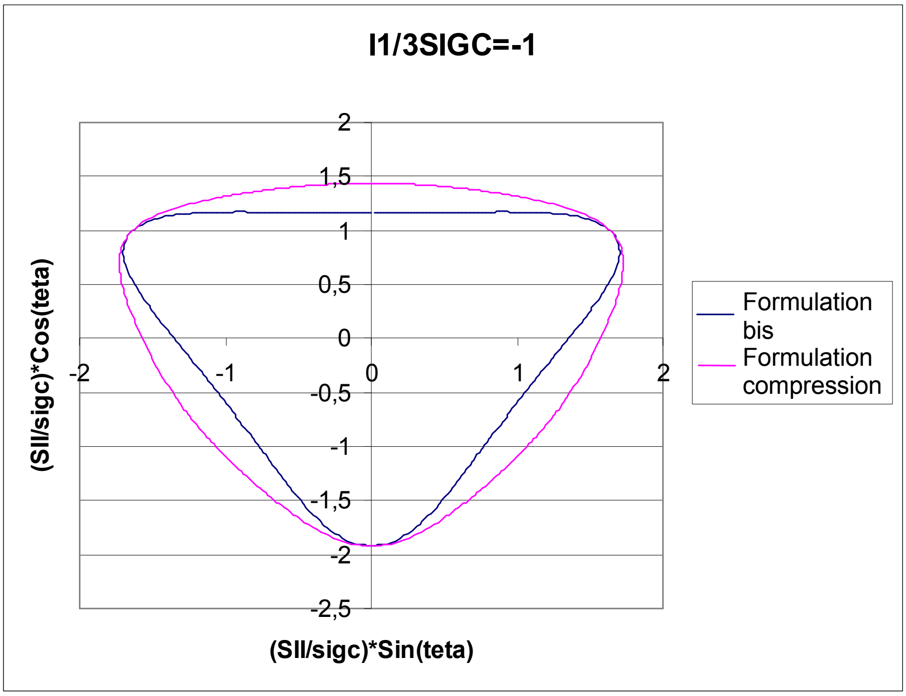


Figure 9.2-b: Pace of the thresholds for a null containment in compression

One notes in these charts that the (a) formulation has the disadvantage of taking a nonconvex form in the plan π .

10 Features and checking

The law of behavior can be defined by the keyword `LAIGLE` (order `STAT_NON_LINE`, keyword factor `BEHAVIOR`). It is associated with material `LAIGLE` (order `DEFI_MATERIAU`).

The law `LAIGLE` is checked by the cases following tests:

SSNV158	[V6.04.158]	Triaxial compression test drained with the model of Laigle
WTNV101	[V7.31.101]	Triaxial compression test not drained with the model of Laigle and hydraulic coupling

11 Bibliography

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12 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
7,4	R.Fernandes, C.Chavant EDF-R&D/AMA	Initial text

Annexe 1Retiming of the criterion on the triaxial one in compression

By taking the general expression of the criterion under the conditions of triaxial in compression, one finds:

$$f=\left(\frac{g(\boldsymbol{s})}{\sigma_c h_c^0}\right)^{\frac{1}{a(\gamma^p)}}-\left(-\frac{m(\gamma^p)k(\gamma^p)}{\sqrt{6}\sigma_c}\frac{g(\boldsymbol{s})}{h_c^0}-\frac{m(\gamma^p)k(\gamma^p)}{3\sigma_c}I_1+s(\gamma_p).k(\gamma^p)\right)$$

$$=\left(\frac{\sqrt{\frac{2}{3}}|\sigma_1-\sigma_3|.h}{\sigma_c h_c^0}\right)^{\frac{1}{a(\gamma^p)}}+\frac{1}{\sigma_c}\left(\frac{m(\gamma^p)k(\gamma^p)}{\sigma_c\sqrt{6}}\frac{\sqrt{\frac{2}{3}}|\sigma_1-\sigma_3|.h}{\sigma_c h_c^0}+\frac{m(\gamma^p)k(\gamma^p)}{3\sigma_c}(\sigma_1+2\sigma_3)-s(\gamma^p).k(\gamma^p)\right)$$

$$=\left(\frac{\sqrt{\frac{2}{3}}|\sigma_1-\sigma_3|}{\sigma_c h_c^0}\right)^{\frac{1}{a(\gamma^p)}}+\left(\frac{m(\gamma^p)k(\gamma^p)}{\sigma_c\sqrt{6}}\sqrt{\frac{2}{3}}|\sigma_1-\sigma_3|+\frac{m(\gamma^p)k(\gamma^p)}{3\sigma_c}(\sigma_1+2\sigma_3)-s(\gamma^p).k(\gamma^p)\right)$$

