Law of behavior of reinforced concrete plates GLRC_DAMAGE

Summary:

This documentation presents the theoretical formulation and the digital integration of the law of behavior `GLRC_DAMAGE` [bib1]. She is written in a total way in efforts and moments resulting for modelings in finite elements from plates. This law integrates the elastic behavior and endommageable in inflection coming from the concrete and the elastoplastic behavior coming primarily from the steel reinforcements starting from the material characteristics of two materials and the composition of the section of the reinforced concrete plate. It results from it an elastoplastic behavior endommageable cyclic, adapted for dynamic studies of structures out of reinforced concrete. The model `GLRC_DAMAGE` current does not take into account the damage out of membrane and is thus not very precise when the requests of the plate are dominated by the effects out of membrane. On the other hand, the rupture of a plate depends especially on the behavior of the steel, modelled by the elastoplastic part of the model. It is thus estimated that the rupture should be represented correctly even out of membrane. A model similar to that one, `GLRC_DM`, is able to better represent the damage out of membrane/inflection, but does not take into account the phase of plasticization of steels and cannot thus be used to simulate the rupture.

The orthotropic elasticity induced by the orthogonal network of reinforcements is not taken into account that within the framework of a linear analysis; for the nonlinear analysis, one simplifies by building an approximate equivalent isotropic elasticity. At the present time, the tangent module coherent general is not yet available for this model.

Contents

1 Introduction

1.1 Models of total behaviors

The total models represent the evolutions of material within the structure studied – beam and plate – on the basis of a relation between the generalized sizes of deformations (extension, curve, distortion) and the generalized efforts (efforts of membrane, inflection, efforts cutting-edges). These models "are fixed" as a preliminary using a fixed local analysis (for example using the results of limiting analysis of the sections), according to the characteristics of the materials concrete and steel constituting the plate and the distribution of those in the section, cf [Figure 1.1-a]. The general diagram is the following:

$$d(\boldsymbol{\epsilon}, \boldsymbol{\kappa}, \boldsymbol{\gamma}) \underset{\substack{\text{loi globale de comportement} \\ \uparrow \\ \text{analyse locale}}}{\rightarrow} d(\boldsymbol{N}, \boldsymbol{M}, \boldsymbol{T}) \quad (1.1.1)$$

This local analysis must take account of the various couplings: for example the evolution in inflection is dependent on the value of the normal effort applied. The nonlinear balance of the structure is treated at the total level on the generalized efforts, via the kinematics of plate considered.

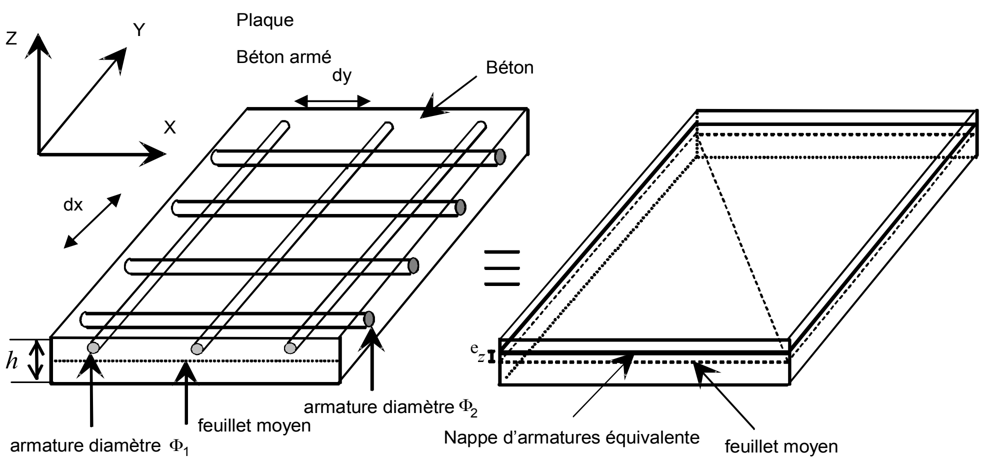

Figure 1.1-a: Reinforced concrete flagstone.

As the local analysis is put in work only in preprocessing (within the framework of an analysis in monotonous load), there is not an immediate means to return in the course of calculation to the local analysis of the constraints starting from the generalized internal efforts. Indeed, the dissipative character of the irreversible laws of behavior requires to store during cycles the evolution of the internal variables in any moment if one wants to calculate the constraints in a particular point. One could plan to launch in parallel the three-dimensional law of behavior according to [éq 1.1-1] and to integrate in the thickness to return to a total behavior, but the cost and the complexity of such a approach seem an obstacle. It should be noted that could be a means of considering the mistake made by a total law of behavior. On the other hand, this approach is not adopted yet in *Code_Aster*.

This kind of model can be usefully validated by comparison with a direct analysis carried out with a local model.

1.2 Objectives of the law GLRC_DAMAGE

One finds the formulation initial of the model total of reinforced concrete of plate GLRC_DAMAGE, established by Koechlin in 2002, in [bib1], [bib2] and [bib3].

This model was first of all developed for applications of dynamics with ruin under impact of works out of reinforced concrete. The elastoplastic answer of the model is essential for this kind of applications. Indeed, the dissipation of energy by plasticization of steels is important. The taking into account of the damage by cracking of the concrete makes it possible to make more precise the first phases of the behavior nonlinear. Within the framework of seismic applications, one can expect a reversed situation: the damage and the answer after cracking are essential, while it is rare to go until mobilizing the generalized plasticization of steels. It seems however advantageous to have the same model to treat these two families of applications.

The formulation of the model is established within the framework of the thermodynamics of the irreversible processes. It combines plasticity with work hardening, in particular brought by steels, and the damage brought by the concrete fissuring at the time of the inflection of the plate. The plastic behavior is built on the basis of analysis limits in inflection of a reinforced concrete plate. It is described using the framework of generalized standard materials. On the basis of experimental result, cf [bib1], a linear kinematic work hardening was selected to treat the cyclic behavior. The damage is introduced to represent the elastic loss of rigidity which takes place by cracking of the concrete before the plasticization of steels. The threshold of damage is supposed to be constant. This behavior is supposed to be independent speeds of requests (dissipations are instantaneous).

2 Formulation of the model

The formulation of the model hereafter is presented GLRC_DAMAGE, under the formalism of the thermodynamics of the irreversible processes.

One must note that the use of this model is associated with that of an element of plate. If one chooses the family of finite elements **DKT** (supported modeling: DKTG), one adopts the theory of **Coil-Kirchhoff**, i.e. one considers no transverse distortion in the thickness of the plate. The model GLRC_DAMAGE could be usable with the finite elements of thick plate Q4G, but this extension was not carried out yet.

The mesh of the finite element is supposed to be placed on the average layer of the flagstone (with $z=0$).

For being able to use the model of behavior GLRC_DAMAGE in two different types of analysis, one chose according to the case:

[1] ***for a linear elastic analysis of reinforced concrete plate:*** to take into account the orthotropism induced by the orthogonal network of steel reinforcements, as well as the coupling inflection-membrane in the event of unequal tablecloths of reinforcements, via a steel-concrete homogenized elastic behavior;

[2] ***for an elastoplastic analysis endommageable nonlinear of reinforced concrete plate:*** to neglect the orthotropism and the coupling inflection-membrane in the phase of elasticity. This assumption makes it possible to simplify the model, by supposing that in the presence of strongly non-linear phenomena, orthotropic elasticity becomes negligible, especially at the time of the modeling of the rupture. Moreover, in practice one expects that the walls, veils as well as the other elements of structure are reinforced about in the same way between the two orthogonal principal directions. That causes to decrease the effect of the elastic orthotropism. On the other hand, one very often chooses asymmetrical reinforcements in order to optimize them according to the direction of the loading due to the actual weight. That tends to induce a coupling membrane-inflection in elasticity besides that in plasticity. However, even if the model neglects asymmetry in elasticity, its influence in

elastoplasticity, the dominating behavior during the rupture, can be controlled through functions threshold, which they can be asymmetrical.

The model is defined by the description of the variables of state, which represent the mechanical system in each material point of the average surface of the plate, the surface density of free energy which includes the form of the relations of behavior and the type of work hardening, the expression of the criteria of plasticity and damage, and the laws of irreversible evolution, deduced from the principle of the maximum work of Drücker.

