Laws of behavior of the joints of the stoppings: JOINT_MECA_RUPT and JOINT_MECA_FROT.

Summary:

This document describes surface laws making it possible to model the rupture and friction between the lips of a crack or a joint. The law JOINT_MECA_RUPT is based on a cohesive formulation of the rupture, the law JOINT_MECA_FROT is an elastoplastic version of the law of Mohr-Coulomb friction in pure mechanics.

In mechanics those are based on modelings of the standard joints XXX_JOINT. The laws are dedicated to the modeling of the stoppings, more precisely of the joints concrete/rock or the joints between the studs of a stopping. According to the type of loading the use of one or other law can be selected for various parts of the work. To be able to simulate the behavior of the real stoppings within the framework of the same modeling, certain specificities of construction were introduced, in particular keying-up and sawed itGe. Keying-up is presented in the form of an injection of the concrete under pressure between the studs of a stopping. It is an intermediate stage of the construction of an arch dam, it is used to reinforce its sealing after the phase of construction of vertical studs. Sawing is a similar procedure during which the stopping is sawn in order to slacken the constraints. This can intervene before or after put out of water. Each procedure is defined via the keywords PRES_CLAVAGE and SAWING in DEFI_MATERIAU. One introduces also the effect of pressure hydrostatic without coupling due to the presence of fluid (option PRES_FLUIDE for the two laws).

These laws also admit a hydraulic coupled modeling (XXX_JOINT_HYME) taking into account the propagation of the uplifts to the interface stopping-rock.

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1 Introduction

The incidents which have occurred on concrete dams (Bouzey 1895, Malpasset 1959), as well as the results of sounding, raised that their stability, and consequently their security, depend very largely on the hydraulic behavior of the weakest zones of the valley-stopping unit. Located on the level of discontinuities in the structure and the rock, these weak points are mainly the faults of the zones of support, the concreting resumptions in the stopping, the contact concrete-rock of the foundation and the joints between the studs of the stopping. The mechanical behavior of these zones at the risk is strongly non-linear, but thanks to their surface character, the industrial studies on the important works are complex but possible. Besides these difficulties, the method of construction of the stoppings, the techniques of keying-up/sawing used and its multiple points of drainage make of them works whose modeling by finite elements is all the more complex in a conventional computer code.

The two laws of behavior described in this document, make it possible to take into account principal non-linearities of the behavior of the works: the phase of opening of crack (`JOINT_MECA_RUPT`) and the phase of slip of its lips (`JOINT_MECA_FROT`).

1.1 Joints

As mentioned previously, the joints of the stoppings have a varied origin (Figure 1.1-1). Generally, one can to represent the joint like a rough discontinuity possibly reinforced by a fill material.

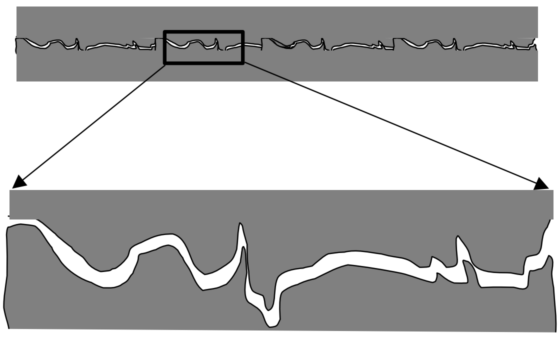

Figure 1.1-1: Physical image of joint

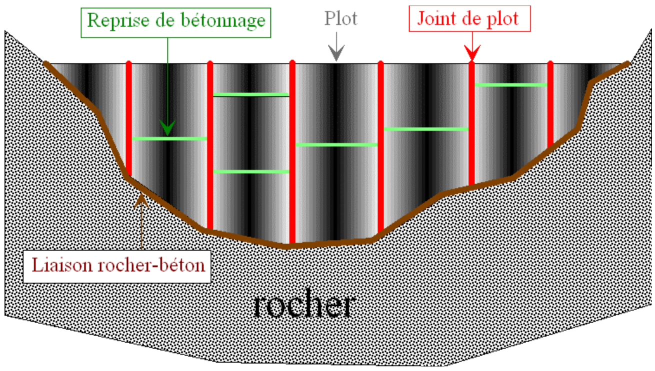

Figure 1.1-2: Various types of joint of a stopping

The law of friction of Coulomb, which uses one parameter (coefficient of friction), collects only one negligible part of the very complex mechanical behavior of such a structure. Indeed, except for friction, the joint displays the following important phenomena: loss of tensile strength, elastic behaviour with very weak displacement, the progressive disappearance of the peak of stress shear for a loading cyclic. The relevance of these phenomena strongly depends on several physical parameters in particular of the level of roughness, the mean size of the asperities, the mechanical properties of fill materials and the rock matrix (as the Young modulus, the Poisson's ratio, or the coefficient of friction).

The complete modeling of the joint thus requires the introduction of a law depending on many parameters. Implementation of such law in an implicit computer code not being possible, we propose simpler laws, dependent on few parameters but which make it possible nevertheless to collect the essential behavior of the joint in most current conditions of use. These last are baptized `JOINT_MECA_RUPT` and `JOINT_MECA_FROT`. They make it possible to model the behavior of the joints which one finds between the studs of an arch dam and/or with the interface between the gravity dam and his foundation. We make a brief description Ci of it below, before detailing them in the parts which follow.

1.2 Laws of mechanical behavior

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The law JOINT_MECA_RUPT is a lenitive elastic law, whose normal behavior is founded on the cohesive formulation of the rupture. It open the possibility of rupture in mode I (traction) and takes into account the coupling between the normal opening and tangential rigidity. This law collects well the behavior of real joints with weak displacements, as long as the mode of slip is not reached.

To identify the modes in which it is applicable one can take again the physical image of the joint. They are two rough interfaces possibly containing a fill material between its lips; this can be either of clay, or the elements of the rock for the cracks stopping-foundation, or of the coulis of keying-up for the joint-studs. A request of the crack puts initially concerned the properties of fill material and the geometry of the asperities which define the behavior of the structure in weak displacements. As long as the fill material is not damaged or as the asperities are not broken, the behavior of the joint remains elastic as well in opening, that in shearing. However parameters of normal and tangential rigidity, noted K_n and K_t, are not equivalent, because they utilize two distinct physical phenomena. The first depends mainly on rigidity of fill material, the second depends on advantage of rigidity in inflection of the asperities. The joint has a tensile strength, noted σ_{max}, which can be connected to fill materials, but also with transverse frictions between the asperities of the two lips of the crack.

The rupture of the joint occurs in a progressive way. Indeed, the joint is damaged initially by partially decreasing its rigidity before breaking completely. To quantify this phenomenon we introduce an adimensional parameter of penalization in rupture P_{rupt}, which represents a relative opening in softening compared to the elastic opening. For the values of P_{rupt} weak, the rupture is brutal. For large values $P_{rupt} \gg 1$ the passage is more progressive, but this will increase the energy of initial dissipation significantly. The details of this behavior will be presented in the following parts.

In order to model Lbe various types of profiles of the joints of stud of the stoppings (represented on the Figure 1.2-1 below), we introduce into the user interface a parameter $\alpha \in [0,2]$ additional. This one binds the normal opening of joint with the fall of its tangential rigidity. Physically it reflects the depth of the asperities and varies continuously enters 0 and 2. The value $\alpha=0$ corresponds to the smooth interface without asperities (tangential rigidity falls to zero as of the normal opening of joint, to see Figure 1.2-1 on the right). The value $\alpha=2$ represent another extreme case, where the interface is very rough with an infinite depth of the asperities, of the profile of a joint of stud in crenel (Figure is the equivalent 1.2-1 on the left). In this case rigidity tangential is not affected by the normal opening. The value by default is fixed at 1, which represents an intermediate situation.

If one requests the joint in pure shearing more, it will end up slipping with a certain coefficient of friction. Before passing in this plastic mode, one observes in experiments a peak of the force of friction. This phenomenon is related to the fact that to be able to slip, the asperities must leave their inserted position (dilatancy). During this phase the rubbing contacts are not inevitably parallel to the surface of the lips, which increases the coefficient of effective friction. This procedure of “exit” is accompanied besides the increase by normal displacement by the joint. If one repeats this cycle several times, the peak of constraint attenuates and disappears completely. One notes that the deeper the asperities are, the more the peak of friction is important. One can even imagine the borderline case where in spite of the normal opening, the joint remains always elastic on the level of the tangential behavior.
The law JOINT_MECA_FROT does not take into account the peak, it represents only the phenomenon of pure slip, characterized by the coefficient of friction μ and adhesion c, which is related to tensile strength $R_t = c/\mu$. The loss of tensile strength is not taken into account in the current version of friction.

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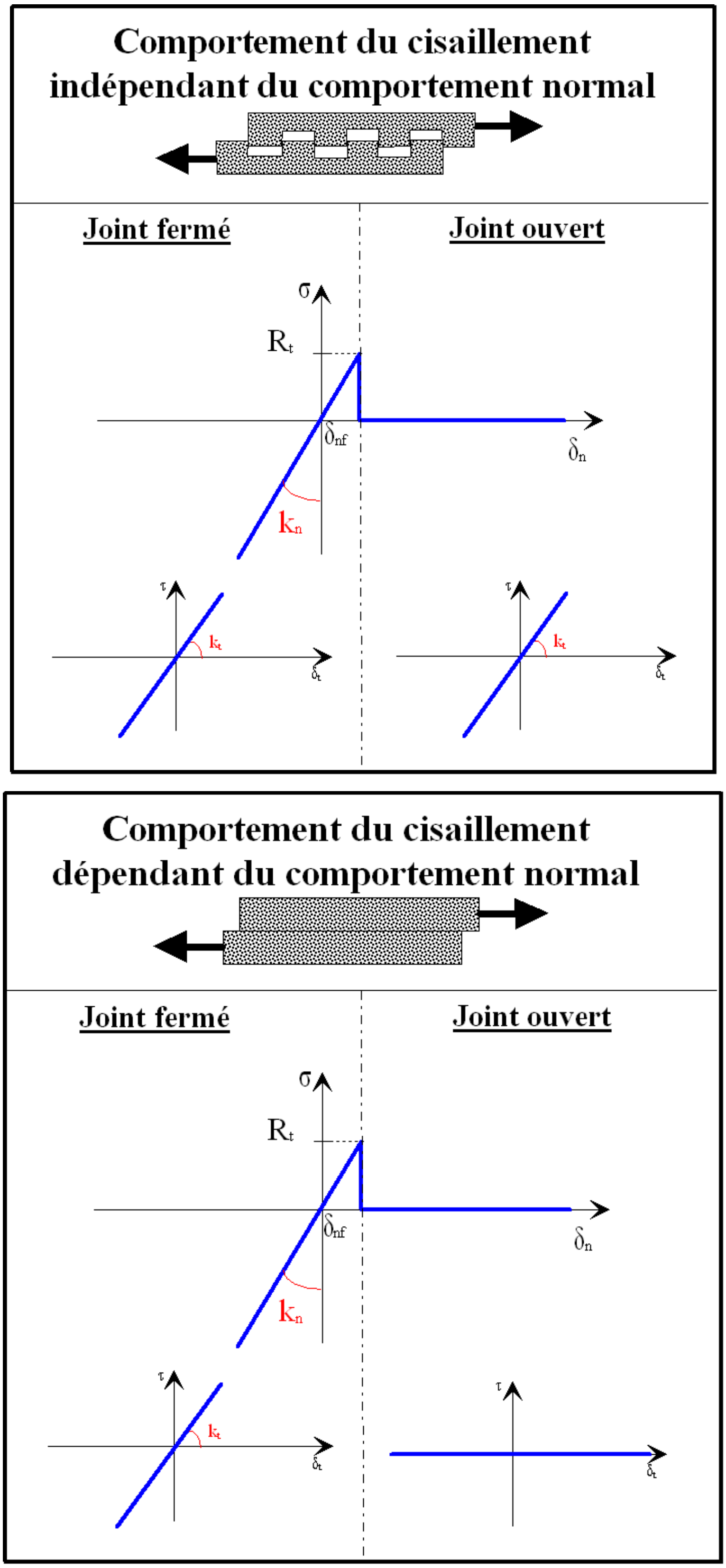

Figure 1.2-1: Normal and tangential behavior according to profile of joint

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1.3 Hydraulic coupling

To evaluate the stability of the stoppings it is important to be able to model the propagation of the uplifts between the rock and the foundation of the stopping. Two opportunities are given to user. Initially, a profile of uplift can be imposed, like a parameter of entry of the loading. This simple possibility makes it possible to study the stability of the stopping for a conservative hydraulic loading (the least favorable). In addition, the speed of calculation has a significant advantage. One can test a hydraulic profile thus extreme without spending more time than in pure mechanical calculation. Moreover, this functionality makes it possible to do a calculation of chaining hydromécanique. This one consists in starting by a mechanical calculation with an initial state of pressure in the joints. According to the damage of the latter, profile of pressure (of which the form is given *a priori*) is updated. Once the modified pressure, the mechanical state of the joints evolves again, which can generate the rupture of some of them. The fluid is propagated then more easily and the profile of pressure undergoes an evolution again. This process is chained thanks to a loop of point fixes in the command file in order to obtain states converged mechanics and hydraulics.

