
Behavior of the steel subjected to corrosion

Summary:

This documentation presents the model of behavior of the steel subjected to the corrosion which makes it possible to describe the mechanical behaviour of reinforcement corroded in the reinforced concrete structures. This model is developed in 1D and 3D, elastoplastic endommageable with isotropic work hardening and is based on the model of Lemaître. This behavior is used for calculations of prediction of the bearing capacity of structures out of reinforced concrete reached by corrosion in the case of unidimensional loading.

It is implemented in *Code_Aster* under the name of `CORR_ACIER` in 3D and 1D (for the elements `BAR` and multifibre beams). It also functions with the option `DEBORST` for the plane constraints (elements hull); the equations are integrated numerically by an implicit scheme of Mr. Ortiz and J.C. Simo [bib1] with plastic multiplier obtained while linearizing function threshold compared to the internal variables and by imposing the respect of the criterion on convergence.

Contents

Contents

1 Introduction.....	3
2 Description of the model.....	3
3 Calculation of the parameters material.....	6
3.1 Law of behavior of steel.....	6
3.2 Taking into account of corrosion.....	6
4 Digital resolution.....	8
4.1 Integration of the model.....	8
4.2 Calculation of the tangent matrix.....	10
4.2.1 Finite element 1D.....	10
4.2.2 Finite element 3D.....	10
4.3 Stored internal variables.....	11
5 Bibliography.....	12
6 Checking.....	12
7 Description of the versions of the document.....	12

1 Introduction

This document presents a law of behavior of the steel subjected to corrosion in the works as a Génie Civil. This law was formulated on the basis of model of Lemaître (plasticity coupled with the damage). Following tensile tests carried out on corroded bars, one notes an influence of corrosion on the plastic deformation with rupture (the plastic deformation of the corroded bars decreases with the increase in the degree of corrosion) [bib2], [bib3]. One connects the plastic deformation to rupture of the model of Lemaître at the rate of corrosion (the diameter of the bar corroded compared to that not corroded in 1D or the thickness of flat reinforcement corroded compared to that not corroded in 3D).

2 Description of the model

From a thermodynamic point of view, to translate the behavior of hammer-hardenable plastic material and endommageable, the model replaces in surface threshold, the constraint σ by the effective constraint [bib4]:

$$\tilde{\sigma} = \frac{\sigma}{1-D} \quad \text{éq 21}$$

where, one a:

- $\tilde{\sigma} = \sigma$ for a virgin material
- $\tilde{\sigma} = 0$ at the instant of the failure

One applies a particular form of the free energy of Helmholtz:

$$Y = \frac{1}{\rho} \left[\frac{1}{2} (1-D) (\varepsilon - \varepsilon^p) E (\varepsilon - \varepsilon^p) + R(r) \right] \quad \text{éq 22}$$

ρ density; Ψ potential of state; E the Young modulus; D the variable of damage; ε total deflection; ε^p plastic deformation; $R(r)$ the isotropic function of work hardening; r the variable associated with R .

Thus posed the potential is separate in two distinct parts. The first corresponds to the classical coupling damage-elasticity, the second term with work hardening.

The laws of state describing this potential are:

$$\begin{aligned} \sigma &= \rho \frac{\partial \Psi}{\partial \varepsilon} \\ R &= \rho \frac{\partial \Psi}{\partial r} \\ Y &= \rho \frac{\partial \Psi}{\partial D} \end{aligned} \quad \text{éq 23}$$

By supposing that the material obeys the criterion of Von Mises, the criterion of flow is expressed by:

$$f = \frac{\sigma_{eq}}{1-D} - R - \sigma_y \leq 0 \quad \text{éq 24}$$

where $\sigma_{eq} = \left(\frac{3}{2} \boldsymbol{\sigma}' : \boldsymbol{\sigma}' \right)^{\frac{1}{2}}$ is the equivalent constraint within the meaning of Settings, with $\boldsymbol{\sigma}' = \boldsymbol{\sigma} - \sigma_H \mathbf{1}$ the deviator of the matrix of constraints and $\sigma_H = \frac{1}{3} Tr(\boldsymbol{\sigma})$ the hydrostatic constraint.

