Coupling élasto-plasticity-damage

Summary:

This document presents a method of coupling of an elastoplastic model with a model of damage, which is established within the framework of KIT_DDI. The method uses an approach of partitioning of the model in three modules: elastoplasticity, damage and coupler. It is particularly this modularity which represents the principal interest of the method. It makes it possible to use the same module “coupler” for couplings between various models of elastoplasticity and damage without intervening in the establishment of the latter. Exactly the same idea is used for the couplings mechanics/creep (voir [R7.01.19]).

For the moment, the establishment makes it possible to couple that the model GLRC_DM for the damage with the models VMIS_ (ISOT/CINE) _LINE for elastoplasticity and this only for modelings DKTG.

Contents

1 Introduction

The document is organized in two great parts: the first presents the general method of the couplings élasto-plasticity-damage and the second describes specificities of the various couplings, which are currently limited to the coupling enters GLRC_DM and VMIS_ (ISOT/CINE) _LINE for the "total" models.

In the first part one considers in particular the following aspects: challenges of this kind of couplings, thermodynamic framework and digital establishment. In the second part one recalls what are the total models (GLRC_) for the plates and hulls out of reinforced concrete and how the general method is declined for this framework.

2 Models of élasto-plasticity-damage in general

2.1 Challenges and difficulties

In the modeling of the behavior of the solids, one notes that the non-linear phenomena most important to take into account, during an analysis of structure under extreme loading, relate to the presence and the evolution of the unelastic deformation (plasticity) and the weakening of the stiffness (damage). The relative importance of these two phenomena can drastically vary from one material to another. Thus, generally one considers only plasticity at the time of the modeling of metals while neglecting the damage, while for a concrete material type, the damage is considered to be more important and plasticity is not taken into account. Nevertheless, often this simplification is not justifiable and one is brought to model the two phenomena, plasticity and the damage, at the same time.

Since already a certain time the thermodynamic framework for the modeling of the two phenomena is well established (e.g to see [bib1], [bib2]) and there does not exist any obstacle to extend it to the couplings élasto-plastic-endommageables. As for the digital resolution of these models within the framework of the finite element method (MEF), there exist very robust algorithms in particular for plasticity (e.g to see [bib13]) and also for the damage (e.g to see [bib14]) provided that the phenomenon of localization does not become dominating. On the other hand, the digital resolution of a coupled model is much less obvious.

The digital difficulties concerning the algorithm of resolution of a coupled model intervene on two aspects primarily:

- satisfactions of the conditions (of Kuhn-Tucker) of the thresholds of plasticity and damage,
- construction of the tangent matrix.

The first difficulty is due to the fact that the plastic variables internal and endommageables act very differently on the level of the constraints, which makes the system of the equations difficult to solve. The second point is a simple consequence of the increased complexity of the model coupled compared to the models with only one phenomenon.

Lately, one proposed in [bib3] and [bib4] an algorithm of resolution of a model élasto-plastic-endommageable based on strict partitioning in a plastic part and with a endommageable part. It is a question of introducing an additional iterative process in order to satisfy plastic admissibility and admissibility endommageable simultaneously. This iterative process is controlled by what is called the deformation of damage, a variable which, contrary to the plastic deformation, is not a variable of state, but rather represents a connection between various models. Thus, in the model introduced into [bib3] the choice of the elastoplastic model being free, one was nevertheless to modify the formulation of damage, which in fact envisaged only the modeling of nonlenitive behaviors. In [bib4] one showed the interest to integrate this kind of laws within the framework of mixed finite elements.

In this document one generalizes the approach presented in [bib3] being able to practically couple any elastoplastic model with any model of damage. The advantage of such a platform actually goes beyond the coupling élasto-plastic-endommageable.

2.2 Tally thermodynamic

In this part one recalls the usual approach to build models of damage and elastoplasticity, based on thermodynamics. The combination of the two types of model can be done in a rather natural way, provided that the coupling is made on the level of the calculation of the constraints. All the approach presented here is carried out under the assumption that the internal variables of damage and plasticity are not coupled directly between them. Moreover, one puts oneself within the framework of the small deformations and without thermal phenomenon.

2.2.1 Plasticity

Free energy ψ can be written as the sum of elastic energy and energy known as stored, which had with plasticity:

$$\psi\left(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^{p}, \boldsymbol{\alpha}\right)=\psi^{e}\left(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{p}\right)+\psi^{p}\left(\boldsymbol{\alpha}\right) \tag{2.2.1-1}$$

where $\boldsymbol{\varepsilon}$ is the total deflection, $\boldsymbol{\varepsilon}^{p}$ plastic deformation and $\boldsymbol{\alpha}$ a vector of internal variables. The essential assumption that one makes in (2.2.1-1) is that elastic energy depends only on the elastic strain, $\psi^{e}=\psi^{e}\left(\boldsymbol{\varepsilon}^{e}\right)$, where $\boldsymbol{\varepsilon}^{e}=\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^{p}$.

In the second phase one defines the plastic rate of dissipation and one makes sure that it remains always positive,

$$D^{p}=\boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}}-\rho\,\dot{\psi}=\left(\sigma-\rho\,\frac{\partial\psi^{e}}{\partial\boldsymbol{\varepsilon}^{e}}\right):\dot{\boldsymbol{\varepsilon}}^{e}+\boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}}^{p}-\boldsymbol{A}*\dot{\boldsymbol{\alpha}}\geq 0 \tag{2.2.1-2}$$

Where

$$\boldsymbol{A}=\rho\,\frac{\partial\psi^{p}}{\partial\boldsymbol{\alpha}} \tag{2.2.1-3}$$

are the thermodynamic forces associated with internal variables and the symbol $*$ refers to the operator of suitable product enters $\boldsymbol{A}$ and $\dot{\boldsymbol{\alpha}}$.

