Viscoplastic law of behavior VISC_DRUC_PRAG

Summary:

This document describes the viscoplastic law of behavior VISC_DRUC_PRAG based on the elastoplastic of Drucker-Prager and fascinating model of account viscosity according to a law power of the Perzyna type. Its scope of application is the mudstone which is the rock host of the concept of storage.

The model suggested comprises only one viscoplastic mechanism. The criterion is hammer-hardened with the viscoplastic deformation cumulated via three thresholds: rubber band, of peak and ultimate. The flow is nonassociated, the potential of flow being a potential of Drucker-Prager being hammer-hardened according to three levels: rubber band, of peak and ultimate. Between the thresholds, work hardenings are linear.

This law can be used in a pure mechanical modeling as it can be used in a modeling THM. It is available in 3D, deformations plane and axisymmetric. It is integrated by the solution of only one scalar equation nonlinear.

Contents

1 Notations

$\boldsymbol{\sigma}$ indicate the tensor of the effective constraints in small disturbances, noted in the shape of the following vector:

$$\begin{matrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sqrt{2}\,\sigma_{12}\\ \sqrt{2}\,\sigma_{13}\\ \sqrt{2}\,\sigma_{23}\end{matrix}$$

One notes:

$\mathbf{D}^e$	tensor of elasticity
$I_1=\mathrm{tr}(\boldsymbol{\sigma})$	first invariant of the constraints
$\mathbf{s}=\boldsymbol{\sigma}-\frac{I_1}{3}\mathbf{Id}$	tensor of the constraints déviatoires
$s_{II}=\sqrt{\mathbf{s}.\mathbf{s}}$	second invariant of the tensor of the constraints déviatoires
$\sigma_{\mathrm{eq}}=\sqrt{\frac{3}{2}s_{ij}s_{ij}}$	equivalent constraint
I_1^{el}	trace of the elastic prediction of the constraints
$\mathbf{s}^{el}=\boldsymbol{\sigma}^{el}-\frac{I_1^{el}}{3}\mathbf{Id}$	tensor of the constraints déviatoires of the elastic prediction of the constraints
$\sigma_{\mathrm{eq}}^{el}=\sqrt{\frac{3}{2}s_{ij}^{el}s_{ij}^{el}}$	equivalent constraint of the elastic prediction of the constraints
$\tilde{\boldsymbol{\varepsilon}}=\boldsymbol{\varepsilon}-\frac{\mathrm{tr}(\boldsymbol{\varepsilon})}{3}\mathbf{Id}$	diverter of the deformations
$\varepsilon_v=\mathrm{tr}(\boldsymbol{\varepsilon})$	voluminal deformation
$\dot{p}=\sqrt{\frac{2}{3}\dot{\tilde{\boldsymbol{\varepsilon}}}_{ij}^{\mathrm{vp}}\dot{\tilde{\boldsymbol{\varepsilon}}}_{ij}^{\mathrm{vp}}}$	cumulated viscoplastic deviatoric deformations
f	viscoplastic surface of load
G	viscoplastic potential of flow
α_0 , R_0 and β_0	parameters of corresponding work hardening with the threshold of elasticity ($p=0$)
α_{pic} , R_{pic} and β_{pic}	parameters of corresponding work hardening with the peak $(p=p_{pic})$
α_{ult} , R_{ult} and β_{ult}	parameters of corresponding work hardening with the ultimate threshold $(p=p_{ult})$
Φ	amplitude the speed of the unrecoverable deformations
A	parameter of creep
n	power of the law of creep
P_{ref}	pressure of reference

2 Introduction

This document describes the integration of the viscoplastic law of behavior VISC_DRUC_PRAG in Code_Aster. This law comprises only one viscoplastic mechanism. The viscoplastic criterion is hammer-hardened with the deviatoric viscoplastic deformation cumulated via three thresholds: rubber band for a worthless viscoplastic deformation, a threshold known as of peak for a viscoplastic deformation known as of peak (parameter of the model) and an ultimate threshold for a viscoplastic deformation known as ultimate (parameter of the model). Between the thresholds, the functions of work hardening are linear. In Code_Aster there exists another law based on the model of Drücker_Prager and used in elastoplasticity in a form associated in the name DRUCK_PRAGER or nonassociated under the name DRUCK_PRAG_N_A (see [R7.01.16]).

3 Formulation of the viscoplastic model VISC_DRUC_PRAG

3.1 Equations model

This model is based on a viscoplastic formulation of the Drücker-Prager type, where the surface of load $f(\boldsymbol{\sigma}, p)$ is defined by:

$$f=\sqrt{\frac{3}{2}}s_{II}+\alpha(p)I_1-R(p)$$

$\alpha(p)$ and $R(p)$ are functions of the cumulated deviatoric viscoplastic deformation p ,

A viscoplastic potential of flow is introduced $g(\boldsymbol{\sigma}, p)$:

$$g=\sqrt{\frac{3}{2}}s_{II}+\beta(p)I_1$$

For the evolution of the criterion f and of the potential g we distinguish three thresholds distinct corresponding to three values from the variable from work hardening: an elastic threshold, a threshold of peak and an ultimate threshold. Between these thresholds, work hardening is linear. Between the elastic threshold and the threshold of peak, work hardening is positive, after the peak work hardening is negative and becomes constant after the ultimate threshold.