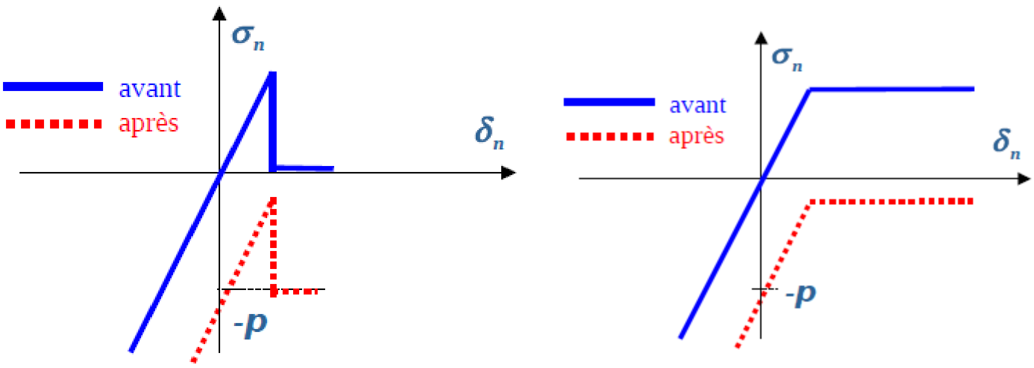

Figure 1.3-1: Taking into account of the hydrostatic pressure. Shift of relation of normal behavior for the law `JOINT_MECA_RUPT` (left) and the law `JOINT_MECA_FROT` (right-hand side)

From the theoretical point of view, the introduction of the fluid into the joint modifies the normal mechanical constraint $\sigma_n \rightarrow \sigma_n - p$. In practice, the law of behavior in question is shifted downwards according to the value of pressure p in each point of integration (cf Figure 3.7).

These laws accept also one modeling hydromechanics ***coupled***, named `xxx_JOINT_HYME`. The difference between the model with ***chaining*** hydromechanics (presented above) relates to the more precise taking into account of the action of mechanics on hydraulics. Indeed, in the model chained the profile is given *a priori*. In the coupled model, the opening of the joint amends the law of flow of the fluid, the profile of pressure is an unknown factor of the problem. At the time of the hydraulic coupling, to model the flow, the mechanical law is enriched by taking of account the cubic flow by One tenth of a poise, which is regularized for very weak openings of crack. Thus, the profile of pressure any more is not imposed, but is calculated during simulation. Besides the standard mechanical equation one solves simultaneously the following equation of flow:

$$\mathrm{div}\,\vec{w} = 0 \; ; \vec{w} = \frac{\rho}{12\mu} \delta_n^3 \vec{\nabla} p \qquad (1)$$

where $\vec{w}$ corresponds to hydraulic flow, δ_n is the normal opening of joint, finally ρ and μ indicate the mass respectively voluminal and the dynamic viscosity of the fluid.

1.4 Procedure of keying-up

Keying-up is a key stage during the construction of an arch dam. It results in a grouting of concrete under pressure between the studs of the stopping. It is thus important to be able to model correctly this process. In practice the casting of the concrete breaks up into several stages where the concrete is injected at various places and varied pressures. From the mechanical point of view keying-up is

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interpreted by a setting in compression of the lips of joint clavé until $\sigma_n = -\sigma_{nc}$ (pressure of the concrete injected, keyword `PRES_CLAVAGE`).

This option is directly included in the parameters of the law of behavior. Physically keying-up is accompanied by a process of solidification of the coulis injected, the procedure is modelled by the modification thickness of the joints concerned. If the joint is in strong compression initially, keying-up does not influence it. So on the other hand the joint is opened or not sufficiently compressed ($\sigma_n > -\sigma_{nc}$), keying-up will result in the change of the parameter thickness of the noted joint δ_{nf} . And, consequently, of a translation of the normal constraint.

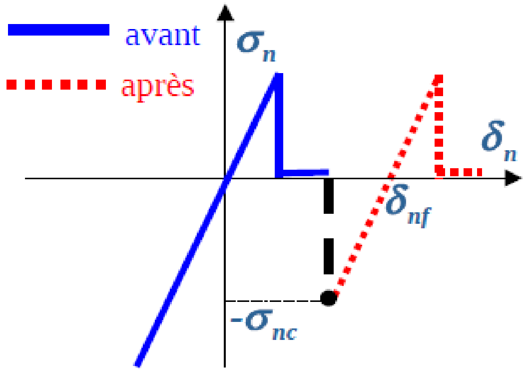

Figure 1.4-1: Illustration of the procedure of keying-up

The normal behavior of the joint after keying-up can be modified. Thus, the tensile strength can be restored (Figure 1.4-1) either partially, or completely according to the damage of the joints before the procedure of keying-up. The implemented procedure consists in not restoring it, this one keeps its current price.

1.5 Procedure of sawing

It was shown by tests *in situ* that the concrete of certain hydraulic works in France suffered damage which had with the not-known chemical reaction alkali-aggregate at the time of the massive construction of the stoppings before war. The most notable example is the dam Chambon located on the Romansh one, which hasclose 80 years of existence was victim of the phenomenon of disease of concrete (the RAG), consequently every fifteen years it must undergo a particular treatment. Operations of microsciage of the structure make it possible to prolong its lifetime. As its name indicates it, this procedure consists in sawing the stopping in order to slacken the compressive stresses, which are developed in the course of time. It is thus important to be able to model this process correctly. From the digital point of view, sawing is very close to the procedure of keying-up: the thickness of joint evolves during the operation. Nevertheless, put except for the fact that this time the thickness of joint decreases, its evolution is also controlled by the displacement which is to be compared with piloting in pressure at the time the procedure of keying-up. The phase of sawing is characterized by the thickness of saw indicated under a keyword `SAWING`.

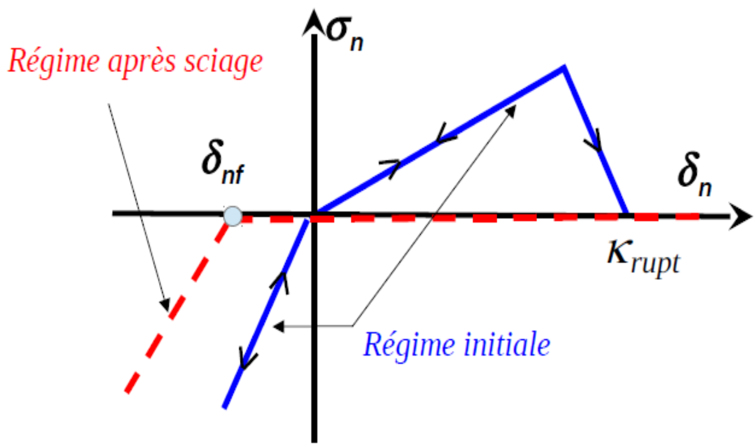

Figure 1.5-1: Illustration of the procedure of sawing

This option is directly included in the parameters of the law of behavior. In practice, so

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to make the phenomenon close to physical reality, the sawn thickness is decreased by the initial opening of joint before the operation. If the joint is in strong initial opening, sawing will not influence it. So on the other hand the joint is compressed, it will undergo sawing complete. Numerically it will result in the change of the parameter thickness of the noted joint δ_{nf}, and, consequently, of a translation of the normal constraint, such as:

$$\delta_{nf}=max(0,\delta_n^-)-\delta_{scie} \quad (2)$$

It is also considered that the joint in its final state is damaged completely by sawing. This last, even if it is put in traction thereafter, will not thus develop any tensile strength. The procedure is explicit, compound with the elastic discharge for the total structure which it generates, results in fast digital convergence from modeling. At more one puts two iterations of Newton, whose first is used to make updated the thickness of the joints and the second is a phase of linear discharge.

Considering their current digital implementation the phenomena of sawing and keying-up cannot be chained on the same joint.

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1.6 Limit of application and vocabulary

The laws of behavior presented in this section are simple, robust, depend on few parameters and have the advantage significant to be based on a theoretical formalism tested in the scientific literature. The principal parameters are the following:

Physical parameter	Denomination Aster	Value advised for the concrete dam
K_n Normal rigidity	K_N	$K_n=3\cdot 10^{12}\,\mathrm{Pa/m}$
K_t Tangential rigidity	K_T	Defect $K_t=K_n$
σ_{max} Threshold of rupture	SIGMA_MAX	$\sigma_{max}=3\,\mathrm{MPa}$
P_{rupt} Penalization rupture	PENA_RUPT	Defect $P_{rupt}=1$
P_{cont} Penalization contacts	PENA_CONTACT	Defect $P_{cont}=1$
α Relative roughness	ALPHA	Defect $\alpha=1$
p_{flu} Internal fluid pressure	PRES_FLUIDE	Defect $p_{flu}=0\,\mathrm{Pa}$ (absence of fluid)
σ_{nc} Pressure of keying-up	PRES_CLAVAGE	Defect $\sigma_{nc}=-1\,\mathrm{Pa}$ (not of keying-up)
δ_{scie} Size of saw	SAWING	Defect $\delta_{\mathrm{scie}}=0\,\mathrm{m}$ (not of sawing)
$\bar{\mu}$ Dynamic viscosity of fluid	VISC_FLUIDE	$\bar{\mu}=10^{-3}\,\mathrm{Pa\cdot s}$ [1]
ρ Density	RHO_FLUIDE	$\rho=1000\ \mathrm{kg/m^3}$
ϵ_{min} Minimal opening of joint	OUV_MIN	$\epsilon_{min}=10^{-8}\,\mathrm{m}$
μ Coefficient of friction	DRIVEN	$\mu=1$
c Adhesion	ADHESION	Defect $c=0\ \mathrm{Pa}$
K Work hardening	PENA_TANG	Defect $K=(K_n+K_t)\cdot 10^{-6}$

These laws currently do not make it possible to model the behavior in the phase of transition rupture-friction. For the law of friction, which has a tensile strength nonworthless, the rupture of joint is not implemented. Moreover modeling of the hydraulic coupling limits itself to the law of rupture.

1 This value being a hydraulic multiplier of flow, its unit can be selected in order to have flows and the mechanical constraints of the same order of magnitude, which simplifies the analysis of error.

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2 Theoretical formulation of JOINT_MECA_RUPT

The law JOINT_MECA_RUPT accept a hydraulic coupled modeling, but these two phenomena can be treated separately. Initially one will describe the mechanical part of the law, which includes rupture, contact, procedure of keying-up and imposed pressure. It is based on modelings XXX_JOINT (R3.06.09).

Notice : The two next sections introduce to the theoretical concept general. In first reading one can omit them and pass directly to the section §2.3 , which provides sufficient elements to understand the digital implementation of the law.

2.1 Cohesive law in mechanics

The selected theoretical framework to model the rupture of the joints of dams concrete is based on the cohesive models (R7.02.11, [Bar62], [Lav04]), because in brittle fracture, to avoid the problem of infinite constraints in bottom of crack, one can introduce forces of cohesion which impose a criterion of starting in constraint. The forces in evanescent matter are then exerted between the particles on both sides of the plan of separation of the crack during its opening (see Figure 2.1-1).