That dissipation does not evolve, i.e is supposed. $\dot{D}=0$, when the plastic variables do not evolve, i.e. $\dot{\boldsymbol{\varepsilon}}^{p}=\dot{\boldsymbol{\alpha}}=0$, which enables us to define the constraint as:

$$\sigma=\rho\,\frac{\partial\psi^{e}}{\partial\boldsymbol{\varepsilon}^{e}} \tag{2.2.1-4}$$

In this document, one is satisfied with the materials which have a linear and isotropic behavior. In this case, the elastic contribution of the free energy is given by:

$$\rho\,\psi\left(\boldsymbol{\varepsilon}^{e}\right)=\frac{1}{2}\boldsymbol{\varepsilon}^{e}:\boldsymbol{D}^{e}:\boldsymbol{\varepsilon}^{e} \tag{2.2.1-5}$$

Where $\boldsymbol{D}^{e}$ is the tensor of standard isotropic elasticity. From the two preceding equations, one deduces the form of the tensor of the constraints:

$$\sigma = \boldsymbol{D}^e : \varepsilon^e \tag{2.2.1-6}$$

Another essential ingredient of an elastoplastic model is the definition of the function of the threshold of flow which depends on the tensor of the constraints and the thermodynamic forces associated with internal variables. This function is negative when the deformations purely elastic and are reached zero when the plastic flow is imminent:

$$\Phi^p = \Phi^p(\sigma, \boldsymbol{A}) \leq 0 \tag{2.2.1-7}$$

The complete characterization of the model general of plasticity requires the definition of the laws of evolution of the internal variables, i.e the varaibles ones associated with the phenomena of dissipation. In our case, the internal variables are the tensor of the plastic deformations and the whole of internal variables α . To establish the laws of evolution of these internal variables have often use the principle of maximum plastic dissipation. However this is not essential in our approach. One will make use of a more general writing:

$$\begin{aligned} \dot{\varepsilon}^p &= \dot{\gamma}^p \boldsymbol{N} = \dot{\gamma}^p \frac{\partial \psi}{\partial \sigma} \\ \dot{\alpha} &= \dot{\gamma}^p \boldsymbol{H} = -\dot{\gamma}^p \frac{\partial \psi}{\partial \boldsymbol{A}} \end{aligned} \tag{2.2.1-8}$$

$\dot{\gamma}$ is the plastic multiplier, $\boldsymbol{N}$ the vector of flow is called and $\boldsymbol{H}$ is the module of work hardening. Moreover any solution must satisfy the conditions with *admissibility figures* , which is:

$$\Phi^p \leq 0, \quad \dot{\gamma} \geq 0, \quad \Phi^p \dot{\gamma} = 0 \tag{2.2.1-9}$$

2.2.2 Damage

For the models of damage it is more difficult to have a construction as general as for that of plasticity. Consequently one will treat a case simple but rather representative of the desired characteristics of the model. Thus, one defines the free energy as:

$$\psi(\varepsilon, d) = \psi^e(\varepsilon)\zeta(d) \tag{2.2.2-1}$$

where ζ is the function of damage and d the variable of damage, which is a scalar but which can as well be a tensor. In the same way, the function ζ can in general depend on ε , in particular of the sign of its trace to distinguish from behaviours typically very different in traction and compression.
While using (2.2.2-1), one defines the dissipation of damage like,

$$D^d = \sigma : \dot{\varepsilon} - \rho \dot{\psi} = \left(\sigma - \rho \frac{\partial \psi}{\partial \varepsilon} \right) : \dot{\varepsilon} - \rho \frac{\partial \psi}{\partial d} \dot{d} \geq 0 \tag{2.2.2-2}$$

Dissipation does not evolve, i.e. $\dot{D}^d = 0$, when the variable of damage does not evolve, i.e. $\dot{d} = 0$, which enables us to define the constraint as:

$$\sigma = \rho \frac{\partial \psi}{\partial \varepsilon} = \rho \frac{\partial \psi^e}{\partial \varepsilon} \zeta(d) = \boldsymbol{D}^e(d) : \varepsilon^e \tag{2.2.2-3}$$

Where $\boldsymbol{D}^e(d)$ represent the tensor of standard isotropic elasticity evolving according to the damage. By making the assumption of an isotropic damage, while taking $\zeta = 1 - d$ the constraint becomes:

$$\sigma = (1-d)\,\boldsymbol{D}^{e} : \varepsilon^{e} \tag{2.2.2-4}$$

As for plasticity one introduces a function threshold of damage:

$$\Phi^{d} = \Phi^{d}(\varepsilon, Y) \leq 0 \tag{2.2.2-5}$$

Where Y is the dual variable with d . This time, the threshold is a direct function of the total deflection, ε , and one seeks to satisfy the conditions with admissibility of damage:

$$\Phi^{d} \leq 0, \quad \dot{d} \geq 0, \quad \Phi^{d}\dot{d} = 0 \tag{2.2.2-6}$$