The functions related to cohesion are written in the following form:

$$\alpha(p)=\left(\frac{\alpha_{pic}-\alpha_0}{p_{pic}}\right)p+\alpha_0 \text{ for } 0<p<p_{pic}$$

$$\alpha(p)=\left(\frac{\alpha_{ult}-\alpha_{pic}}{p_{ult}-p_{pic}}\right)(p-p_{pic})+\alpha_{pic} \text{ for } p_{pic}<p<p_{ult}$$

$$\alpha(p)=\alpha_{ult} \text{ for } p>p_{ult}$$

Functions dependent on dilatancy are written in the following form:

$$\beta(p)=\left(\frac{\beta_{pic}-\beta_0}{p_{pic}}\right)p+\beta_0 \text{ for } 0<p<p_{pic}$$

$$\beta(p)=\left(\frac{\beta_{ult}-\beta_{pic}}{p_{ult}-p_{pic}}\right)(p-p_{pic})+\beta_{pic} \text{ for } p_{pic}<p<p_{ult}$$

$\beta(p)=\beta_{ult}$ for $p>p_{ult}$

The functions of work hardening are written:

$R(p)=\left(\frac{R_{pic}-R_0}{p_{pic}}\right)p+R_0$ for $0<p<p_{pic}$

$R(p)=\left(\frac{R_{ult}-R_{pic}}{p_{ult}-p_{pic}}\right)(p-p_{pic})+R_{pic}$ for $p_{pic}<p<p_{ult}$

$R(p)=R_{ult}$ for $p>p_{ult}$

The constraints are connected to the deformations by the law of Hooke:

$$\boldsymbol{\sigma}=\mathbf{D}^e\left(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{vp}\right)$$

When the viscoplastic threshold is reached, of the viscoplastic unrecoverable deformations are generated and expressed according to the theory of Perzyna by:

$$d\,\varepsilon_{ij}^{vp}=A\left\langle\frac{f}{P_{ref}}\right\rangle^n\frac{\partial g}{\partial\sigma_{ij}}dt$$

f being the criterion of viscoplasticity; A and n are parameters of the model; P_{ref} a pressure of reference.

$$\frac{\partial g}{\partial\sigma_{ij}}=\sqrt{\frac{3}{2}}\frac{\partial s_{II}}{\partial\sigma_{ij}}+\beta(p)\frac{\partial I_1}{\partial\sigma_{ij}} \quad\text{and}\quad \dot{p}=\sqrt{\frac{2}{3}\dot{\tilde{\varepsilon}}_{ij}^{vp}\dot{\tilde{\varepsilon}}_{ij}^{vp}}$$

with, $\tilde{\varepsilon}_{ij}^{vp}$ the diverter of the viscoplastic tensor of deformation,

$$\frac{\partial s_{II}}{\partial\sigma_{ij}}=\frac{\partial s_{II}}{\partial s_{kl}}\frac{\partial s_{kl}}{\partial\sigma_{ij}}=\frac{s_{kl}}{s_{II}}\left(\delta_{ik}\delta_{jl}-\frac{1}{3}\delta_{ij}\delta_{kl}\right)=\frac{s_{ij}}{s_{II}}$$

and

$$\frac{\partial I_1}{\partial\sigma_{ij}}=\frac{\partial\,\mathrm{tr}\left(\sigma_{ij}\right)}{\partial\sigma_{ij}}=\delta_{ij}$$

from where

$$\frac{\partial g}{\partial\sigma_{ij}}=\sqrt{\frac{3}{2}}\frac{s_{ij}}{s_{II}}+\beta(p)\,\delta_{ij}$$

Summary of the equations:

The criterion:

$$f=\sqrt{\frac{3}{2}}s_{II}+\left[\left(\frac{\alpha_{pic}-\alpha_0}{p_{pic}}\right)p+\alpha_0\right]I_1-\left[\left(\frac{R_{pic}-R_0}{p_{pic}}\right)p+R_0\right] \text{ for } 0<p<p_{pic}$$

$$f=\sqrt{\frac{3}{2}}s_{II}+\left[\left(\frac{\alpha_{ult}-\alpha_{pic}}{p_{ult}-p_{pic}}\right)(p-p_{pic})+\alpha_{pic}\right]I_1-\left[\left(\frac{R_{ult}-R_{pic}}{p_{ult}-p_{pic}}\right)(p-p_{pic})+R_{pic}\right] \text{ for } p_{pic}<p<p_{ult}$$

$$f=\sqrt{\frac{3}{2}}s_{II}+\alpha_{ult}I_1-R_{ult} \text{ for } p\geq p_{ult}$$:

Potential of flow:

$$g=\sqrt{\frac{3}{2}}s_{II}+\left[\left(\frac{\beta_{pic}-\beta_0}{p_{pic}}\right)p+\beta_0\right]I_1 \text{ for } 0<p<p_{pic}$$

$$g=\sqrt{\frac{3}{2}}s_{II}+\left[\left(\frac{\beta_{ult}-\beta_{pic}}{p_{ult}-p_{pic}}\right)(p-p_{pic})+\beta_{pic}\right]I_1 \text{ for } p_{pic}<p<p_{ult}$$

$$g=\sqrt{\frac{3}{2}}s_{II}+\beta_{ult}I_1 \text{ for } p\geq p_{ult}$$

α_0 , R_0 and β_0 : parameters of work hardening corresponding to the threshold of elasticity ($p=0$)
α_{pic} , R_{pic} and β_{pic} : parameters of work hardening corresponding to the parameter p_{pic}
α_{ult} , R_{ult} and β_{ult} : parameters of work hardening corresponding to the parameter p_{ult}

The law of Hooke:

$$\boldsymbol{\sigma}=\mathbf{D}^e(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{vp})$$

$f(\boldsymbol{\sigma},p)\leq 0$ field of elasticity; $\dot{\varepsilon}_{ij}^{vp}=0$

$f(\boldsymbol{\sigma},p)>0$ viscoplasticity ; $\dot{\varepsilon}_{ij}^{vp}=A\left\langle\frac{f}{P_{ref}}\right\rangle^n\frac{\partial g}{\partial\sigma_{ij}}$; $\dot{p}=\sqrt{\frac{2}{3}\dot{\varepsilon}_{ij}^{vp}\dot{\varepsilon}_{ij}^{vp}}$

4 Integration in Code_Aster

4.1 Decomposition of the tensor of deformation

The decomposition of the increment of total deflection is written:

$$\Delta\boldsymbol{\varepsilon}=\Delta\boldsymbol{\varepsilon}^e+\Delta\boldsymbol{\varepsilon}^{vp}$$

where $\Delta\boldsymbol{\varepsilon}^e$ and $\Delta\boldsymbol{\varepsilon}^{vp}$ are the increments of the elastic and viscoplastic tensors.