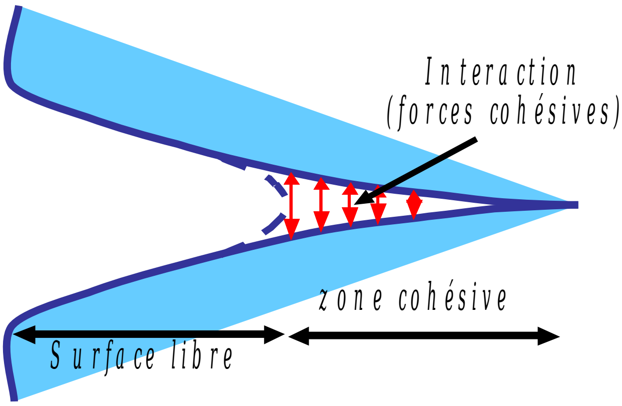

Figure 2.1-1: Diagram of a cohesive crack

From the physical point of view one considers that the opening of the crack costs an energy proportional to its length in 2D and on its surface in 3D. It is called energy of surface which one expresses using the density of energy $\Psi=\int_\Gamma \psi(\vec{\delta})\,d\Gamma$ [2]. field of displacement to balance $\mathbf{u}$ is obtained by minimizing the sum of elastic energy Φ , energy of surface, and work of the external efforts $W^{ext.}$. The solution is obtained by using a variational approach of the rupture. The state which carries out the minimum of total energy corresponds in a state of mechanical balance:

$$\min_{\mathbf{u}}\left(\Phi(u)+\Psi(\vec{\delta}(u))+W^{ext}\right) \quad (3)$$

The surface of discontinuity is discretized in 2D or 3D by finite elements of joint (see R3.06.09 documentation). The jump of displacement in the element $\vec{\delta}=(\delta_n,\delta_{t1},\delta_{t2})$ is a linear function of nodal displacements. The force[3] of cohesion which is exerted on the lips of the crack is noted $\vec{\sigma}$, it is defined by the derivative of the density of energy of surface compared to the jump of displacement. One calls cohesive law a relation enters $\vec{\sigma}$ and the jump of displacement $\vec{\delta}$. Parameters materials most relevant, which describes the joint of a stopping are:

- two rigidities in requests normal K_n and tangential K_t , which characterizes the surface and fill materials of the crack;
- and the constraint criticizes with the rupture σ_{max}

2 Γ represent the contour of the crack
3 force per unit of area, homogeneous with a constraint.

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One introduces, in addition, three adimensional digital parameters: P_{rupt}, P_{cont} and α. The first controls the regularization of the slope of softening in rupture, the second the penalization of the contact and the third ensures a progressive resumption of tangential stresses according to the normal opening. This last can be associated with the relative size of the asperities of surfaces in contact: $\alpha \in [0,2]$.
In the cohesive laws standards (R7.02.11) the energy of surface depends on the vector on displacement[4] and the constraints are defined as a derivative first of energy:

$$\sigma_n = \frac{\partial \psi_n(\delta_n)}{\partial \delta_n} \text{ and } \vec{\sigma}_t = \frac{\partial \psi_t(\vec{\delta}_t)}{\partial \vec{\delta}_t} \quad (4)$$

With energy:

$$\Psi(\vec{\delta}) \equiv \int_\Gamma \psi(\delta_n, \vec{\delta}_t) d\Gamma \quad (5)$$

Unlike these cohesive laws standards, in this model, only the normal part of the law is derived starting from energy from surface, whereas the tangential component of the law is given in an explicit way[5].
In both cases, the irreversibility of cracking is taken into account via a condition of increase in maximum normal opening of joint.

2.2 Energy of surface for the normal behavior

Density of energy of surface ψ, in a given point of the crack, depends explicitly on the jump of normal displacement between the lips of the crack δ_n. It also varies according to the state of the joint, which is taken into account via a variable interns threshold $\kappa \geqslant 0$, which manages the irreversibility of cracking. The latter memorizes the greatest standard of the jump reached during the opening. Its law of evolution between two successive increments of loading - and + is written:

$$\kappa^+ = \max(\kappa^-, \delta_n^+) \quad (6)$$

According to the value of opening of the joint, one will be able to find oneself in one of these three situations:

1. The compressed joint is in the mode of contact;
2. For a positive normal opening, if the latter exceeds the threshold one speaks about mode dissipative (dissipation of energy during cracking);
3. In the intermediate case the joint is in a linear mode (discharge or linear refill without dissipation of energy).

The normal part of energy is written:

$$\psi_n(\delta_n, \kappa) = \begin{cases} \psi_n^{con}(\delta_n, \kappa) & si \quad \delta_n < 0 \\ \psi_n^{lin}(\delta_n, \kappa) & si \quad 0 \leqslant \delta_n < \kappa \\ \psi_n^{dis}(\delta_n) & si \quad \delta_n \geqslant \kappa \end{cases} \quad (7)$$

And the tangential part:

$$\psi_t(\vec{\delta}_t) = \begin{cases} \psi_t^{fer}(\vec{\delta}_t) & si \quad \delta_n < 0 \\ \psi_t^{ouv}(\vec{\delta}_t) & si \quad \delta_n \geqslant 0 \end{cases} \quad (8)$$

In a synthetic way energy of surface is written in the following way:

$$\psi(\delta_n, \kappa) = H(\delta_n - \kappa)\psi_n^{dis}(\delta_n) + H(\delta_n) \cdot H(\kappa - \delta_n)\psi_n^{lin}(\delta_n, \kappa) + H(-\delta_n) \cdot \psi_n^{con}(\delta_n) \quad (9)$$

Where $H(x)$ is the function of Heaviside ($H(x)=0 \text{ si } x<0, H(x)=1 \text{ si } x \geqslant 0$).

4 The vector with two components $\vec{\delta}_t = (\delta_{t1}, \delta_{t2})$ indicate the tangential jump.

5 Our formulation thus leaves the energy formalism of the rupture. We can imagine a future improvement of the law by introducing a derivable function of regularization there.

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One of the characteristics of the law `JOINT_MECA_RUPT` is that, some is the mode, the behavior is always linear on the level of the constraints. On the level of energies, we obtain quadratic functions, which are data in following an additive constant close, which it depends on the threshold.
The normal behavior of joint is separate in three modes: contact, linear and dissipative. The function $\psi_n^{con}(\delta_n)=\frac{P_{con} K_n \delta_n^2}{2}$ ensure the condition of contact (not interpenetration of the lips of the crack) it is regularized so to obtain a better digital convergence. While varying the parameter P_{con}, one can change normal rigidity for the closed joint.
In linear mode, if an existing crack evolves without dissipating energy, the density of corresponding energy depends only on the parameter of normal rigidity: $\psi_n^{lin}(\delta_n,\kappa)=\frac{K_a(\kappa)\delta_n^2}{2}$, where $K_a(\kappa)=(P_{rupt}^{-1}+1)\sigma_{max}/\kappa-K_n P_{rupt}^{-1}$ is a decreasing function of the threshold of rupture κ.
In the dissipative mode, so to obtain a linear softening, the density of corresponding energy has a quadratic form according to the opening:

$$\psi_n^{dis}(\delta_n)=\begin{cases}\sigma_{max}(1+P_{rupt}^{-1})\delta_n - K_n P_{rupt}^{-1}\delta_n^2/2 & si \quad \delta_n<\sigma_{max}(1+P_{rupt})/K_n \\ \sigma_{max}^2(1+P_{rupt})^2/(2P_{rupt}K_n) & si \quad \delta_n\geqslant\sigma_{max}(1+P_{rupt})/K_n\end{cases} \quad (10)$$

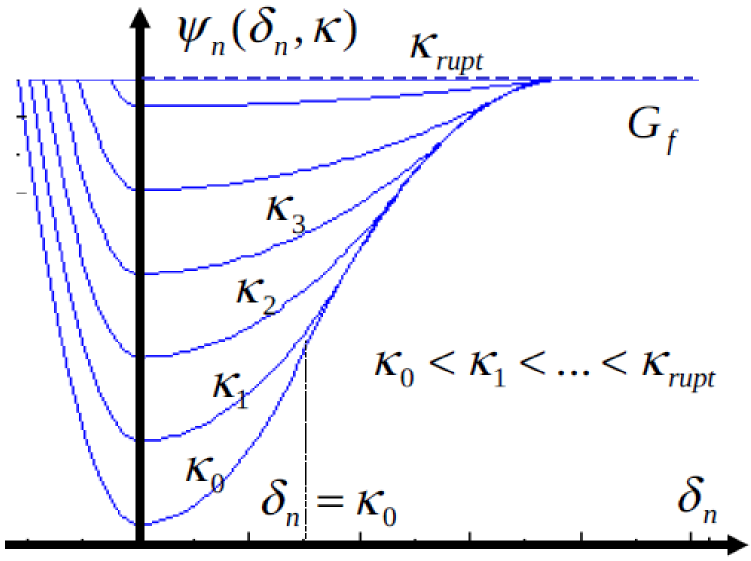

Figure 2.2-1: Density of energy of surface according to the jump of displacement for various values of the threshold of damage κ

The additive constant makes relocate elastic energies ψ_n^{lin} and ψ_n^{con} defined previously so that it does not matter the state of damage of the joint one always obtains the same rate of refund of the energy (constant of Griffith G_f, to see Figure 2.2-1 and eq. 3.2-1). It depends only on the threshold and does not affect the expression of the constraints.
P_{rupt} is introduced so that more it increases more the critical opening to the rupture is important and by consequence plus that made increase the dissipative energy of Griffith G_f.

In short : *the normal behavior of the law* `JOINT_MECA_RUPT` *is controlled by the evolution of the density of energy surface, that Ci appears itself as a potential well. It comprises three principal modes: contact, elastic linear traction, damage/softening, whose profiles corresponding are approximated by quadratic functions in opening. The joint starts to be damaged when the normal constraint reaches the breaking value* $\sigma_n=\sigma_{max}$ *. More joint is damaged more the energy well is flattened to see (Figure 2.2-1). The parameter of damage (the threshold) evolves to him only in this last mode on the basis of its*

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initial value for the operational joint $\kappa_0=\sigma_{max}/K_n$ up to the ultimate value for the completely damaged joint $\kappa_{rupt}=\sigma_{max}(1+P_{rupt})/K_n$. P_{con} is a constant defined by the user who changes the level of penalization into contact (see Figure 2.3-2). P_{rupt} change the energy dissipated per unit unit of area $G_f=\sigma_{max}^2(1+P_{rupt})/(2\mathrm{K}_n)$. With final, one obtains the following expression for normal energy:

$$\psi_n(\delta_n,\kappa)=\begin{cases}\sigma_{max}(1+P_{rupt}^{-1})(\kappa-\kappa_0)/2\ +\ P_{con}K_n\delta_n^2/2 & si\quad \delta_n<0\\ \sigma_{max}(1+P_{rupt}^{-1})(\kappa-\kappa_0)/2\ +\ \left[\sigma_{max}(1+P_{rupt}^{-1})/\kappa-K_nP_{rupt}^{-1}\right]\delta_n^2/2 & si\quad 0\leqslant\delta_n<\kappa\\ \sigma_{max}(1+P_{rupt}^{-1})(\delta_n-\kappa_0/2)\ -\ K_nP_{rupt}^{-1}\delta_n^2/2 & si\quad \kappa\leqslant\delta_n<\kappa_{rupt}\\ G_f & si\quad \delta_n\geqslant\kappa_{rupt}\end{cases}\tag{11}$$

This function is continuous and derivable, which ensures the continuity of the constraints.

2.3 Vector forced

The vector forced in the element is noted $\vec{\sigma}=(\sigma_n,\vec{\sigma}_t)$ [6], it can be separate in several modes. For the normal part it vector-constraint is equal to the sum of the derivative of the density of energy of surface and the density of energy of penalization in contact compared to the jump.

$$\sigma_n=H(\delta_n-\kappa)\sigma_n^{dis}+H(\kappa-\delta_n)H(\delta_n)\sigma_n^{lin}+H(-\delta_n)\sigma_n^{con}\tag{12}$$

It is enough thus to derive the expressions given in the preceding section (§ 2.2) to obtain the normal component of the constraints [7] . The tangential part $\vec{\sigma}_t^{fer}=f(K_t,\vec{\delta}_t)$ is written:

$$\vec{\sigma}_t=H(\delta_n)\vec{\sigma}_t^{ouv}+H(-\delta_n)\vec{\sigma}_t^{fer}¿\tag{13}$$

It is a function of the tangential opening, it brings into play tangential rigidity for the closed joint. According to the profile of the interface the tangential behavior for the open joint can be varied $\vec{\sigma}_t^{ouv}=\vec{\sigma}_t^{fer}$ for surfaces in crenel (such as for example on the Figure 1.2-1 on the right), or $\vec{\sigma}_t^{ouv}\equiv 0$ for very smooth surfaces (Figure 1.2-1 on the left).

