Relation of behavior BETON_BURGER for the creep of the concrete

Summary:

This document presents the model of creep BETON_BURGER, who is a way of modelling the creep of the concrete (clean and of desiccation). This model is strongly inspired by the structure already installation in the model BETON_UMLV.

This model is implemented under MFront and is integrated directly in code_aster.

Contents

1 Introduction

Within the framework of the studies of the long-term behavior of structures out of concrete, a dominating share of the deformations measured on structure relates to the differed deformations which appear in the concrete during its life. One speaks about "withdrawal" when the deformation is measured on a test-tube not subjected to loading and of "creep" when the deformation takes place under loading. The exchanges of water between the test-tube and the environment influence the final deformation in a considerable way. This observation causes classically later distinctions on the differed deformations. One then distinguishes classically the withdrawals with the young age, the withdrawal of desiccation, clean creep and the creep of desiccation. One points out the deformations differed from a concrete structure to locate the share of the deformation calculated in this document:

- with the young age without load:
  1) endogenous withdrawal (1 day - 1 year)
  2) thermal withdrawal (1 hour – 1 day)

  caused by a reaction of thermohydration.

- in the medium term without load: withdrawal of desiccation (a few months – a few years) according to dimensions of the structure caused by the drying which results in an evaporation of part of the water not used in the process of hydration.
- long-term under load :
  1) The clean creep, which is the share of creep of the concrete that one would observe during a test without exchange of water with outside.
  2) The creep of desiccation in complement to clean creep is the share of total creep directly related to the water departure affecting the concrete which undergoes a mechanical loading on the one hand and drying on the other hand.

The model presented here is dedicated to the modeling of the differed deformation associated with creep, clean and desiccation. In *Code_hasster*, the model is used under the name of BETON_BURGER.

Clean creep. The first model of creep of the concretes introduces into Code_hasster (see [R7.01.01] and [bib4]) was developed in optics to predict the longitudinal deflections of creep under uniaxial constraints. The generalization of this model, in order to take into account a state of multiaxial stresses, is done then via a Poisson's ratio of creep arbitrary, constant and equal, or close, elastic Poisson's ratio. However, determination *a posteriori* Poisson's ratio of effective creep shows its dependence with respect to the way of loading. In addition, concrete of certain works of parc EDF, the such containment systems of nuclear reactor, is subjected in a state of biaxial stresses. This report led to the clarification of the law of deformations of clean creep UMLV (University of Marne-the-Valley, partner in the development of this model) for which the Poisson's ratio of creep is a direct consequence of the calculation of the principal deformations.
The model BETON_UMLV suppose speeds of in the long run constant deformation for its part, rheology which seems not very probable within sight of the experimental results resulting from work of Brooks [bib7]. By preserving the structure of the model BETON_UMLV, one adds nona linearity on speeds of long-term deformation to correct this point, methodology also employed by Saddler and al. [bib6]. The new developed model is described like phenomenologic.
The clean deformation of creep is strongly affected by the hygroscopy of the concrete. This dependence is taken into account by the law BETON_BURGER (as it was the case for BETON_UMLV). The effect of the temperature in the deformations of clean creep is also taken into account in the law BETON_BURGER via a law of the Arrhenius type.
In experiments the deformations of clean creep present also a growing old behavior: deformation after a time Δt depends on the age of material at the moment of loading. This aspect of the behavior of the concrete is not taken into account in this model.

Creep of desiccation. The model suggested here is that of Bazant [bib10]. It is a purely viscous law.

2 Assumptions

Assumption 1 (H.P.P.)
The law is written within the framework of the small disturbances.

Assumption 2 (partition of the deformations)
In small deformations, the tensor of the total deflections is broken up into several terms relating to the processes considered. As regards the description of the various mechanisms of deformations differed from the concretes, one admits that the total deflection is written:

$$\varepsilon = \underbrace{\varepsilon^{e}}_{\text{déformation élastique}} + \underbrace{\varepsilon^{fp}}_{\text{fluage propre}} + \underbrace{\varepsilon^{fdess}}_{\text{fluage de dessiccation}} + \underbrace{\varepsilon^{re}}_{\text{retrait endogène}} + \underbrace{\varepsilon^{rd}}_{\text{retrait de dessiccation}} + \underbrace{\varepsilon^{th}}_{\text{déformation thermique}} \quad (1)$$

In this document, one does not describe the taking into account of the various types of withdrawals (for that, to see the documentation of *Code_hasster* [R7.01.12]), so that (1) is reduced to:

$$\varepsilon = \varepsilon^{e} + \varepsilon^{fp} + \varepsilon^{fdess} \quad (2)$$

Assumption 3 (decomposition of the components of clean creep)