One must also formulate the law of evolution of d :

$$\dot{d} = \dot{d}(Y) \tag{2.2.2-7}$$

2.2.3 Coupling

Lastly, one combines the formulations of damage and plasticity to arrive at a coupled model. The free energy of the coupled model is thus written like:

$$\psi(\varepsilon, \varepsilon^{p}, \alpha, d) = \psi^{e}(\varepsilon - \varepsilon^{p})\zeta(d) + \psi^{p}(\alpha) \tag{2.2.3-1}$$

Dissipation is written:

$$D = \sigma : \dot{\varepsilon} - \rho\dot{\psi} = \left(\sigma - \rho\frac{\partial \psi^{e}}{\partial \varepsilon^{e}}\right) : \dot{\varepsilon}^{e} + \sigma : \dot{\varepsilon}^{p} - \boldsymbol{A} * \dot{\alpha} - \frac{\partial \psi^{d}}{\partial d}\dot{d} \geq 0 \tag{2.2.3-2}$$

The tensor of the constraints is worth:

$$\sigma = \rho\frac{\partial \psi^{d}}{\partial \varepsilon^{e}} = \rho\frac{\partial \psi^{e}}{\partial \varepsilon^{e}}\zeta(d) = \boldsymbol{D}^{e}(d) : (\varepsilon - \varepsilon^{p}) \tag{2.2.3-3}$$

If elasticity is linear isotropic and the damage is isotropic:

$$\sigma = (1-d)\,\boldsymbol{D}^{e} : (\varepsilon - \varepsilon^{p}) \tag{2.2.3-4}$$

There are then two functions threshold: for the plastic flow and the other for the damage:

$$\begin{aligned} \Phi^{p} &= \Phi^{p}(\sigma, \boldsymbol{A}) \leq 0 \\ \Phi^{d} &= \Phi^{d}(\varepsilon^{e}, Y) \leq 0 \end{aligned} \tag{2.2.3-5}$$

The laws of evolution of the variables of damage are given by:

$$\dot{\varepsilon}^p = \dot{\gamma}^p \boldsymbol{N} = \dot{\gamma}^p \frac{\partial \psi}{\partial \boldsymbol{\sigma}}$$
$$\dot{\alpha} = \dot{\gamma}^p \boldsymbol{H} = -\dot{\gamma}^p \frac{\partial \psi}{\partial \boldsymbol{A}} \quad (2.2.3\text{-}6)$$
$$\dot{d} = \dot{d}(Y)$$

The conditions of admissibility are:

$$\Phi^p \leq 0, \quad \dot{\gamma} \geq 0, \quad \Phi^p \dot{\gamma} = 0$$
$$\Phi^d \leq 0, \quad \dot{d} \geq 0, \quad \Phi^d \dot{d} = 0 \quad (2.2.3\text{-}7)$$

2.3 Digital integration

2.3.1 Recall of the classical resolution of the problems non-linear by finite elements

In the method suggested one adds an additional iterative process compared to a classical calculation by the MEF, which one recalls in this chapter. In a total resolution of a problem of structure one seeks the state of balance, defined by:

$$f^{int}(u(t)) = f^{ext}(t) \quad (2.3.1\text{-}1)$$

Where f^{int} is the vector of the internal forces and f^{ext} the vector of the external forces (the loading). The first is a function of the vector of displacements, the basic variable of the system, and the second a function of the parameter *T* (pseudo-time), defining the concept of trajectory as well for the loading as for the evolution of the field of displacements. Within the framework of the MEF the internal force is determined by:

$$f^{int}(u) = \int_\Omega \boldsymbol{B}^T \boldsymbol{\sigma}(\boldsymbol{\varepsilon}) d\Omega \quad (2.3.1\text{-}2)$$

where Ω the field considered defines, $\boldsymbol{\varepsilon}$ the tensor of the deformation, $\boldsymbol{\sigma}$ the tensor of the constraint and $\boldsymbol{B}$ is the classical matrix corresponding to the discretized operator of the symmetrical gradient which makes it possible to pass from the deformations to displacements.

As we considered the assumption of the small deformations, the tensor $\boldsymbol{\varepsilon}$ is defined like:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \quad (2.3.1\text{-}3)$$

By using the definition of $\boldsymbol{B}$, one a:

$$\varepsilon_{ij} = B_{ijk} u_k \quad (2.3.1\text{-}4)$$

After having discretized the field of displacements by the MEF. When the behavior of material is non-linear, one generally resorts to the method of Newton to solve (2.3.1-1). With this intention, one is brought to linearize (2.3.1-1), which gives us:

$$f^{int,(k+1)} = f^{int,(k)} + \frac{\partial f^{int,(k)}}{\partial u^{(k)}} \Delta u^{(k)} \quad (2.3.1\text{-}5)$$

Where the index k represent the number of the iteration in progress. With convergence one must have:

$$f^{int,(k)} \overset{k\to\infty}{\to} f^{ext}(t) \text{ and } \Delta u^{(k)} \overset{k\to\infty}{\to} 0 \tag{2.3.1-6}$$

In order to be able to calculate the correction of displacements from one iteration to another, $\Delta u^{(k)}$, one must determine:

$$K = \frac{\partial f^{int,(k)}}{\partial u^{(k)}} = \int_{\Omega} \boldsymbol{B}^{T} \boldsymbol{D}^{epd} \boldsymbol{B}\, d\Omega \tag{2.3.1-7}$$

Where $\boldsymbol{D}^{epd}$ is the coherent tangent module for the law of behavior used:

$$\boldsymbol{D}^{epd} = \frac{\partial \boldsymbol{\sigma}(\boldsymbol{\varepsilon})}{\partial \boldsymbol{\varepsilon}} \tag{2.3.1-8}$$

The method detailed below positions on the level of a material point (even at the point of Gauss) and intervenes on the level of the calculation of $\boldsymbol{\sigma}$ and $\boldsymbol{D}^{epd}$ starting from the deformation $\boldsymbol{\varepsilon}$, given for each call of the total process of the iterations of Newton.