4.2 Update of the constraints

The following notations are adopted: A^-, A and ΔA respectively indicating the quantities at the beginning, the step of time and its increment lasting the step.
One expresses the constraints brought up to date at the moment t^+ compared to those calculated at the moment t^- :

$$\boldsymbol{\sigma}=\boldsymbol{\sigma}^-+\mathbf{D}^e\Delta\boldsymbol{\varepsilon}^e \;;\; \mathbf{s}=\mathbf{s}^-+2\mu\Delta\tilde{\boldsymbol{\varepsilon}}^e \;;\; I_1=I_1^-+3K\Delta\varepsilon_v^e$$

$$\sigma_{ij}=s_{ij}+\frac{I_1}{3}\delta_{ij}$$

$$\Delta\varepsilon_{ij}=\Delta\tilde{\varepsilon}+\mathrm{tr}\frac{(\Delta\varepsilon)}{3}\delta_{ij}=\Delta\tilde{\varepsilon_{ij}}+\frac{\Delta\varepsilon_v}{3}\delta_{ij}$$

$$I_1=\mathrm{tr}(\boldsymbol{\sigma}) \;;\; \varepsilon_v=\mathrm{tr}(\Delta\boldsymbol{\varepsilon})$$

Elastic prediction:

$$\boldsymbol{\sigma}^{el}=\boldsymbol{\sigma}^-+\mathbf{D}^e\Delta\boldsymbol{\varepsilon} \;;\; \mathbf{s}^{el}=\mathbf{s}^-+2\mu\Delta\tilde{\boldsymbol{\varepsilon}} \;;\; I_1^{el}=I_1^-+3K\Delta\boldsymbol{\varepsilon}_v$$

4.2.1 Elastic solution

Calculation of the increment of the constraints in elastic mode:

$$\Delta\sigma_{ij}=\Delta s_{ij}+\frac{\Delta I_1}{3}\delta_{ij} \;;\; \Delta\varepsilon_{ij}=\Delta\tilde{\varepsilon}_{ij}+\frac{\Delta\varepsilon_v}{3}\delta_{ij}$$

$$\Delta\sigma_{ij}=2\mu\Delta\tilde{\varepsilon_{ij}}+3K\frac{\Delta\varepsilon_v}{3}\delta_{ij}=2\mu\Delta\tilde{\varepsilon_{ij}}+K\Delta\varepsilon_v\delta_{ij}=2\mu\left(\Delta\epsilon_{ij}-\frac{\mathrm{tr}(\Delta\boldsymbol{\varepsilon})}{3}\delta_{ij}\right)+K\,\mathrm{tr}(\Delta\boldsymbol{\varepsilon})\delta_{ij}$$

$$\Delta\sigma_{ij}=2\mu\Delta\varepsilon_{ij}+\left(K-\frac{2G}{3}\right)\mathrm{tr}(\Delta\boldsymbol{\varepsilon})\delta_{ij}$$

$$\begin{Bmatrix}\Delta\sigma_{11}\\ \Delta\sigma_{22}\\ \Delta\sigma_{33}\\ \sqrt{2}\Delta\sigma_{12}\\ \sqrt{2}\Delta\sigma_{13}\\ \sqrt{2}\Delta\sigma_{23}\end{Bmatrix}=\underbrace{\begin{bmatrix}\frac{4\mu}{3}+K & K-\frac{2\mu}{3} & K-\frac{2\mu}{3} & 0 & 0 & 0\\ K-\frac{2\mu}{3} & \frac{4\mu}{3}+K & K-\frac{2\mu}{3} & 0 & 0 & 0\\ K-\frac{2\mu}{3} & K-\frac{2\mu}{3} & \frac{4\mu}{3}+K & 0 & 0 & 0\\ 0 & 0 & 0 & 2\mu & 0 & 0\\ 0 & 0 & 0 & 0 & 2\mu & 0\\ 0 & 0 & 0 & 0 & 0 & 2\mu\end{bmatrix}}_{\mathbf{D}^e}\cdot\begin{Bmatrix}\Delta\varepsilon_{11}\\ \Delta\varepsilon_{22}\\ \Delta\varepsilon_{33}\\ \sqrt{2}\Delta\varepsilon_{12}\\ \sqrt{2}\Delta\varepsilon_{13}\\ \sqrt{2}\Delta\varepsilon_{23}\end{Bmatrix}$$

4.2.2 Viscoplastic solution

One expresses the stress field at the moment t^+ :

$$\sigma_{ij}=\sigma_{ij}^-+D_{ijkl}^e\Delta\varepsilon_{kl}^e=\sigma_{ij}^-+D_{ijkl}^e\left(\Delta\varepsilon_{kl}-\Delta\varepsilon_{kl}^{vp}\right)=\sigma_{ij}^{el}-D_{ijkl}^e\Delta\varepsilon_{kl}^{vp}$$

$s_{ij}=s_{ij}^{el}-2\mu\,\Delta\,\tilde{\varepsilon}_{ij}^{vp}$ and $I_1=I_1^{el}-3\,K\,\Delta\,\varepsilon_v^{vp}$

$$\sigma_{ij}=s_{ij}+\frac{I_1}{3}\delta_{ij}$$

who is written by replacing the increase in the viscous deformations by their expressions in the form:

$\sigma_{ij}=\sigma_{ij}^{el}-D_{ijkl}^{el}\langle\Phi\rangle\frac{\partial g}{\partial\sigma_{ij}}(\boldsymbol{\sigma},p)\Delta t$ with $\Phi=A\left(\frac{f(\boldsymbol{\sigma},p)}{P_{ref}}\right)^n$ where

Φ and $\frac{\partial g}{\partial\boldsymbol{\sigma}}$ the amplitude and the direction the speed of the unrecoverable deformations characterize.

$f(\boldsymbol{\sigma},p)$ being the criterion of viscoplasticity, A and n are parameters of the model.