$$=\left(\frac{\sqrt{\frac{2}{3}}}{\sigma_c}\right)^{\frac{1}{a(\gamma^p)}}(|\sigma_1-\sigma_3|)^{\frac{1}{a(\gamma^p)}}+\left(\frac{m(\gamma^p).k(\gamma^p)}{3\sigma_c}|\sigma_1-\sigma_3|+\frac{m(\gamma^p).k(\gamma^p)}{3\sigma_c}(\sigma_1+2\sigma_3)-s(\gamma^p).k(\gamma^p)\right)$$

$$=\left(\frac{\sqrt{\frac{2}{3}}}{\sigma_c}\right)^{\frac{1}{a(\gamma^p)}}(|\sigma_1-\sigma_3|)^{\frac{1}{a(\gamma^p)}}+\left(\frac{m(\gamma^p).k(\gamma^p)}{3\sigma_c}(\sigma_3-\sigma_1)+\frac{m(\gamma^p).k(\gamma^p)}{3\sigma_c}(\sigma_1+2\sigma_3)-s(\gamma^p).k(\gamma^p)\right)$$

$$=\left(\frac{\sqrt{\frac{2}{3}}}{\sigma_c}\right)^{\frac{1}{a(\gamma^p)}}(|\sigma_1-\sigma_3|)^{\frac{1}{a(\gamma^p)}}+\left(\frac{m(\gamma^p).k(\gamma^p)}{\sigma_c}(\sigma_3)-s(\gamma^p).k(\gamma^p)\right)$$

$$=\left(\frac{\sqrt{\frac{2}{3}}}{\sigma_c}\right)^{\frac{1}{a(\gamma^p)}}(|\sigma_1-\sigma_3|)^{\frac{1}{a(\gamma^p)}}-\sqrt{\frac{2}{3}}^{\frac{1}{a(\gamma^p)}}\left(\frac{m(\gamma^p)}{\sigma_c}(-\sigma_3)+s(\gamma^p)\right)$$

$$=\left(\frac{\sqrt{\frac{2}{3}}}{\sigma_c}\right)^{\frac{1}{a(\gamma^p)}}\left[(|\sigma_1-\sigma_3|)^{\frac{1}{a(\gamma^p)}}-(\sigma_c)^{\frac{1}{a(\gamma^p)}}\left(\frac{m(\gamma^p)}{\sigma_c}(-\sigma_3)+s(\gamma^p)\right)\right]$$

Annexe 2Standardisation of Q

$$\boldsymbol{Q}=\frac{1}{h(\theta)^5}\left[\left(1+\frac{\gamma_{\text{cjs}}}{2}\cos(3\theta)\right)\frac{\boldsymbol{s}}{s_{\text{II}}}+\frac{\gamma_{\text{cjs}}\sqrt{54}}{6.s_{\text{II}}^2}\text{dev}\left(\frac{\partial\det(\underline{\underline{s}})}{\partial\boldsymbol{s}}\right)\right]$$

One poses $\boldsymbol{t}=\frac{\partial\det(\underline{\underline{s}})}{\partial\boldsymbol{s}}$ and $\boldsymbol{t}=\text{dev}\left(\frac{\partial\det(\underline{\underline{s}})}{\partial\boldsymbol{s}}\right)$ (cf reference document CJS R7.01.13)

$$Q_{\text{II}}^2=\boldsymbol{Q}.\boldsymbol{Q}=\frac{1}{h(\theta)^{10}}\left[1+\frac{\gamma_{\text{cjs}}}{2}\cos(3\theta)^2+\frac{3}{2}.\frac{\gamma_{\text{cjs}}{}^2}{s_{\text{II}4}}\boldsymbol{t}^d.\boldsymbol{t}^d+\frac{\gamma_{\text{cjs}}\sqrt{54}}{3.s_{\text{II}}^3}\left(1+\frac{\gamma_{\text{cjs}}}{2}\cos(3\theta)\right)\boldsymbol{s}.\boldsymbol{t}^d\right]$$

To evaluate this expression, one places oneself if $\mathbf{s}$ is diagonal by preoccupations with a simplification of calculations.

