

Law of behavior (in 2D) for the steel-concrete connection: JOINT_BA

Summary:

The law of behavior JOINT_BA described the phenomenon of degradation and rupture of the connection between the steel bars and the concrete, in the reinforced concrete structures. This documentation presents the theoretical writing in the thermodynamic framework and the digital integration of the law, as well as the parameters which manage the model.

For his use, one will be pressed on the finite elements of joint type (see the document [R3.06.09]) already existing in the code.

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1 Introduction

The law of behavior `JOINT_BA` described the phenomenon of degradation and rupture of the existing connection between the steel bars (smooth or ribbed) and the concrete surrounding it. Key point for the structural design out of reinforced concrete, the purpose of the modeling of the steel-concrete connection is the representation as well as simplification of this complex phenomenon of interaction between the two materials which develops in the interface and which undergoes an increasing degradation when certain thresholds of resistance are exceeded, specific for each material. The structural models which do not take into account the linkage effects, are generally unable to predict the localization of the cracks as well as the networks created. In addition, the degradation of the rigidity of the connection increases the period of vibration, reduced the capacity of dissipation of energy and conduit to a significant redistribution of the forces interns (according to *Bertero*, 1979, cf [bib2]).

The law of behavior `JOINT_BA` is described within the framework of the thermodynamics of the irreversible processes: the writing and the use of a “classical” material model coupling cracking and friction make it possible to integrate in a robust way of the fine mechanisms nonlinear concomitant into the particular description of the kinematics of slip. This last point enables us not to resort to the classical modelings of type ‘contact’ very often used in this context in spite of the many sources of digital instabilities. Thus, in monotonous loading the taking into account of the coupling normal effort – shearing makes it possible to treat cases of strong multiaxial pressures; into cyclic, the behavior hysteretic and corresponding dissipations are expressed thanks to the coupling between the state of damage and kinematic work hardening. The use of an implicit scheme makes it possible to obtain a robust implementation.

The paragraph [§2] described in short form the phenomenon of the connection steel concrete. The paragraph [§3] presents the thermodynamic writing of the law of behavior, while the paragraph [§4] specifies the digital stage of integration of the law. The parameters which manage the model and which could be obtained starting from the properties of implied materials, are described in the paragraph [§5].

2 Short description of the steel-concrete connection

Conceptually, the phenomenon of connection corresponds to the physical interaction of two different materials, which occurs on a zone of interface by allowing the transfer and the continuity of the efforts and the constraints between the two bodies in contact. In the case of the reinforced concrete structures, this phenomenon is also known as the “rigidity of tension” which develops around an element of reinforcement, partially or completely drowned in a volume of concrete. The forces of traction which appear inside the reinforcement are transformed into shear stresses on surface, and are transmitted directly to the concrete in contact which will balance them finally, and vice versa. The answer of the unit will depend on the capacity of the concrete to become deformed as much as steel, since steel will tend to slip inside the concrete surrounding it. The phenomenon of connection corresponds to this capacity of the concrete to become deformed and to be degraded locally by creating a species of layer, or wraps, around the reinforcement, whose properties kinematics and material differ from those moreover concrete or reinforcement employed.

The phenomenon can be broken up into three well defined mechanisms:

- a chemical adherence of origin,
- a mechanism of friction between two rough surfaces (steel-concrete or concrete-concrete),
- a mechanical action created by the presence of the veins of the steel bar on the neighbouring concrete.

According to this decomposition, one can deduce clearly that for a bar smoothes, the dominating mechanism is friction between two materials, while for a bar ribbed (in French usually called “ha braces: High Adherence”), the mechanism dominating is the mechanical interaction between surfaces.

When the reinforcement is consisted the strands with steel wire ropes, it is possible to control or combine the various mechanisms since they are function directly of the surface of the cables.

The connection will undergo a different degradation according to the type of loading applied, either monotonous, or cyclic. In addition, among the most important parameters which influence the behavior of the connection, one can quote:

- 1) characteristics of the loading,
- 2) geometrical characteristics of the steel bar,
- 3) spacing between active bars,
- 4) characteristics of the concrete,
- 5) containment by passive reinforcement,
- 6) side pressure.

At the time of the study of a cylindrical bar drowned in an infinite medium, one can identify the surface of discontinuity where one will place the linkage effects, which develops in a certain zone of concrete fissured and crushed around the steel bar. At a given moment, this surface will correspond to the cylindrical crack created during the coalescence of the cracks of shearing. By looking at the network of the cracks, one can suppose that, in ideal conditions, the plan of cracking is always perpendicular (normal direction) to the surface of the bar and parallel (tangential direction) to its longitudinal axis (see [Figure 2-a]). That enables us to project the components of displacement on the normal and tangential direction of the plan of cracking, and consequently to obtain the corresponding strains and stresses.

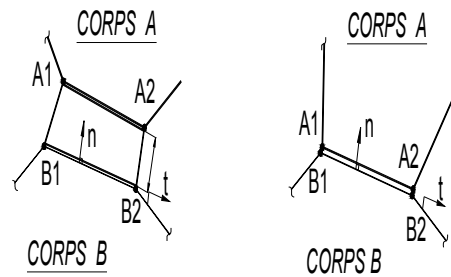


Figure 2-a: real description of the phenomenon of connection and simplification finite elements: coordinates in the local reference mark of the element of interface used like support of the law JOINT_BA

3 Theoretical writing

The formulation presented here was developed within the framework of the thermodynamics of the irreversible processes; it gives the constitutive relation between the normal effort, the shear stress and the slip by considering the influence of the cracking of the concrete, friction and the various couplings in the phenomenon. For that, the constitutive relations which connect the tensor of the constraints and the tensor of the deformations must include:

- the cracking of material of interface by shearing
- inelastic deformations because of slip
- the behavior hysteretic due to friction
- coupling between the tangential answer and the normal constraints

3.1 Presentation of the model

One places oneself within the framework of a formulation planes in 2D, in the definite local reference mark [Figure 2-a]. Tensors of the constraints σ and of the deformations ε are written:

$$\varepsilon = \begin{pmatrix} \varepsilon_N & \varepsilon_\tau \\ \varepsilon_\tau & 0 \end{pmatrix} \quad \text{and} \quad \sigma = \begin{pmatrix} \sigma_N & \sigma_\tau \\ \sigma_\tau & 0 \end{pmatrix} \quad \text{éq 3.1-1}$$

where σ_N is the normal constraint and σ_τ is the tangential constraint of the element of interface; ε_N corresponds to the normal deformation and ε_τ with the tangential deformation. The normal deformation in the tangential direction with the interface is regarded as worthless. This mode of deformation for an element of adherence is with worthless deformation energy.

The normal and tangential behaviors being regarded as uncoupled on the level from the state, the thermodynamic potential obtained starting from the free energy of Helmholtz is expressed in the following way:

$$\rho \cdot \Psi = \frac{1}{2} [\langle \varepsilon_N \rangle_- E \langle \varepsilon_N \rangle_- + \langle \varepsilon_N \rangle_+ E \cdot (1 - D_N) \langle \varepsilon_N \rangle_+ + \varepsilon_T G (1 - D_T) \varepsilon_T + (\varepsilon_T - \varepsilon_T^f) G \cdot D_T (\varepsilon_T - \varepsilon_T^f) + \gamma \alpha^2] + H(z) \quad \text{éq 3.1-2}$$

where ρ is the density, E is the Young modulus, D_N is the internal variable of normal damage and D the variable interns tangential damage, both being related to the cracking and ranging between 0 and 1. G is the module of rigidity or shearing, ε_T^f is the unrecoverable deformation induced by slip with friction of the cracks, α is the internal variable of kinematic work hardening, γ is a parameter material and z , the variable of pseudonym "isotropic work hardening" by damage, with its function of consolidation $H(z)$. $\langle \cdot \rangle_-$ and $\langle \cdot \rangle_+$ define respectively the positive and negative parts tensor considered.