2.3.2 Approach with the coupled problem

In this document one does not detail digital integration in time for each model. It is supposed quite simply that one can do it and this with an implicit scheme of Euler (back). Let us say, that in an incremental resolution by the finite element method, for each iteration of Newton of the total iterative process, one wants to calculate, for each point of Gauss, the constraints $\boldsymbol{\sigma}_{n+1}$ and internal variables $\boldsymbol{\varepsilon}^{p}_{n+1}$, $\boldsymbol{\alpha}_{n+1}$ and d_{n+1} starting from the values of the total deflection, $\boldsymbol{\varepsilon}_{n+1}$, while supposing that one converged at the previous moment and that one knows $\boldsymbol{\sigma}_{n}$, $\boldsymbol{\varepsilon}^{p}_{n}$, $\boldsymbol{\alpha}_{n}$ and d_{n} . Indices n or $n+1$ indicate that the value of the variable corresponds to the moment t_n or t_{n+1} , respectively.

Thus it is supposed that the two models can be integrated in uncoupled mode and that one is thus able to write:

$$\boldsymbol{\sigma}^{p}_{n+1} = \boldsymbol{\sigma}^{p}\left(\tilde{\boldsymbol{\varepsilon}}_{n+1}\right) \text{ in the module of plasticity} \tag{2.3.2-1}$$

and

$$\boldsymbol{\sigma}^{d}_{n+1} = \boldsymbol{\sigma}^{d}\left(\tilde{\boldsymbol{\varepsilon}}_{n+1}\right) \text{ in the module of plasticity} \tag{2.3.2-2}$$

Exhibitors d or p in $\boldsymbol{\sigma}^{d}$ and $\boldsymbol{\sigma}^{p}$ mean that the constraint is calculated with the model of damage and the model of plasticity, respectively. $\tilde{\boldsymbol{\varepsilon}}$ is a variable of the standard deformation.

Into the algorithm presented in the continuation one introduces an additional iterative process so that the constraints $\boldsymbol{\sigma}^{d}$ and $\boldsymbol{\sigma}^{p}$ converge towards only one value. He similar to that is proposed in [bib3] except that here the construction of the model of damage is more general, which require to consider another variable that the deformation of damage (used in [bib3]) on which one reiterates.

2.3.3 Construction of the non-linear system

Compared to a classical resolution of the law of behavior, one must make converge here two values of constraints by introducing another unknown factor of the standard deformation. This partitionné system is built more easily by introducing the following decomposition of the total deflection,

$$\varepsilon = \varepsilon^e + \varepsilon^p + \varepsilon^d \tag{2.3.3-1}$$

Where the deformation of damage was introduced, which enables us to write the relation of elasticity like:

$$\sigma = \boldsymbol{D}^e : \left(\varepsilon - \varepsilon^p - \varepsilon^d\right) \tag{2.3.3-2}$$

or:

$$\varepsilon^d\left(\sigma, \varepsilon, \varepsilon^p\right) = \varepsilon - \varepsilon^p - \left(\boldsymbol{D}^e\right)^{-1} : \sigma \tag{2.3.3-3}$$

One can summarize the equations to be solved in three groups:

Module plasticity

$$\begin{aligned} \sigma^p_{n+1} &= \sigma^p\left(\varepsilon_{n+1} - \varepsilon^d_{n+1}\right)_e \\ \varepsilon^p_{n+1} &= \varepsilon^p\left(\varepsilon_{n+1} - \varepsilon^d_{n+1}\right) \end{aligned} \tag{2.3.3-4}$$

The index e mean that the elastoplastic law is calculated with the operator of elasticity $\boldsymbol{D}^e$

Module damage

$$\sigma^d_{n+1} = \sigma^d\left(\varepsilon_{n+1} - \varepsilon^p_{n+1}\right)_e \tag{2.3.3-5}$$

Module coupling

$$\begin{aligned} \varepsilon^d_{n+1} &= \varepsilon_{n+1} - \varepsilon^p_{n+1} - \left(\boldsymbol{D}^e\right)^{-1} : \sigma^d_{n+1} \\ \sigma^p_{n+1} &= \sigma^d_{n+1} \end{aligned} \tag{2.3.3-6}$$

In [bib3] one chose ε^d_{n+1} like the additional unknown factor, which is not possible in our case, since σ^d is not a function of ε^d but of $\bar{\varepsilon}^{ed} = \varepsilon - \varepsilon^p$, the variable which is finally used to control the iterative processes described below. In addition it is worth too $\bar{\varepsilon}^{ed} = \varepsilon^e + \varepsilon^d = \varepsilon - \varepsilon^p$.