The viscoplastic criterion at the moment t^+ is written:

$$f(\boldsymbol{\sigma},p)=f\left(\sigma_{ij}^{el}-D_{ijkl}^{e}\langle\Phi\rangle\frac{\partial g}{\partial\sigma_{ij}}(\boldsymbol{\sigma},p)\Delta t,p\right)$$

The increment of the viscoplastic deformation being:

$$\Delta\,\varepsilon_{ij}^{vp}=\langle\Phi\rangle\frac{\partial g}{\partial\sigma_{ij}}\Delta t=\langle\Phi\rangle\left(\sqrt{\frac{3}{2}}\frac{s_{ij}}{s_{II}}+\beta(p)\delta_{ij}\right)\Delta t$$

Viscoplastic voluminal deformation being:

$$\Delta\,\varepsilon_v^{vp}=3\langle\Phi\rangle\beta(p)\Delta t$$

The deviatoric component of the viscoplastic deformation is written in the form:

$\Delta\,\tilde{\varepsilon}_{ij}^{vp}=\langle\Phi\rangle\sqrt{\frac{3}{2}}\frac{s_{ij}}{s_{II}}\Delta t$ or $\Delta\,\tilde{\varepsilon}_{ij}^{vp}=\langle\Phi\rangle\frac{3}{2}\frac{s_{ij}}{\sigma_{eq}}\Delta t$

like $\sigma_{eq}=\sqrt{\frac{3}{2}}s_{II}$, $s_{II}=\sqrt{s_{ij}s_{ij}}$ and $\sigma_{eq}^{el}=\sqrt{\frac{3}{2}s_{ij}^{el}s_{ij}^{el}}$

One writes also the following equalities:

$$s_{ij}\frac{\sigma_{eq}^{el}}{\sigma_{eq}}=s_{ij}^{el}$$

$$\Delta\,p=\sqrt{(\frac{2}{3}\Delta\,\tilde{\epsilon}_{ij}^{vp}\Delta\,\tilde{\epsilon}_{ij}^{vp})}$$

$$\frac{\Delta\,p}{\Delta\,t}=\langle\Phi\rangle=A\left\langle\frac{f(\boldsymbol{\sigma},p)}{P_{ref}}\right\rangle^n \quad \textbf{éq 1}$$

from where: $\Delta\,p=\langle\phi\rangle\Delta\,t$

By using these equalities one can find an expression for s_{ij} , σ_{eq} and I_1 according to s_{ij}^{el} , σ_{eq}^{el} , I_1^{el} and $\Delta\,p$:

$$s_{ij}=s_{ij}^{el}-2\mu\,\Delta\,\tilde{\varepsilon}_{ij}^{vp}=s_{ij}^{el}-3\,\mu\,\langle\Phi\rangle\frac{s_{ij}}{\sigma_{eq}}\Delta\,t=s_{ij}^{el}-3\,\mu\,\langle\Phi\rangle\frac{s_{ij}^{el}}{\sigma_{eq}^{el}}\Delta\,t$$

$$s_{ij}=s_{ij}^{el}\left(1-\frac{3\,\mu\langle\Phi\rangle}{\sigma_{eq}^{el}}\Delta\,t\right)=s_{ij}^{el}\left(1-\frac{3\,\mu}{\sigma_{eq}^{el}}\Delta\,p\right)$$

$$\sigma_{eq}=\sigma_{eq}^{el}-3\,\mu\,\langle\Phi\rangle\Delta\,t=\sigma_{eq}^{el}-3\,\mu\,\Delta\,p$$ **éq 2**

$$I_1=I_1^{el}-3\,K\,\Delta\,\varepsilon_v^{vp}=I_1^{el}-9\,K\,\beta\,\langle\Phi\rangle\Delta\,t=I_1^{el}-9\,K\,\beta\,\Delta\,p$$ **éq 3**

4.2.3 Calculation of the unknown factor

The viscoplastic increment of cumulated deformation $\Delta\,p$ is the only unknown factor of the problem. To determine it, one writes the viscoplastic law of flow (éq 1):

$$\frac{\Delta\,p}{\Delta\,t}=A\left\langle\frac{\sigma^{eq}+\alpha(p)I_1-R(p)}{P_{ref}}\right\rangle^n$$

$$R(p)=R(p^-+\Delta\,p)=R^-+R_{const}\Delta\,p\ \ ;\ R_{const}=\frac{\partial R}{\partial p}$$

$$\alpha(p)=\alpha(p^-+\Delta\,p)=\alpha^-+\alpha_{const}\Delta\,p\ \ ;\ \alpha_{const}=\frac{\partial\alpha}{\partial p}$$

$$\beta(p)=\beta(p^-+\Delta\,p)=\beta^-+\beta_{const}\Delta\,p\ \ ;\ \beta_{const}=\frac{\partial\beta}{\partial p}$$

By preoccupation with a simplification of the writing of the equation in $\Delta\,p$, one poses:

$$C=\frac{(A\,\Delta\,t)}{{P_{ref}}^n}$$

Maybe, while replacing σ_{eq} and I_1 by their expressions (éq 2 and éq 3), one obtains:

$$F(\Delta\,p)=C\left\langle\begin{array}{c}(\sigma_{eq}^{el}+\alpha\,I_1^{el}-R^-)-(3\,\mu+R_{const}-\alpha_{const}\,I_1^{el}+9\,K\,\alpha^-\beta^-)\Delta\,p\\ -\\ (9\,K\,\alpha^-\beta_{const}+9\,K\,\alpha_{const}\,\beta^-)\Delta\,p^2-(9\,K\,\alpha_{const}\,\beta_{const})\Delta\,p^3\end{array}\right\rangle^n-\Delta\,p=0$$

One seeks $\Delta\,p$ such as $F(\Delta\,p)=0$.

$F(\Delta\,p)=0$ is a nonlinear scalar equation. The lower limit being $x_{\inf}=0$ and the upper limit can be fixed at:

$$x_{\sup}=A\left\langle\frac{\sigma_{eq}^{el}+\alpha\,I_1^{el}-R^-}{P_{ref}}\right\rangle^n\Delta\,t$$

One uses the method of the cords with a control of the interval of research while taking as a starting point the document [R5.03.04].