One can notice in the equation [éq 3.1-2] that in the normal direction, the damage will be activated during the appearance of the positive deformations produced by forces of traction, while if the deformations are negative because of effects of compression, the behavior will remain elastic. With regard to the tangential part of the behavior, one can recognize a classical coupling elasticity-damage as well as a new term allowing to associate with the state elasticity-endommageable, a state of slip with friction. The coupling between slip and cracking is possible thanks to the presence of the variable of damage like multiplier in the second element of the right part of the equation [éq 3.1-2].

The laws of state are obtained classically by derivation of the thermodynamic potential, and thus make it possible to define the associated thermodynamic variables. The normal constraint is expressed like:

$$\sigma_N = \rho \frac{\partial \Psi}{\partial \varepsilon_N} = \begin{cases} E \cdot \varepsilon_N & \text{si } \varepsilon_N \leq 0 \\ (1 - D_N) \cdot E \cdot \varepsilon_N & \text{si } \varepsilon_N > 0 \end{cases} \quad \text{éq 3.1-3}$$

and the total tangential constraint like:

$$\sigma_T = \rho \frac{\partial \Psi}{\partial \varepsilon_T} = G (1 - D_T) \varepsilon_T + G \cdot D_T (\varepsilon_T - \varepsilon_T^f) \quad \text{éq 3.1-4}$$

One can also define the tangential constraint due to the slip with friction (deformation ε_T^s):

$$\sigma_T^f = -\rho \frac{\partial \Psi}{\partial \varepsilon_T^f} = G \cdot D_T (\varepsilon_T - \varepsilon_T^f) \quad \text{éq 3.1-5}$$

Note:

Such a formulation moves away amply from a classical formulation of coupling plasticity – damage. The assumption bringing to the introduction of the damage into the constraint by slip is based on an

experimental observation which is that all the inelastic phenomena in a fragile material come from the growth of the cracks.

The rate of energy restored by damage-friction can be written like:

$$-Y = -\rho \frac{\partial \psi}{\partial D_T} = \frac{1}{2} \varepsilon_T \cdot G \cdot \varepsilon_T - \frac{1}{2} (\varepsilon_T - \varepsilon_T^f) \cdot G \cdot (\varepsilon_T - \varepsilon_T^f) = -(Y_{DT} + Y_{fT}) \quad \text{éq 3.1-6}$$

In this last expression, Y_{DT} corresponds to the rate of energy restored by damage and Y_{fT} at the rate of energy restored by friction of the cracks.

The law of state of kinematic work hardening brings to the definition of the constraint of recall:

$$X = \rho \frac{\partial \psi}{\partial \alpha} = \gamma \alpha \quad \text{éq 3.1-7}$$

Concerning the law of work hardening of the isotropic damage, it is expressed by:

$$Z = \rho \frac{\partial \psi}{\partial z} = H'(z) \quad \text{éq 3.1-8}$$

It is necessary for us now to clarify in a more detailed way the evolution of the damage mechanism in the connection, in other words to specify the expression of $H(z)$. For one low value of damage, the mechanism which prevails is the interaction of the concrete with the veins of the steel bar, while for a value much larger, it is the friction between the concrete and the steel which prevails. During the evolution of the damage, 2 principal phases could be identified:

- 1) the first phase corresponds to a stable growth of transverse cracks related to the presence of veins on steel (positive apparent work hardening of the law of evolution),
- 2) the second does not utilize any more but the coalescence of these transverse cracks bringing not to consider but the mechanisms of friction any more (negative work hardening towards a residual constraint of friction).

3.2 Analysis of the damage in the tangential direction

The law of evolution of the damage is divided into three stages:

- area of perfect adherence,
- area of passage of small deformations to the great slips,
- area of maximum resistance of the connection and degradation until ultimate residual resistance.

To identify these areas, two thresholds are established:

- 1) the threshold of perfect adherence ε_T^1 ,
- 2) the threshold of continuity before coalescence of the cracks ε_T^2 .

Thus, by taking again the expressions related to the damage with knowing that of the rate of refund of energy [éq 3.1-6] and that of the variable interns associated with isotropic work hardening [éq 3.1-8], one can note:

- 1) a true separation between L 'damage and the friction of the cracks (what makes it possible to amend only the law D' evolution of the damage without affecting the part "friction"),
- 2) the partition in two parts of isotropic work hardening since one has two different stages in the damage.

From now on we will write for work hardening related to the variable of damage:

$$Z_T = \rho \frac{\partial \Psi}{\partial z} = H(z) = \begin{cases} Z_{T1}, & \text{si } \varepsilon_T^1 < \varepsilon_T \leq \varepsilon_T^2 \\ Z_{T1} \cdot Z_{T2}, & \text{si } \varepsilon_T^2 < \varepsilon_T \end{cases} \quad \text{éq 3.2-1}$$

Components Z_{T1} and Z_{T2} express themselves in the following way:

$$Z_{T1} = \left[\sqrt{Y_{T1}} + \frac{1}{A_{1br}} \cdot \sqrt{\frac{G}{2}} \cdot \ln \left(\left(1 + z_T \right) \frac{\varepsilon_T}{\varepsilon_T^1} \right) \right]^2 \quad \text{éq 3.2-2}$$

$$Z_{T2} = \left[y_{T2} + \frac{1}{A_{2br}} \left(\frac{-z_T}{1 + z_T} \right) \right] \quad \text{éq 3.2-3}$$

The function threshold is also defined Φ_{DT} who depends on Y_{DT} and which is written like:

$$\Phi_{DT} = Y_{DT} - (Y_{T1} + Z_T) \leq 0 \quad \text{éq 3.2-4}$$

The thresholds which manage the law of evolution of the damage are also expressed in terms of Y_{DT} (see [Figure 3.2-a]). The first expression corresponds to the threshold of perfect adherence and is written:

$$Y_{T1} = Y_{elas}|_T = \frac{1}{2} \varepsilon_T^1 \cdot G \cdot \varepsilon_T^1 \quad \text{éq 3.2-5}$$

Where Y_{T1} is the initial threshold of damage defined according to the limiting deformation of perfect adherence ε_T^1 , which will correspond to the limiting deformation of shearing – or traction – concrete before the initialization of the damage. In addition, Y_{T2} is the threshold of initiation of coalescence of microcracks - cracks which is defined according to the initial tangential deformation of the great slips ε_T^2 :

$$Y_{T2} = \frac{1}{2} \varepsilon_T^2 \cdot G \cdot \varepsilon_T^2 \quad \text{éq 3.2-6}$$