2.3.4 Algorithm of resolution

To solve the system non-linear, defined in the equations (2.3.3-4 with 2.3.3-6) by allotting them to the various modules (plasticity, damage and coupling), one applies the classical method of Newton. In order to linearize the system, one must define the derivative following:

$$\begin{aligned} \boldsymbol{D}^{ep}\left(\bar{\varepsilon}^{ed}\right) &= \frac{\partial \sigma^p_{n+1}}{\partial \bar{\varepsilon}^{ed}} \\ \boldsymbol{D}^d\left(\bar{\varepsilon}^{ed}\right) &= \frac{\partial \sigma^d_{n+1}}{\partial \bar{\varepsilon}^{ed}} \\ \frac{\partial \varepsilon^d_{n+1}}{\partial \bar{\varepsilon}^{ed}} &= \mathbf{1} - \left(\boldsymbol{D}^e\right)^{-1} \boldsymbol{D}^d\left(\bar{\varepsilon}^{ed}_{n+1}\right) \end{aligned} \tag{2.3.4-1}$$

The non-linear system (2.3.3-4 with 2.3.3-6) is first of all brought back to the last of the equations, i.e. with $\boldsymbol{\sigma}_{n+1}^{p}=\boldsymbol{\sigma}_{n+1}^{d}$, since one can express all the unknown factors compared to the value of $\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed}$. One can thus define the residue as:

$$R_{\sigma}^{(k)}=\boldsymbol{\sigma}^{p}\left(\boldsymbol{\varepsilon}_{n+1}-\boldsymbol{\varepsilon}_{n+1}^{d,(k)}\right)-\boldsymbol{\sigma}^{d}\left(\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k)}\right) \tag{2.3.4-2}$$

where the exhibitor k indicate the current iteration and where:

$$\boldsymbol{\varepsilon}_{n+1}^{d,(k)}=\boldsymbol{\varepsilon}_{n+1}-\boldsymbol{\varepsilon}_{n+1}^{p,(k)}-\left(\boldsymbol{D}^{e}\right)^{-1}:\boldsymbol{\sigma}^{d}\left(\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k)}\right) \tag{2.3.4-3}$$

By applying the method of Newton, one calculates the new approximation of $\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k+1)}$ like:

$$\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k+1)}=\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k)}+\Delta\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k)} \tag{2.3.4-4}$$

Where

$$\Delta\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k)}=-\left(\frac{\partial R_{\sigma}^{(k)}}{\partial\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed}}\right)^{-1}R_{\sigma}^{(k)} \tag{2.3.4-5}$$

The tangent module correspondent can be entirely expressed by the uncoupled modules (2.3.4-1):

$$\left(\frac{\partial R_{\sigma}^{(k)}}{\partial\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed}}\right)=-\boldsymbol{D}^{ep,(k)}\frac{\partial\boldsymbol{\varepsilon}_{n+1}^{d,(k)}}{\partial\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k)}}-\boldsymbol{D}^{d,(k)}=-\left(\boldsymbol{D}^{d,(k)}+\boldsymbol{D}^{ep,(k)}\left(\mathbf{1}-\left(\boldsymbol{D}^{e}\right)^{-1}\boldsymbol{D}^{d,(k)}\right)\right) \tag{2.3.4-6}$$

It is pointed out here that the total deflection $\boldsymbol{\varepsilon}_{n+1}$, exit of the total process of the iterations of Newton, is constant for the local iterative process determining $\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed}$. The algorithm of the calculation of the variables to the iteration $k+1$ be carried out in the following way:

0) Initialization: $\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k=0)}=\boldsymbol{\varepsilon}_{n+1}-\boldsymbol{\varepsilon}_{n}^{p}$

1) Buckle on $k=0,\ldots,N_{max}$

2) Calculation of the increment:

$\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k+1)}=\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k)}+\Delta\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k)}$ where $\Delta\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k)}=\left(\boldsymbol{D}^{d,(k)}+\boldsymbol{D}^{ep,(k)}\left(\mathbf{1}-\left(\boldsymbol{D}^{e}\right)^{-1}\boldsymbol{D}^{d,(k)}\right)\right)^{-1}R_{\sigma}^{(k)}$

3) $\boldsymbol{\sigma}_{n+1}^{d,(k+1)}=\boldsymbol{\sigma}^{d}\left(\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k+1)}\right)_{e}$

4) $\boldsymbol{\varepsilon}_{n+1}^{d,(k+1)}=\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k+1)}-\boldsymbol{\varepsilon}_{n+1}^{el,(k+1)}$ where $\boldsymbol{\varepsilon}_{n+1}^{el,(k+1)}=\left(\boldsymbol{D}^{e}\right)^{-1}:\boldsymbol{\sigma}^{d}\left(\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k+1)}\right)$

5) $\boldsymbol{\sigma}_{n+1}^{p,(k+1)}=\boldsymbol{\sigma}^{p}\left(\boldsymbol{\varepsilon}_{n+1}-\boldsymbol{\varepsilon}_{n+1}^{d,(k+1)}\right)_{e}$

6) $R_{\sigma}^{(k+1)}=\boldsymbol{\sigma}_{n+1}^{p,(k+1)}-\boldsymbol{\sigma}_{n+1}^{d,(k+1)}$

7) If $\left|\Delta\bar{\boldsymbol{\varepsilon}}_{n+1}^{ed,(k)}R_{\sigma}^{(k+1)}\right|>tol$ begins again the loop with 1), if not END

Notice 1 *: the only places where physical specificities of the coupled models are taken into account are the points 3) and 5), where one calls on the modules of plasticity and damage, respectively.*