The potential of damage is selected in function power of the variable associated with the damage $-Y$:

$$\Phi_D^* = \left(\frac{S_0}{s_0 + 1} \right) \left(\frac{1}{1 - D} \right) \left(-\frac{Y}{S_0} \right)^{s_0 + 1} \quad \text{éq 25}$$

where s_0 and S_0 are coefficients characteristic of material.

The rule of generalized normality provides the law of flow and the evolutions of the internal variables:

$$\begin{cases} \dot{\varepsilon}^p = \dot{\lambda} \frac{\partial f}{\partial \boldsymbol{\sigma}} \\ \dot{r} = -\dot{\lambda} \frac{\partial f}{\partial R} \\ \dot{D} = -\dot{\lambda} \frac{\partial \Phi_D^*}{\partial Y} \end{cases} \quad \text{éq 26}$$

$\dot{\lambda}$ is the plastic multiplier.

In this model CORR_ACIER, internal variables introduced into Code_Aster are:

- p : cumulated plastic deformation, such as $\dot{p} = \left(\frac{2}{3} \dot{\varepsilon}^p : \dot{\varepsilon}^p \right)^{1/2}$;
- D : scalar variable of isotropic damage.

If $f = \frac{\sigma_{eq}}{1 - D} - R - \sigma_y > 0$, one is in the plastic range:

$$\begin{cases} \dot{\varepsilon}^p = \frac{3}{2} \left(\frac{\dot{\lambda}}{1 - D} \right) \frac{\boldsymbol{\sigma}'}{\sigma_{eq}} \\ \dot{r} = \dot{\lambda} = \dot{p} (1 - D) \\ \dot{D} = \left(-\frac{Y}{S_0} \right)^{s_0} \dot{p} \end{cases} \quad \text{éq 27}$$

In the unidimensional case, the equivalent constraint within the meaning of Settings is $\sigma_{eq} = |\sigma|$ and the cumulated plastic deformation is equal to the absolute value of the unidimensional plastic deformation: $p = |\varepsilon^p|$

To formulate an isotropic criterion of damage, one applies that the damage mechanism is controlled by the total elastic deformation energy (energy of distortion + voluminal deformation energy). By analogy with the equivalent constraint in plasticity, by writing that the energy of a three-dimensional state is equal to that of the unidimensional state are equivalent, definite by an equivalent constraint of damage σ_{eq}^* , one finds [bib5]:

$$\sigma_{eq}^* = \sigma_{eq} \left[\frac{2}{3}(1+\nu) + 3(1-2\nu) \left(\frac{\sigma_H}{\sigma_{eq}} \right)^2 \right]^{1/2} \quad \text{éq 28}$$

In addition it is shown that the variable associated with the damage is expressed in the isotropic case by $-Y = \frac{\tilde{\sigma}_{eq}^{*2}}{2E}$, therefore:

$$-Y = \frac{\sigma_{eq}^2}{2E(1-D)^2} \left[\frac{2}{3}(1+\nu) + 3(1-2\nu) \left(\frac{\sigma_H}{\sigma_{eq}} \right)^2 \right] \quad \text{éq 29}$$

from where:

$$\dot{D} = \left(\frac{\sigma_{eq}^2}{2ES_0(1-D)^2} \right)^{s_0} \left[\frac{2}{3}(1+\nu) + 3(1-2\nu) \left(\frac{\sigma_H}{\sigma_{eq}} \right)^2 \right]^{s_0} \dot{p} \quad \text{éq 210}$$

The ductile damage intervenes only beyond of a threshold:

$$f_{end} = p - p_D > 0 \quad \text{éq 211}$$

where p_D is the deformation which corresponds to the beginning of damage (see it [Figure 2-a]). One can then consider that work hardening is saturated in this field, i.e. the behavior of material not damaged equivalent would be perfectly plastic. This simplifying assumption allows the analytical integration of the model and to lead to a linear evolution according to the plastic deformation. Indeed, one has in this case:

$$\frac{\sigma_{eq}}{1-D} = \tilde{\sigma}_{eq} = \sigma_y = Cte \quad \text{éq 212}$$