2.1 Variables of state

The total variables of state are the following ones. First of all aggregate variables of deformation:

[1] a membrane tensor of deformation: $\boldsymbol{\epsilon}$ defined in the tangent plan in the plate.

[2] a tensor of curve: $\boldsymbol{\kappa}$ defined in the tangent plan in the plate.

Then internal variables:

[1] two variables of damage associated with the parts higher, d_1 and lower, d_2 plate. They are put a ceiling to each one with a value, d_1^{max} and d_2^{max}.

[2] two tensors of plastic curve associated with the plasticization of the beds of steel, superior and inferior $\boldsymbol{\kappa}_1^p$, $\boldsymbol{\kappa}_2^p$.

[3] two tensors of plastic membrane deformation associated with the plasticization of the beds of steel, superior and inferior, $\boldsymbol{\epsilon}_1^p$, $\boldsymbol{\epsilon}_2^p$.

[4] tensors of order 2 of internal variables of work hardening kinematics $\boldsymbol{\alpha}$.

2.2 Free energy: linear elastic case

The surface density of free energy is an additive expression of the contributions rubber band of membrane and inflection:

$$\Phi_e^S=\frac{1}{2}\boldsymbol{\epsilon}:\boldsymbol{H}_m:\boldsymbol{\epsilon}+\frac{1}{2}\boldsymbol{\kappa}:\boldsymbol{H}_f:\boldsymbol{\kappa}+\boldsymbol{\epsilon}:\boldsymbol{H}_{mf}:\boldsymbol{\kappa} \qquad (2.2.1)$$

Tensors $\boldsymbol{H}_m$, $\boldsymbol{H}_f$, $\boldsymbol{H}_{mf}$ (coupling inflection-membrane in the event of unequal tablecloths of reinforcements in the thickness) are described with [§3.1]. In the actual position of the model, one supposes that:

$$\boldsymbol{H}_{mf}=\boldsymbol{0}$$

thus that the plate is symmetrical and that there is no elastic coupling membrane-inflection. In the model, the coupling membrane-inflection can appear only because of one evolution towards elastoplasticity (cf §2.5).

2.3 Free energy: elastoplastic case endommageable

In this case, one neglects the elastic orthotropism induced by reinforcement in the two directions of the plan, as well as the elastic coupling inflection-membrane (for dissymmetrical tablecloths of reinforcements). One thus compares the steel reinforcements to an isotropic elastic membrane, cf [Figure 1.1-a].

The surface density of free energy is an additive expression of the contributions elastoplastic of membrane, elastoplastic endommageable of inflection, and kinematic work hardening:

$$\Phi_{epd}^S(\boldsymbol{\epsilon},\boldsymbol{\epsilon}^p,\boldsymbol{\kappa},\boldsymbol{\kappa}^p,d_1,d_2,\boldsymbol{\alpha})=\Phi_{e,m}^S(\boldsymbol{\epsilon}-\boldsymbol{\epsilon}^p)+\Phi_{ed,f}^S(\boldsymbol{\kappa}-\boldsymbol{\kappa}^p,d_1,d_2)+\Phi_p^S(\boldsymbol{\alpha})+H(d_j-d_j^{max}) \qquad (2.3.1)$$

with the energy of work hardening:

$$\Phi_p^S(\boldsymbol{\alpha})=\frac{1}{2}\boldsymbol{\alpha}:\boldsymbol{C}:\boldsymbol{\alpha} \tag{2.3.2}$$

where $\boldsymbol{C}$ is a kinematic tensor of work hardening of Prager. In practice the tensor $\boldsymbol{C}$ is diagonal, with a coefficient C_m out of membrane and another C_f in inflection, one thus has:

$$\boldsymbol{C}=\begin{pmatrix} C_m & 0 & 0 & 0 & 0 & 0 \\ 0 & C_m & 0 & 0 & 0 & 0 \\ 0 & 0 & C_m & 0 & 0 & 0 \\ 0 & 0 & 0 & C_f & 0 & 0 \\ 0 & 0 & 0 & 0 & C_f & 0 \\ 0 & 0 & 0 & 0 & 0 & C_f \end{pmatrix}$$

In [éq 2.3.1], H indicate an indicating function of the field of admissibility of the thermodynamic potential. Concretely, it is used to limit the evolution of the damage to the top of d_j^{max}. d_j^{max} are identified by [éq 3.2.11].

The densities of energy of membrane and inflection are given by:

[1] Out of membrane:

$$\Phi_{e,m}^S(\boldsymbol{\epsilon}-\boldsymbol{\epsilon}^p)=\frac{1}{2}(\boldsymbol{\epsilon}-\boldsymbol{\epsilon}^p):\boldsymbol{H}_m:(\boldsymbol{\epsilon}-\boldsymbol{\epsilon}^p) \tag{2.3.3}$$

[2] In inflection:

$$\Phi_{ed,f}^S(\boldsymbol{\kappa}-\boldsymbol{\kappa}^p,d_1,d_2)=\frac{\lambda_f}{2}tr(\boldsymbol{\kappa}-\boldsymbol{\kappa}^p)^2\xi_f(tr(\boldsymbol{\kappa}-\boldsymbol{\kappa}^p),d_1,d_2)+\mu_f\sum_{i=1}^{2}\widetilde{(\kappa-\kappa^p)}_i^2\xi_f(\widetilde{(\kappa-\kappa^p)}_i,d_1,d_2) \tag{2.3.4}$$

where one introduces the parameters of Lamé in inflection λ_f and μ_f :

$$\lambda_f=\frac{h^3}{12}\lambda$$
$$\mu_f=\frac{h^3}{12}\mu$$

h being the thickness of the plate and λ , μ coefficients of Lamé of homogenized material.
$\widetilde{(\kappa-\kappa^p)}_i$ indicate $i^{\text{ème}}$ eigenvalue of $\boldsymbol{\kappa}-\boldsymbol{\kappa}^p$. One will note thereafter $\boldsymbol{\kappa}^e$ the tensor of variation of elastic curve definite according to the assumption of partition of the deformations in inflection by: $\boldsymbol{\kappa}^e=\boldsymbol{\kappa}-\boldsymbol{\kappa}^p$.
And finally, one defines the function characteristic of the damage in inflection ξ_f :

$$\xi_f(x,d_1,d_2)=\frac{1+\gamma d_1}{1+d_1}H(x)+\frac{1+\gamma d_2}{1+d_2}H(-x) \tag{2.3.5}$$

In this expression H is the Heaviside function and γ a parameter of the damage ranging between 0 and 1. This function ξ_f characterize the weakening of the stiffnesses by damage. It is decreasing for d_1, d_2 positive. It is convex (thanks to the choice of γ, identified by the procedure described in [§3.2.2]) ensuring the stability of "material" reinforced concrete of the flagstone.

2.4 Elastoplastic law of behavior endommageable

The elastoplastic law of behavior endommageable (law of state) provides the dual variables: efforts of membrane, the bending moments which are of the tensors of order 2 definite on the tangent plan of the plate, the forces of damage and the irreversible tensors of work hardening. They are written:

[1] Effort of membrane:

$$\boldsymbol{N}=\frac{\partial\Phi^S_{epd}}{\partial\boldsymbol{\epsilon}}=\boldsymbol{H}_m:(\boldsymbol{\epsilon}-\boldsymbol{\epsilon}^p) \tag{2.4.1}$$

[2] Bending moment:

$$\boldsymbol{M}=\frac{\partial\Phi^S_{epd}}{\partial\boldsymbol{\kappa}}=\boldsymbol{H}^d_f(\boldsymbol{\kappa}-\boldsymbol{\kappa}^p,d_{1,}d_2):(\boldsymbol{\kappa}-\boldsymbol{\kappa}^p) \tag{2.4.2}$$

The membrane tensor of elasticity $\boldsymbol{H}_m$ is given in [§3.1], while $\boldsymbol{H}^d_f$ is the tensor of elasticity endommageable which depends on the variables of damage d_1, d_2 and also of the signs of certain components of $\boldsymbol{\kappa}^e=\boldsymbol{\kappa}-\boldsymbol{\kappa}^p$ (trace and eigenvalues, in particular). One recalls that in [éq. 2.4.1] and [éq. 2.4.2] the elastic coupling membrane-inflection is neglected (see [§2.2]). Moreover, because of the presence of the eigenvalues of the elastic curves in the expression of the free energy (see [éq. 2.3.4]), one calculates the constraints generalized by using the equations [éq. 2.4.1], [éq. 2.4.2] in the clean reference mark. The details of the transformation between the reference marks are available in [R7.01.32]. It is also specified that according to the isotropic assumption of elasticity the coupling membrane-inflection is due only to the elastoplastic process through $\boldsymbol{\epsilon}^p$ and $\boldsymbol{\kappa}^p$ (cf §2.5).