2.3.1 Normal constraints

Let us consider the expression the normal constraint:

$$\sigma_n(\delta_n,\kappa)=\begin{cases}P_{con}K_n\delta_n & si\quad \delta_n<0\\ \left[\sigma_{max}(1+P_{rupt}^{-1})/\kappa-K_nP_{rupt}^{-1}\right]\delta_n & si\quad 0\leqslant\delta_n<\kappa\\ \sigma_{max}(1+P_{rupt}^{-1})\ -\ K_nP_{rupt}^{-1}\delta_n & si\quad \kappa\leqslant\delta_n<\kappa_{rupt}\\ 0 & si\quad \delta_n\geqslant\kappa_{rupt}\end{cases}\tag{14}$$

The evolution of this constraint in the zone of traction according to the jump is represented on the Figure 2.3-1 . The arrows represent the possible direction of evolution of the constraint according to whether the process of opening is reversible (linear mode) or not (dissipative mode). With starting, the joint behaves initially in an elastic way linear, then as soon as the normal constraint reaches the breaking value $\sigma_n=\sigma_{max}$, it has a lenitive behavior: it loses its rigidity gradually, which gives the mode linear but not elastic, it is characterized by a slope of softening of rupture $-K_n/P_{rupt}$. Elastic rigidity for the operational joint defines the initial value of the threshold of damage $\kappa_0=\sigma_{max}/K_n$.

6 In our case $\sigma_n=\partial\psi_n(\delta_n)/\partial\delta_n$;

7 The expressions of the constraints can be applied directly without passing by the energy formulation

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The threshold of the rupture is given by $\kappa_{rupt}=\sigma_{max}(1+P_{rupt})/K_n$. More P_{rupt} more energy of dissipation is important increases $G_f=\sigma_{max}^2(1+P_{rupt})/(2K_n)$.

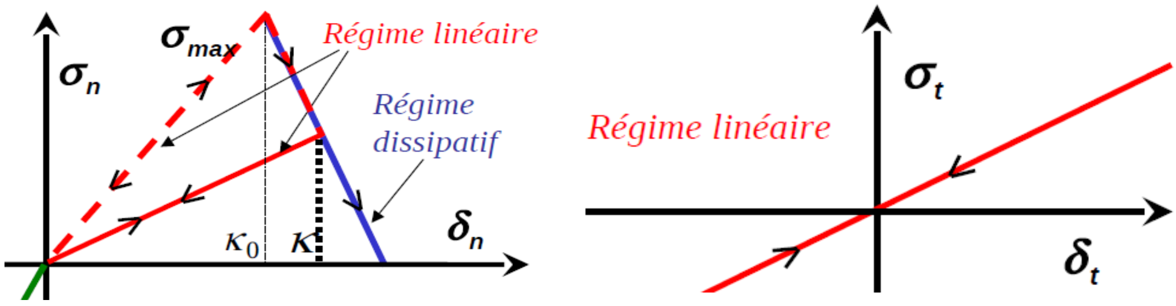

Figure 2.3-1: Dependence of the constraints according to the opening

2.3.2 Constraint of penalization of the contact

The value of the slope of penalization in contact is given by the following relation:

$$\sigma_n(\delta_n)=P_{con}K_n\delta_n \text{ if } \delta_n<0 \quad (15)$$

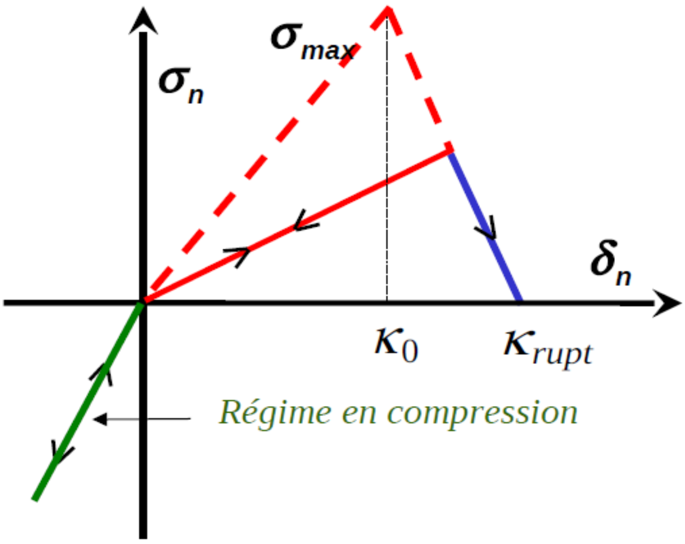

Figure 2.3-2: Normal cohesive constraint according to the normal jump for the partially damaged joint

The digital parameter PENA_CONTACT, given by the user, allows to exploit the slope of the penalization of the contact (see Figure 2.3-2). It is worth by default 1, which corresponds if the slope of the contact is identical to that of rigidity in opening. If one chooses a value higher than 1, the penalization is increased. This allows to model, for example, the resumption of the efforts by the concrete partially damaged in traction. For a value lower than 1, the penalization is decreased, which makes it possible to simulate the concrete partially damaged in compression.

2.3.3 Tangential constraint

For the joints of stopping to weak opening, one observes that independently of the mode of normal loading, the tangential constraint varies always linearly, tangential rigidity is function of the normal opening. In the extreme case of a perfectly smooth surface of contact, tangential rigidity falls brutally to zero with the positive normal opening. Consequently, the surface energy of the law is not continuous

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any more, which generates in theory a peak in the normal constraint (function delta $\delta(x)$) with the opening. For this reason we give up in the current version of the law of to keep the complete energy formalism. The tangential law is then applied in an empirical way in form linear with tangential rigidity dependent on the normal opening :

$$\vec{\sigma}_t(\delta_t,\delta_n)=\begin{cases} K_t\cdot(\vec{\delta}_t-\vec{\delta}_{shift}) & si \quad \delta_n<0 \\ (1-\delta_n/\kappa_{rupt}^{\tan})K_t\cdot(\vec{\delta}_t-\vec{\delta}_{shift}) & si \quad 0\leqslant\delta_n<\kappa_{rupt}^{\tan} \\ 0 & si \quad \delta_n\geqslant\kappa_{rupt}^{\tan} \end{cases} \quad (16)$$

The variable of history δ_{shift} represent the shift of the point of balance in tangential slip with open joint: δ_{shift} is equal to the last value of δ_t for which joint was open $\delta_n\geq\kappa_{rupt}^{\tan}$. We introduce the threshold of tangential rupture $\kappa_{rupt}^{\tan}=\kappa_{rupt}\tan(\alpha\pi/4)$, of which the value can be modified by the user with $\alpha\in[0,2]$ and the keyword ALPHA . For a zero value the tangential slope changes brutally with the opening, for the value $\alpha=2$ tangential rigidity does not evolve. By preoccupation with a compatibility with the laws of behavior developed in the Gefdyn code, we do not make correction of the normal component of constraints in the phase of transition, that Ci is always given by (14), which gives a nonsymmetrical tangent matrix and by consequence the mode with not controlled dissipation. The evolution of the tangential constraint separates in three modes: elastic joint in compression; partially open elastic joint with a decreased rigidity; joint completely broken (see equation (16) and Appears 2.3-3 .)

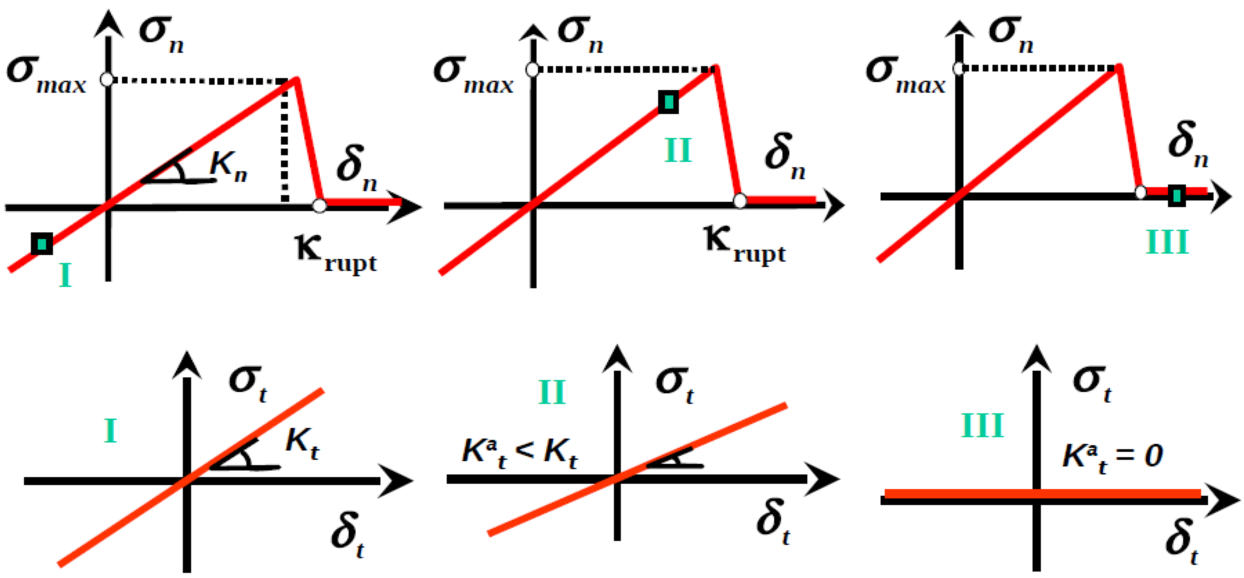

Figure 2.3-3: Illustration of the coupling between the shearing and the normal opening of joint: I in the mode in compression; II in the partial mode of opening; III in the mode of complete opening. One notes $K_t^a\equiv(1-\delta_n/\kappa_{rupt}^{\tan})K_t$

2.4 Tangent operator

As the behavior in each mode is linear, the calculation of the tangent matrix is easy:

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$$\frac{\partial\sigma_n(\delta_n)}{\partial\delta_n}=\begin{cases} P_{con}K_n & si \quad \delta_n<0 \\ \sigma_{max}(1+P_{rupt}^{-1})/\kappa-K_nP_{rupt}^{-1} & si \quad 0\leqslant\delta_n<\kappa \\ -K_nP_{rupt}^{-1} & si \quad \kappa\leqslant\delta_n<\kappa_{rupt} \\ 0 & si \quad \delta_n\geqslant\kappa_{rupt}\end{cases} \tag{17}$$

$$\frac{\partial\sigma_n(\delta_n)}{\partial\delta_t}=0 \tag{18}$$

$$\frac{\partial\vec{\sigma}_t(\vec{\delta}_t,\delta_n)}{\partial\vec{\delta}_t}=\begin{cases} K_t\boldsymbol{Id} & si \quad \delta_n<0 \\ (1-\delta_n/k_{rupt}^{\tan})K_t\boldsymbol{Id} & si \quad 0\leqslant\delta_n<\kappa_{rupt}^{\tan} \\ \boldsymbol{0} & si \quad \delta_n\geqslant\kappa_{rupt}^{\tan}\end{cases} \tag{19}$$

$$\frac{\partial\vec{\sigma}_t(\vec{\delta}_t,\delta_n)}{\partial\delta_n}=\begin{cases} 0 & si \quad \delta_n<0 \\ -K_t(\vec{\delta}_t-\vec{\delta}_{shift})/k_{rupt}^{\tan} & si \quad 0\leqslant\delta_n<\kappa_{rupt}^{\tan} \\ 0 & si \quad \delta_n\geqslant\kappa_{rupt}^{\tan}\end{cases} \tag{20}$$

Let us note that the tangent matrix is not symmetrical. This results from nonthe repercussion on the normal constraints of the regularization of the evolution of the tangential constraint to the opening of the joint (the singular terms are not taken into account) [5].