If one restricts oneself with the case radial loading for which the rate of triaxiality σ_H/σ_{eq} is constant, one obtains, while taking as initial condition $D=0$ for $p < p_D$:

$$D = \left(\frac{\sigma_y^2}{2ES_0} \right)^{s_0} \left[\frac{2}{3}(1+\nu) + 3(1-2\nu) \left(\frac{\sigma_H}{\sigma_{eq}} \right)^2 \right]^{s_0} \langle p - p_D \rangle \quad \text{éq 213}$$

One can simplify this expression by introducing the condition of rupture $p = p_R$ who is the rate of plastic deformation cumulated with rupture => $D = D_c$ (damage criticizes) [bib4]

$$D_c = \left(\frac{\sigma_y^2}{2ES_0} \right)^{s_0} \left[\frac{2}{3}(1+\nu) + 3(1-2\nu) \left(\frac{\sigma_H}{\sigma_{eq}} \right)^2 \right]^{s_0} \langle p_R - p_D \rangle \quad \text{éq 214}$$

While paying in the expression of D , one obtains:

$$D = \frac{D_c}{p_R - p_D} (p - p_D) \quad \text{éq 215}$$

The law of work hardening of steel integrated in the model of steel is the following one (except damage):

$$\sigma_{eq} - \sigma_y = K p^{1/m} \quad \text{éq 2 16}$$

K , m , σ_y are the parameters of material, provided in `DEFI_MATERIAU/CORR_ACIER` by the keywords `ECRO_K`, `ECRO_M`.

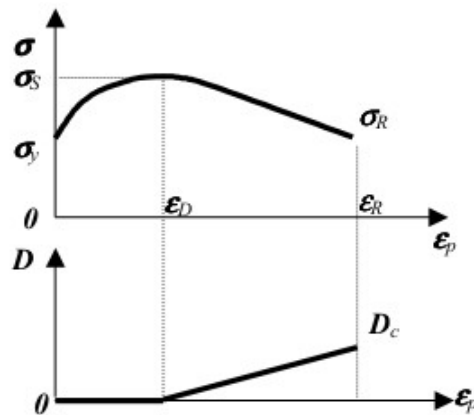


Figure 2-a: Evolution of the damage

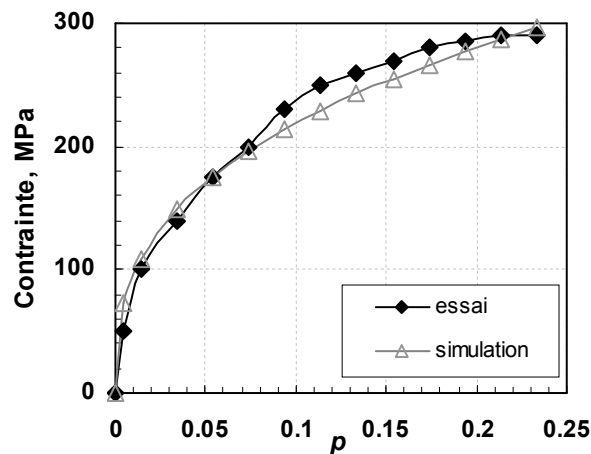


Figure 2-b: Law of work hardening of steel not corroded

3 Calculation of the parameters material

The results of the tensile tests of A.A. Almusallam [bib2] were used to identify the law of behavior of steel not corroded and the dependence of the plastic deformation with rupture according to the rate of corrosion.

3.1 Law of behavior of steel

Nonalloyed steel is the type of principal steel used in the civil engineer. The model of unidimensional behavior of nonalloyed steel in one-way monotonous loading must be given starting from the results of a tensile test on a bar or a not corroded flat test-tube. An example of digital simulation necessary to determine this law of behavior is presented on [Figure 3.2-a].