Note:
It is noted that the clean reference mark of the moments is the same one as that of the elastic curves. In the same way the clean reference mark of the efforts of membrane is the same one as that of elastic strain. In the absence of damage, $\xi_f(x,d_{1,}d_2)=1$ *: one finds well a behavior of elastic plate isotropic.*

[1] Forces of damage, for $j=1,2$:

$$Y_j=-\frac{\partial\Phi_{epd}}{\partial d_j}=\frac{1-\gamma}{(1+d_j)^2}\left(\frac{\lambda_f}{2}tr(\boldsymbol{\kappa}^e)^2H((-1)^j tr(\boldsymbol{\kappa}^e))+\mu_f\sum_i(\tilde{\kappa}^e_i)^2H((-1)^j\tilde{\kappa}^e_i)\right) \tag{2.4.3}$$

It is noted that them Y_j defined by [éq. 2.4.3] are positive (it is a surface restitution of energy, of which the unit IF is J/m ²) if $\gamma\in[0,1]$.

[2] Efforts and irreversible moments of plasticity:

$$\boldsymbol{N}^p=\frac{\partial\Phi^S_{epd}}{\partial\boldsymbol{\epsilon}^p}=\boldsymbol{H}_m:(\boldsymbol{\epsilon}-\boldsymbol{\epsilon}^p)=\boldsymbol{N} \text{ and } \boldsymbol{M}^p=\frac{\partial\Phi^S_{epd}}{\partial\boldsymbol{\kappa}^p}=\boldsymbol{H}^d_f:(\boldsymbol{\kappa}-\boldsymbol{\kappa}^p)=\boldsymbol{M} \tag{2.4.4}$$

[3] Tensors of kinematic recall of work hardening:

$$\boldsymbol{X}^m=\frac{\partial \Phi_{epd}^S}{\partial \boldsymbol{\alpha}_m}=-\boldsymbol{C}_m:\boldsymbol{\alpha}_m \text{ and } \boldsymbol{X}^f=\frac{\partial \Phi_{epd}^S}{\partial \boldsymbol{\alpha}_f}=-\boldsymbol{C}_f:\boldsymbol{\alpha}_f \quad (2.4.5)$$

2.5 Criteria – surfaces thresholds

2.5.1 Criterion of plasticity

criterion of plasticity of Johansen with kinematic work hardening is duplicated for the plasticity of the upper part (index 1) and the lower part (index 2) of the plate. This criterion couples plasticity out of membrane with that in inflection. If x and y the directions of the orthogonal reinforcement of the concrete plate indicate, for $j=1,2$, the criterion is written:

$$\begin{aligned} f_j^p(\boldsymbol{N}-\boldsymbol{X}^m, \boldsymbol{M}-\boldsymbol{X}^f) &= -(M_{xx}-X_{xx}^f-M_{jx}^p(N_{xx}-X_{xx}^m)) \\ &\times \quad (M_{yy}-X_{yy}^f-M_{jy}^p(N_{yy}-X_{yy}^m))+(M_{xy}-X_{xy}^f)^2\leq 0 \end{aligned} \quad (2.5.1)$$

f_j^p define a convex field (cf [bib4]) of reversibility, parameterized by 4 functions: $M_{jx}^p(N_{xx})$ and $M_{jy}^p(N_{yy})$. These functions are built using the limiting analysis of sections of reinforced concrete beam representative of the section of the studied plate, taken in the direction of the reinforcements of reinforcement, cf [bib1, bib2]. It is noted that only the differences between the state of requests and the tensors of recall in work hardening intervene in the expression of the criterion. This is characteristic of the models with kinematic work hardening.

The potential of dissipation associated with this criterion is given by:

$$\begin{gathered} \Psi_{tot}=\boldsymbol{N}:\dot{\boldsymbol{\epsilon}}+\boldsymbol{M}:\dot{\boldsymbol{\kappa}}-\dot{\Phi}_{epd}=\Psi_p+\Psi_d \\ \Psi_p=\boldsymbol{N}:\dot{\boldsymbol{\epsilon}}^p+\boldsymbol{M}:\dot{\boldsymbol{\kappa}}^p-\dot{\boldsymbol{\alpha}}_m:\boldsymbol{C}_m:\boldsymbol{\alpha}_m-\dot{\boldsymbol{\alpha}}_f:\boldsymbol{C}_f:\boldsymbol{\alpha}_f \\ \Psi_d=Y_1\dot{d}_1+Y_2\dot{d}_2 \end{gathered} \quad (2.5.2)$$

2.5.2 Criterion of damage

criterion of damage fragile without work hardening is defined by a scalar. This criterion is duplicated to differentiate the positive inflections from the negative inflections. He is written:

$$f_j^d(Y_j)=Y_j(\boldsymbol{\kappa}, d_j)-k_j\leq 0 \quad (2.5.3)$$

This criterion represents a convex field (cf [bib2]) of reversibility parameterized by the thresholds k_1 and k_2 who define the appearance of the first cracks in inflection of the reinforced concrete plate. Their unit IF is it J/m^2 . They correspond to a limitation of the surface density of elastic energy. This criterion is associated with the positive potential of dissipation:

$$\Psi_d(d_j, \dot{d}_j)=k_j\dot{d}_j\geq 0 \text{ and } \dot{d}_j\geq 0 \quad (2.5.4)$$

This criterion of damage is basic, but it is its combination with the effect of the damage on the elastic stiffness, cf the function $\xi_f(x, d_1, d_2)$ [éq. 2.3.5], which exploits the answer of the model.

2.6 Laws of plastic flow

The plastic law of flow is written (according to the rule of normality to the criterion [éq. 2.5.1]):

$$\dot{\boldsymbol{\epsilon}}^p=\dot{\boldsymbol{\epsilon}}_1^p+\dot{\boldsymbol{\epsilon}}_2^p=\lambda_1^p\frac{\partial f_1^p}{\partial \boldsymbol{N}}+\lambda_2^p\frac{\partial f_2^p}{\partial \boldsymbol{N}} \tag{2.6.1}$$

$$\dot{\boldsymbol{\kappa}}^p=\dot{\boldsymbol{\kappa}}_1^p+\dot{\boldsymbol{\kappa}}_2^p=\lambda_1^p\frac{\partial f_1^p}{\partial \boldsymbol{M}}+\lambda_2^p\frac{\partial f_2^p}{\partial \boldsymbol{M}} \tag{2.6.2}$$

$$\dot{\boldsymbol{\alpha}}^m=\dot{\boldsymbol{\alpha}}_1^m+\dot{\boldsymbol{\alpha}}_2^m=\lambda_1^p\frac{\partial f_1^p}{\partial \boldsymbol{X}^m}+\lambda_2^p\frac{\partial f_2^p}{\partial \boldsymbol{X}^m}=-(-\dot{\boldsymbol{\epsilon}}_1^p-\dot{\boldsymbol{\epsilon}}_2^p)=\dot{\boldsymbol{\epsilon}}^p \tag{2.6.3}$$

$$\dot{\boldsymbol{\alpha}}^f=\dot{\boldsymbol{\alpha}}_1^f+\dot{\boldsymbol{\alpha}}_2^f=\lambda_1^p\frac{\partial f_1^p}{\partial \boldsymbol{X}^f}+\lambda_2^p\frac{\partial f_2^p}{\partial \boldsymbol{X}^f}=-(-\dot{\boldsymbol{\kappa}}_1^p-\dot{\boldsymbol{\kappa}}_2^p)=\dot{\boldsymbol{\kappa}}^p \tag{2.6.4}$$

where them λ_j^p are the plastic, positive or worthless multipliers, for the positive inflections and the negative inflections. They are divided by the flow out of membrane and that in inflection. One deduces from [éq. 2.6.3] and [éq. 2.6.4], in a usual way in linear kinematic work hardening, that the internal variables of work hardening out of membrane and inflection are equal respectively to the deformations and the plastic curves. It results from this the following relations on the tensors from recall out of membrane and inflection:

$$\boldsymbol{X}^m=-\boldsymbol{C}_m:\boldsymbol{\epsilon}^p \tag{2.6.5}$$

$$\boldsymbol{X}^f=-\boldsymbol{C}_f:\boldsymbol{\kappa}^p \tag{2.6.6}$$

It should be noted that this choice of a tensor of Prager $\boldsymbol{C}_m$ identical at the same time in traction and compression is criticizable. Indeed, in plastic compression, the concrete and steel intervene, while in traction, only steel contributes (concrete being broken).
Criteria of plasticity [éq. 2.4.1] can be reached at the same time (for specific schemes of bi--inflection), therefore the flow can take place since the two criteria reached at the same time. The condition of coherence gives two additional relations:

$$\lambda_j^p\dot{f}_j^p(\boldsymbol{N},\boldsymbol{M})=0 \quad \text{si} \quad \lambda_j^p>0 \quad \text{alors} \quad \dot{f}_j^p(\boldsymbol{N},\boldsymbol{M})=0 \tag{2.6.7}$$