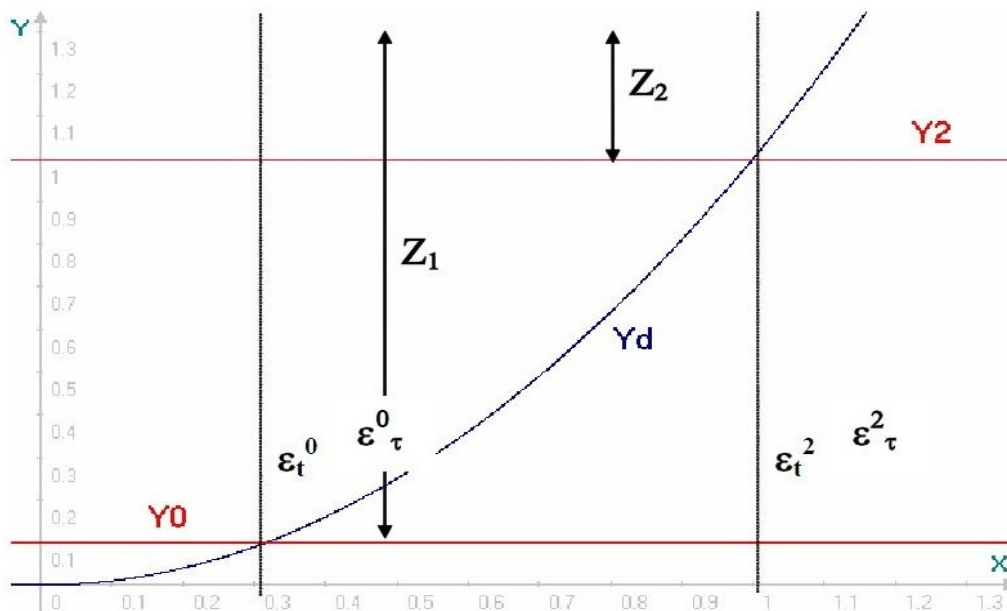


Figure 3.2-a: construction of the functions thresholds in terms of energy

The laws of evolution of the internal variables within the framework of the standard associated laws make it possible to obtain the derivative of the multiplier of damage λ_D :

$$D = \lambda_D \cdot \frac{\partial \Phi_D}{\partial Y_D} = \lambda_D \quad \text{et} \quad \dot{z} = \lambda_D \cdot \frac{\partial \Phi_D}{\partial Z} = -\lambda_D \quad \text{éq 3.2-7}$$

By using the condition of consistency in addition, one obtains the expression of the damage:

$$D_T = 1 - \sqrt{\frac{Y_{TI}}{Y_{DT}}} \cdot \exp \left\{ A_{1_{DT}} \cdot \left[\sqrt{\frac{2}{G}} \cdot (\sqrt{Y_{DT}} - \sqrt{Y_{TI}}) \right]^{B_{1_{DT}}} \right\} * \left\{ \frac{1}{1 + A_{2_{DT}} \cdot \langle Y_{DT} - Y_{TI} \rangle_{+}^{B_{2_{DT}}}} \right\} \quad \text{éq 3.2-8}$$

In this expression, one can identify the part which corresponds to the area of the passage of $A_{1_{DT}}$ small deformations with the great slips with two parameters: and $B_{1_{DT}}$, as well as the final part of damage in mode 2, with the parameters $A_{2_{DT}}$ and $B_{2_{DT}}$. It should be noted that the relation $\langle Y_{DT} - Y_{TI} \rangle$ is managed by a function of *Macaulay*, i.e. this difference in energy must be always positive or worthless.

The functions which manage isotropic work hardening in the tangential direction are expressed like:

$$Z_{T1} = Y_{DT} - Y_{TI} ; \quad \text{éq 3.2-9}$$

$$Z_{T2} = \begin{cases} 0, & \text{si } Y_{TI} < Y_{DT} \leq Y_{T2} \\ Y_{DT} - Y_{T2}, & \text{si } Y_{T2} < Y_{DT} \end{cases} \quad \text{éq 3.2-10}$$

According to these expressions, one can notice that Z_{T2} is not taken into account in the area of transition from the small deformations to great slips.

3.3 Analysis of the damage in the normal direction

The two most important mechanisms which can appear on the normal direction are the detachment between the concrete and the bars of steel, and the penetration of the reinforcement in the body of the concrete. These two conditions can be interpreted respectively like an opening or a closing of crack, and can be described by a particular law of behavior in the normal direction uncoupled from the tangential behavior.

In order to simplify the resolution for compression between surfaces, one decided to allow a small penetration between those, which implies that $\varepsilon_N \leq 0$, and by adopting an elastic law of behavior, one will have:

$$\sigma_N = E \cdot \langle \varepsilon_N \rangle^- \quad \text{si } \varepsilon_N \leq 0 \quad \text{éq 3.3-1}$$

The case of the decoherence of the interface can be described by a behavior endommageable in the normal direction, that is to say:

$$\sigma_N = (1 - D_N) \cdot E \cdot \langle \varepsilon_N \rangle^+ \quad \text{si } \varepsilon_N > 0 \quad \text{éq 3.3-2}$$

with D_N scalar variable of the damage in the normal direction, calculated with the following expression:

$$D_N = \begin{cases} 0 & \text{si } \varepsilon_N \leq \varepsilon_N^1 \\ \frac{1}{1 + A_{DN} \cdot \langle Y_{DN} - Y_{NI} \rangle_{+}^{B_{DN}}} & \text{si } \varepsilon_N^1 < \varepsilon_N \end{cases} \quad \text{éq 3.3-3}$$

In this expression, two parameters material, A_{DN} and B_{DN} , control decoherence by the damage in traction of the concrete. In addition, Y_{NI} is the threshold of damage defined in term of energy, equivalent to the elastic threshold in the normal direction $Y_{elas|N}$ and which is expressed like:

$$Y_{NI} = Y_{elas|N} = \frac{1}{2} \varepsilon_N^1 \cdot E \cdot \varepsilon_N^1 \quad \text{éq 3.3-4}$$

ε_N^1 being limiting deformation of perfect adherence, which corresponds to the limiting deformation of the concrete in traction before the initialization of the damage. It should be mentioned that when the detachment – or the opening of crack – reached the maximum value of resistance to traction, no force of shearing must be transmitted between two materials: it is the single condition under which the scalar variable of damage in the tangential direction becomes 1 because of the damage in the normal direction

3.4 Analysis of the contribution of the friction of cracks by slip

With regard to the part “slip” of the formulation, one supposes that it has a behavior pseudo-plastic, with nonlinear kinematic work hardening. Initially introduced by *Armstrong & Frederick*, 1966, cf [bib1], nonlinear kinematic work hardening makes it possible the formulation to overcome the principal disadvantage of the kinematic law of work hardening of *Prager*, namely, the linearity of the law of state who connects the forces associated with kinematic work hardening. Here, the nonlinear terms are added in the potential of dissipation. The criterion of slip takes the classical shape of the function threshold of *Drucker-Prager* who takes into account the effect of radial containment on the slip:

$$\varphi_f = |\sigma_T^f - X| + c \cdot I_1 \leq 0 \quad \text{éq 3.4-1}$$

Here X is the constraint of recall, c is a parameter related to material, translating the influence of containment, while I_1 corresponds to the first invariant of the tensor of the constraints, which for our case is expressed like:

$$I_1 = \frac{1}{3} \text{Tr}[\sigma] = \frac{1}{3} \sigma_N \quad \text{éq 3.4-2}$$

In addition, the initial threshold for the slip is 0. Moreover, by considering the principle of maximum plastic dissipation, the laws of evolution can be derived from the expression of the plastic potential which is:

$$\varphi_f^p = |\sigma_T^f - X| + c \cdot I_1 + \frac{3}{4} \cdot a \cdot X^2 \quad \text{éq 3.4-3}$$

Where a is a parameter material. It should be mentioned that the quadratic term in X allows to introduce the non-linearity of kinematic work hardening. The laws of evolution for the deformation of slip as for kinematic work hardening take the following shapes:

$$\dot{\varepsilon}_T^f = \dot{\lambda}_f \cdot \frac{\partial \varphi_f^p}{\partial \sigma_T^f} \quad \text{et} \quad \dot{\alpha} = \dot{\lambda}_f \cdot \frac{\partial \varphi_f^p}{\partial X} \quad \text{éq 3.4-4}$$

The multiplier of slip $\dot{\lambda}_f$ is calculated numerically by imposition of the condition of consistency.