3.2 Taking into account of corrosion

The presence of corrosion has two effects on the reinforcement in the reinforced concrete structures:

- a reduction of the section;
- a reduction of ϵ_R according to T_c :

The reduction of section results by a reduction in the diameter for the bars or in a reduction thickness for sheets:

$$T_c = 100 \left(\frac{d_{\text{corrodé}}^2}{d_{\text{noncorrodé}}^2} \right) \quad \text{or} \quad T_c = 100 \left(\frac{e_{\text{corrodé}}}{e_{\text{noncorrodé}}} \right) \quad \text{éq 3.2- 1}$$

Note: The reduction of section is not treated with the level of the model of behavior, it must be taken into account on the level of the command file in AFPE_CARA_ELEM for example.

In the uniaxial case, the plastic deformation with rupture ϵ_R depends on the rate of corrosion. This evolution is presented on [Figure 3.2-b].

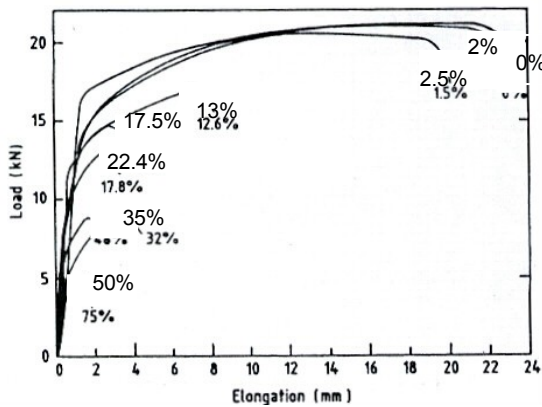


Figure 3.2-a: Influence of corrosion on the behavior of steel according to the rate of corrosion

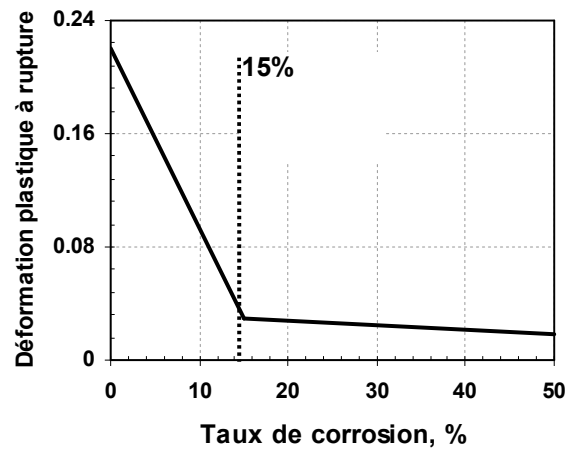


Figure 3.2-b: Evolution of the plastic deformation with rupture according to the rate of corrosion

One deduces from these experimental data (tensile tests) the variation of the plastic deformation with rupture according to the rate of corrosion:

$$T_c < 15\% \Rightarrow \epsilon_R = -0.0111 T_c + 0.2345 \quad \text{éq 3.2- 2}$$

$$T_c > 15\% \Rightarrow \epsilon_R = -0.0006 T_c + 0.051 \quad \text{éq 3.2- 3}$$

By analysis of various tensile tests, one notes that the behaviour of the corroded reinforcement is quasi-fragile and $\epsilon_D = 0.8 \epsilon_R$ (ϵ_D with the peak) .

In order to integrate the model in 3D, the critical cumulated plastic deformation is calculated while using $p_R = \left(\frac{2}{3} \epsilon_R : \epsilon_R \right)^{1/2}$ while taking into account that the uniaxial state is defined by a unidimensional in constraint but three-dimensional state in deformation [bib5]:

$$\epsilon_R = \begin{bmatrix} \epsilon_R & 0 & 0 \\ 0 & -\nu^* \epsilon_R & 0 \\ 0 & 0 & -\nu^* \epsilon_R \end{bmatrix} \quad \text{éq 3.2-4}$$

where ν^* is the coefficient of contraction, equal to the Poisson's ratio ν in elasticity:

$$\nu^* = \nu \frac{\varepsilon^e}{\varepsilon} + \frac{1}{2} \frac{\varepsilon^p}{\varepsilon} = \frac{1}{2} - \frac{\varepsilon^e}{\varepsilon} \left(\frac{1}{2} - \nu \right) \quad \text{éq 3.2-5}$$

here $\varepsilon = \varepsilon_R$ and one approximates ε^e by: $\varepsilon_y = \frac{\sigma_y}{E}$

$$\nu^* = \frac{1}{2} - \frac{\varepsilon_y}{\varepsilon_R} \left(\frac{1}{2} - \nu \right) \quad \text{éq 3.2-6}$$