2.7 Law of evolution of the variables of damage

The law of evolution of the damage in inflection is written, for the positive inflections and the negative inflections (according to the rule of normality to the criterion [éq. 2.5.3]):

$$\dot{d}_j=\lambda_j^d\frac{\partial f_j^d}{\partial Y_j} \tag{2.7.1}$$

λ_j^d are the multipliers of damage, positive or worthless. It is pointed out that them Y_j , defined by [éq. 2.4.3], are positive by construction (restitution of energy).

The condition of coherence gives two additional relations:

$$\lambda_j^d\dot{f}_j^d(Y_j)=0 \quad \text{si} \quad \lambda_j^d>0 \quad \text{alors} \quad \dot{f}_j^d(Y_j)=0\Leftrightarrow\dot{Y}_j=0 \tag{2.7.2}$$

The two variables of damage can evolve simultaneously.

3 Parameters of the law

With the total models such as GLRC_DAMAGE one seeks to have a simpler representation of the non-linear phenomena, by using more effective and more robust digital methods. Consequently, it is difficult to allot a physical significance to all the parameters of the model, because most between them include several phenomena. Thus, it is strongly recommended that the parameters of the model are validated by a comparative study between the approach GLRC_DAMAGE and a finer approach, such modelings by multifibre beams, multi-layer hulls or 3D, on a part sufficiently representative of the structure to be analyzed. Or else, the error of an analysis using the model GLRC_DAMAGE cannot be estimated nor controlled.

In any case, the parameters of the law are given in a way simplified using the analysis of the monotonous behavior of a reinforced concrete section, except for the linear elastic behavior where it is also possible to resort to an approach homogenized in plate, cf for example [bib4]. It is supposed that the set of parameters describing the elastic behavior is identifiable independently of the parameters of plasticity and damage. Moreover, the methods of homogenisation enable us to determine the total elastic behavior with a very good precision. One considers in [§3.1] two approaches to homogenize the elastic behavior: in one one makes the assumption of an isotropic equivalent medium and in the other the orthotropism is taken into account. Now, only the isotropic approach is available in *Code_Aster*. Moreover, it is only the isotropic approximation which can be used in combination with plasticity and the damage. In theory, an extension to the orthotropic behaviors into linear and non-linear could be under consideration within a theoretical framework are equivalent. The limitation with the isotropic cases was selected on the one hand because the phenomenon of the orthotropism was considered to be negligible during the rupture, of which simulation is the principal goal of the model, and on the other hand to reduce the formulation of the model and the identification of the parameters.

The parameters of the non-linear behaviors are more delicate to determine than the parameters of elasticity. For the damage that is simpler, because their number is smaller. On the other hand, for plasticity, the user must inform a function, determining the limiting moment of flow according to the membrane effort. Moreover, kinematic work hardening is given by four tensors of Drucker, each one having three parameters. In this version their number is artificially tiny room by supposing that these tensors are the same ones for the two thresholds of plasticity. If the behaviors in elasticity and with damage can be identified without having of a modeling (or a test) of reference, for the plastic behavior that is strongly disadvised.

One notes h the height of the section (thickness of the plate). One notes $\Omega_x^{sup}=A_x^{sup}/d_x^{sup}$ and $\Omega_y^{sup}=A_y^{sup}/d_y^{sup}$ densities of reinforcement in the two directions, cf [Figure 1.1-a], [Figure 3-a]. A_x^{sup} (*resp*. A_y^{sup}) is the surface of the section of a steel bar in the direction x (*resp*. y) higher tablecloth. One makes in the same way for the lower tablecloth: $\Omega_x^{inf}=A_x^{inf}/d_x^{inf}$, $\Omega_y^{inf}=A_y^{inf}/d_y^{inf}$. In general, all quantities having *sup* while exposing correspond to the upper part of the plate, while that with *inf* correspond at its lower part.

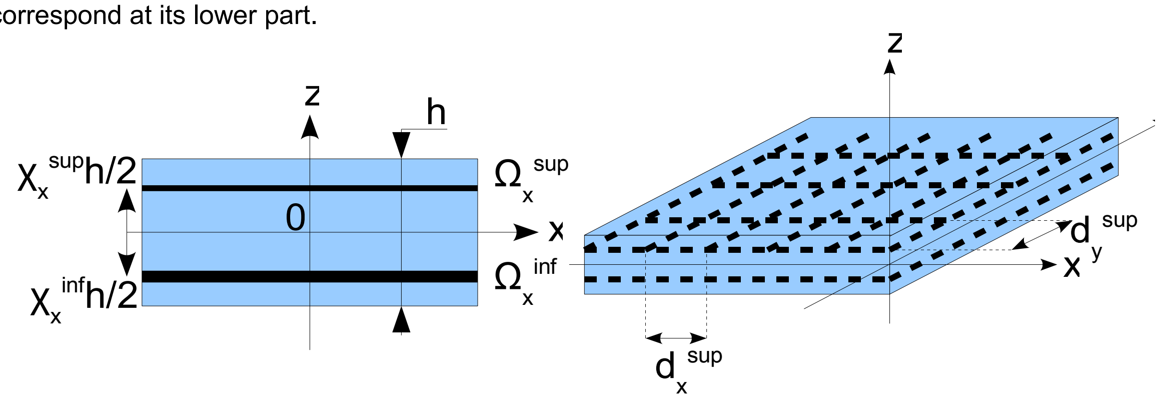

Figure 3-a: Cut of the reinforced concrete flagstone; sight in prospect.

The adimensional positions of the tablecloths of reinforcement in the thickness check:

$$\chi_{x/y}^{sup} \in]0,1[\text{ and } \chi_{x/y}^{inf} \in]-1,0[$$

The equivalent density of the reinforced concrete plate is defined by a simple average balanced by the densities ρ_a, ρ_b respective proportions of the two materials (law of the mixtures). It is used to establish the kinetic energy of the plate.

$$\rho_{éq} = \rho_b + \frac{\rho_a}{h}\left(\Omega_x^{sup} + \Omega_x^{inf} + \Omega_y^{sup} + \Omega_y^{inf}\right) \quad (3.1)$$

This equivalent density must be indicated under the keyword `ELAS` of the operator `DEFI_MATERIAU` of definition of the material concrete, with Young and the Poisson's ratio modulus of the concrete. This last data is used to draw up an estimation the speed of the waves, used for the control of the step of time in explicit integration (condition of Current):

	E	NAKED	RHO
parameter	E_b	ν_b	$\rho_{éq}$
Units IF	[Pa]	without	[kg/m^3]

3.1 Identification of the parameters of linear elastic behavior

The linear elastic behavior is *a priori* orthotropic and a coupling membrane – inflection integrates. To carry out an elastic design preliminary to a nonlinear analysis, one wishes to represent the best possible this kind of behavior of the reinforced concrete structure.

One proposes to identify the coefficients of linear elastic behavior in two manners:

- by **orthotropic approach** where one builds the elastic matrix membrane-inflection starting from the elastic characteristics of the concrete (E_b , ν_b), steel (E_a) and of the geometrical characteristics of the section of reinforced concrete, cf (Figure 3-a).
- by **isotropic approach** where one determines the elastic parameters of the medium homogenized equivalent.