$\Delta p \in [x_{\inf}, x_{\sup}]$;
$x = \Delta p$

If $|F(x_{\inf})| < \eta$ then $\Delta p = x_{\inf}$

If $|F(x_{\sup})| < \eta$ then $\Delta p = x_{\sup}$

If $F(x_{\inf}) > 0$ then $x_2 = x_{\inf}$ and $y_2 = F(x_{\inf})$

If $F(x_{\sup}) < 0$ then one makes a loop while cutting out $x_{\sup}$ by 10 until obtaining a value of $x_{\sup}$ for which $F(x_{\sup}) > 0$ in this case one multiplies the last solution by 10 and one fixes $x_1 = x_{\sup}$ and $y_1 = F(x_{\sup})$

If $F(x_{\sup}) > 0$ then one makes a loop while multiplying $x_{\sup}$ by 10 until obtaining a value of $x_{\sup}$ for which $F(x_{\sup}) < 0$ and one fixes $x_1 = x_{\sup}$ and $y_1 = F(x_{\sup})$

If $F(x_{\inf}) < 0$ then $x_1 = x_{\inf}$ and $y_1 = F(x_{\inf})$

If $F(x_{\sup}) > 0$ then one makes a loop while cutting out $x_{\sup}$ by 10 until obtaining a value of $x_{\sup}$ for which $F(x_{\sup}) < 0$ in this case one multiplies the last solution by 10 and one fixes $x_2 = x_{\sup}$ and $y_2 = F(x_{\sup})$

If $F(x_{\sup}) < 0$ then one makes a loop while multiplying $x_{\sup}$ by 10 until obtaining a value of $x_{\sup}$ for which $F(x_{\sup}) > 0$ and one fixes $x_2 = x_{\sup}$ and $y_2 = F(x_{\sup})$

Checks are made on the values which the terminals can take and in particular if they are weaker than a tolerance fixed at 1.E-12, they will be considered equal to 0. and thus the solution Δp also. If the terminals are equal, one makes a recutting of the step of time.

Values x_1 , x_2 , y_1 and y_2 will be the values to be given as starter to the routine `zeroco` who bases himself on the method of the cords. The solution is calculated by the following formula:

$$x^{n+1} = x^{n-1} - F(x^{n-1}) \frac{x^n - x^{n-1}}{F(x^n) - F(x^{n-1})}$$

With the following values, one represents the scalar function to solve.

σ_{eq}^{el}	$6{,}315\ MPa$	α^-	$6{,}86\ 10^{-2}$
I_1^{el}	$-21{,}061\ MPa$	β^-	$-0{,}147$
N	$4{,}5$	R^-	$1{,}394\ MPa$
Δt	$10\ s$	α_{const}	$13.$
A	$1{,}5\ 10^{-12}$	β_{const}	$10.$
P_{ref}	$0{,}1\ MPa$	R_{const}	$329{,}732\ MPa$

The unknown factor x for which $F(x)$ cancel yourself is located between 6.10^{-5} and 7.10^{-5} who is located well between the lower limit x_{inf} and the upper limit $x_{\sup}$ who is worth in this precise case $1{,}2913\,10^{-4}$.

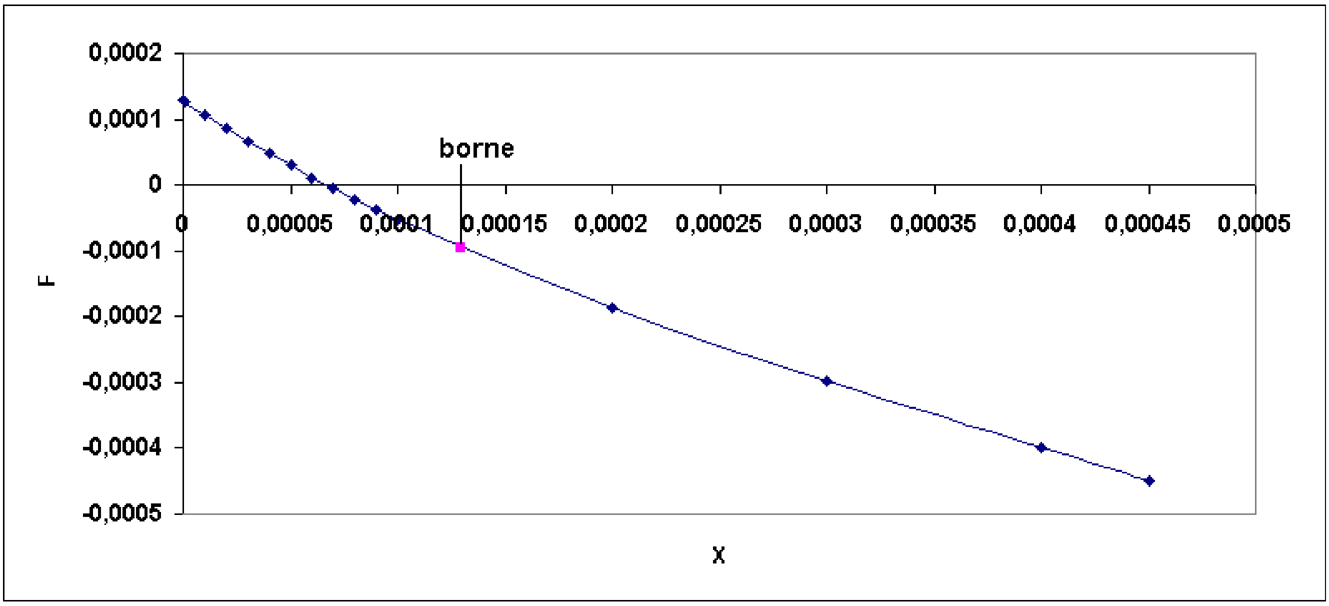

Figure 4-1: Pace of the scalar function

4.3 Coherent tangent operator

One seeks to calculate: $\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}}=\frac{\partial \mathbf{s}}{\partial \boldsymbol{\varepsilon}}+\frac{1}{3}\mathbf{Id}\otimes\frac{\partial I_1}{\partial \boldsymbol{\varepsilon}}$