Notice 2 *: the value of* ε_{n+1} *is considered constant in the process of balancing of the constraints and during the variation of* $\bar{\varepsilon}^{ed}_{n+1}$ *one varies actually the value of* ε^{p}_{n+1} *. This plastic deformation is in theory given by the module of elastoplasticity like a function of* ε_{n+1} *(see (2.3.3-4)). Actually,* $\bar{\varepsilon}^{ed}_{n+1}$ *is regarded here as a variable and its link with* ε^{p}_{n+1} *is restored only with the convergence of the iterative process. As follows:*

$$\bar{\varepsilon}^{ed,(k)}_{n+1} \neq \varepsilon_{n+1} - \varepsilon^{p}\left(\varepsilon_{n+1} - \varepsilon^{d,(k)}_{n+1}\right) \text{ but } \bar{\varepsilon}^{ed,(k)}_{n+1} \overset{k\to\infty}{\to} \varepsilon_{n+1} - \varepsilon^{p}\left(\varepsilon_{n+1} - \varepsilon^{d,(k)}_{n+1}\right) \tag{2.3.4-7}$$

2.3.5 Construction of the tangent module cohéren T

Lastly, in order to obtain an optimal performance in the total iterative process one must calculate the coherent tangent module, once the process describes above converged, i.e. $\sigma_{n+1} = \sigma^{p}_{n+1} = \sigma^{d}_{n+1}$:

$$\boldsymbol{D}^{epd}_{n+1} = \frac{\partial \sigma_{n+1}}{\partial \varepsilon_{n+1}} \tag{2.3.5-1}$$

Initially it is observed that if one derives (2.3.3-4) compared to σ^{p}_{n+1} one can write:

$$\boldsymbol{I} = \boldsymbol{D}^{ep}_{n+1}\left(\boldsymbol{D}^{epd}_{n+1} - \frac{\partial \varepsilon^{d}_{n+1}}{\partial \sigma^{p}_{n+1}}\right) \tag{2.3.5-2}$$

and that same manner, while deriving (2.3.3-5) compared to σ^{d}_{n+1} one obtains:

$$\boldsymbol{I} = \boldsymbol{D}^{d}_{n+1}\left(\boldsymbol{D}^{epd}_{n+1} - \frac{\partial \varepsilon^{p}_{n+1}}{\partial \sigma^{d}_{n+1}}\right) \tag{2.3.5-3}$$

with, in the end of the iterative process $\sigma_{n+1} = \sigma^{p}_{n+1} = \sigma^{d}_{n+1}$, one obtains finally:

$$\frac{\partial \varepsilon^{d}_{n+1}}{\partial \sigma_{n+1}} = (\boldsymbol{D}^{epd}_{n+1})^{-1} - (\boldsymbol{D}^{ep}_{n+1})^{-1} \text{ and } \frac{\partial \varepsilon^{p}_{n+1}}{\partial \sigma_{n+1}} = (\boldsymbol{D}^{epd}_{n+1})^{-1} - (\boldsymbol{D}^{d}_{n+1})^{-1} \tag{2.3.5-4}$$

While using then $\varepsilon_{n+1} = \varepsilon^{e}_{n+1} + \varepsilon^{p}_{n+1} + \varepsilon^{d}_{n+1}$ and thus that:

$$\frac{\partial \varepsilon_{n+1}}{\partial \sigma_{n+1}} = \frac{\partial \varepsilon^{e}_{n+1}}{\partial \sigma_{n+1}} + \frac{\partial \varepsilon^{p}_{n+1}}{\partial \sigma_{n+1}} + \frac{\partial \varepsilon^{d}_{n+1}}{\partial \sigma_{n+1}} \tag{2.3.5-5}$$

one obtains the coherent tangent module directly like:

$$\boldsymbol{D}^{epd}_{n+1} = \left(\left(\boldsymbol{D}^{epd}_{n+1}\right)^{-1} + \left(\boldsymbol{D}^{epd}_{n+1}\right)^{-1} + \left(\boldsymbol{D}^{epd}_{n+1}\right)^{-1}\right)^{-1} \tag{2.3.5-6}$$

2.4 Limitations of the coupler

There exist some conditions to satisfy for the use of this method of coupling concerning at the same time physical and digital aspects:

- **Distinct internal variables:**

As specified in the introduction of the thermodynamic framework, the two coupled models should not share the same internal variables. The method is conceived to be able to combine models representing of the physical phenomena uncoupled, which is judged to be the case in most target applications.

- **Existence of the coherent tangent modules:**

So that the approach functions, it is necessary absolutely that one has the coherent tangent modules for the two modules, elastoplastic and endommageable. Under “coherent tangent module” one implies that it is the coherent derivative with the temporal discretization employed of the tensor of constraints at the moment given compared to the tensor of deformation at the same moment.

- **Not-singularity of the tangent modules:**

One is obliged several time to reverse the tangent matrices of individual modules. If the behavior of a module has a non-linear slope punt (perfect elastoplasticity for example) it is necessary to give him a not-worthless value. This not-worthless value must be selected such that it is sufficiently large so that the algorithm functions and sufficiently small so that it is coherent with the studied physical model.

3 Specific models

3.1 Total models

3.1.1 Recall of the total models

In this context, a model of behavior of plate known as total or element of structure in general, means that the law of behavior is written directly in terms of relation between the generalized constraints and the generalized deformations. The comprehensive approach of modeling of the behavior of the structures applies in particular to the composite structures, for example the reinforced concrete (see Figure 1), and represents an alternative to the approaches known as local or semi-total, which are finer but more expensive modelings (see [bib6] and [bib7]). In the local approach one uses a fine modeling for each phase (steel, concrete) and their interactions (adherence) and in the semi-total approach one exploits the twinge of the structure to simplify the description of kinematics, which leads to models PMF (Multifibre Beam) or multi-layer hulls.