The total elastic law of the reinforced concrete flagstone with coupling membrane and inflection is written with the tensors $\boldsymbol{H}_m$, $\boldsymbol{H}_f$, $\boldsymbol{H}_{mf}$ and is given in the orthogonal local reference mark related to reinforcement by:

$$\begin{pmatrix} N_{xx} \\ N_{yy} \\ 2N_{xy} \\ M_{xx} \\ M_{yy} \\ 2M_{xy} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} H_{1111}^m & H_{1122}^m & 0 \\ H_{1122}^m & H_{2222}^m & 0 \\ 0 & 0 & H_{1212}^m \end{pmatrix} & \begin{pmatrix} H_{1111}^{mf} & H_{1122}^{mf} & 0 \\ H_{1122}^{mf} & H_{2222}^{mf} & 0 \\ 0 & 0 & H_{1212}^{mf} \end{pmatrix} \\ \begin{pmatrix} H_{1111}^{mf} & H_{1122}^{mf} & 0 \\ H_{1122}^{mf} & H_{2222}^{mf} & 0 \\ 0 & 0 & H_{1212}^{mf} \end{pmatrix} & \begin{pmatrix} H_{1111}^{f} & H_{1122}^{f} & 0 \\ H_{1122}^{f} & H_{2222}^{f} & 0 \\ 0 & 0 & H_{1212}^{f} \end{pmatrix} \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2\epsilon_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ 2\kappa_{xy} \end{pmatrix} \quad (3.1.1)$$

The orthogonal local reference mark related to reinforcement is defined with `AFFE_CARA_ELEM` (keyword factor `HULL`, keyword `ANGL_REP`).

In this expression, H^m_{ijkl} are the stiffnesses of membrane, them H^f_{ijkl} are the stiffnesses of inflection and them H^{mf}_{ijkl} are the stiffnesses of coupling membrane-inflection. The orthotropism imposes in this reference mark that terms H_{ij12} are worthless. In the case of two grids of symmetrical reinforcements, there is decoupling membrane-inflection: $H^{mf}_{ijkl}=0$.

It is checked that necessarily: $H^m_{1111} H^f_{1111} - \left(H^{mf}_{1111}\right)^2 > 0$, in the same way in the other direction. Same manner one must have: $H^m_{1111} H^m_{2222} - \left(H^m_{1212}\right)^2 > 0$, $H^f_{1111} H^f_{2222} - \left(H^f_{1212}\right)^2 > 0$, always because of definite-positive character of the tensor of elasticity.

- **Orthotropic approach (**inalienable in `Code_Aster` now)

One directly builds the coefficients by the approximate following relations:

$$\begin{cases} H^m_{1111} = \dfrac{E_b h}{1-\nu_b^2} + E_a \langle \Omega \rangle_x \\ H^m_{2222} = \dfrac{E_b h}{1-\nu_b^2} + E_a \langle \Omega \rangle_y \end{cases}, \begin{cases} H^m_{1122} = \dfrac{\nu_b E_b h}{1-\nu_b^2} \\ H^m_{1212} = \dfrac{E_b h}{1+\nu_b} \end{cases}$$

$$\begin{cases} H^f_{1111} = \dfrac{E_b h^3}{12(1-\nu_b^2)} + \dfrac{E_a h^2}{4} \langle \chi^2 \Omega \rangle_x \\ H^f_{2222} = \dfrac{E_b h^3}{12(1-\nu_b^2)} + \dfrac{E_a h^2}{4} \langle \chi^2 \Omega \rangle_y \end{cases}, \begin{cases} H^f_{1122} = \dfrac{\nu_b E_b h^3}{12(1-\nu_b^2)} \\ H^f_{1212} = \dfrac{E_b h^3}{12(1+\nu_b)} \end{cases}$$

$$\begin{cases} H^{mf}_{1111} = \dfrac{E_a h}{2} \langle \chi \Omega \rangle_x \\ H^{mf}_{2222} = \dfrac{E_a h}{2} \langle \chi \Omega \rangle_y \end{cases}, \begin{cases} H^{mf}_{1122} = 0 \\ H^{mf}_{1212} = 0 \end{cases} \quad (3.1.2)$$

where one posed, to simplify, the expressions:

$$\langle \Omega \rangle_x = \Omega_x^{\sup} + \Omega_x^{\inf}, \langle \Omega \rangle_y = \Omega_y^{\sup} + \Omega_y^{\inf}$$

$$\langle \chi \Omega \rangle_y = \chi_y^{\sup} \Omega_y^{\sup} + \chi_y^{\inf} \Omega_y^{\inf}, \langle \chi \Omega \rangle_y = \chi_y^{\sup} \Omega_y^{\sup} + \chi_y^{\inf} \Omega_y^{\inf}$$

$$\langle \chi^2 \Omega \rangle_x = \chi_x^{\sup^2} \Omega_x^{\sup} + \chi_x^{\inf^2} \Omega_x^{\inf}, \langle \chi^2 \Omega \rangle_x = \chi_x^{\sup^2} \Omega_x^{\sup} + \chi_x^{\inf^2} \Omega_x^{\inf}$$

One supposes thus that steels do not bring stiffness in membrane distortion of the plate, nor in torsion.

Note:

The orthotropic approach is not available in the current version. It is planned to introduce it into the next evolutions of the model.

- **Isotropic approach**

One builds the total elastic matrix of éq. 3.1.1 while supposing:

$$\begin{pmatrix} H^m_{1111} & H^m_{1122} & 0 \\ H^m_{1122} & H^m_{2222} & 0 \\ 0 & 0 & H^m_{1212} \end{pmatrix} = \frac{E^m_{éq} h}{1-(\nu^m_{éq})^2} \begin{pmatrix} 1 & \nu^m_{éq} & 0 \\ \nu^m_{éq} & 1 & 0 \\ 0 & 0 & 1-\nu^m_{éq} \end{pmatrix} \quad (3.1.3)$$

for the membrane part and

$$\begin{pmatrix} H^f_{1111} & H^f_{1122} & 0 \\ H^f_{1122} & H^f_{2222} & 0 \\ 0 & 0 & H^f_{1212} \end{pmatrix} = \frac{E^f_{éq} h^3}{12(1-(\nu^f_{éq})^2)} \begin{pmatrix} 1 & \nu^f_{éq} & 0 \\ \nu^f_{éq} & 1 & 0 \\ 0 & 0 & 1-\nu^f_{éq} \end{pmatrix} \quad (3.1.4)$$

for the inflection part. The coupling membrane-inflection is also neglected:

$$\begin{pmatrix} H^{mf}_{1111} & H^{mf}_{1122} & 0 \\ H^{mf}_{1122} & H^{mf}_{2222} & 0 \\ 0 & 0 & H^{mf}_{1212} \end{pmatrix} = \mathbf{0} \quad (3.1.5)$$

By comparison with [éq. 3.1.3] and [éq. 3.1.4], one chooses the following relations, by privileging the average behavior in the plan and while realising on the directions *X* and *there*, which gives the four elastic coefficients necessary, starting from the notations [éq 3.1.2]:

$$\nu^m_{éq} = \nu_b \frac{2 E_b h}{2 E_b h + E_a (1-\nu_b^2)(\langle \Omega \rangle_x + \langle \Omega \rangle_y)} \quad (3.1.6)$$

$$\nu^f_{éq} = \nu_b \frac{2 E_b h}{2 E_b h + 3 E_a (1-\nu_b^2)(\langle \chi^2 \Omega \rangle_x + \langle \chi^2 \Omega \rangle_y)} \quad (3.1.7)$$

$$E^m_{éq} = E_b \frac{\nu_b (1-(\nu^m_{éq})^2)}{\nu^m_{éq}(1-\nu_b^2)}, \; E^f_{éq} = E_b \frac{\nu_b (1-(\nu^f_{éq})^2)}{\nu^f_{éq}(1-\nu_b^2)}, \; D_{éq} = \frac{E^f_{éq} h^3}{12(1-(\nu^f_b)^2)} \quad (3.1.8)$$

Among the two elastic approaches only the second (*II)* is currently available.

Elastic coefficients of the concrete (E_b , ν_b) like those of steels E_a are well informed under the keyword ELAS of DEFI_MATERIAU. Characteristics of the provision of steels in the concrete plate (Ω_x^{inf}, Ω_x^{sup}, Ω_y^{inf}, Ω_y^{sup}, χ_x^{inf}, χ_x^{sup}, χ_y^{inf}, χ_y^{sup}) are well informed under the keyword TABLECLOTH of DEFI_GLRC.