With:

$$\frac{\partial \mathbf{s}}{\partial \boldsymbol{\varepsilon}}=\frac{\partial \mathbf{s}^{el}}{\partial \boldsymbol{\varepsilon}}\left(1-\frac{3\mu}{\sigma_{eq}^{el}}.\Delta p\right)+\frac{3\mu}{\left(\sigma_{eq}^{el}\right)^2}.\Delta p\left(\mathbf{s}^{el}\otimes\frac{\partial \sigma_{eq}^{el}}{\partial \boldsymbol{\varepsilon}}\right)-\frac{3\mu}{\sigma_{eq}^{el}}.\left(\mathbf{s}^{el}\otimes\frac{\partial \Delta p}{\partial \boldsymbol{\varepsilon}}\right)$$

$$\frac{\partial I_1}{\partial \boldsymbol{\varepsilon}}=\frac{\partial I_1^{el}}{\partial \boldsymbol{\varepsilon}}-9K\beta(p)\frac{\partial \Delta p}{\partial \boldsymbol{\varepsilon}}$$

Calculation of $\frac{\partial \mathbf{s}^{el}}{\partial \boldsymbol{\varepsilon}}$:

$$\frac{\partial \mathbf{s}^{el}}{\partial \boldsymbol{\varepsilon}}=2\mu\left(\mathbf{Id}_4-\frac{1}{3}\mathbf{Id}\otimes\mathbf{Id}\right)$$

$$\frac{\partial s_{ij}}{\partial \varepsilon_{pq}}=2\mu\left(\delta_{ip}\delta_{jq}-\frac{1}{3}\delta_{ij}\delta_{pq}\right)$$

Calculation of $\frac{\partial I_1^{el}}{\partial \boldsymbol{\varepsilon}}$:

$\frac{\partial I_1^{el}}{\partial \boldsymbol{\varepsilon}}=3K\,\mathbf{Id}$ that is to say:

$$\frac{\partial I_1^{el}}{\partial \varepsilon_{pq}}=3K\,\delta_{pq}$$

Calculation of $\frac{\partial \sigma_{eq}^{el}}{\partial \boldsymbol{\varepsilon}}$:

$$\frac{\partial \sigma_{eq}^{el}}{\partial \boldsymbol{\varepsilon}}=\frac{\partial \sigma_{eq}^{el}}{\partial \boldsymbol{\sigma}^{el}}\frac{\partial \boldsymbol{\sigma}^{el}}{\partial \boldsymbol{\varepsilon}}=\frac{\partial \sigma_{eq}^{el}}{\partial \mathbf{s}^{el}}\frac{\partial \mathbf{s}^{el}}{\partial \sigma^{el}}\frac{\partial \sigma^{el}}{\partial \boldsymbol{\varepsilon}}=\frac{3}{2}\frac{\mathbf{s}^{el}}{\sigma_{eq}^{el}}\left(\mathbf{Id}_4-\frac{1}{3}\mathbf{Id}\otimes\mathbf{Id}\right)D^e=\frac{3}{2}\frac{\mathbf{s}^{el}}{\sigma_{eq}^{el}}\mathbf{D}^e$$

Calculation of $\frac{\partial \Delta p}{\partial \boldsymbol{\varepsilon}}$:

$$\frac{\Delta p}{\Delta t}=A\left\langle\frac{f(\boldsymbol{\sigma},p)}{P_{ref}}\right\rangle^n$$

that is to say $F(\Delta p)=\frac{A\Delta t}{P_{ref}^n}\langle f(\boldsymbol{\sigma},p)\rangle^n-\Delta p$

$$\frac{\partial \Delta p}{\partial \boldsymbol{\varepsilon}}=\frac{\partial \Delta p}{\partial \boldsymbol{\sigma}^{el}}\frac{\partial \boldsymbol{\sigma}^{el}}{\partial \boldsymbol{\varepsilon}}$$

to calculate $\frac{\partial \Delta p}{\partial \boldsymbol{\sigma}^{el}}$, one uses $F\left(\boldsymbol{\sigma}^{el},p\right)=0$

$\frac{\partial F\left(\boldsymbol{\sigma}^{el},p\right)}{\partial \boldsymbol{\sigma}^{el}}\delta\boldsymbol{\sigma}^{el}+\frac{\partial F\left(\boldsymbol{\sigma}^{el},p\right)}{\partial \Delta p}\delta\Delta p=0$ from where:

$$\frac{\delta\Delta p}{\delta\boldsymbol{\sigma}^{el}}=-\frac{\frac{\partial F\left(\boldsymbol{\sigma}^{el},p\right)}{\partial \boldsymbol{\sigma}^{el}}}{\frac{\partial F\left(\boldsymbol{\sigma}^{el},p\right)}{\partial \Delta p}}$$

$$F(\Delta p)=C\left\langle\begin{matrix}(\sigma_{eq}^{el}+\alpha I_1^{el}-R^-)-(3\mu+R_{const}-\alpha_{const}I_1^{el}+9K\alpha^-\beta^-)\Delta p\\ -\\ (9K\alpha^-\beta_{const}+9K\alpha_{const}\beta^-)\Delta p^2-(9K\alpha_{const}\beta_{const})\Delta p^3\end{matrix}\right\rangle^n-\Delta p=0$$

$$\frac{\partial F\left(\boldsymbol{\sigma}^{el},p\right)}{\partial \boldsymbol{\sigma}^{el}}=C.n.\langle f(\boldsymbol{\sigma}^{el},p)\rangle^{n-1}\frac{\partial f\left(\boldsymbol{\sigma}^{el},p\right)}{\partial \boldsymbol{\sigma}^{el}}$$

where $\frac{\partial f\left(\boldsymbol{\sigma}^{el},p\right)}{\partial \boldsymbol{\sigma}^{el}}=\left(\frac{\partial \sigma_{eq}^{el}}{\partial \boldsymbol{\sigma}^{el}}+\alpha\frac{\partial I_1^{el}}{\partial \boldsymbol{\sigma}^{el}}\right)+\alpha_{const}\left(\frac{\partial I_1^{el}}{\partial \boldsymbol{\sigma}^{e}}\right)\Delta p$

$$\frac{\partial F\left(\boldsymbol{\sigma}^{el},p\right)}{\partial \Delta p}=C.n.\left\langle f\left(\boldsymbol{\sigma}^{el},p\right)\right\rangle^{n-1}\frac{\partial f\left(\boldsymbol{\sigma}^{el},p\right)}{\partial \Delta p}-1$$

$$\frac{\partial f\left(\boldsymbol{\sigma}^{el},p\right)}{\partial \Delta p}=-\left(3\mu+R_{const}-\alpha_{const}I_1^{el}+9K\alpha^-\beta^-\right)$$
$$-2\Delta p\,9K\left(\alpha^-\beta_{const}+\alpha_{const}\beta^-\right)-3\Delta p^2\,9K\left(\alpha_{const}\beta_{const}\right)$$