3.5 Summary of the equations

We show here, a summary of the equations which constitute the law of behavior of the connection steel - concrete:

Free energy of Helmholtz	$\rho \cdot \Psi = \frac{1}{2} [\langle \varepsilon_N \rangle^- E \langle \varepsilon_N \rangle^- + \langle \varepsilon_N \rangle^+ E \cdot (1 - D_N) \langle \varepsilon_N \rangle^+ + \varepsilon_T G (1 - D_T) \varepsilon_T + (\varepsilon_T - \varepsilon_T^f) G \cdot D_T (\varepsilon_T - \varepsilon_T^f) + \gamma \alpha^2] + H(z)$
Function threshold	$\Phi_{DT} = Y_{DT} - (Y_{TI} + Z_T) \leq 0 ;$ $\Phi_f = \sigma_T^f - X + c \cdot I_1 \leq 0$
Laws of state	$\sigma_N = \begin{cases} E \cdot \varepsilon_N & \text{si } \varepsilon_N \leq 0 \\ (1 - D_N) \cdot E \cdot \varepsilon_N & \text{si } \varepsilon_N > 0 \end{cases} ;$ $\sigma_T = G (1 - D_T) \varepsilon_T + G \cdot D_T (\varepsilon_T - \varepsilon_T^f) ;$ $\sigma_T^f = G \cdot D_T (\varepsilon_T - \varepsilon_T^f)$
Dissipation	$-Y = -\rho \frac{\partial \Psi}{\partial D} = -(Y_D + Y_f) ;$ $X = \rho \frac{\partial \Psi}{\partial \alpha} = \gamma \alpha ;$ $Z = \rho \frac{\partial \Psi}{\partial z} = H'(z)$
Laws evolution of	$\dot{D} = \dot{\lambda}_D \cdot \frac{\partial \Phi_D}{\partial Y_D} = \dot{\lambda}_D ; \quad \dot{z} = \dot{\lambda}_D \cdot \frac{\partial \Phi_D}{\partial Z} = -\dot{\lambda}_D ;$ $\dot{\varepsilon}_T^f = \dot{\lambda}_f \cdot \frac{\partial \varphi_f^p}{\partial \sigma_T^f} ; \quad \dot{\alpha} = \dot{\lambda}_f \cdot \frac{\partial \varphi_f^p}{\partial X}$

3.6 Form of the tangent matrix

In order to ensure the robustness and the effectiveness of the model in the digital establishment and for the total analysis of the massive structures, it is necessary to calculate the tangent matrix, which can be given starting from the following expression:

$$\dot{\sigma}_T = H \cdot \dot{\varepsilon}_T \quad \text{éq 3.6-1}$$

After some analytical calculations, one can deduce the expression from the tangent module by using the condition of consistency and the respective laws of evolution:

$$H = \frac{G \cdot \left(1 - \left(\frac{\partial g(\varepsilon_T)}{\partial \varepsilon_T}\right) \cdot \varepsilon_T^f\right)}{1 + G \cdot D_T \left(\frac{\left(\frac{\partial \Phi_f}{\partial \sigma_T}\right) \cdot \left(\frac{\partial \Phi_f^p}{\partial \sigma_T}\right)}{\left(\frac{\partial \Phi_f}{\partial X}\right) \left(\frac{\partial^2 \rho \psi}{\partial \alpha^2}\right) \left(\frac{\partial \Phi_f^p}{\partial X}\right)} \right)} \quad \text{éq 3.6-2}$$

With

$$\frac{\partial g(\varepsilon_T)}{\partial \varepsilon_T} = \frac{\partial D_T}{\partial Y_{DT}} \cdot \frac{\partial Y_{DT}}{\partial \varepsilon_T} = \left[\frac{f \cdot g \cdot h' - f' \cdot g \cdot h - f \cdot g' \cdot h}{h^2} \right] \cdot G \cdot \varepsilon_T \quad \text{éq 3.6-3}$$

Where f , g and h are the following functions, obtained thanks to [éq 3.2-8]:

$$f = \sqrt{\frac{Y_{TI}}{Y_{DT}}} \quad \text{éq 3.6-4}$$

$$g = \exp \left\{ A_{1_{DT}} \cdot \left[\sqrt{\frac{2}{G}} \cdot \left(\sqrt{Y_{DT}} - \sqrt{Y_{TI}} \right) \right]^{B_{1DT}} \right\} \quad \text{éq 3.6-5}$$

$$h = 1 + A_{2_{DT}} \cdot \langle Y_{DT} - Y_{TI} \rangle_{+}^{B_{2DT}} \quad \text{éq 3.6-6}$$

Note:

In practice in Aster, the tangent matrix was not established, only the secant matrix is used either

$$H = \begin{pmatrix} E(1 - D_N) & 0 \\ 0 & G(1 - D_T) \end{pmatrix} .$$

4 Digital integration

The separation in two parts in the formulation: damage – slip, enables us to treat each one of it separately. Thus, the integration of the damage part is carried out explicitly by the definition of two surfaces threshold. On the other hand, the part “slip” is solved in an implicit way by a classical method with knowing the algorithm of the type “return-mapping” suggested by *Ortiz & Simo*, Cf [bib4], which will ensure the effective convergence of way.

4.1 Calculation of the part “friction of the cracks” with a method of integration implicit

The effects on the connection associated with the phenomenon of friction with the cracks can be calculated within the framework of a behavior pseudo-plastic with a nonlinear kinematic work hardening. For the establishment with the method of integration suggested, we will carry out a linearization of the function threshold around the current values of the associated internal variables. With the iteration $(i+1)$, surface threshold is written:

$$\varphi_f = \varphi_f^{(i)} + \frac{\partial \varphi_f^{(i)}}{\partial \sigma_T^f} : (\sigma_T^{f(i+1)} - \sigma_T^{f(i)}) + \frac{\partial \varphi_f^{(i)}}{\partial X} : (X^{(i+1)} - X^{(i)}) \approx 0 \quad \text{éq 4.1-1}$$

According to the equations [éq 3.1-7], [éq 3.1-8], and [éq 3.6-5], one a:

$$\dot{X} = \gamma \cdot \dot{\alpha} = -\gamma \cdot \dot{\lambda}_f \cdot \frac{\partial \varphi_f^p}{\partial X} \quad \text{éq 4.1-2}$$

$$\dot{\sigma}_T^f = -G \cdot D_T \cdot \dot{\varepsilon}_T^f = -G \cdot D_T \cdot \dot{\lambda}_f \cdot \frac{\partial \varphi_f^p}{\partial \sigma_T^f} \quad \text{éq 4.1-3}$$

That one can discretize in the following way:

$$\Delta X = X^{(i+1)} - X^{(i)} = \gamma \cdot \Delta \alpha = -\gamma \cdot \Delta \lambda_f \cdot \frac{\partial \varphi_f^p}{\partial X} \quad \text{éq 4.1-4}$$

$$\Delta \sigma_T^f = \sigma_T^{f(i+1)} - \sigma_T^{f(i)} = -G \cdot D_T \cdot \Delta \varepsilon_T^f = -G \cdot D_T \cdot \Delta \lambda_f \cdot \frac{\partial \varphi_f^p}{\partial \sigma_T^f} \quad \text{éq 4.1-5}$$

By combining these expressions with the expression of surface threshold and by writing that is equal φ_f to zero, one can deduce the increment from multiplier $\Delta \lambda_f$ with each iteration i :

$$\Delta \lambda_f = \frac{\varphi_f^{(i)}}{\frac{\partial \varphi_f^{(i)}}{\partial \sigma_T^f} \cdot G \cdot D_T \cdot \frac{\partial \varphi_f^p(i)}{\partial \sigma_T^f} + \frac{\partial \varphi_f^{(i)}}{\partial X} \cdot \gamma \cdot \frac{\partial \varphi_f^p(i)}{\partial X}} \quad \text{éq 4.1-6}$$

After obtaining the value of $\Delta \lambda_f$, one can substitute it in the equations [éq 4.1-4] and [éq 4.1-5] in order to bring up to date the thermodynamic forces σ_T^f and X . The iterations will have to continue until the moment when the condition of consistency is checked.