For the calculation of p_D , it is considered that the rate of triaxiality to the threshold of damage is identical to that of the rupture:

$$p_D = 0,8 p_R \quad \text{éq 3.2-7}$$

4 Digital resolution

4.1 Integration of the model

The integration of the model is carried out in two stages, first of all an elastic prediction of the constraint:

$$\text{1st stage: elasticity} \quad \begin{cases} \dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p \\ \dot{\sigma} = E \dot{\varepsilon}^e \\ \dot{\varepsilon}^p = 0 \\ \dot{p} = 0 \\ \dot{D} = 0 \end{cases} \quad \text{éq 4.1-1}$$

Let us notice in particular that the damage D does not intervene in the elastic relation stress-strain (not of coefficient $(1 - D)$ in the moduli of elasticity).

The second stage consists of an implicit actualization of the internal variables of the model by corrections made to worthless total deflection. In this stage, one makes the iteration between plasticity and the damage.

$$\begin{cases} \dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p = 0 \\ \dot{\sigma} = E \dot{\varepsilon}^p \\ \dot{\varepsilon}^p = \dot{\lambda} \frac{\partial f}{\partial \sigma} \\ \dot{r} = -\dot{\lambda} \frac{\partial f}{\partial R} \end{cases} \quad \text{éq 4.1-2}$$

The expression of the plastic multiplier is obtained by linearizing the function threshold compared to the internal variables and by imposing the respect of the criterion on convergence:

$$f = f^- + \frac{\partial f^-}{\partial \sigma} (\sigma - \sigma^-) + \frac{\partial f^-}{\partial R} (R - R^-) = 0 \quad \text{éq 4.1-3}$$

Let us consider the discretization of the equations of relieving:

$$\left\{ \begin{array}{l} \sigma - \sigma^- = -\Delta\lambda \mathbf{E} \frac{\partial f^-}{\partial \sigma} \\ R - R^- = R'(r) \Delta r \\ \Delta p = \frac{\Delta\lambda}{1-D^-} \\ R'(p) = \frac{K}{m} \left[\frac{\sigma_{eq}}{(1-D^-)K} - \frac{\sigma_y}{K} \right]^{1-m} \end{array} \right. \quad \text{éq 4.1-4}$$

The expression of the plastic multiplier is obtained while replacing [éq 4.1-3] in [éq 4.1-4], ($\Delta r = \Delta\lambda$ and for D fixed at D^- , $R'(p) = R'(r)$)

$$\Delta\lambda = \frac{f^-}{\frac{\partial f^-}{\partial \sigma} \mathbf{E} \frac{\partial \phi^-}{\partial \sigma} - \frac{\partial f^-}{\partial R} \frac{K}{m} \left[\frac{\sigma_{eq}}{K(1-D^-)} - \frac{\sigma_y}{K} \right]^{1-m}} \quad \text{éq 4.1-5}$$

The expressions of different the derivative from surface threshold and the potential of dissipation are given in the following equations:

$$\begin{aligned} \frac{\partial f}{\partial \sigma} &= \frac{3}{2} \frac{\sigma'}{\sigma_{eq}(1-D)} \\ \frac{\partial \phi}{\partial \sigma} &= \frac{3}{2} \frac{\sigma'}{\sigma_{eq}(1-D)} \\ \frac{\partial f}{\partial R} &= -1 \end{aligned} \quad \text{éq 4.1-6}$$

In the case 1D:

$$\frac{\partial f}{\partial \sigma} = \frac{\partial \phi}{\partial \sigma} = \frac{\text{sgn}(\sigma)}{1-D} \quad \text{éq 4.1-7 B}$$

The value obtained for the plastic multiplier with each integration is reinjected in the equations of relieving until convergence (return of the constraint on surface threshold). The criterion selected consists in stopping when the value of the threshold to the following iteration became sufficiently small compared to σ_y :

$$|f| \leq \text{tol} \cdot \sigma_y \quad \text{éq 4.1-8}$$

For a direct use of the properties of normality of surfaces thresholds, calculation to be carried out with each iteration is reduced only to the calculation of the multiplier of plasticity allowing the corrections so necessary (not calculation of Jacobien as for the methods of Newton) [bib1], [bib6].