Calculation of $\frac{\partial \sigma_{eq}^{el}}{\partial \boldsymbol{\sigma}^{el}}$:

$$\frac{\partial \sigma_{eq}^{el}}{\partial \boldsymbol{\sigma}^{el}}=\frac{\partial \sigma_{eq}^{el}}{\partial \mathbf{s}^{el}}\frac{\partial \mathbf{s}^{el}}{\partial \boldsymbol{\sigma}^{el}}=\frac{3}{2}\frac{\mathbf{s}^{el}}{\sigma_{eq}^{el}}.\left(\mathbf{I}_4-\frac{1}{3}\mathbf{Id}\otimes\mathbf{Id}\right)=\frac{3}{2}\frac{\mathbf{s}^{el}}{\sigma_{eq}^{el}}$$

Calculation of $\frac{\partial I_1^{el}}{\partial \boldsymbol{\sigma}^{el}}$:

$$\frac{\partial I_1^{el}}{\partial \boldsymbol{\sigma}^{el}}=\frac{\partial \operatorname{tr}\left(\boldsymbol{\sigma}^{el}\right)}{\partial \boldsymbol{\sigma}^{el}}=\mathbf{Id}$$

4.4 Data materials

The 16 parameters of the model are:

under `ELAS`

E	:	Young modulus (Pa or MPa)
ν	:	Poisson's ratio

under `VISC_DRUC_PRAG`

P_{ref}	:	pressure of reference (Pa or MPa)
A	:	viscoplastic parameter (in s^{-1})
n	:	power of the law creep
p_{pic}	:	rate of work hardening on the level of the threshold of peak
p_{ult}	:	rate of work hardening on the level of the ultimate threshold
α_0 , α_{pic} and α_{ult}	:	parameters of the function of cohesion $\alpha(p)$
R_0 , R_{pic} and R_{ult}	:	parameters of the function of work hardening $R(p)$
β_0 , β_{pic} and β_{ult}	:	parameters of the function of dilatancy $\beta(p)$

4.5 Internal variables

$v_1=p$: cumulated deviatoric viscoplastic deformation;

$v_2=(0\text{ ou }1)$: indicator of plasticity;

$v_3=pos$: position of the point of load compared to the threshold:

($\mathrm{pos}=1$ si $0<p<p_{\mathrm{pic}}$; $\mathrm{pos}=2$ si $\mathrm{p_{pic}}<p<p_{\mathrm{ult}}$; $\mathrm{pos}=3$ si $\mathrm{p}>p_{\mathrm{ult}}$)

v_4 : iteration count local.

4.6 Summary of the algorithm of resolution

The algorithm of resolution such as it is established in Code_Aster:

$$\boldsymbol{\sigma}^{el}=\boldsymbol{\sigma}^{-}+\mathbf{D}^{e}\Delta\,\boldsymbol{\varepsilon}$$

The criterion: $f\left(\boldsymbol{\sigma}^{el},p^{-}\right)=\sigma_{eq}^{el}+\alpha\left(p^{-}\right)I_1^{el}-R\left(p^{-}\right)$

Elasticity: if $f\left(\boldsymbol{\sigma}^{el},p^{-}\right)\leq 0$ then $\Delta\,p=0$;

Viscoplasticity: if $f\left(\boldsymbol{\sigma}^{el},p^{-}\right)=0$ then $\Delta\,p\geq 0$ with $\Delta\,p$ solution of the equation $F\left(\Delta\,p\right)=0$

where:

$$\frac{\Delta\,p}{\Delta\,t}=A\left\langle\frac{\sigma^{eq}+\alpha\left(p\right)I_1-R\left(p\right)}{P_{ref}}\right\rangle^{n}=\frac{A}{P_{ref}^{n}}\left\langle f\left(\boldsymbol{\sigma},p\right)\right\rangle^{n}$$

and

$$F=\frac{A\,\Delta\,t}{P_{ref}^{n}}\left\langle f\left(\boldsymbol{\sigma},p\right)\right\rangle^{n}-\Delta\,p$$

Update of the tensor of the constraints:

$$\boldsymbol{\sigma}=\boldsymbol{\sigma}^{el}-\mathbf{D}^{e}\Delta\,\boldsymbol{\varepsilon}^{vp}$$

$$\mathbf{s}=\mathbf{s}^{el}\left(1-3\frac{\mu}{\sigma_{eq}^{el}}\Delta\,p\right)$$

$$\sigma_{eq}=\sigma_{eq}^{el}-3\,G\,\Delta\,p$$

$$I_1=I_1^{el}-9\,K\,\beta\,\Delta\,p$$

$$\boldsymbol{\sigma}=\mathbf{s}+\frac{1}{3}I_1\,\mathbf{Id}$$

Once $\Delta\,p$ is calculated, the constraints and the up to date put internal variables, one checks the position of p compared to p^{-} and signs it $f\left(\sigma,p\right)$:

If $0<p^{-}<p_{pic}$; to test 1) if not 2) if not 3)
If $p_{pic}<p^{-}<p_{ult}$; to test 2) if not 3)
If $p^{-}>p_{ult}$; to test 3)

If $p^{-}+\Delta\,p<p_{pic}$;

one checks $f\left(\boldsymbol{\sigma},p\right)>0$ with R , α and β corresponding to $0<p<p_{pic}$,

if $f(\boldsymbol{\sigma}, p) > 0$ then one updates the fields of constraints
and of internal variables,
if not, it is considered that Δp is not valid and one Re-cutting the step of time