4.2 The algorithm of resolution

In a general way, one seeks to check the balance of the structure at every moment, in an incremental form. As clarified previously, for the damage a simple scalar equation makes it possible to obtain the corresponding value, which makes it possible to avoid a recourse to the iterative methods. On the other hand, an iterative method is applied for the integration of the friction part of the cracks. Then, the algorithm is the following:

1) Geometrical reactualization:

$$(\boldsymbol{\varepsilon}_T)_{n+1} = (\boldsymbol{\varepsilon}_T)_n + \nabla^s \mathbf{u}_T$$

2) Elastic prediction:

$$(\boldsymbol{\varepsilon}_T^f)^{(0)} = (\boldsymbol{\varepsilon}_T^f)_n ;$$

$$(\boldsymbol{\varepsilon}_T^e)^{(0)} = (\boldsymbol{\varepsilon}_T)_{n+1} - (\boldsymbol{\varepsilon}_T^f)_{n+1} ;$$

$$\alpha_{n+1}^{(0)} = \alpha_n$$

3) Evaluation of the threshold:

$$(\varphi_f)_{n+1}^{(0)} \leq 0 ?$$

if SO, end of the cycle; so NOT, beginning of the iterations

YES:

$$(\boldsymbol{\varepsilon}_T^f)_{n+1} = (\boldsymbol{\varepsilon}_T^f)^{(0)} ; (\boldsymbol{\varepsilon}_T^e)_{n+1} = (\boldsymbol{\varepsilon}_T^e)^{(0)} ; \alpha_{n+1} = \alpha_{n+1}^{(0)} ; \boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}_{n+1}^{(0)}$$

NOT:

$$i = 0$$

$$\Delta \lambda_f = \frac{(\varphi_f)_{n+1}^{(i)}}{\left[\partial \varphi_f / \partial \sigma_T^f \right]_{n+1}^{(i)} G \cdot D_T \left[\partial \varphi_f^p / \partial \sigma_T^f \right]_{n+1}^{(i)} + \left[\partial \varphi_f / \partial X \right]_{n+1}^{(i)} \cdot \gamma \cdot \left[\partial \varphi_f^p / \partial X \right]_{n+1}^{(i)}}$$

4) Plastic correction:

$$\boldsymbol{\sigma}_{n+1}^{(i+1)} = \boldsymbol{\sigma}_{n+1}^{(i)} - G \cdot D_T \cdot \Delta \lambda_f \cdot \left[\partial \varphi_f^p / \partial \sigma_T^f \right]_{n+1}^{(i)} - \gamma \cdot \Delta \lambda_f \cdot \left[\partial \varphi_f^p / \partial X \right]_{n+1}^{(i)}$$

$$\alpha_{n+1}^{(i+1)} = \alpha_{n+1}^{(i)} + \Delta \lambda_f \cdot \left[\partial \varphi_f^p / \partial X \right]_{n+1}^{(i)}$$

5) Checking of convergence:

$$(\varphi_f)_{n+1}^{(i+1)} \leq TOL \left| (\varphi_f)_{n+1}^{(0)} \right| ?$$

if SO, end of the cycle; so NOT, to continue them iterations in (iv)

YES:

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}_{n+1}^{(i+1)} ;$$

$$\alpha_{n+1} = \alpha_{n+1}^{(i+1)} ;$$

$$(\boldsymbol{\varepsilon}_T^e)_{n+1} = \boldsymbol{\varepsilon}_T^e(\boldsymbol{\sigma}_{n+1}, \alpha_{n+1}) ;$$

$$(\boldsymbol{\varepsilon}_T^f)_{n+1} = (\boldsymbol{\varepsilon}_T)_{n+1} - (\boldsymbol{\varepsilon}_T^e)_{n+1}$$

NOT:

$$i = i + 1$$

4.3 Internal variables of the model

We show here the internal variables stored in each point of Gauss in the implementation of the model:

Internal number of variable	Physical direction
1	D_N : Scalar variable of the damage in the normal direction
2	D_T : Scalar variable of the damage in the tangential direction
3	z_{TI} : Scalar variable of isotropic work hardening for the damage in mode 1
4	z_i : Scalar variable of isotropic work hardening for the damage in mode 2
5	ε_T^f : Deformation of slip cumulated by friction of the cracks
6	α : Value of kinematic work hardening by friction of the cracks

5 Parameters of the law

The law of behavior presented here is controlled by 14 parameters, of which 3 manage the answer in the normal direction and the others affect the answer in the tangential direction. In addition, the Young modulus is recovered starting from the elastic data provided by the operator ELAS, which must always appear in the command file.

These parameters, or the analytical expressions which make it possible to obtain them, were obtained or determined starting from the digital simulation of the experimental tests carried out by *Eligehausen et al.*, 1983, cf [bib3]. The realization of multiple simulations made it possible to determine a relation between the geometrical and material characteristics of materials in question (steel and concrete) and the parameters which manage the model of the interface.

5.1 Initial parameters

5.1.1 The parameter "hpen"

The element joint functioning on the concept of jump of displacement, it is necessary to introduce a dimension characteristic of the zone of degraded interface making it possible to define the concept of deformation in the interface. With this intention it was introduced the principle of penetration between surfaces: the parameter "hpen" allows to define this zone surrounding the bar of steel. This parameter corresponds to the possible maximum penetration which depends on the thickness of the compressed concrete - crushed. At the same time, "hpen" the dissipation of energy in the element as well as the kinematics of the slip manages.

In order to give a reference to the user for the choice of this parameter, one proposes to calculate it starting from the diameter of the bar d_b and the relative surface of the veins α_{sR} defined by:

$$\alpha_{sR} = \frac{k \cdot F_R \cdot \sin \beta}{\pi \cdot d_b \cdot c} \quad \text{éq 5.1.1-1}$$

where k is the number of veins on the perimeter; F_R the transverse surface of a vein; β is the angle between the vein and the axis longitudinal of the steel bar; and c between veins center in center is the measured distance. Finally, "hpen" will be calculated with the expression:

$$h_{pen} = d_b \cdot \alpha_{sR} \quad \text{éq 5.1.1-2}$$

According to *Eligehausen et al.*, the reinforcements usually used in the United States have values of α_{SR} enter 0.05 and 0.08 . For the smooth bars, since one needs a small value for “*hpen*”, one proposes values of α_{SR} enter 0.005 and 0.02 .

The following table gives the values of “*hpen*” according to the diameter of the bar:

Diameter (mm)	Relative surface	<i>Hpen</i> (mm)	Description
8	0.01	(0.08) □ 0.1	Bar commercial smooth
8	0.08	0.64	Bar commercial ribbed
20	0.08	1.50	Bar commercial ribbed
25	0.08	2.00	Bar commercial ribbed
32	0.08	2.54	Bar commercial ribbed

The unit of “*hpen*” must of course correspond to the unit used for the grid.

5.1.2 The parameter G or modulates rigidity of the connection

Generally, because of difficulty in measuring the deformations by shearing, the module of rigidity of a material is calculated starting from Young and the Poisson's ratio modulus, current parameters obtained in experiments. However, for our case, the interface is a pseudo-material whose characteristics must depend on the properties corresponding to materials in contact, steel and concrete. Since the material which one expects to damage is the concrete, one proposes to initially use for the connection the same value of G that for the studied concrete but it can be higher up to a value similar to the value of the Young modulus E , when one increases the value of “*hpen*”. In the case of the reinforcements with rigidities higher than those of the current commercial bars (because of a provision or special geometry of the veins), one can make a correction of the value chosen, by multiplying the module of rigidity by a coefficient of correction calculated starting from the relative surfaces of the commercial bars, with the expression:

$$C_{arm} = \frac{(\alpha_{SR})_{barre}}{(\alpha_{SR})_{barre_{comm}}} \quad \text{éq 5.1.2-1}$$

Then, the module of rigidity of the connection G will be:

$$G_{liai} = C_{arm} \cdot G_{beton} \quad \text{éq 5.1.2-2}$$

In the last expressions, G_{liai} is the module of rigidity of the connection; G_{beton} is the module of rigidity of the concrete; C_{arm} is the coefficient of correction per reinforcement; $(\alpha_{SR})_{barre}$, relative surface of the veins of the bar concerned; and $(\alpha_{SR})_{barre_{com}}$, relative surface of the veins of the commercial bar of the same diameter (preferably, 0.08).