2.5 Digital realization of keying-up

Keying-up corresponds physically to a procedure of grouting of concrete under pressure between the studs of the stopping, it is characterized by a simple parameter the local pressure of coulis injected $\sigma_{nc}\geqslant 0$. To set up keying-up the user must define a function of pressure of keying-up, key word PRES_CLAVAGE, which depends with the faith on space (keying-up at the various places with different pressure) and of time (several successive keying-up). The places where the pressure of keying-up is negative are not clavés.
The procedure is modelled by the modification thickness of the joints concerned. If the joint is in strong compression initially, keying-up does not influence it. So on the other hand the joint is opened or not sufficiently compressed (i.e. if $\sigma_n>-\sigma_{nc}$), keying-up will result in the change of the parameter total thickness of the noted joint $\delta_{nf}^{+}=\delta_{nf}^{-}+\delta_n^{+}+\sigma_{nc}/(P_{con}K_n)$.
Keying-up then preserves the normal opening of each joint while relocating the law of behavior according to the x-axis. The new position of local balance has a normal constraint equal to the pressure of the concrete injected $\sigma_n=-\sigma_{nc}$. This procedure is applied locally. Although each joint subjected to keying-up is found in the linear mode in compression, the selection it even of the joints with claver is “non-linear”. The total procedure becomes, also non-linear so and it is not enough to just modify the normal constraints of the joints clavés to again obtain mechanical balance after keying-up. Consequently a mechanical calculation of balance is carried out after this “local” keying-up to give the system in its state of balance. If the game of the joint with claver is modified, the update thickness of the joints is carried out and so on as long as there exist the points where $\sigma_n>-\sigma_{nc}$. The joints concerned are closed and are put gradually in compression while following the curve of normal behavior. This procedure is made only once during the phase of digital “construction” of stopping. An illustration is given on the Figure 2.5-2 above. The points “in” and “out” correspond respectively to the values of the constraint before and after keying-up.

The normal behavior of the joint after keying-up can be modified. Thus, the tensile strength can be restored either partially, or completely (it is the case on the Figure 2.5-2) according to the damage of

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the joints before the procedure of keying-up. *In the procedure such as it was developed we made the choice not to restore the tensile strength, this one keeps its current price.*

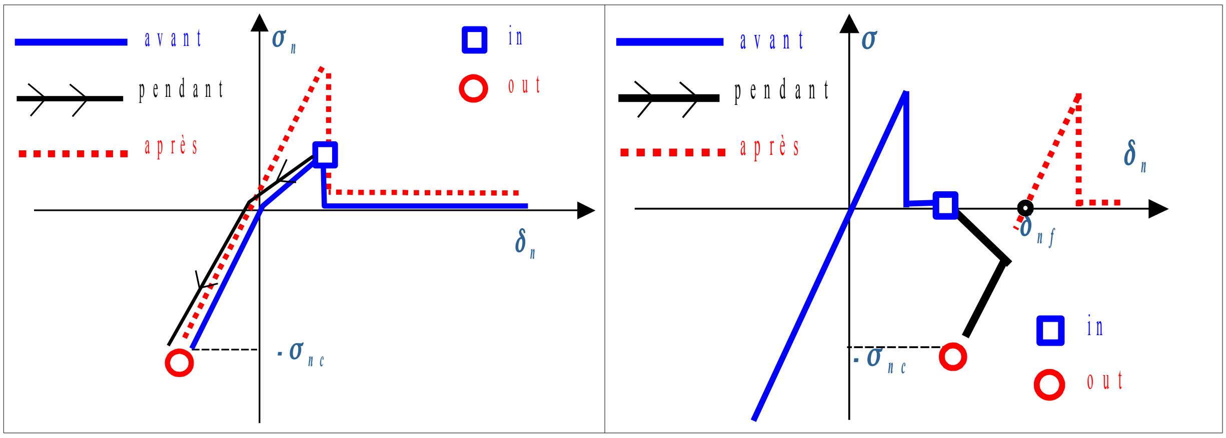

Figure 2.5-1: . Evolution of the normal constraint during keying-up: joint partially damaged on the left and completely damaged on the right

In order to identify the tangent matrix for keying-up it is useful to represent this procedure like a temporary modification of the initial law of behavior by an incremental law. For the tangential mode the procedure of keying-up does not import modification: the associated constraint always depends in a linear way of tangential displacement and the slope of loading varies according to the opening of joint. The normal coupling deserves to be analyzed more in detail. Let us suppose that one is in a state of loading given δ_{n+} and one tries to find the state of stress infinitésimalement close. Let us look at initially the case where δ_n decrease. Considering the thickness of joint was already put up to date at the preceding iteration of Newton the joint returns in the field of compression and the activation of keying-up does not play any part with this stage. One thus finds the slope of compression of the initial law. On the other hand if one charges the joint in traction the procedure with keying-up activates oneself and one carries out the adaptation thickness of joint in kind to make put a ceiling to the normal constraint by the pressure of keying-up. De facto the behavior becomes bilinear with a slope of compression of the initial law in compression and the slope in the zone of traction, which is completely cancelled (see Figure 2.5-2).

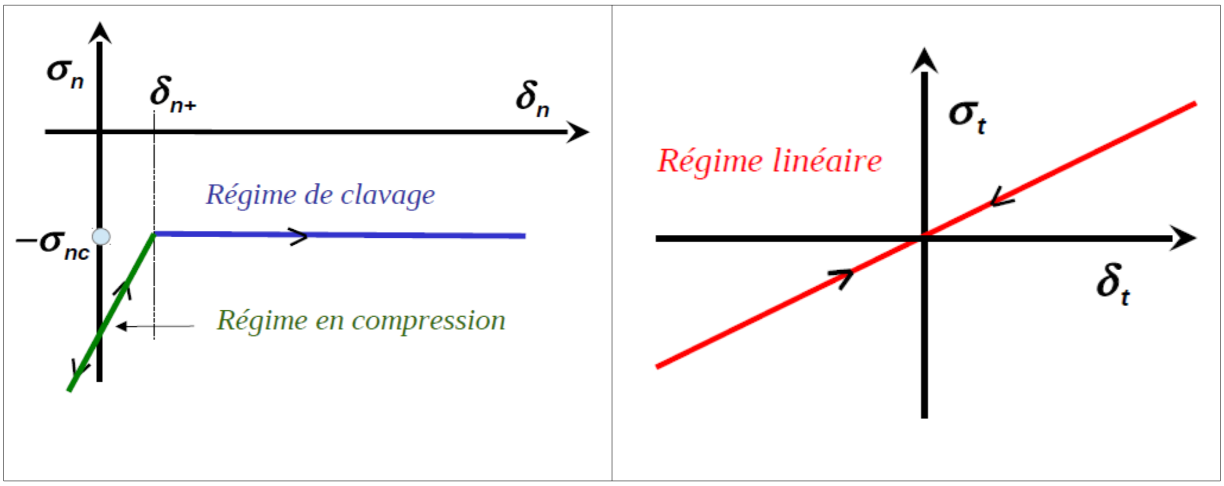

Figure 2.5-2: . Modification of the law of behavior during the phase of keying-up

As the behavior in each mode remains linear, the calculation of the tangent matrix is always easy:

$$\frac{\partial \sigma_n(\delta_n)}{\partial \delta_n} = \begin{cases} P_{con} K_n & si \quad \delta_n < \delta_{nf} \\ 0 & si \quad \delta_n \geq \delta_{nf} \end{cases} \tag{21}$$

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$$\frac{\partial \sigma_n(\delta_n)}{\partial \delta_t}=0 \quad (22)$$

$$\frac{\partial \vec{\sigma}_t(\vec{\delta}_t,\delta_n)}{\partial \vec{\delta}_t}=\begin{cases} K_t \boldsymbol{I_d} & si \quad \delta_n<\delta_{nf} \\ (1-\delta_n/k_{rupt}^{\tan})K_t \boldsymbol{I_d} & si \quad \delta_{nf}\leqslant\delta_n<\kappa_{rupt}^{\tan} \\ \mathbf{0} & si \quad \delta_n\geqslant\kappa_{rupt}^{\tan} \end{cases} \quad (23)$$

$$\frac{\partial \vec{\sigma}_t(\vec{\delta}_t,\delta_n)}{\partial \delta_n}=\begin{cases} 0 & si \quad \delta_n<\delta_{nf} \\ -K_t(\vec{\delta}_t-\vec{\delta}_{shift})/k_{rupt}^{\tan} & si \quad \delta_{nf}\leqslant\delta_n<\kappa_{rupt}^{\tan} \\ 0 & si \quad \delta_n\geqslant\kappa_{rupt}^{\tan} \end{cases} \quad (24)$$

2.6 Internal variables

The law JOINT_MECA_RUPT have eighteen internal variables. From the point of view of the law of behavior, only the first and it tenth are *stricto sensu* internal variables. The others provide indications on the hydraulic state of the joint to a given moment.

$V1=\kappa$: threshold in jump (greater standard reached).
$V2$: indicator of dissipation $=0$ if linear mode, $=1$ if dissipative mode.
Mechanical indicators:
$V3$: indicator of normal damage $=0$ healthy, $=1$ damaged, $=2$ broken
$V4\in[0,1]$: percentage of normal damage (in the lenitive zone)
$V5$: indicator of tangential damage $=0$ healthy, $=1$ damaged, $=2$ broken
$V6\in[0,1]$: percentage of tangential damage
Value of the jump in the local reference mark:
$V7=\delta_n$: normal jump, $V8=\delta_{t1}$ tangential jump, $V9=\delta_{t2}$ tangential jump (no one in 2D)
$V10=\delta_{nf}$: thickness of the clavé joint
$V11=\sigma_n$: normal mechanical constraint (without pressure of fluid)
Hydraulic indicators:
Components of the gradient of pressure in the total reference mark (only for xxx_JOINT_HYME) :
$V12=\partial_x p$, $V13=\partial_y p$, $V14=\partial_z p$ three components in space
Components of hydraulic flow in the total reference mark (only for xxx_JOINT_HYME):
$V15=w_x$, $V16=w_y$, $V17=w_z$ three components in space
$V18=p$: pressure of fluid imposed by the user (PRES_FLUIDE) in the case of modelings xxx_JOINT or pressure of fluid interpolated from that calculated (degree of freedom of the problem) with the nodes mediums of the elements of joint of modelings: xxx_JOINT_HYME.
$V19\,et\,V20$, two component S vector of tangential slip δ_{shift}

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3 Theoretical formulation of JOINT_MECA_FROT

One considers the law of friction of Coulomb, which depends only on one parameter $\mu \in [0,\infty]$. It carries out the condition of not-interpenetration of the lips in contact (condition of Signorini) by establishing a local link between the tangential and normal constraint in the phase of slip: $\|\vec{\sigma}_t\| = \mu\,\sigma_n$. Several regularizations of this law are made in order to facilitate its digital implementation. Firstly the condition of Signorini must be made derivable, which is easy if it is supposed that the behavior of surfaces in contact follows an elastic law. In the same way for the slope of change of management of slip in the tangential behavior. Moreover for modeling of the concrete dams one observes in experiments a tensile strength considerable between the joints. All these considerations bring back for us towards a law of Mohr-Coulomb of which the representation in the plan of Mohr is given on the Figure 3-1 , it describes the phase of slip of joints between the foundation and the stopping or the studs of a stopping (in a simplified way) while taking of account the most relevant effects.