The height h section is also provided by the operator DEFI_GLRC, keyword CONCRETE, operand THICK. The directions of the local reference mark of orthotropism are defined by the operator AFFE_CARA_ELEM, keyword HULL with the operand ANGL_REP. The linear elastic behavior is usable in nonlinear analysis under the keyword BEHAVIOR with the operand RELATION = 'GLRC_DAMAGE'.

3.2 Identification of the parameters of elastoplastic behavior endommageable

The model of damage, [§2.3], is formulated under the assumption of isotropy (see [§3.1, II]), which is a reasonable approximation in most case. Moreover, it is admitted that the coupling inflection-membrane (terms H_{ijkl}^{mf}) in the elastic phase of the behavior is negligible.

According to [§3.1, II] one identifies the tensor of elasticity inflection-membrane (cf the tensors $\boldsymbol{H}_m$ and $\boldsymbol{H}_f$ defined by [éq 2.3.3] and [éq. 2.3.4]):

$$\begin{pmatrix} N_{xx} \\ N_{yy} \\ 2\mathrm{N}_{xy} \\ M_{xx} \\ M_{yy} \\ 2\mathrm{M}_{xy} \end{pmatrix} = \begin{pmatrix} \frac{E_{éq}^m h}{1-(\nu_{éq}^m)^2} \begin{pmatrix} 1 & \nu_{éq}^m & 0 \\ \nu_{éq}^m & 1 & 0 \\ 0 & 0 & 1-\nu_{éq}^m \end{pmatrix} & (\mathbf{0}) \\ (\mathbf{0}) & \frac{E_{éq}^f h^3}{12(1-(\nu_{éq}^f)^2)} \begin{pmatrix} 1 & \nu_{éq}^f & 0 \\ \nu_{éq}^f & 1 & 0 \\ 0 & 0 & 1-\nu_{éq}^f \end{pmatrix} \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2\epsilon_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ 2\kappa_{xy} \end{pmatrix} \quad (3.2.1)$$

3.2.1 Identification of the thresholds of damage

One must identify the thresholds of damage defined by [éq. 2.4.3] starting from the limits of cracking in monoaxial pure traction and monoaxial pure inflection (in the directions positive M_1^d and negative M_2^d) flagstone out of reinforced concrete, themselves definite starting from the threshold of resistance in traction of the concrete $\sigma_{ft} \geq 0$ (cf [bib2]). One with this intention uses the analytical resolution of the case of a concrete beam reinforced in the same way as the flagstone. One preserves the approximation consisting in considering tablecloths of reinforcements realised according to the directions x and y. It will thus be admitted that one has $H_{1111}^m \equiv H_{2222}^m$, $H_{1111}^f \equiv H_{2222}^f$ and $H_{1111}^{mf} \equiv H_{2222}^{mf}$. Not to take into account this approximation leads to calculations heavy and not very necessary.

In positive monoaxial pure inflection M_{xx}, with $N_{xx}=0$, $\epsilon_{\alpha y}=0$, $\kappa_{xy}=0$ and $\kappa_{yy}=-\nu_{éq}^f \kappa_{xx}$, $\alpha=\{x, y\}$, the damage of the concrete is reached initially in lower skin of the reinforced concrete flagstone: $\sigma_{ft}=\frac{E_b}{1-\nu_b^2}(\epsilon_{xx}+\kappa_{xx} h/2)$. There are thus the following relations, cf [éq. 3.1.2], [éq. 3.1.3] and [éq. 3.1.4]:

$$\epsilon_{xx}=-\frac{H_{1111}^{mf}}{H_{1111}^m}\kappa_{xx}$$

then

$$\begin{aligned} M_{xx} &= \left(H^f_{1111} - H^f_{1122} \nu^f_{éq} - \frac{(H^{mf}_{1111})}{H^m_{1111}} \right) \kappa_{xx} \\ &= \left(H^f_{1111} (1 - (\nu^f_{éq})^2) - \frac{(H^{mf}_{1111})}{H^m_{1111}} \right) \kappa_{xx} \end{aligned} \tag{3.2.2}$$

$$\sigma_{ft} = \frac{E_b \kappa_{xx}}{1-\nu_b^2} \left(\frac{h}{2} - \frac{H^{mf}_{1111}}{H^m_{1111}} \right) = \frac{E_b M_{xx}}{1-\nu_b^2} \frac{H^m_{1111}}{(1-(\nu^f_{éq})^2) H^m_{1111} H^f_{1111} - (H^{mf}_{1111})^2} \left(\frac{h}{2} - \frac{H^{mf}_{1111}}{H^m_{1111}} \right) \tag{3.2.3}$$

from where:

$$M^d_1 = (1-\nu_b^2) \frac{\sigma_{ft}}{E_b} \frac{(1-(\nu^f_{éq})^2) H^m_{1111} H^f_{1111} - (H^{mf}_{1111})^2}{H^m_{1111}} \left(\frac{h}{2} - \frac{H^{mf}_{1111}}{H^m_{1111}} \right)^{-1} \tag{3.2.4}$$

If the coupling inflection-membrane is neglected, $H^{mf}_{1111}=0$, one has simply:

$$M^d_1 = \frac{\sigma_{ft} h^2}{6} \frac{\nu_b}{\nu^f_{éq}} (1-(\nu^f_{éq})^2) \tag{3.2.5}$$

In the same way, in negative monoaxial pure inflection M_{xx} , with $N_{xx}=0$, $\epsilon_{\alpha y}=0$, $\kappa_{xy}=0$ and, $\kappa_{yy}=-\nu^f_{éq}\kappa_{xx}$, $\alpha=\{x, y\}$, the damage of the concrete is reached initially in higher skin of the reinforced concrete flagstone: $\sigma_{ft}=\frac{E_b}{1-\nu_b^2}(\epsilon_{xx}-\kappa_{xx} h/2)$. There are thus the following relations:

$$\sigma_{ft} = -\frac{E_b \kappa_{xx}}{1-\nu_b^2} \left(\frac{h}{2} - \frac{H^{mf}_{1111}}{H^m_{1111}} \right) = -\frac{E_b M_{xx}}{1-\nu_b^2} \frac{H^m_{1111}}{(1-(\nu^f_{éq})^2) H^m_{1111} H^f_{1111} - (H^{mf}_{1111})^2} \left(\frac{h}{2} + \frac{H^{mf}_{1111}}{H^m_{1111}} \right) \tag{3.2.6}$$

from where:

$$M^d_2 = -(1-\nu_b^2) \frac{\sigma_{ft}}{E_b} \frac{(1-(\nu^f_{éq})^2) H^m_{1111} H^f_{1111} - (H^{mf}_{1111})^2}{H^m_{1111}} \left(\frac{h}{2} + \frac{H^{mf}_{1111}}{H^m_{1111}} \right)^{-1} \tag{3.2.7}$$

If the coupling inflection-membrane is neglected, one has simply:

$$M^d_1 = -\frac{\sigma_{ft} h^2}{6} \frac{\nu_b}{\nu^f_{éq}} (1-(\nu^f_{éq})^2) \tag{3.2.8}$$

Note:

It is checked that: $M^d_1 \geq 0$ and $M^d_2 \leq 0$.