5.2 Parameters of damage

5.2.1 Limit of elastic strain \square^1_{τ} or threshold of perfect adherence

To define the threshold of perfect adherence, it is considered that the damage by shearing must be initiated at the time of the going beyond a certain threshold of deformation. So one proposes to adopt the limiting deformations of the concrete in traction, i.e., enters 1×10^{-4} and 0.5×10^{-3} , which corresponds to shear stresses enters 0.5 and 4 MPa in perfect adherence.

5.2.2 The parameter of damage A_{DT} for the passage of the small deformations to the great slips

In this area, the law of evolution of the damage is expressed in term of deformations and its construction depends on the definite elastic slope for the linear behavior (shear stress versus deformation) in the area of perfect adherence: this parameter controls the value of the constraint compared to the slip in the passage of small deformations to the great slips.

The determination of the value of this parameter is a key and delicate point model, since the evolution of the damage must be carried out with certain conditions noticed by several researchers; for example:

- the resistance of the connection is directly proportional to the compressive strength of the concrete. However, as the resistance of the concrete is increased, the behavior becomes more rigid, bringing to the brittle fracture of the connection,
- the particular rigidity of the reinforcement, which is related on the diameter and the quantity of the veins on surface, must increase the resistance of the connection,
- the relation between the moduli of elasticity of two materials concerned must manage the kinematics of the connection directly.

From the digital simulations that one carried out, one observed that this value is located between a minimum of 1 and a maximum of 5, and that it will have to be adjusted according to the selected test of reference. Optionally, one proposes an expression which makes it possible to adopt an initial value and which depends on the particular characteristics of materials:

$$A_{DT} = \frac{1}{(1 + \alpha_{SR})} \cdot \sqrt{\frac{f'c}{30}} \cdot \sqrt{\frac{E_a}{E_b}} \quad \text{éq 5.2.2-1}$$

In the last expression, E_b will be calculated with the expression provided in the section A.2.1, 2 of BAEL' 91:

$$E_b = 11000 \times (f'c)^{1/3} \quad \text{éq 5.2.2-2}$$

In the two last expressions, one a:

- 1) $f'c$, compressive strength of the concrete in MPa ;
- 2) E_a , modulus of elasticity of steel, in MPa ;
- 3) E_b , modulus of elasticity of the concrete, in MPa ;
- 4) α_{SR} , relative surface of the veins of the bar concerned.

[the Figure 5.2.2-a] in gives a graphic comparison.

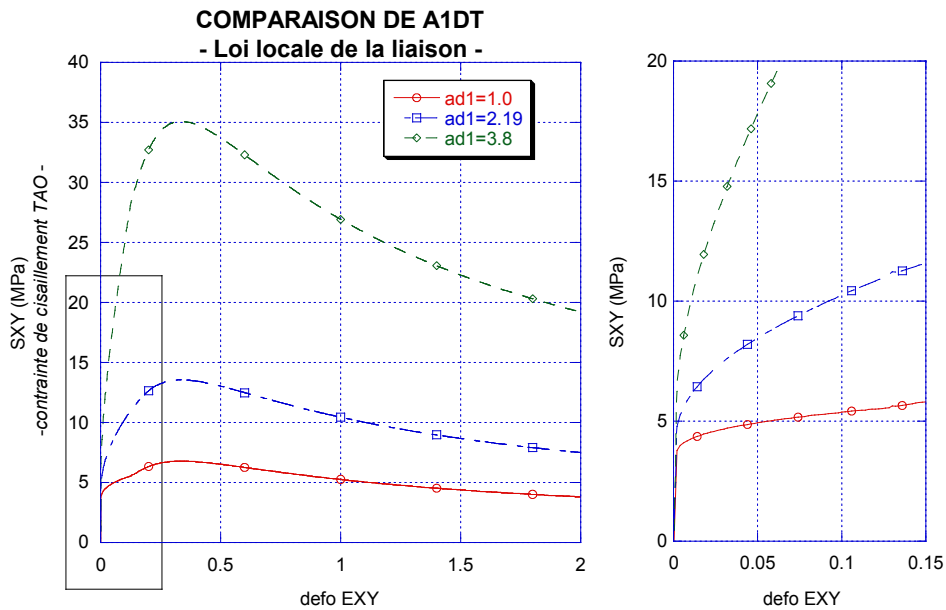


Figure 5.2.2-a: Appearance of A_{IDT} : growth of the resistance of the connection

5.2.3 The parameter of damage B_{IDT}

The purpose of this parameter is to soften the shape of the curve of behavior, like facilitating the transition from the elastic slope towards the nonlinear area. It can have a value understood enters 0.1 and 0.5 (never higher than 0.5 since it is the equivalent of the square root of the formula). One can advise to adopt the value of 0.3 for ordinary calculations. (See [Figure 5.2.3-a]).

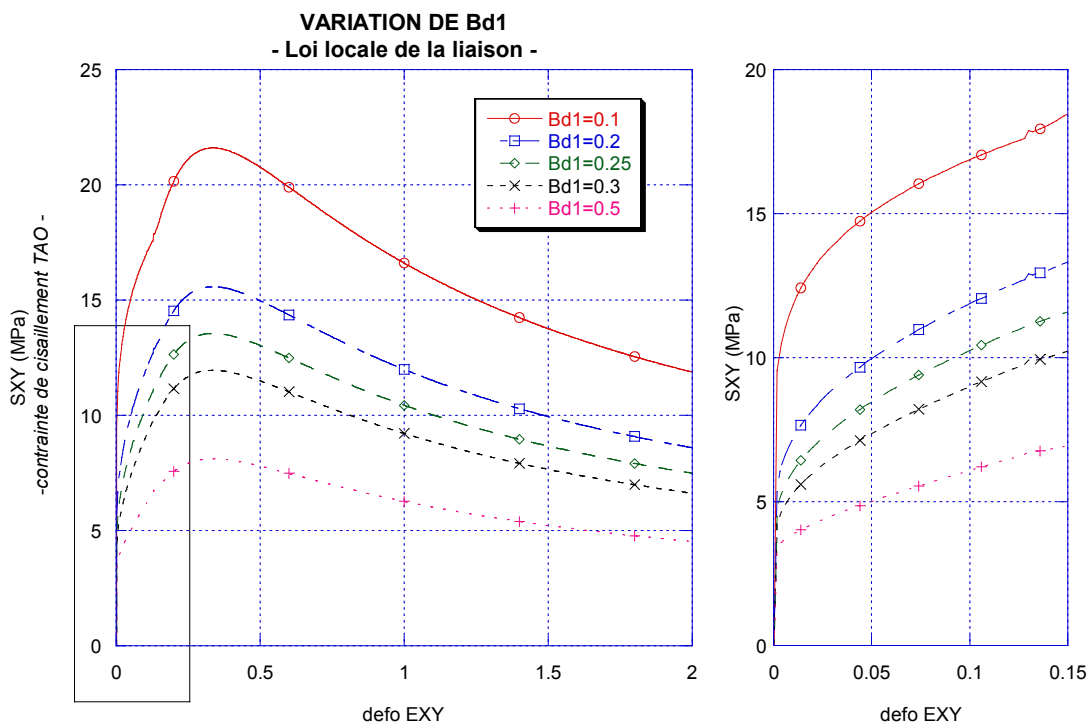


Figure 5.2.3-a: Comparison of B_{IDT} : Modification of the curve

5.2.4 Limit of deformation ε_T^2 or threshold of the great slips

According to several authors, the great slips are overall higher than 1 mm of displacement, but that is an indicator which depends on the form and dimensions of the specimens tested; therefore, it is proposed that this deformation never exceeds 1.00 (adimensional value). In a more precise way, one proposes to apply the following expression:

$$\varepsilon_T^2 = \frac{1}{(h_{pen})^2} \cdot \left(1 - \frac{(A_{IDT})^n}{C + (A_{IDT})^n} \right) = \frac{1}{(h_{pen})^2} \cdot \left(1 - \frac{(A_{IDT})^4}{9 + (A_{IDT})^4} \right) \leq 1.0 \quad \text{éq 5.2.4-1}$$

In this expression, a sigmoid function was applied of which coefficients C and n allow to adjust the kinematic effect of A_{IDT} on the slip, i.e., when the connection becomes more resistant because of an increase in rigidity, the slip is reduced gradually. The values were adopted 9.0 and 4.0 respectively, but they are always optional.