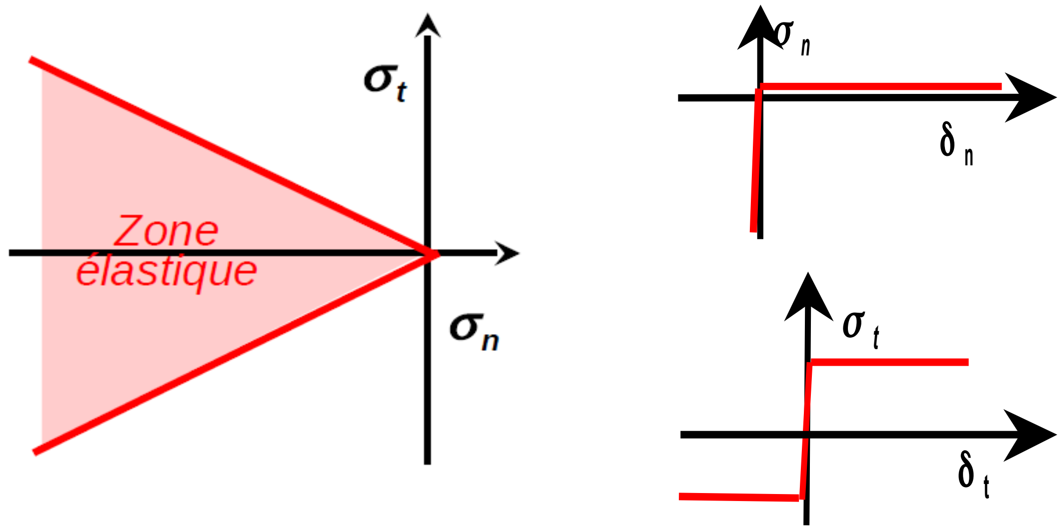

Figure 3-1 : Law of friction of Coulomb in 2D

The law JOINT_MECA_FROT is an elastoplastic alternative of the law of Mohr-Coulomb, it depends on four parameters: normal rigidity K_n , tangential rigidity K_t , adhesion c (which is related to the tensile strength maximum $R_t = c/\mu$) and the coefficient of friction of the joint μ . In addition we introduce an isotropic parameter of work hardening, which makes it possible to regularize the tangential slope in the phase of slip, one notes it K . The introduced elastoplastic model relates only to the tangent part of the law of behavior. There is no plastic part of displacement for the normal part: this one is always elastic. The jump of tangential displacement is broken up into an elastic part $\vec{\delta}_t^{el}$ and a plastic part $\vec{\delta}_t^{pl}$, one indicates by λ the jump of cumulated tangential displacement. The law of flow is orthogonal with the plan of cut of the cone of slip $\sigma_n = \text{const}$ (circle 2D for a cone 3D). What gives, strictly speaking, a non-aligned law of flow total. The mechanical formulation of speed of such a law gives the following equations:

$$\begin{cases} \vec{\delta}_t = \vec{\delta}_t^{el} + \vec{\delta}_t^{pl} \\ \vec{\sigma}_t = K_t \vec{\delta}_t^{el} \equiv K_t (\vec{\delta}_t - \vec{\delta}_t^{pl}) \\ \sigma_n = min(K_n \delta_n, R_t) \end{cases} \quad (25)$$

With:

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$$\begin{cases} f(\vec{\sigma},\lambda)=\|\vec{\sigma}_t\|+\mu\sigma_n-c-K\lambda\leqslant 0 \\ f\cdot\dot{\lambda}=0;\ \dot{\lambda}\geqslant 0 \\ \dot{\vec{\delta}}_t^{pl}=\dot{\lambda}\dfrac{\vec{\sigma}_t}{\|\vec{\sigma}_t\|} \end{cases} \quad (26)$$

As long as one is in the elastic zone $f(\vec{\sigma},\lambda)<0$ the relations between the jumps of opening of joint and the constraints are linear and the parameter of the plastic tangential jump does not evolve $\vec{\delta}_t^{pl}=const$. As soon as one touches the edges of the cone of slip defined by $f(\vec{\sigma},\lambda)=0$, the evolution of tangential jump plastic is governed by the non-aligned law of flow (26). The regularization of the function threshold of flow with the term of work hardening [8] $K\lambda>0$ is necessary in order to make invertible the tangent matrix of the law and to avoid thus the problem of multiple solutions in the case of loading in imposed forces. The tensile strength of the joint varies in the interval $(0,R_t)$, it is a function of the tangential constraint, it is worthless for a tangential constraint higher than the parameter of adhesion c and the maximum is worth if the tangential constraint is worthless (see fig. 3.1-1). In the current version of the law, the tensile strength maximum is not affected by the phenomenon of slip, it does not evolve because of term of work hardening (Figure 3-2). It is also supposed that once reached the value of the tensile strength maximum, the normal constraint does not evolve any more. This last assumption remains valid as long as the number of joints broken by plastic shearing and solicited thereafter in traction is negligible. In order to be able to take into account this phenomenon more rigorously, it is necessary to introduce a regularization of rupture-friction into the phase of slip in traction, which represents a rather important theoretical difficulty [CR10357] .

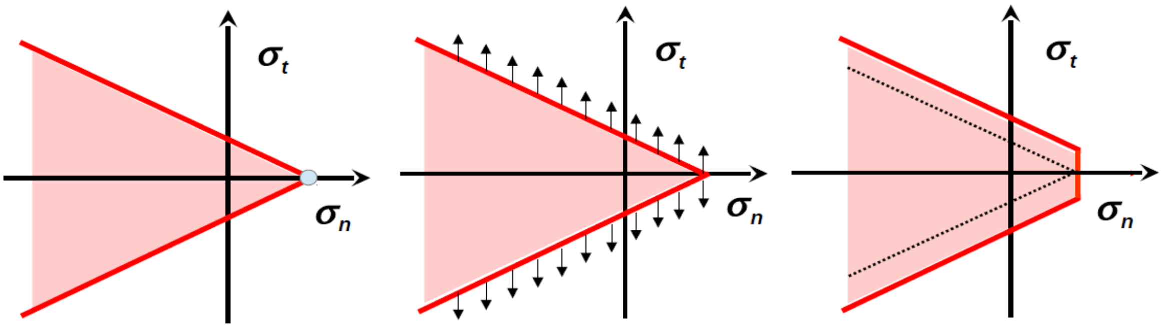

Figure 3-2: Evolution of the cone of slip due to work hardening

3.1 Dimplicit iscretisation of the law of friction

The elastoplastic version of the law of friction is formulated of speed, which facilitates its digital discretization. The incremental version of the law remains strictly equivalent to the version continue on condition that having infinitesimal steps of change. In order to limit the number of steps of loading, we adapt the version continues law with finished increments, in an implicit way, C. - AD. that the conditions of slip are written in the state of final balance. The algorithm used is that of the radial return with elastic prediction. By convention we note by a sign "-" the variables with the state of preceding balance, the current state is noted by usual variables without additional sign (see [R5.03.02]).

8 Isotropic work hardening on the tangent level

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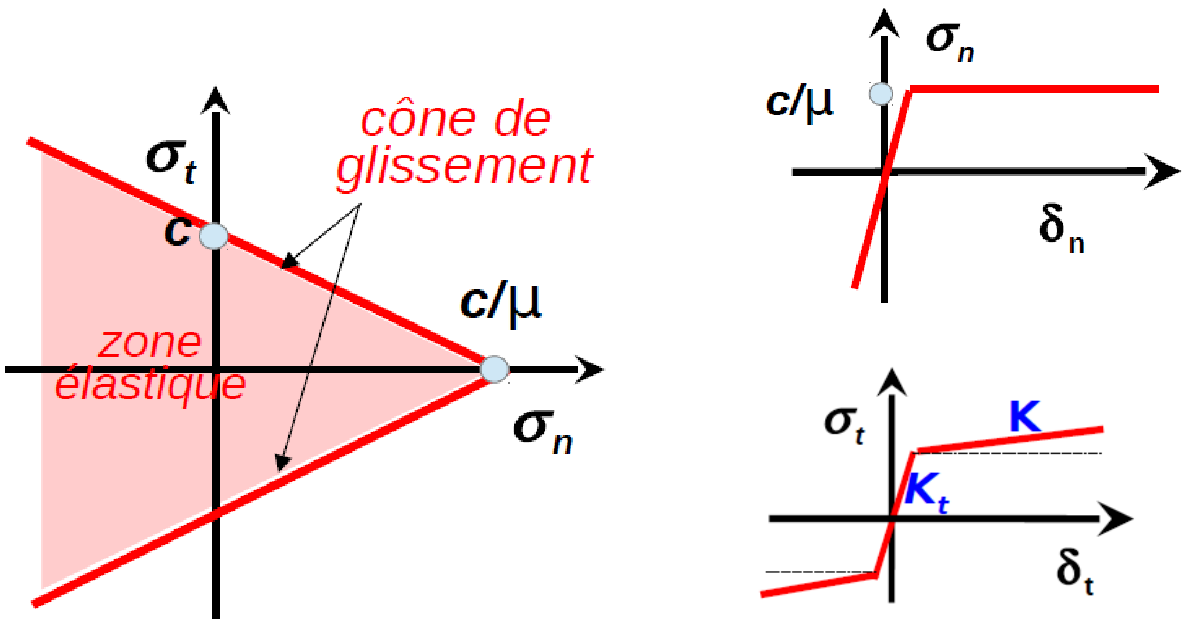

Figure 3.1-1: Law of friction of Coulomb

The continuous equations of the law are written in a discretized way:

$$\begin{cases}\lambda=\lambda^{-}+\Delta\lambda\\ \vec{\delta}_t^{pl}=\vec{\delta}_t^{pl-}+\Delta\vec{\delta}_t^{pl}\\ \vec{\sigma}_t=K_t(\vec{\delta}_t-\vec{\delta}_t^{pl})\\ \sigma_n=min(K_n\delta_n,R_t)\end{cases}\quad(27)$$

With the following law:

$$\begin{cases}f(\vec{\boldsymbol{\sigma}},\lambda)=\|\vec{\sigma}_t\|+\mu\sigma_n-c-K\lambda\leqslant 0\\ f(\vec{\boldsymbol{\sigma}},\lambda)\cdot\Delta\lambda=0;\ \ \Delta\lambda\geqslant 0\\ \Delta\vec{\delta}_t^{pl}=\Delta\lambda\dfrac{\vec{\sigma}_t}{\|\vec{\sigma}_t\|}\end{cases}\quad(28)$$

In an algorithm of Newton jumps of displacement $\vec{\boldsymbol{\delta}}=(\delta_n,\vec{\delta}_t)$ as well as the internal variables at the previous moment $\vec{\delta}_t^{pl-}$ and λ^{-} being known, to solve the law it is enough to obtain the values of constraints $\vec{\boldsymbol{\sigma}}=(\sigma_n,\vec{\sigma}_t)$ and all internal variables at the moment running (in our case $\vec{\delta}_t^{pl}$ and λ). The law the equation of evolution for the normal component is completely uncoupled from the tangential movement, one can thus solve it immediately:

$$\sigma_n=min(K_n\delta_n,R_t)\quad(29)$$

We thus obtain a set of five of equations and an inequality, to obtain five scalar unknown factors:

$$\vec{\sigma}_t=K_t(\vec{\delta}_t-\vec{\delta}_t^{pl-}-\Delta\vec{\delta}_t^{pl})\quad(30)$$

And:

$$\begin{cases}f(\vec{\boldsymbol{\sigma}},\lambda)\equiv\|\vec{\sigma}_t\|+\mu\sigma_n-c-K\lambda^{-}-K\Delta\lambda\leqslant 0\\ f(\vec{\boldsymbol{\sigma}},\lambda)\cdot\Delta\lambda=0;\ \ \Delta\lambda\geqslant 0\\ \Delta\vec{\delta}_t^{pl}=\Delta\lambda\dfrac{\vec{\sigma}_t}{\|\vec{\sigma}_t\|}\end{cases}\quad(31)$$

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To simplify this mathematical problem let us look at more in detail the condition of Karush-Kuhn-Tucker $f(\vec{\boldsymbol{\sigma}},\lambda)\cdot\Delta\lambda=0$. There are two possibilities: either one slips $\Delta\lambda>0$, one is in the elastic range $\Delta\lambda=0$. If one is in the elastic range, then $\Delta\vec{\delta}_t^{pl}=0$ and the elastic solution is obtained if, and only if:

$$f_{el}(\sigma_n,\vec{\sigma}_\tau^-,\lambda^-)\equiv K_t\|\vec{\delta}_\tau-\vec{\delta}_\tau^{pl-}\|+\mu\sigma_n-c-\mathrm{K}\lambda^-\leqslant 0 \quad (32)$$

In practice if (32) is satisfied then the elastic prediction is the solution of the problem:

$$\begin{cases}\lambda=\lambda^-\\ \vec{\delta}_t^{pl}=\vec{\delta}_t^{pl-}\\ \vec{\sigma}_t=K_t(\vec{\delta}_t-\vec{\delta}_t^{pl-})\\ \sigma_n=min(K_n\delta_n,R_t)\end{cases} \quad (33)$$