The interest of the total model lies in the fact that the finite element corresponding requires only one point of integration in the thickness and especially in obtaining a homogenized behavior. This advantage is even more important in the analysis of the reinforced concrete, since one circumvents the problems of localization encountered at the time of the modeling of the concrete without reinforcements. Obviously, a total model represents the local phenomena in a coarse way.

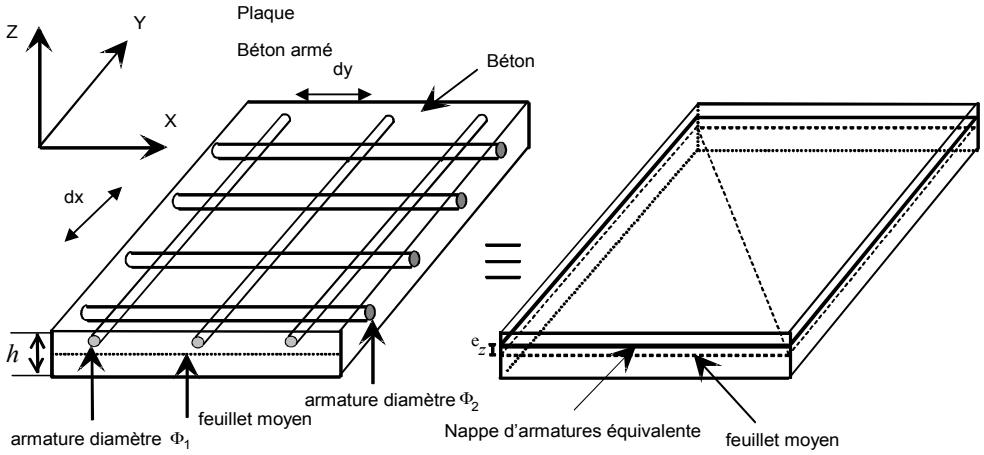

Figure 3.1-a. Reinforced concrete flagstone.

3.1.2 Establishment of the coupling GLRC_DM/VMIS_

The most important assumption which is made in this particular coupling is that elastoplasticity can be activated only by requesting the diaphragm disc. In other words, in pure inflection the structure will never plasticize. On the one hand, this simplification is justified by the target applications, where the requests are more important out of membrane than in inflection and where one thus does not expect dominant influences of the inflection. In addition, the assumption is essential for technical reasons, because currently one does not have a “total” model elastoplastic suitable for the coupling with a “total” model of damage.

The total models are formulated in term of constraints and deformations generalized, which imposes to us, in the coupler (see §2), to replace the constraints, σ , and deformations, ε , by:

$$\sigma \to \Sigma = \begin{pmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{pmatrix} \text{ and } \varepsilon \to E = \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{pmatrix} \quad (3.1.2\text{-}1)$$

where generalized deformations, E, break up into membrane extensions ϵ and curves κ while the generalized constraints are made up of the membrane efforts N and of the bending moments M.

It is important to note that within this framework one makes the assumption of thin sections where the transverse distortion and the shearing action are negligible. Below one points out the kinematics of Hencky-Mindlin (see [bib9] for the details) for the hulls and the plates as well as the definition of the generalized constraints:

$$\begin{pmatrix} U_1(x_1,x_2,z) \\ U_2(x_1,x_2,z) \\ U_z(x_1,x_2,z) \end{pmatrix} = \underbrace{\begin{pmatrix} u_1(x_1,x_2) \\ u_2(x_1,x_2) \\ u_z(x_1,x_2) \end{pmatrix} + z \begin{pmatrix} \theta_2(x_1,x_2) \\ -\theta_1(x_1,x_2) \\ 0 \end{pmatrix}}_{\substack{\text{cinématique de plaque} \\ \mathbf{u}^s \in V_s}} + \underbrace{\begin{pmatrix} u_1^c(x_1,x_2,z) \\ u_2^c(x_1,x_2,z) \\ u_z^c(x_1,x_2,z) \end{pmatrix}}_{\substack{\text{déplacement complémentaire} \\ \mathbf{u}^c \in V_c}} \quad (3.1.2\text{-}2)$$

where $\mathbf{U} = \begin{pmatrix} U_1 & U_2 & U_z \end{pmatrix}^T$ is the field of displacement in 3D, $\mathbf{u} = \begin{pmatrix} u_1 & u_2 & u_z \end{pmatrix}^T$ the displacement of the average layer and θ_x, θ_y its rotations. Thus, the tensor of deformation, definite like:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right), \quad i, j = 1..3 \quad (3.1.2\text{-}3)$$

is also written like:

$$\begin{aligned} \varepsilon_{11} &= \underbrace{\epsilon_{11} + z\kappa_{11}}_{\varepsilon_{11}^s} + u_{1,1}^c \\ \varepsilon_{22} &= \underbrace{\epsilon_{22} + z\kappa_{22}}_{\varepsilon_{22}^s} + u_{2,2}^c \\ \varepsilon_{12} &= \underbrace{\epsilon_{12} + z\kappa_{12}}_{\varepsilon_{12}^s} + \frac{1}{2}\left(u_{2,1}^c + u_{1,2}^c \right) \\ \varepsilon_{1z} &= \varepsilon_{1z}^c = \frac{1}{2} u_{3,1}^c \\ \varepsilon_{2z} &= \varepsilon_{2z}^c = \frac{1}{2} u_{3,2}^c \\ \varepsilon_{zz} &= \varepsilon_{zz}^c = u_{3,z}^c \end{aligned} \quad (3.1.2\text{-}4)$$