The following stage is the damage which takes into account the variation of the variable of damage. The function threshold of the damage is the following one:

$$f_{end} = p - p_D \quad \text{éq 4.1-9}$$

if $f_{end} > 0$ one calculates the damage then one turns over to the correction of plasticity:

$$D = \frac{D_C}{p_R - p_D} (p - p_D) \quad \text{éq 4.1-10}$$

Note: in practice, D evolve even if $p > p_R$, one limits D with 0.99 maximum. D_C is defined by the keyword `D_CORR` of `DEFI_MATERIAU/CORR_ACIER`

The same approach for convergence is adopted:

$$|f| \leq \text{tol} \cdot \sigma_y \quad \text{éq 4.1- 11}$$

One will refer to reference [bib1] and to [bib6] for all the details concerning this method and this algorithm employed.

4.2 Calculation of the tangent matrix

One seeks to calculate the tangent matrices (continuous and consistent) for the plastic part and the part of damage. Options RIGI_MECA_TANG and FULL_MECA tangent matrix are calculated for the finite element 1D and 3D:

4.2.1 Finite element 1D

In the plastic range:

$$\delta \varepsilon^p = \delta \lambda = \frac{\delta f^-}{E + \frac{K}{m} p^{\frac{1}{m}-1}} = \frac{E \delta \varepsilon}{E + \frac{K}{m} p^{\frac{1}{m}-1}} \quad \text{éq 4.2.1- 1}$$

$$\frac{\partial \sigma}{\partial \Delta \varepsilon} = E(\delta \varepsilon - \delta \varepsilon^p) = \frac{\frac{K}{m} p^{\frac{1}{m}-1}}{1 + \frac{K}{mE} p^{\frac{1}{m}-1}} \quad \text{éq 4.2.1-2}$$

In the field of damage:

$$\begin{aligned} \frac{\delta \sigma}{1-D} + \frac{\sigma}{(1-D)^2} \delta D - R'(p) \delta p &= -\delta f^- \\ \frac{-E}{(1-D)^2} \delta \lambda + \frac{\sigma}{(1-D)^2} \frac{D_c}{p_R - p_D} \frac{\delta \lambda}{(1-D)} - \frac{K}{m} p^{\frac{1}{m}-1} \frac{\delta \lambda}{(1-D)} &= -\delta f^- = \frac{E}{(1-D)} \delta \varepsilon \\ \frac{\partial \sigma}{\partial \Delta \varepsilon} = E(\delta \varepsilon - \delta \varepsilon^p) &= E \frac{\frac{K}{m} (1-D) p^{\frac{1}{m}-1} - \frac{\sigma}{(1-D)} \frac{D_c}{p_R - p_D}}{E + \frac{K}{m} (1-D) p^{\frac{1}{m}-1} - \frac{\sigma}{(1-D)} \frac{D_c}{p_R - p_D}} \end{aligned}$$

4.2.2 Finite element 3D

The absence of damage in the plastic part makes it possible to use the tangent matrix calculated for elastoplastic behaviour with linear and isotropic work hardening kinematic nonlinear [bib7]:

$$\frac{\partial \sigma}{\Delta \varepsilon} = \lambda^* \vec{1} \otimes \vec{1} + 2\mu^* \text{Id} - \xi \frac{9\mu^2}{H(p)} \left(1 - \frac{R'(p) \cdot \Delta p}{(R(p) + \sigma_y)} \right) \frac{1}{R'(p) + 3\mu} \left(\frac{\sigma^{dev}}{(R(p) + \sigma_y)} \otimes \frac{\sigma^{dev}}{(R(p) + \sigma_y)} \right) \quad \text{éq 4.2.2-1}$$

$$\text{with } \lambda^* = K - \frac{2\mu}{3H(\Delta p)} \quad 2\mu^* = \frac{2\mu}{H(\Delta p)} \quad H(\Delta p) = 1 + \frac{3\mu \xi \cdot \Delta p}{(R(p) + \sigma_y)}$$