If the condition (32) is not satisfied, then $\Delta\lambda>0$ and one is in the phase of slip. One then obtains a system of three non-linear equations with three unknown factors $\Delta\vec{\delta}_t^{pl}$ and $\Delta\lambda$:

$$\begin{cases}f(\vec{\boldsymbol{\sigma}},\lambda)\equiv\|K_t(\vec{\delta}_t-\vec{\delta}_t^{pl-}-\Delta\vec{\delta}_t^{pl})\|+\mu\sigma_n-c-\mathrm{K}\lambda^--\mathrm{K}\Delta\lambda=0\\ \Delta\vec{\delta}_t^{pl}=\Delta\lambda\dfrac{(\vec{\delta}_t-\vec{\delta}_t^{pl-}-\Delta\vec{\delta}_t^{pl})}{\|\vec{\delta}_t-\vec{\delta}_t^{pl-}-\Delta\vec{\delta}_t^{pl}\|}\end{cases} \quad (34)$$

This equation perhaps solved while eliminating $\Delta\vec{\delta}_t^{pl}$, according to the procedure usually used for the plastic designs:

$$\Delta\vec{\delta}_t^{pl}\{\|\vec{\delta}_t-\vec{\delta}_t^{pl-}-\Delta\vec{\delta}_t^{pl}\|+\Delta\lambda\}=\Delta\lambda(\vec{\delta}_t-\vec{\delta}_t^{pl-}) \quad (35)$$

By taking the standard of this last equation and by noting that $\|\Delta\vec{\delta}_t^{pl}\|=\Delta\lambda$, one obtains the standard of the vector of the brought up to date tangential constraint, which one inserts in the equation (34) in order to obtain a scalar equation for $\Delta\lambda$:

$$\|\vec{\delta}_t-\vec{\delta}_t^{pl-}-\Delta\vec{\delta}_t^{pl}\|=\|\vec{\delta}_t-\vec{\delta}_t^{pl-}\|-\Delta\lambda \quad (36)$$

And:

$$\|K_t(\vec{\delta}_t-\vec{\delta}_t^{pl-})\|-K_t\Delta\lambda+\mu\sigma_n-c-\mathrm{K}\lambda^--\mathrm{K}\Delta\lambda=0 \quad (37)$$

Once $\Delta\lambda$ known, it is enough to notice the colinearity of the vectors according to $\Delta\vec{\delta}_t^{pl}\uparrow\uparrow\vec{\delta}_t-\vec{\delta}_t^{pl-}-\Delta\vec{\delta}_t^{pl}$, from where $\Delta\vec{\delta}_t^{pl}\uparrow\uparrow\vec{\delta}_t-\vec{\delta}_t^{pl-}$. What makes it possible to rewrite the equation (37) in a simplified form, which gives the value of the second unknown factor $\Delta\vec{\delta}_t^{pl}$:

$$\Delta\vec{\delta}_t^{pl}=\Delta\lambda\frac{\vec{\delta}_t-\vec{\delta}_t^{pl-}-\Delta\vec{\delta}_t^{pl}}{\|\vec{\delta}_t-\vec{\delta}_t^{pl-}-\Delta\vec{\delta}_t^{pl}\|}=\Delta\lambda\frac{\vec{\delta}_t-\vec{\delta}_t^{pl-}}{\|\vec{\delta}_t-\vec{\delta}_t^{pl-}\|} \quad (38)$$

This solution corresponds in fact to the slips in the direction of tangential constraint in elastic prediction. This implies that the change of the direction of slip will be done primarily in the elastic zone provided that the steps of loading are small. The final solution in the event of slip, obtained starting from the equation (38), is written like:

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$$\begin{cases} \sigma_n & = & min(K_n \delta_n, R_t) \\ f_{el}(\sigma_n, \vec{\sigma}_t^-, \lambda^-) & = & K_t \|\vec{\delta}_t - \vec{\delta}_t^{pl-}\| + \mu\sigma_n - c - \mathrm{K}\lambda^- \\ \lambda & = & \lambda^- + \dfrac{f_{el}(\sigma_n, \vec{\sigma}_t^-, \lambda^-)}{K_t + \mathrm{K}} \\ \vec{\delta}_t^{pl} & = & \vec{\delta}_t^{pl-} + \dfrac{f_{el}(\sigma_n, \vec{\sigma}_t^-, \lambda^-)}{K_t + \mathrm{K}} \cdot \dfrac{\vec{\delta}_t - \vec{\delta}_t^{pl-}}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|} \\ \vec{\sigma}_t & = & K_t(\vec{\delta}_t - \vec{\delta}_t^{pl}) \end{cases} \quad (39)$$

In short, starting from the elastic prediction one checks initially the inequality (32), if it is satisfied, then the solution is given by (33), if not the solution is given by the equation (39).

3.2 Tangent matrix

For the law `JOINT_MECA_FROT` the mtangent atrice is calculated into implicit, which reinforces the robustness of calculations[9]. As it is shown in the ref. [Ngu77], such a digital diagram is unconditionally stable for the laws with positive work hardening $K \geqslant 0$. In the case of the elastic mode (the inequality (41) is satisfied), the tangent matrix takes a simple form, it is diagonal:

$$\begin{pmatrix} K_n & 0 & 0 \\ 0 & K_t & 0 \\ 0 & 0 & K_t \end{pmatrix} \quad (40)$$

In the case of slip (the inequality (41) is not satisfied) the tangent matrix is obtained by derivation of equations (39). The derivative compared to the normal opening depend on the state of the joint. For the joint closed that gives:

$$\text{si } \delta_n < \frac{c}{\mu K_n} \Rightarrow \begin{cases} \dfrac{\partial \sigma_n}{\partial \delta_n} & = & K_n \\ \dfrac{\partial \vec{\sigma}_t}{\partial \delta_n} & = & -\mu \dfrac{\vec{\delta}_t - \vec{\delta}_t^{pl-}}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|} \cdot \dfrac{K_n K_t}{K_t + \mathrm{K}} \end{cases} \quad (41)$$

For the open joint all the derivative corresponding are worthless:

$$\text{si } \delta_n \geqslant \frac{c}{\mu K_n} \Rightarrow \begin{cases} \dfrac{\partial \sigma_n}{\partial \delta_n} & = & 0 \\ \dfrac{\partial \vec{\sigma}_t}{\partial \delta_n} & = & 0 \end{cases} \quad (42)$$

The derivative compared to the tangential opening do not depend on the opening of joint:

$$\begin{cases} \dfrac{\partial \sigma_n}{\partial \vec{\delta}_t} & = & 0 \\ \dfrac{\partial \vec{\sigma}_t}{\partial \vec{\delta}_t} & = & \dfrac{\mathrm{K} K_t}{K_t + \mathrm{K}} \mathbf{Id} + \dfrac{-\mu\sigma_n + c + \mathrm{K}\lambda^-}{K_t + \mathrm{K}} \cdot \dfrac{K_t}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|} \cdot \left(\mathbf{Id} - \dfrac{(\vec{\delta}_t - \vec{\delta}_t^{pl-}) \otimes (\vec{\delta}_t - \vec{\delta}_t^{pl-})}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|^2} \right) \end{cases} \quad (43)$$

9 In the preceding attempts [Kol00],[CR09039] of introduction in explicit version [Div97], it appeared that these modelings prove generally not very powerful [Div97]

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$$\begin{cases} \dfrac{\partial \sigma_n}{\partial \vec{\delta}_t} = 0 \\ \dfrac{\partial \vec{\sigma}_t}{\partial \vec{\delta}_t} = \dfrac{\mathrm{K} K_t}{K_t+\mathrm{K}} \mathbf{Id} + \dfrac{-\mu \sigma_n + c + \mathrm{K}\lambda^-}{K_t+\mathrm{K}} \cdot \dfrac{K_t}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|} \cdot \left(\mathbf{Id} - \dfrac{(\vec{\delta}_t - \vec{\delta}_t^{pl-}) \otimes (\vec{\delta}_t - \vec{\delta}_t^{pl-})}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|^2} \right) \end{cases}$$ **éq 3.2-1**

Notice 1 : The tangent matrix in the plastic phase (in slip) is not-symmetrical, it is degenerated if work hardening is null ($\mathrm{K}=0$). One can display a clean vector associated with the worthless eigenvalue:

$$\begin{pmatrix} K_n & 0 \\ \mu K_n \dfrac{\vec{\delta}_t - \vec{\delta}_t^{pl-}}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|} & \dfrac{-\mu \sigma_n + c}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|} \left(\mathbf{Id} - \dfrac{(\vec{\delta}_t - \vec{\delta}_t^{pl-}) \otimes (\vec{\delta}_t - \vec{\delta}_t^{pl-})}{\|\vec{\delta}_t - \vec{\delta}_t^{pl-}\|^2} \right) \end{pmatrix} \cdot \begin{pmatrix} 0 \\ \vec{\delta}_t - \vec{\delta}_t^{pl-} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (44)$$

For this reason the isotropic parameter of work hardening is introduced.

Notice 1 : In the studies one notes a difficulty of convergence related to the fact that one passes from the state of plasticization (for example: work hardening due to the loading in force imposed) towards the elastic solution. The tangent matrix in the phase of prediction is not "good" to recover the elastic phase and calculation converges only when the step is very cut out. To have convergence it is just necessary to take a step with the elastic tangent matrix. It is enough to use the key word PAS_MINI_ELAS in the keyword factor NEWTON of STAT_NON_LINE or DYNA_NON_LINE.

3.3 Internal variables

The law JOINT_MECA_FROT have eighteen internal variables. From the point of view of the law of behavior, only the first, the third and the fourth are *stricto sensu* internal variables. The others provide indications on the state of hydromechanics of the joint to a given moment.

__Internal variables__ :

$V1=\lambda$: parameter growing indicating cumulated plastic tangential displacement (without orientation).
$V2$: slip meter $=0$ if linear mode, $=1$ if mode is plastic
$V3, V4=\vec{\delta}^{pl}$: vector of tangential displacement plastic compared to the starting point (indicates the current position of balance). $V4$ is put at zero in 2D

Mechanical indicators:

$V5$: indicator of complete opening $=0$ closed $(\sigma_n < c/\mu)$, $=1$ opened $(\sigma_n = c/\mu)$
$V6=\|\vec{\sigma}_\tau\|$: normalizes tangent constraint
Value of the jump in the local reference mark:
$V7=\delta_n$: normal jump, $V8=\delta_{t1}$ tangential jump, $V9=\delta_{t2}$ tangential jump (no one in 2D)
Value of the jump in the local reference mark:
$V10$: unused variable
$V11=\sigma_n$: normal mechanical constraint (without pressure of fluid)

Hydraulic indicators:

Components of the gradient of pressure in the total reference mark (only for xxx_JOINT_HYME) :
$V12=\partial_x p$, $V13=\partial_y p$, $V14=\partial_z p$ three components in space
Components of hydraulic flow in the total reference mark (only for xxx_JOINT_HYME):
$V15=w_x$, $V16=w_y$, $V17=w_z$ three components in space
$V18=p$: pressure of fluid imposed by the user (PRES_FLUIDE) in the case of modelings xxx_JOINT or pressure of fluid interpolated from that calculated (degree of freedom of the problem) with the nodes mediums of the elements of joint of modelings: xxx_JOINT_HYME.

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3.4 Psmall channel in depreciation account in dynamics

Dynamic damping in each element of joint `3D_JOINT` is represented by a matrix of damping $\mathbf{C}$, calculated with each step of calculation by the option `AMOR_MECA` for this kind of modeling. The assembled operator $\mathbf{C}$ intervenes then in the first member of the equilibrium equation of the dynamic problem and of this fact it is invariant diagrams of integration.