As follows: $\boldsymbol{s}=\begin{bmatrix}s_1\\s_2\\s_3\\0\\0\\0\end{bmatrix}$ and $\boldsymbol{t}^d=\frac{1}{3}\begin{bmatrix}2s_2s_3-s_1s_2-s_1s_3\\2s_1s_3-s_1s_2-s_2s_3\\2s_1s_2-s_1s_3-s_2s_3\\0\\0\\0\end{bmatrix}$

By using the property of $\mathbf{s}$: , one $\boldsymbol{s}=s_1+s_2+s_3$ watch that $s_{\text{II}}^4=\left(s_1^2s_2^2+s_1^2s_3^2+s_2^2s_3^2\right)$ and consequently:

$$\boldsymbol{t}^{d.}\boldsymbol{t}^d=\frac{1}{9}\begin{vmatrix}2s_2s_3-s_1s_2-s_1s_3\\2s_1s_3-s_1s_2-s_2s_3\\2s_1s_2-s_1s_3-s_2s_3\\0\\0\\0\end{vmatrix}\begin{vmatrix}2s_2s_3-s_1s_2-s_1s_3\\2s_1s_3-s_1s_2-s_2s_3\\2s_1s_2-s_1s_3-s_2s_3\\0\\0\\0\end{vmatrix}=\frac{s_{\text{II}}{}^4}{6}$$

One also shows starting from the property $s_1+s_2+s_3=0$ that $s_1^3s_2^3s_3^3=3s_1s_2s_3=3.\det(\boldsymbol{s})$ and consequently:

$$\frac{\gamma_{\text{cjs}}.\sqrt{54}}{3.s_{\text{II}}^3}\boldsymbol{s}.\boldsymbol{t}^d=\frac{\gamma_{\text{cjs}}.\sqrt{54}}{9.s_{\text{II}}^3}\begin{vmatrix}s_1\\s_2\\s_3\\0\\0\\0\end{vmatrix}.\begin{vmatrix}2s_2s_3-s_1s_2-s_1s_3\\2s_1s_3-s_1s_2-s_2s_3\\2s_1s_2-s_1s_3-s_2s_3\\0\\0\\0\end{vmatrix}=\frac{\gamma_{\text{cjs}}.\sqrt{54}}{s_{\text{II}}^3}\det(\boldsymbol{s})=\gamma_s.\cos(3\theta)$$

One from of deduced as follows:

$$Q_{\mathrm{II}}^2=\frac{1}{h(\theta)^{10}}\left[\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos(3\theta)\right)^2+\frac{\gamma_{\mathrm{cjs}}{}^2}{4}+\gamma_{\mathrm{cjs}}\cos(3\theta)\left(1+\frac{\gamma_{\mathrm{cjs}}}{2}\cos(3\theta)\right)\right]$$

Annexe 3Framing of the jetting angle

It is pointed out that $\cos\varphi_s \underset{s\to 0}{\to} \dfrac{3}{(\beta'^2+3)\sqrt{\left(\dfrac{3}{\beta'^2+3}\right)^2-\dfrac{1}{4}+\dfrac{1}{2(1+\gamma_{\mathrm{cjs}}\cos(3\theta))}+\dfrac{\gamma_{\mathrm{cjs}}^2-1}{4(1+\gamma_{\mathrm{cjs}}\cos(3\theta))^2}}}$

One poses: $X(\psi)=\dfrac{1}{2(1+\gamma_{\mathrm{cjs}}\cos(\psi))}+\dfrac{\gamma_{\mathrm{cjs}}^2-1}{4(1+\gamma_{\mathrm{cjs}}\cos(\psi))^2}$ where $\psi\in[\,0{,}2\pi[$

It is noted that: $X(-\psi)=X(\psi)$, the function X being even one restricts the interval of study at $\psi\in[\,0{,}2\pi[$.

The resolution of $\dfrac{dX}{d\psi}=0$ give $\dfrac{\gamma_{\mathrm{cjs}}\sin(\psi)}{2(1+\gamma_{\mathrm{cjs}}\cos(\psi))^3}\cdot\gamma_{\mathrm{cjs.}}(\gamma_{\mathrm{cjs}}+\cos(3\psi))=0$

One from of deduced that limits lower and higher from the function X are:

$$\begin{cases} X(\psi=0)=\dfrac{1}{4} \\ X(\psi_{\mathrm{cjs}})=\dfrac{1}{4(1-\gamma_{\mathrm{cjs}}^2)}\ \text{où}\ \psi_{\mathrm{cjs}}\ \text{est tel que}\ \cos(\psi_{\mathrm{cjs}})=-\gamma_{\mathrm{cjs}} \end{cases}$$

One can thus give the framing of $\cos\varphi_s$ according to: $\cos\varphi_s^{\min}\le co\varphi_s\le\cos\varphi_s^{\max}$ with:

$$\begin{cases} \cos\varphi_s^{\min}=\dfrac{3}{(\beta'^2+3)\sqrt{\left(\dfrac{3}{\beta'^2+3}\right)^2+\dfrac{\gamma_{\mathrm{cjs}}{}^2}{4(1-\gamma_{\mathrm{cjs}}^2)}}} \\ \cos\varphi_s^{\max}=1 \end{cases}$$