It any more but does not remain to connect these moments of cracking to the thresholds k_1, k_2 defined in [éq. 2.5.3]. Since the loading is exerted starting from a virgin state, $d_1=d_2=0$. From where forces of damage (restitution of energy), cf [éq. 2.4.3]:

$$Y_j=\frac{1-\gamma}{(1+d_j)^2}\left(\frac{\lambda_f}{2}tr(\boldsymbol{\kappa}^e)^2 H((-1)^j tr(\boldsymbol{\kappa}^e))+\mu_f\sum_i(\tilde{\kappa}_i^e)^2 H((-1)^j\tilde{\kappa}_i^e)\right)$$

where one applies $d_j=0$, $\boldsymbol{\kappa}^p=0$, $\boldsymbol{\kappa}^e=\boldsymbol{\kappa}$, $\kappa_{xy}=0$ and $\kappa_{yy}=-\nu_{éq}^f\kappa_{xx}$ in order to obtain:

$$Y_j=\frac{1}{2}(1-\gamma)\left(\lambda_f(1-\nu_{éq}^f)^2+\mu_f\right)\kappa_{xx}^2$$

By using [éq. 3.1.1] of the document [R7.01.32]:

$$\lambda_f=\frac{h^3\nu_f E_{éq}^f}{12(1-\nu_f^2)}\ ,\ \mu_f=\frac{h^3 E_{éq}^f}{24(1+\nu_f)}$$

one obtains:

$$\begin{aligned}Y_j &= \frac{h^3}{24}(1-\gamma)E_{éq}^f\frac{1+\nu_{éq}^f(1-\nu_{éq}^f)}{1+\nu_{éq}^f}\kappa_{xx}^2\\ &= \frac{h^3}{24}(1-\gamma)E_{éq}^f\frac{1+\nu_{éq}^f(1-\nu_{éq}^f)}{1+\nu_{éq}^f}\left(\frac{M_{xx}}{H_{1111}^f(1-(\nu_{éq}^f)^2)}\right)^2\\ &= \frac{h^3}{24}(1-\gamma)E_{éq}^f\frac{1+\nu_{éq}^f(1-\nu_{éq}^f)}{1+\nu_{éq}^f}\left(\frac{12}{E_{éq}^f h^3}M_{xx}\right)^2\\ &= \frac{6}{h^3}\frac{(1-\gamma)}{E_{éq}^f}\frac{1+\nu_{éq}^f(1-\nu_{éq}^f)}{1+\nu_{éq}^f}M_{xx}^2\end{aligned}$$

then:

$$k_1=\frac{6}{h^3}\frac{(1-\gamma)}{E_{éq}^f}\frac{1+\nu_{éq}^f(1-\nu_{éq}^f)}{1+\nu_{éq}^f}(M_1^d)^2\ ,\ k_2=\frac{6}{h^3}\frac{(1-\gamma)}{E_{éq}^f}\frac{1+\nu_{éq}^f(1-\nu_{éq}^f)}{1+\nu_{éq}^f}(M_2^d)^2 \tag{3.2.9}$$

These thresholds have as units IF the Joule.

The linear elastic behavior endommageable is usable in nonlinear analysis under the keyword `BEHAVIOR` with the operand `RELATION = 'GLRC_DAMAGE'`. Parameters of the operator `DEFI_MATERIAU MF1` and `MF2` correspond to M_1^d and M_2^d.

3.2.2 Identification of the slope of damage in inflection

According to the relations developed in [§3.2] of [R7.01.32], the damaging slope is proportional to the parameter γ :

$$p_f=\gamma\, p_{élas} \tag{3.2.10}$$

The parameter γ corresponds to the parameter `GAMMA` informed in the operator `DEFI_GLRC`.

3.2.3 Identification of the maximum level of damage in inflection

In [éq. 2.3.1], one envisaged to limit the level of damage in inflection of the reinforced concrete plate, using the values of d_j^{max} . One associates these values with the slopes moment-curve, for the two directions of loading $j=1,2$, and compared to the elastic slope, to also see [Figure 3.2.4-a]:

$$\frac{p_{2,j}}{p_{élas}}=\frac{1+\gamma\, d_j^{max}}{1+d_j^{max}}$$

from where, for $j=1,2$:

$$d_j^{max}=\frac{p_{élas}-p_{2,j}}{p_{2,j}-p_f} \quad (3.2.11)$$

In the operator `DEFI_GLRC` one informs `QP1` and `QP2` for $p_{2,1}$ and $p_{2,2}$.

3.2.4 Identification of the parameters of plastic behavior

For the behaviors in elasticity and with damage, it is possible analytically to obtain the values of the parameters starting from the properties materials and geometrical of the reinforced concrete. To characterize the parameters of the plastic behavior, it is imperative to refer to a finer modeling (beam multifibre, hull multi-layer or 3D). A software of type MOCO (see [bib10]) is recommended for the automatic identification of the models `GLRC`. In the long term, it is expected that such a tool is integrated in *Code_Aster*. In the current version, the identification of the elastoplastic part is completely left with the care of the users.

For the identification, it is more reasonable to identify the parameters of non-linear behaviour starting from tests, digital or experimental, with a monotonous loading. For example, one can use a test with the curves and the homogeneous times bending. Such a test is pseudo-unidimensional and can be entirely represented with only one graph, on which one can identify the thresholds of damage and plasticity just as the slopes corresponding to the various phases of loading, to see Figure 3.2.4-a. To measure the effect of the membrane effort the monotonous test of inflection must be combined with a loading out of membrane.

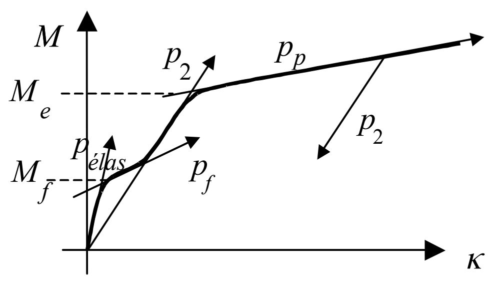

Figure 3.2.4-a: Monotonous uniaxial inflection.

On Figure 3.2.4-a, five phases are distinguished:

i) elastic phase characterized by the slope $p_{élas}$
ii) phase corresponding to the damage of the concrete (slope p_f),
iii) resumption of the stiffness due to steels after the attack of the maximum damage (slope p_2)
iv) plasticization of steels (slope p_p).
v) elastic discharge: the value of slopes of discharge is in the interval $[p_{élas}, p_f]$ for phases I) to III) and is worth p_2 for phase iv).

To describe the plastic behavior the functions should be informed $M^p_{jx}(N_{xx} - X^m_{xx})$ and $M^p_{jy}(N_{yy} - X^m_{yy})$, $j=1,2$, as functions "aster", FMEX1, FMEX2, FMEY1, FMEY2 in DEFI_GLRC. It is recommended to define functions symbolic systems by the order FORMULA, who must then be transformed into functions discretized by the order CALC_FONC_INTERP. Besides these functions, one must also inform their derivative first and seconds, which, on the other hand, can be calculated by the operator CALC_FONCTION. Typically, they are functions close to parabolic functions. For a given direction, let us say x , two functions M^p_{1x} and M^p_{2x} define the elastic range, as in Figure 3.2.4-b.

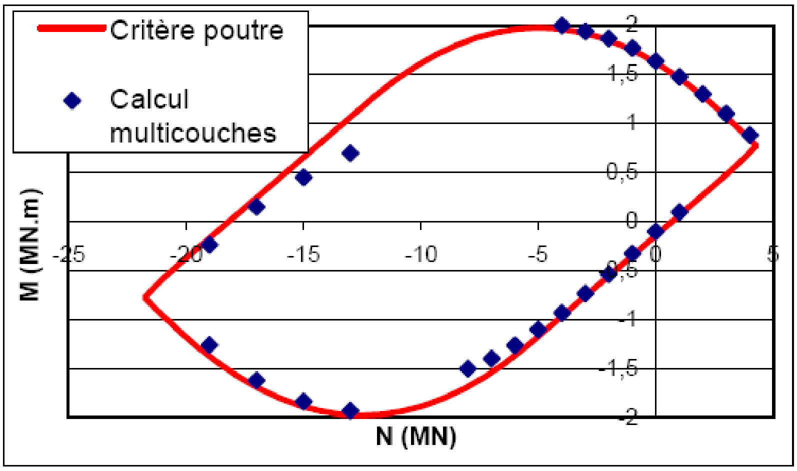

Figure 3.2.4-b: The elastic range is between the two curves bending moment/membrane effort. The graph presents a comparison between the thresholds used by the model GLRC_DAMAGE and those obtained by a calculation multi-layer on the case of a beam.

4 Digital integration of the law of behavior

The law GLRC_DAMAGE was initially conceived for analyses of fast dynamics having recourse to the explicit diagrams of temporal integration. The version of the model is almost identical to the initial version and is thus not optimized for calculations in statics or implicit dynamics. Consequently, the model is likely to be not very robust and little performing for an analysis with great steps of time. It is expected that the digital integration of the model is improved.

4.1 Evaluation of the damage

The model of damage used in GLRC_DAMAGE was extended to the coupling membrane-inflection and put in the model GLRC_DM (see [R7.01.32]). One thus returns the reader to [R7.01.32] for the details concerning the digital integration of the damage part. Calculation in GLRC_DAMAGE is simpler, because one neglects the influence of membrane energy on the evolution of the damage.