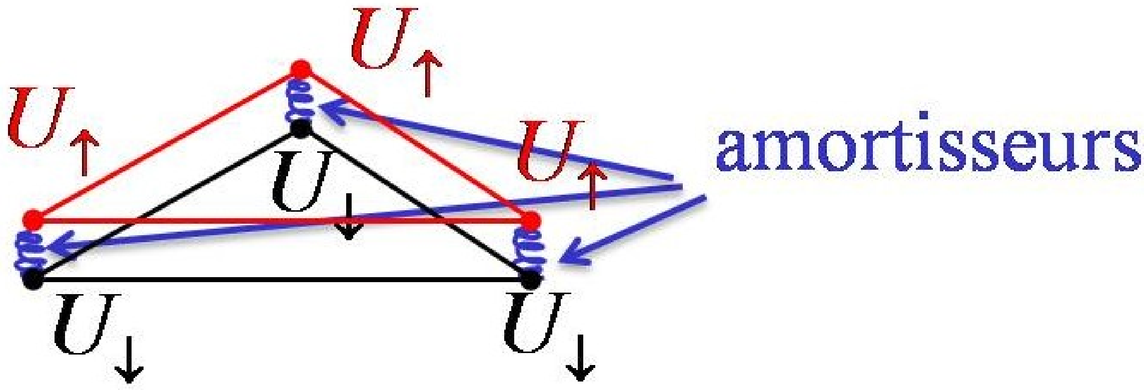

Figure 3.4-1: Catch in depreciation account

The terms of this matrix are obtained by the integration of surface densities of normal damping A_n (informed by the keyword `AMOR_NOR` behavior `JOINT_MECA_FROT`) or tangential A_t (informed by the keyword `AMOR_TAN` behavior `JOINT_MECA_FROT`) integrated on surface $Surf$ of a face of element `3D_JOINT`.
These terms are equivalent to the distribution in parallel of characteristics of discrete of type `DIS_T` on each segment uniting each couple of nodes tops in with respect to vis-a-vis the other of the element (see the figure 3.4-1). These characteristics are affected with their full value only if the element of joint is in compression: maybe if the seventh component of variable interns behavior `JOINT_MECA_FROT`, value of the jump in the local reference mark, $V7=\delta_n$ is negative.
Elements of the matrix $\mathbf{C}$ are directly calculated for the degrees of freedom of the nodes tops. Consequently, if the number of nodes tops of a face of element of joint is n_s, one will affect for the matrix of mechanical cushioning the local characteristics $A_n.Surf/n_s$ or $A_t.Surf/n_s$ distributed in the local reference mark of the element on each diagonal term. One affects their values opposed under the terms of coupling. It is thus as if one had n_s discrete of damping of the type `DIS_T` affected on meshs `SEG2`.
If the element of joint is not in compression: maybe if the seventh component of variable interns behavior `JOINT_MECA_FROT`, value of the jump in the local reference mark, $V7=\delta_n$ is not negative, then one affects the preceding computed values of a coefficient indicated by the keyword `COEF_AMOR` (no by default) in the law of behavior `JOINT_MECA_FROT`.

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4 Catch in account of the hydrostatic pressure without coupling

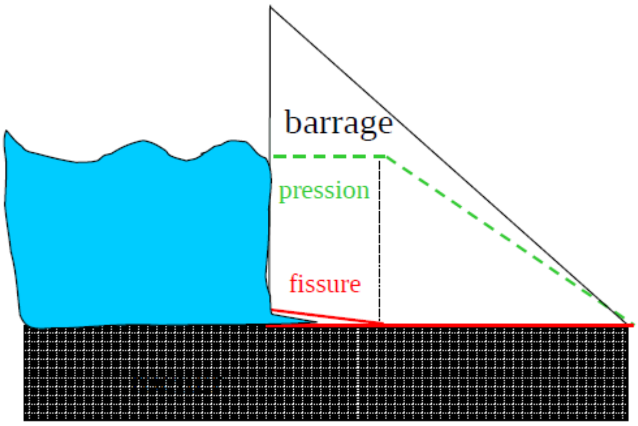

Figure 4-1: Illustration of a possible calculation of stability of a stopping with the profile of pressure imposed

Although modeling `XXX_JOINT` does not couple mechanics and hydraulics, one can however explicitly introduce the influence of a fluid on mechanics via an imposed pressure. The presence of the fluid in the joint modifies the normal mechanical constraint $\sigma_n \rightarrow \sigma_n - p$. By putting an important pressure one is able to make break the joint by a simple hydraulic effect. To take into account the hydrostatic effects the mechanical law is shifted to the bottom (Figure 1.3-1) according to the value of pressure p in each point of integration.
The digital implementation is easy in the event of complete writing of the mechanical laws in form clarifies nonincremental according to displacements and of the internal variables (it is necessary to exclude the dependence from the constraints at the moment more according to constraints at the previous moment). In this case the only modification of normal curve is sufficient to introduce the coupling:

$$\sigma_n = \sigma_n^{meca}(\delta_n, \delta_t) - p \quad (45)$$

While being limited to this kind of physical phenomenon, it is possible to make studies where the profile of pressure is imposed by user, for example a study of stability of stopping under conservative assumption (Figure 4-1), i.e. in the presence of uplift, whose form is very penalizing. In order to make to a calculation with an imposed pressure the user must define a function, by the keyword `PRES_FLUIDE`, which depends at the same time on space (profile of not-homogeneous pressure) and on time (evolution of the profile of pressure).

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5 Theoretical formulation of the coupling hydromechanics

The introduced laws can be based on a coupled hydraulic modeling, by the elements `XXX_JOINT_HYME`. In this part one will speak about the hydraulic part of the law, as well as coupling him even; all the details on the mechanical part of the law were described previously.

5.1 Hydraulic modeling

The fluid runs out of the zones of high pressure towards those basic pressure. A theoretical manner to take into account the steady flow is to associate at the hydraulic state given an energy[10] $H(p(x))$ depending on the distribution of pressure. The first assumption consists in supposing that energy depends explicitly on the variation of pressure and not on the pressure itself $H=H(\nabla p(x))$. By taking the convex shape simplest possible of this dependence in gradient, one obtains energy thus $H=C(\vec{\nabla} p)^2/2$ where C is a parameter of the law, which does not depend on the pressure.
By calculating the generalized efforts corresponding to the field of gradient of pressure one obtains the first law of Fick. Hydraulic flow is proportional to the gradient of pressure:

$$\vec{w}=\frac{\partial H}{\partial \vec{\nabla} p}=C\,\vec{\nabla} p \qquad (46)$$

In this energy formalism one seeks the field of pressure to balance by minimization of the hydraulic power $\min_{p(\vec{x})}\int_{\Omega} H(\vec{\nabla} p(\vec{x}))\,d\Omega$. What gives an equilibrium equation resembling that of mechanics $\mathrm{div}\,\vec{w}=0$. Within the framework of this model the solution of hydraulic equilibrium equation is equivalent to a resolution of mechanical problem into quasi-static, where hydraulic flow is equivalent to the constraint $\vec{w}\Leftrightarrow\boldsymbol{\sigma}$, the field of pressure corresponds to the field of displacement $p(\vec{x})\Leftrightarrow u(\vec{x})$ and finally the gradient of pressure is connected with the field of deformation $\vec{\nabla} p\Leftrightarrow\varepsilon$.

5.2 Influence of hydraulics on mechanics

The presence of the fluid in the joint adds a hydrostatic constraint and this fact modifies the normal mechanical constraint $\sigma_n\rightarrow\sigma_n-p$. By putting an important pressure one is able to make break the joint by a simple hydraulic effect. One can downwards shift the mechanical law according to the value of pressure p in each point to take into account the effects of the pressure, to see Figure 1.3-1.

5.3 Influence of mechanics on hydraulics

In the case of flow of fluid through a crack hydraulic flow must increase with the opening (δ_n) of the latter ($\vec{w}\sim O(\delta_n)\vec{\nabla} p$). In the law of One tenth of a poise, which was found empirically for the laminar flow of a viscous and incompressible fluid, the dependence of flow in opening is cubic (the law is often called the cubic law). The hydraulic part of the law uses this kind of coupling. The equations to be solved are written in the following way:

$$\mathrm{div}\,\vec{w}=0\,;\,\vec{w}=\frac{\rho}{12\bar{\mu}}\delta_n^3\vec{\nabla} p \qquad (47)$$

In the case of a flow of fluid through junctions of a stopping, one notes significant flows even for the closed joints. It is necessary then to define a minimal thickness ϵ_{min} , key word `OUV_MIN`, below which flow reaches its minimal value. We regularize the equations of flow in the following way:

$$\vec{w}=\frac{\rho}{12\bar{\mu}}\max(\epsilon_{min},\epsilon_{min}+\delta_n)^3\vec{\nabla} p \qquad (48)$$

For a gradient of pressure not-no one flow never reaches the zero value, $\min\vec{w}\sim\epsilon_{min}^3\vec{\nabla} p$, which corresponds to the flow through permeable walls of the closed joint.

10 We use a simplified notation, the exact term would be: rate of density of energy.

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5.4 Hydraulic coupling

The hydraulic coupling utilizes the two mechanisms described previously: of with dimensions fluid acts by pressure on the lips of joint, other with dimensions plus the crack is open plus the flow of fluid is easy. In absence of the external forces hydraulic calculation arises schematically in this form:

$$\begin{cases} \vec{w}=\vec{w}(\vec{\boldsymbol{\delta}}(u),\vec{\nabla}p); & \operatorname{div}\vec{w}=0 \\ \vec{\boldsymbol{\sigma}}=\vec{\boldsymbol{\sigma}}(\vec{\boldsymbol{\delta}}(u),p); & \operatorname{div}\vec{\boldsymbol{\sigma}}=0 \end{cases} \Rightarrow \vec{Y}=\vec{Y}(\vec{X});\operatorname{div}\vec{Y}=0 \quad (49)$$

The resolution of the equilibrium equations hydraulic is equivalent to the resolution of the mechanical problem into quasi-static, where the generalized constraints are introduced $\vec{Y}=(\vec{w},\vec{\boldsymbol{\sigma}})$, and the vector field of the unknown factors $\vec{X}=(p,u)$.

5.5 Tangent matrix

Considering the generalized efforts do not depend on u that through $\vec{\boldsymbol{\delta}}(u)$, to calculate the tangent matrix of hydraulic coupling, it is necessary to know only the four following terms:

$$\frac{\partial\vec{\boldsymbol{\sigma}}}{\partial\vec{\boldsymbol{\delta}}},\ \frac{\partial\vec{\boldsymbol{\sigma}}}{\partial p},\ \frac{\partial\vec{\boldsymbol{w}}}{\partial\vec{\nabla}p} \text{ and } \frac{\partial\vec{w}}{\partial\vec{\boldsymbol{\delta}}} \quad (50)$$

The first term is the same one as in pure mechanics, it is given in the equation (17). The second term is commonplace, because the only not-worthless component is equal to $\partial\sigma_n/\partial p=-1$. The diagonal hydraulic term takes a simple form because hydraulic flow depends only on the gradient of pressure:

$$\frac{\partial\vec{\boldsymbol{w}}}{\partial\vec{\nabla}p}=\frac{\rho}{12\bar{\mu}}\max(\epsilon_{min},\epsilon_{min}+\delta_n)^3 \quad (51)$$

In the last term only the derivative compared to the normal opening is not worthless:

$$\frac{\partial\vec{\boldsymbol{w}}}{\partial\delta_n}=\frac{\rho}{4\bar{\mu}}(\epsilon_{min}+\delta_n)^2\vec{\nabla}p \quad (52)$$

This quantity is equal to zero for a closed crack $\delta_n<0$. The tangent matrix thus formulated is not symmetrical.

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6 Features and validation

Two laws of behavior JOINT_MECA_RUPT and JOINT_MECA_FROT are introduced. They are validated on the elementary cases tests **ssnp162** and the pseudonym gravity dam **ssnp142**. The procedure of keying-up is validated on the simulation of injection of the coulis between two rectangular blocks embedded on the ground **ssnp143**. The procedures of sawing is validated by the sawing of two rectangular blocks with various types of embedding **ssnp143**.

Validation in pure mechanics, modelings standard XXX_JOINT	
Law: JOINT_MECA_RUPT	Law: JOINT_MECA_FROT
Tests: ssnp162a/b/c; ssnp142a/b; Keying-up: ssnp143a/b Sawing: ssnp143c/d/e/f	Tests: ssnp162d/e/f; ssnp142c/d Sawing: ssnp143c/d/g/h

Coupled hydraulic validation, modelings standard XXX_JOINT_HYME	
Law: JOINT_MECA_RUPT	Law: JOINT_MECA_FROT
Tests: ssnp162g/h/i; ssnp142e/f	Tests: ssnp162j/k/l; ssnp142g/h

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