The choice of the value of the limit of deformation ε_T^2 is very important because it introduces a more or less great brittleness of the response by translation of the threshold of passage of the small deformations to the great slips. This brittleness is related to the stiffness of the concrete via the parameter A_{IDT} . It should be noted that the following parameters which manage the damage must be also adjusted at the local level to ensure the correct continuity of behaviour in shearing of the connection and to thus be able to obtain the desired or expected answer of a system real steel – connection – concrete.

5.2.5 The parameter of damage A_{2DT}

The damage, such as it was conceived in the model, obeys two laws of evolution which are expressed using one only variable classical scalar which will ensure the coherence of the damage. The parameters of each of the 2 laws are independent and numerically stable, but they are likely to generate serious errors in the continuity of the behavior if one does not pay attention to the shape of the local curve stress-strain: to see the case of the curve shown in graphics of [Figure 5.2.5-a], with a value $A_{2DT} = 1 \times 10^{-3} \text{ MPa}^{-1}$. We are not able to propose an analytical relation for the choice of this parameter, but the gained experience enables us to affirm that the value of this parameter must be understood enters 1×10^{-3} and $9 \times 10^{-2} \text{ MPa}^{-1}$ roughly.

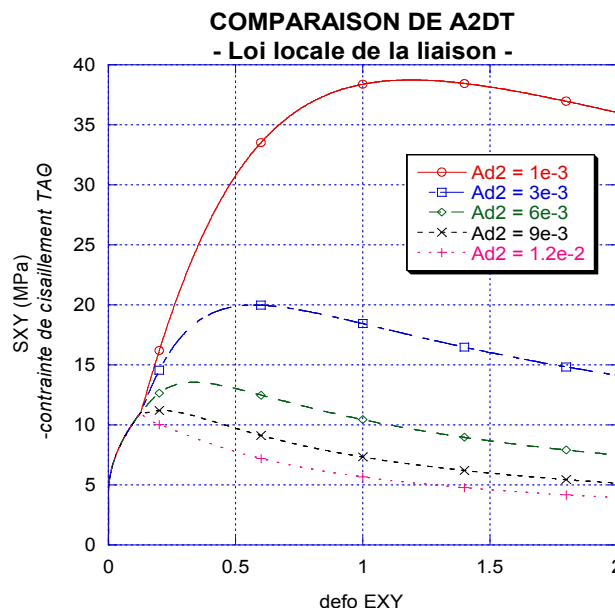


Figure 5.2.5-a: Comparison of A_{2DT} : damage and rupture of the connection

5.2.6 The parameter of damage B_{2DT}

This parameter, which supplements the law of evolution of damage in great slips, controls not only the growth of the resistance of the connection or the shape of the curve of behavior to the peak and in the area post-peak, but also the kinematics of the answer, which implies the determination of the slip for the maximum shear stress as well as the amplitude of the curve to the peak of the behavior. Then, although values of the parameters of damage A_{2DT} and B_{2DT} will have to adjust itself at the same time when one builds the curve of behavior of the connection in order to respect the continuity of the pace, one can say that the value of B_{2DT} is inversely proportional to the amplitude of slip at the top, i.e., a value of 0.8 allows great broader slips in the top than a value of 1.2, for example.

For practical cases, one recommends to use a value understood enters 0.8 and 1.1 to reproduce a coherent curve of behavior (See [Figure 5.2.6-a]).

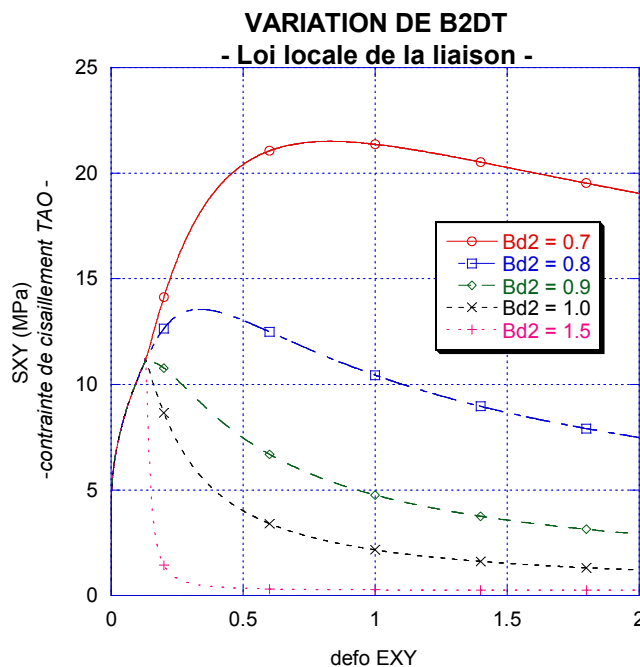


Figure 5.2.6-a: Comparison A_{2DT} of : damage and rupture of the connection

5.3 Parameters of damage on the normal direction

5.3.1 Limit of deformation \square^1_{NR} or threshold of great displacements

In a way similar to the elastic behavior in the tangential direction, it is considered that decoherence must be initiated at the time of the going beyond a certain threshold of deformation. We propose to adopt a value enters 10^{-4} and 10^{-3} .

5.3.2 The parameter of damage A_{DN}

This parameter controls primarily the slope of degradation of the normal constraint compared to the deformation due to the opening of the interface. We propose to use a minimal value of $1 \times 10^{-1} MPa^{-1}$, which corresponds to a degradation similar to that of the concrete. Nevertheless, if one wish to have a behavior of the connection even more fragile, it is enough to increase this value.

5.3.3 The parameter of damage B_{DN}

In combination with the preceding parameter, this parameter controls the damage of the connection, in particular the shape of the curve of behavior in phase post-peak.

We propose to use a value equalizes with 1, or 1,2 for more marked curves.

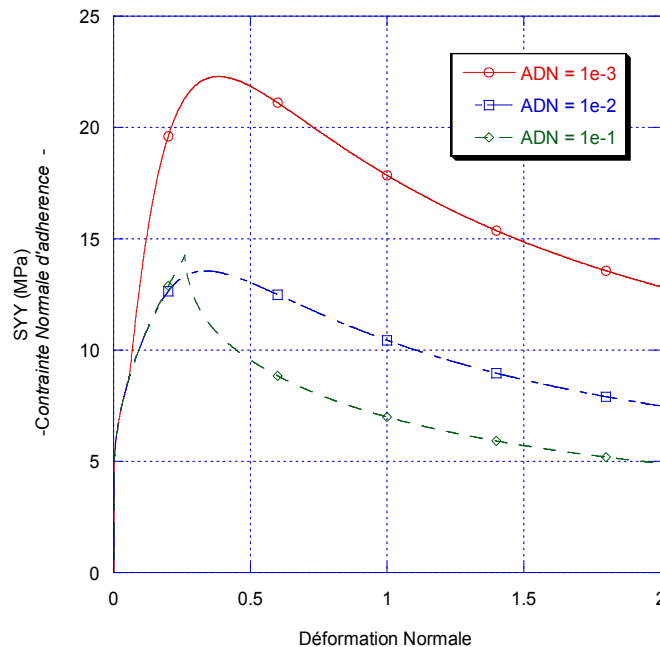


Figure 5.3.3-a: behavior of the connection on the normal direction at the time of the opening interface (normal traction on the connection).