4.2 Evaluation of the plastic flow

The integration of the elastoplastic part is the most delicate part of the model. For the moment, one does not have method having a satisfactory robustness for great increments of (pseudonym) - time. One summarizes below the characteristics of the model presented in [§2], that one can solve with difficulty by the classical approaches, based on the method of Newton.

- **Double cones:** The elastic range of each function threshold defined in [§2.5.1] is not convex and can be represented by a double cone within the space of constraints generalized (see Figure 4.2-a). Obviously, it is only the cone close to the origin which represents the true elastic range. Thus, the resolution of plastic admissibility must be carried out by adding two inequations:

$$g_1^p(\boldsymbol{N}-\boldsymbol{X}^m,\boldsymbol{M}-\boldsymbol{X}^f)=(M_{xx}-X_{xx}^f)-M_{1\mathrm{x}}^p(N_{xx}-X_{xx}^m)+(M_{yy}-X_{yy}^f)-M_{1\mathrm{y}}^p(N_{yy}-X_{yy}^m)\leq 0 \quad (4.2.1)$$

$$g_2^p(\boldsymbol{N}-\boldsymbol{X}^m,\boldsymbol{M}-\boldsymbol{X}^f)=(M_{xx}-X_{xx}^f)-M_{2\mathrm{x}}^p(N_{xx}-X_{xx}^m)+(M_{yy}-X_{yy}^f)-M_{2\mathrm{y}}^p(N_{yy}-X_{yy}^m)\leq 0 \quad (4.2.2)$$

With the inequations [éq. 4.2.1] and [éq. 4.2.2], one eliminates the solutions not-physics from the plastic equations of admissibilities, $f_j^p(\boldsymbol{N}-\boldsymbol{X}^m,\boldsymbol{M}-\boldsymbol{X}^f)\leq 0$, $\lambda_j^p\geq 0$. Functions g_j^p the two plans define, $g_j^p(\boldsymbol{N}-\boldsymbol{X}^m,\boldsymbol{M}-\boldsymbol{X}^f)=0$, separating the cones not-physics from the cones determining the elastic range. On the other hand, the introduction of the inequations into the system prevents us from using the algorithms of the Newton type without important modifications.

- **Summits of the cones** : the other disadvantage, also related to the form of the elastic range within the space of generalized constraints, comes from the two tops of the cones of the elastic range (see Figure 4.2-a). This property can also return the convergence of the difficult iterative algorithm.

Because of the two disadvantages mentioned above one cannot apply the algorithm of the radial return, generally used for the resolution of the problems of plasticity. With his place one implemented an algorithm of “cutting planes”, combined with dichotomy. The details of the algorithm are available in [bib1].

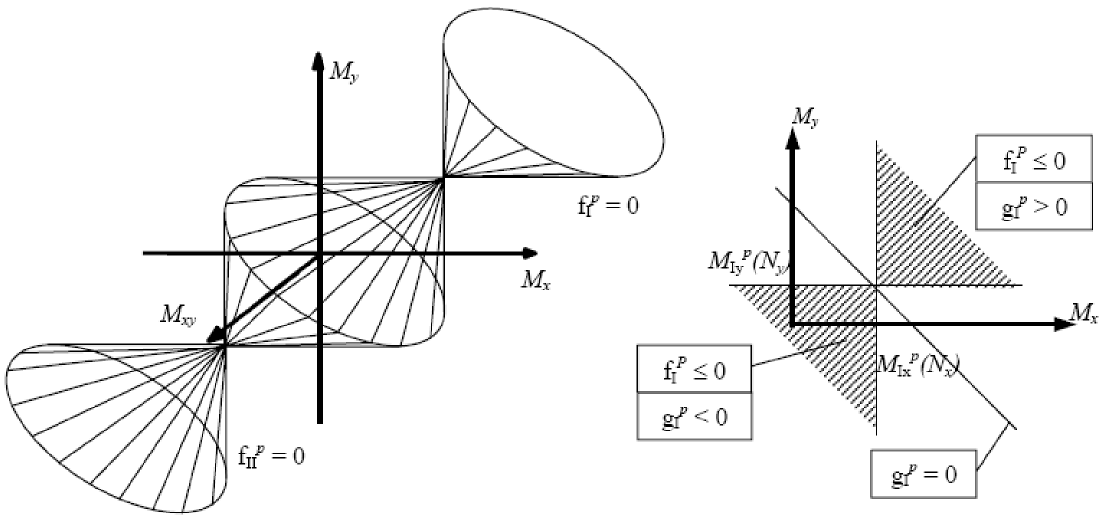

Figure 4.2-a: Plastic threshold within the space of constraints generalized in form of two double cones.

4.3 Evaluation of the tangent operator

Currently, the tangent operator of the model `GLRC_DAMAGE` is not coherent and does not guarantee the quadratic convergence of the total process of Newton. Its establishment is based on an older version of the model. This part of the model is still in building site.

4.4 Internal variables of the model

Us listels here internal variables stored in each point of Gauss in the establishment of the model.

Internal number of variable	physical direction
V1	EXXP plastic membrane extension
V2	EYYP plastic membrane extension
V3	EXYP plastic membrane extension
V4	KXXP cumulated plastic curve
V5	KYYP cumulated plastic curve
V6	KXYP cumulated plastic curve
V7	Cumulated plastic dissipation
V8	D1 variable of endom. higher face
V9	D2 variable of endom. lower face
V10	Dissipation of damage
V11	angle of orthotropism
V12	angle of orthotropism
V13	angle of orthotropism
V14	NXX effort of kinematic membrane of recall
V15	NYY effort of kinematic membrane of recall
V16	NXY effort of kinematic membrane of recall
V17	MXX kinematic moment of recall
V18	MYY kinematic moment of recall
V19	MXY kinematic moment of recall

5 Verification

The law of behavior `GLRC_DAMAGE` is checked by the cases following tests:

linear statics	SSLS126	Inflection of a reinforced concrete flagstone (model `GLRC_DAMAGE`) supported on two with dimensions: elastic mode of beam	[V3.03.126]
linear statics	SSLS127	Inflection of a reinforced concrete flagstone (model `GLRC_DAMAGE`) supported on 4 with dimensions: elastic mode of plate	[V3.03.127]
explicit dynamics non-linear	SDNS106	Transitory answer of a reinforced concrete flagstone: model `GLRC_DAMAGE`	[V5.06.106]

6 Bibliography

- P. KOECHLIN, “Model of behavior membrane-inflection and criterion of perforation for the analysis of mean structures out of reinforced concrete under soft shock”, Doctorate Paris VI, 2007, http://www.lamsid.cnrs-bellevue.fr/productions/theses.htm.
- P.KOECHLIN, S.MILL. Model of total behavior of the reinforced concrete plates under dynamic loading in inflection: improved law GLRC: modeling of cracking by damage. Note HT-62/02/021/A, 11/2002.
- P.KOECHLIN, S.POTAPOV. With total constitutive model for reinforced concrete punts. ASCE J. Eng. Mech. 2006.
- F.VOLDOIRE. Homogenisation of the heterogeneous structures. Note EDF/DER/MMN HI-74/93/055, 10/27/1993.
- [V3.03.126] SSLS126 – Inflection of a reinforced concrete flagstone (model GLRC) pressed on 2 sides.
- [V3.03.127] SSLS127 – Inflection of a reinforced concrete flagstone (model GLRC) pressed on 4 sides.
- [V5.06.106] SDNS106 – Answer transitory of a reinforced concrete flagstone (model GLRC_DAMAGE).
- F.VOLDOIRE. Homogenisation of the heterogeneous structures. Note EDF/DER/MMN HI-74/93/055, 10/27/1993.
- P. KOECHLIN, MOCO 2007: Use and reference material, Notes EDF/R & D HT-T62-2007-00635-FR, 4/12/2007.

7 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
8.4	D.Markovic F.Voldoire EDF-R&D/AMA	Initial text
9.5	S.Fayolle EDF-R&D/AMA	Introduction of `DEFI_GLRC`, rewriting of the equations, reformulations of certain sentences,…
10.2	S.Fayolle EDF-R&D/AMA	Cleaning and consistency with `GLRC_DM`