If $p_{pic} < p^{-} + \Delta p < p_{ult}$;

one checks $f(\boldsymbol{\sigma}, p) > 0$ with R, α and β corresponding to $p_{pic} < p < p_{ult}$
if $f(\boldsymbol{\sigma}, p) > 0$ then one updates the fields of constraints
and of internal variables,
if not, it is considered that Δp is not valid and one redécoupe the step of time

If $p^{-} + \Delta p \geq p_{ult}$;

one checks $f(\boldsymbol{\sigma}, p) > 0$ with R, α and β corresponding to $p \geq p_{ult}$
if $f(\boldsymbol{\sigma}, p) > 0$ then one puts up to date fields of constraints
and of internal variables,
if not, it is considered that Δp is not valid and one redécoupe the step of time

5 Results of a triaxial compression test

It is a question of simulating a triaxial compression test (see the case test ssnv211) while imposing like a constraint of containment of $5\,MPa$. A uniaxial deformation is imposed in compression and which evolves in time.

The speed of the loading is fixed at $10^{-5}\,m/s$. The diverter of the constraints and the voluminal deformation according to the imposed axial deformation are represented Ci against.

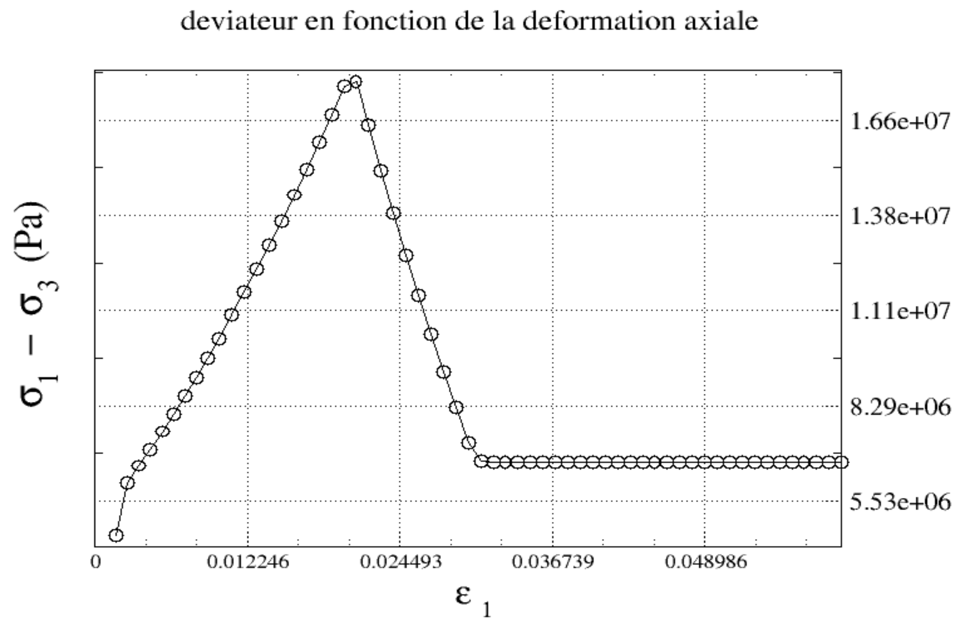

Figure 5-1: Diverter of the constraints according to the uniaxial deformation

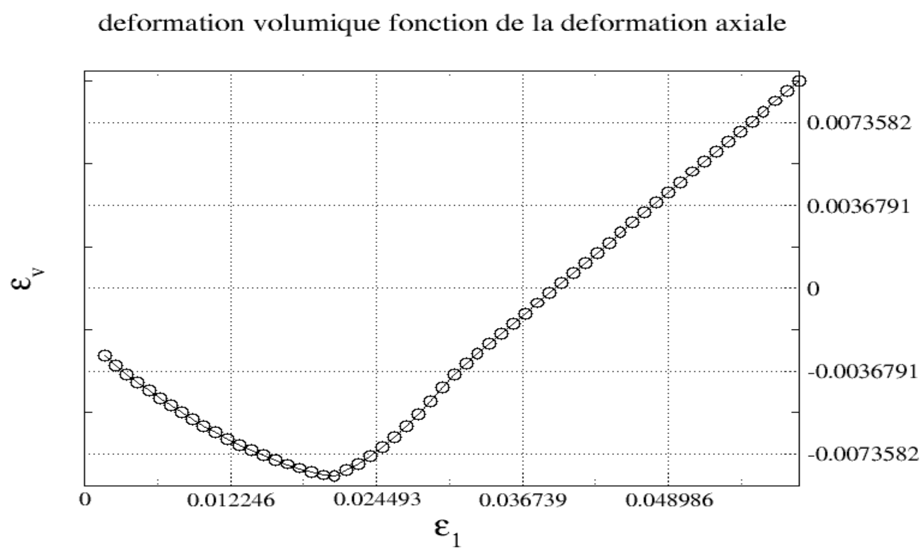

Figure 5-2: Voluminal deformation according to the uniaxial deformation

6 Features and checking

The law of behavior can be defined by the keyword VISC_DRUC_PRAG (order STAT_NON_LINE, keyword factor BEHAVIOR). It is associated with material VISC_DRUC_PRAG (order DEFI_MATERIAU).

The law VISC_DRUC_PRAG is checked by the cases following tests:

SSNV211	[V6.04.211]	Triaxial compression test drained with the model VISC_DRUC_PRAG
WTNV137	[V7.31.137]	Triaxial compression test drained with the model VISC_DRUC_PRAG
WTNV138	[V7.31.138]	Triaxial compression test not drained with the model VISC_DRUC_PRAG

7 Références

[1] J. EL GHARIB and C. CHAVANT, “Chock on triaxial compression tests of a viscoplastic law of behavior for the mudstone based on the Drucker_Prager model”, H-T64-2008-04194-FR,

[2] J. EL GHARIB and C. CHAVANT, “Put in work in Code_Aster of a simplified viscoplastic model”, H-T64-2007-01800-FR,

8 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
10.0	J. EL GHARIB, C.CHAVANT EDF R & D/AMA	Initial text
14.4	F.Voldoire EDF R & D/ERMES	Small corrections of form of the equations