5.4 Parameters of friction

5.4.1 The parameter material γ of friction of the cracks

One of the assets of the model suggested here is that it is able to take into account the effects of friction of the cracks, which, in the case of monotonous loading, appears by a positive contribution to the shear strength of the connection; in addition, in the cyclic cases of loadings, it is obvious that the pace of the loops of hysteresis depends directly on the choice of the value of this parameter material. However, the corresponding values were not gauged, since we did not simulate tests with cyclic loadings yet to validate them. Temporarily, one proposes to use values lower than 10 MPa , with a maximum value of α equal to 1.0 MPa^{-1} .

5.4.2 The parameter material α of kinematic work hardening

On [the Figure 5.4.2-a], one can appreciate that the reduction in the value of α increase hysteretic dissipation, but also the resistance of shearing and the residual deformation pseudo-plastic. That is very important for the cyclic modeling of the connection since in reality, when one exceeds the peak of maximum resistance, one notices that at the time of the discharge there is no more elastic contribution of the slip, i.e. the residual deformation pseudo-plastic corresponds exactly to the total slip reached. In other words, once connected all the cracks in the potential of rupture, longitudinal and tangential layer with the steel bar, the single resistance which will prevent the displacement of the reinforcement is the friction resistance of the connection, produced by the contact and the tangle of the asperities between surfaces concrete – concrete.

As previously, our experiment is limited: one proposes to use a maximum value of 0.1 MPa^{-1} who gives correct results for applications in monotonous loading, and who seems suitable for cyclic loadings.

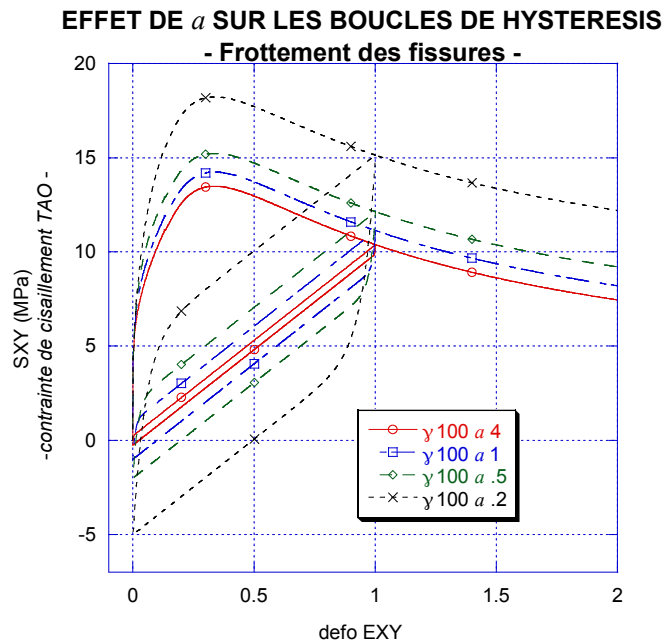


Figure 5.4.2-a: Comparison of α : effects on the loops of hysteresis into cyclic

5.4.3 The parameter of influence of containment c

In our model, the influence of containment was taken into account thanks to the application of this parameter which controls these effects on the connection, and which appears by an increase in the maximum shear stress as by the increase in maximum displacement to the peak when containment increases.

For the calibration, we carried out simulations with containments of 0, 5, 10 and 15 MPa, by always using a value of 1.0 for this parameter. It was noticed that if one wants to produce a kinematic translation of the slip caused by containment, it is enough to adopt a value of 1.2 or 1.5 (adimensional). Optionally, it is advised to maintain the value of 1.0 for ordinary calculations.

5.5 Summary of the parameters

To facilitate the use of the law, the following table presents a synthesis of the whole of the parameters of the model of behavior.

It is pointed out that the values or the expressions suggested have only one indicative value, and that the arbitrary combination can give inaccurate and unexpected results compared to the hoped behavior of the connection; in other words, a bad choice of the parameters can produce a strong rigidity or a weak answer of the steel-concrete interface.

Parameter	Unit	Value suggested	Analytical expression	Variables concerned	
h_{pen}	mm	-	$h_{pen} = d_b \cdot \alpha_{sR}$	d_b	Diameter of the bar
				α_{sR}	Relative surface of the veins
G_{liai}	MPa	-	$G_{liai} = C_{arm} \cdot G_{beton}$	C_{arm}	Coefficient of correction by reinforcements
				G_{beton}	Module of rigidity of the concrete
ϵ_T^1	-	min 1.0×10^{-4} max 1.5×10^{-3}			
A_{1DT}	-	min 1.0 max 5.0	$A_{1DT} = \frac{1}{(1 + \alpha_{sR})} \cdot \sqrt{\frac{f'c}{30}} \cdot \sqrt{\frac{E_a}{E_b}}$	$f'c$	Compressive strength of the concrete (MPa)
				E_{has}	Modulus of elasticity of steel
				E_B	Modulus of elasticity of the concrete
B_{1DT}	-	min 0.1 max 0.5			
ϵ_T^2	-	-	$\epsilon_T^2 = \frac{1}{(h_{pen})^2} \cdot \left(1 - \frac{(A_{1DT})^4}{9 + (A_{1DT})^4} \right) \leq 1.0$		
A_{2DT}	MPa ⁻¹	min 1.0×10^{-4} max $9. \times 10^{-2}$			
B_{2DT}	-	min 0.8 max 1.5			
γ	MPa	max 10.0			
a	MPa ⁻¹	min 0.01 max 1.0			
c	-	1.0	(value recommended)		
α	-	min 10^{-4} max $0.9 \cdot 10^{-3}$			
A_{DN}	MPa ⁻¹	min 1.0×10^{-1}	(value recommended, not gauged)		
B_{DN}	-	1.	(value recommended, not gauged)		

6 Bibliography

1. ARMSTRONG, P.J. & FREDERICK, C.O. : In Mathematical Representation of the Multiaxial Bauschinger Effect. G.E.G.B. ; Carryforward RD/B/N, 731.1966.
2. BERTERO V.V.: Concrete Seismic behavior of structural linear elements (beams and columns) and to their connections. *Euro-International committee of Concrete (CEB)*; News bulletin No 131; Paris, France, 1979.
3. ELIGEHAUSEN R., POPOV E.P. & BERTERO V.V.: Local jump stress-slipway relationships of deformed bars under generalised excitations. *University of California; Carryforward No UCB/EERC - 83/23 of the National Science Foundation* , 1983.
4. ORTIZ MR. & SIMO J.C. : Year analysis of has new class of constitutive integration algorithms for elastoplastic relations. *International Newspaper for Numerical Methods in Engineering* ; Vol. 23, pp. 353 – 366.1986.

7 Checking

The law of behavior JOINT_BA is checked by the cases following tests:

SSNA112	Axisymmetric test of wrenching (Borderie & Pijaudier- Pooch) for the study of the steel-concrete connection: law JOINT_BA	[V6.01.112]
SSNP126	Validation of the law of behavior JOINT_BA (steel-concrete connection) in 2D plan	[V6.03.126]

8 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
7,4	S. MICHEL-PONNELLE, NR. DOMINGUEZ EDF- R&D/AMA	Initial text