where ϵ is the tensor of the extension membrane, defined in the plan:

$$\epsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right),\quad i,j=1..2 \tag{3.1.2-5}$$

and κ the tensor of variation of curve, defined in the plan:

$$\kappa_{11}=\frac{\partial \theta_2}{\partial x_1},\ \kappa_{22}=\frac{-\partial \theta_1}{\partial x_2},\ \kappa_{12}=\frac{1}{2}\left(\frac{\partial \theta_2}{\partial x_2}-\frac{\partial \theta_1}{\partial x_1}\right) \tag{3.1.2-6}$$

relations to which the assumption of plane constraints is added $\sigma_{zz}=0$, $\sigma_{1z}=0$, $\sigma_{2z}=0$ who will determine the complementary field of displacement $\mathbf{u}^c\in V_c$. Into the theory used here, one introduces only two components of rotation θ_x and θ_y, which implies that the tensor of variation of curve is 2D and has only 3 independent components.

Among the two coupled models, GLRC_DM is a model total and thus formulated directly in term of $\Sigma=\Sigma(E)$, but models VMIS_ISOT_LINE and VMIS_CINE_LINE are the laws of behavior 3D classics. Considering one wishes especially that the elastoplastic part be represented out of membrane, one does not apply the law of behavior VMIS_ that on the relation $N=N(\epsilon)$, then it is supposed that the behavior in inflection remains elastic linear. One thus has:

Damage :

$$\Sigma^d(\epsilon,\kappa)=\Sigma_{\text{GLRC\_DM}}(\epsilon,\kappa) \tag{3.1.2-7}$$

Elastoplasticity :

$$\Sigma^p(\epsilon,\kappa)=\begin{pmatrix} N_{VMIS}(\epsilon) \\ H_{ELAS}:\kappa \end{pmatrix} \tag{3.1.2-8}$$

where $\Sigma_{\text{GLRC\_DM}}(\epsilon,\kappa)$ represent the total law of behavior GLRC_DM, $N_{VMIS}(\epsilon)$ the law of Von Mises in plane constraints and H_{ELAS} the elastic tensor acting on the curve (see [bib9] for its exact form). These two modules, of damage and elastoplasticity, are then used in the coupler (see §2) while replacing $\sigma^p\to\Sigma^p$ and $\sigma^d\to\Sigma^d$.

3.1.3 Internal variables

The internal variables of the coupled model are stored in series: the first correspond to the model of damage, which are followed of that of the elastoplastic model. The coupler needs six (6) variable interns to store the tensor of the standard deformation, $\bar{\varepsilon}^{ed}$, which controls the internal loop (see §2.3). In the case of coupling GLRC_DM/VMIS_ one adds of them four (4) because of the algorithm allowing to satisfy the condition with the plane constraints. These four internal variables have the same meaning as those used for the method DEBORST (see [bib12]).
For GLRC_DM/VMIS_ there are thus 24 internal variables: V1-V7 : GLRC_DM; V8-V14: VMIS_ (one makes use only of V5 and V6 for VMIS_ISOT_LINE); V15-V20 : tensor of deformation $\bar{\varepsilon}^{ed}$;V21-V24 : method DEBORST.

3.1.4 Validation

This model is validated by the tests SSNS106F, G (to see [bib8]).

4 Bibliography

[1] J. LUBLINER, “Plasticity theory”, MacMillan, New York, 1990.

[2] J.LEMAITRE, J.L.CHABOCHE, “Mechanics d'endommagement”,

[3] A. IBRAHIMBEGOVIC, D. MARKOVIC, F. GATUINGT, “Constitutive model of coupled ramming-plasticity and its finite element implementation”, Eur. J. Finite Elem., 12 (4), 2003, p. 381-405.

[4] D. MARKOVIC, A. IBRAHIMBEGOVIC, “Complementary energy based FE modelling of coupled elasto-plastic and ramming behavior for continuum microstructure computations”, Comp. Methods Appl. Engrg., 195.2006, p. 5077-5093.

[5] D. MARKOVIC, “Modeling heterogeneous multi-scales of structures to the non-linear unelastic behaviors”, thesis ENS of Cachan, 2004, available on http://tel.archives-ouvertes.fr/tel-00133643 (version pdf) and in Bureau V015 (version paper).

[6] S.MILL. Modeling of the reinforced concrete structures under seismic loading. Note HT-62/04/025/A, 12/2004.

[7] S.MILL. F. VOLDOIRE Study of a reinforced concrete beam under loading of inflection. Note HT-62/05/013/A, 9/2006.

[8] [V6.05.106] SSNS106 – Damage of a plate planes under requests varied with the law of behavior GLRC_DM.

[9] [R3.07.03] – Elements of plate DKT, DST, DKQ, DSQ and Q4G.

[10] [R7.01.31] – Law of behavior of reinforced concrete plate GLRC_DAMAGE.

[11] [R7.01.32] – Law of behavior of reinforced concrete plate GLRC_DM.

[12] [R5.03.03] – Taking into account of the assumption of plane constraints in the non-linear behaviors.

[13] [R5.03.02] – Integration of the relations of elastoplastic behavior of von Mises.

[14] [R7.01.09] – Law of behavior ENDO_ORTH_BETON.

[15] [R7.01.19] – Modeling of the coupling creep/plasticity for the concrete.

5 History of the versions of the document

Version Aster	Author (S) or contributor (S), organization	Description of the modifications
9.3	D.MARKOVIC EDF-R&D/AMA	Initial text