The initial tangent matrix, used by the option RIGI_MECA_TANG is obtained by adopting the behavior of the preceding step (elastic or plastic, meant by internal variable ξ being worth 0 or 1) and while taking $\Delta p = 0$ in the equation éq 4.2.2-2 .

By taking of account the presence of the damage one obtains the following tangent matrix:

$$\frac{\partial \sigma}{\partial \Delta \varepsilon} = \lambda^* \bar{\mathbf{1}} \otimes \bar{\mathbf{1}} + 2\mu^* \mathbf{Id} - \xi \frac{9\mu^2}{H(p)} \left(1 - \frac{\Delta p ((1-D)R'(p) - R(p)D'(p))}{(1-D)(R(p) - \sigma_y)} \right) \frac{1}{(3\mu + (1-D)R'(p) - R(p)D'(p))} \left(\frac{\sigma^{dev}}{(1-D)(R(p) + \sigma_y)} \otimes \frac{\sigma^{dev}}{(1-D)(R(p) + \sigma_y)} \right)$$

éq 4.2.2-2

$$\text{with } \lambda^* = K - \frac{2\mu}{3H(\Delta p)} \quad 2\mu^* = \frac{2\mu}{H(\Delta p)} \quad H(\Delta p) = 1 + \frac{3\mu\xi \cdot \Delta p}{(1-D)(R(p) - \sigma_y)}$$

for the option FULL_MECA : $\sigma^{dev} = \tilde{\sigma}$

for the option RIGI_MECA_TANG : $\sigma^{dev} = \tilde{\sigma}$

4.3 Stored internal variables

We indicate in the table according to the internal variables stored in each point of Gauss for the model of the steel subjected to corrosion:

Internal variable	Physical direction
V1	p : equivalent plastic deformation
V2	D : variable of damage
V3	Indicator of plasticity (0 so elastic, 1 if plasticized i.e. as soon as p is not null)

5 Bibliography

- 1 MR. ORTIZ, J.C. SIMO: "Year analysis of has new class of constitutive integration algorithms for elastoplastic relations ", Int. J. Numer. Meth. Engng., vol. 21, pp 1561-1576, 1986.
- 2 A.A. ALMUSALLAM: "Effect of dismantles of corrosion one the properties of reinforcing steel bars", Construction and Building Materials 15.2001.
- 3 A. OUGLOVA, Y. BERTHAUD, I. PETRE-LAZAR: "Experimental characterization of the corrosion of steels in the concrete on old analogues. First approach of modeling ", HT 2/25/030 /A, EDF, 2002.
- 4 J. LEMAITRE: "A Chases one Ramming Mechanics ", Springer-Verlag, 1992.
- 5 J. LEMAITRE, J.L. CHABOCHE: "Mechanical of solid materials ", Dunod, 1996.
- 6 F. RAQUENEAU: "Dynamic operation of the structures in concrete-influence of the local behaviors hysteretic ", thesis of the University Paris 6, LMT-Cachan, 1999.
- 7 "Elastoplastic relation of behaviour to linear and isotropic work hardening kinematic nonlinear. Modeling plane 3D and constraint ", [R5.03.16]

6 Checking

The law of behavior CORR_ACIER is checked by the case following test:

SSNL127	Tensile test with the model CORR_ACIER	[V6.02.127]
---------	--	-------------

7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
21/12/11	L.DAVENNE (NECS)	Correction of the matrix 1D
08/04/11	I.PETRE-LAZAR, A. OUGLOVA (EDF - R&D/MMC) L.DAVENNE, B.ZUBER (NECS)	Initial text