In a general way, clean creep can be modelled by combining the elastic behavior of the solid and the viscous behavior of the fluid. For the law presented, clean creep is described like the combination of the elastic behavior of the hydrates and the aggregates and the viscous behavior of water.
In the case of the model `BETON_BURGER`, the assumption is carried out that clean creep can be broken up into a process uncoupling a spherical part and a deviatoric part. The tensor of the total deflections of clean creep is written then:

$$\underline{\underline{\varepsilon}}^{fp} = \underbrace{\varepsilon^{fs} \cdot \underline{\underline{1}}}_{\text{partie sphérique}} + \underbrace{\underline{\underline{\varepsilon}}^{fd}}_{\text{partie déviatorique}} \quad \text{with} \quad \varepsilon^{fs} = \frac{1}{3} \cdot tr\, \underline{\underline{\varepsilon}}^{fp} \quad (3)$$

The tensor of the constraints can be developed according to a similar form:

$$\underline{\underline{\sigma}} = \underbrace{\sigma^{s} \cdot \underline{\underline{1}}}_{\text{partie sphérique}} + \underbrace{\underline{\underline{\sigma}}^{d}}_{\text{partie déviatorique}} \quad (4)$$

The model `BETON_BURGER` suppose a total decoupling between the spherical and deviatoric components of clean creep: the deformations induced by the spherical constraints are purely spherical and the deformations induced by the deviatoric constraints are purely deviatoric. On the other hand, the cumulated viscous deformations have an effect on the viscous properties of the fluid, whatever its source (spherical or deviatoric). To take account of the effect of internal moisture, the deformations are multiplied by internal relative moisture:

$$\varepsilon^{s} = h \cdot f\left(\sigma^{s}\right) \quad \text{and} \quad \underline{\underline{\varepsilon}}^{d} = h \cdot f\left(\underline{\underline{\sigma}}^{d}\right) \quad (5)$$

Or h indicate internal relative moisture.
The condition (5) allows to check *a posteriori* that the deformations of clean creep are proportional to the relative humidity.

3 Description of the model

3.1 Clean creep

To model the clean phenomenon of creep, the model suggested is based on simple rheological models (figure 3.1-1) Comprenant in series an elastic body (described by the behavior ELAS), a solid of linear Kelvin Voigt for the modeling of reversible creep (recouvrance), and a liquid of Maxwell with a nonlinear viscosity to model long-term creep. The chains spherical and deviatoric are equivalent in their construction.
The stage of Kelvin Voigt has a limit of deformation managed by the modulus of elasticity. The characteristic of the model rests on the choice of the non-linearity assigned to the viscosity of the body of Maxwell.

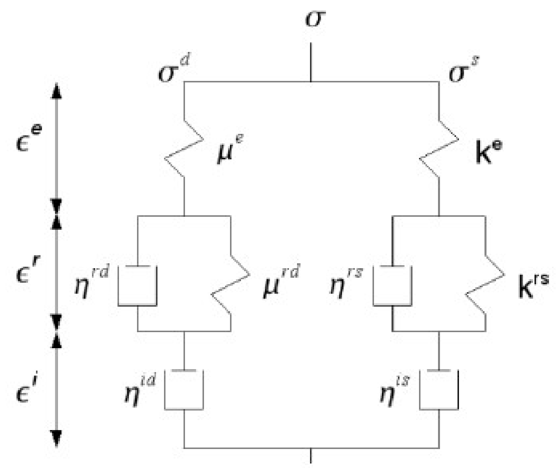

Figure 3.1-1: Rheological diagram distinguishing the spherical and deviatoric part of the tensor from the constraints

3.1.1 Description of the spherical part

The spherical deformation of clean creep is written as the sum of a reversible part and an irreversible part:

$$\varepsilon^{fs} = \underbrace{\varepsilon_r^{fs}}_{\substack{partie\\réversible}} + \underbrace{\varepsilon_i^{fs}}_{\substack{partie\\irréversible}} \quad (6)$$

The process of deformation spherical of creep is controlled by the following equations:

$$h \cdot \sigma^s = k^{rs} \cdot \varepsilon_r^{fs} + \eta^{rs} \cdot \dot{\varepsilon}_r^{fs} \quad (7)$$

and:

$$h \cdot \sigma^s = \eta^{is} \cdot \dot{\varepsilon}_i^{fs} \quad (8)$$

with:

- k_r^s the module of compressibility associated with reversible spherical clean creep,
- η_r^s the viscosity of the stage of Kelvin Voigt associated with reversible spherical clean creep,
- η_i^s the nonlinear spherical viscosity of the fluid of Maxwell.

The indicator $\dot{}$ associated with any variable the speed of evolution of this variable describes.

3.1.2 Description of the deviatoric part

The deviatoric deformation of clean creep is also written as the tensorial sum of a reversible part and an irreversible part:

$$\underbrace{\underline{\varepsilon}^{fd}}_{\substack{\text{déformation}\\\text{déviatorique}\\\text{totale}}} = \underbrace{\underline{\varepsilon}_r^{fd}}_{\substack{\text{contribution}\\\text{réversible}}} + \underbrace{\underline{\varepsilon}_i^{fd}}_{\substack{\text{contribution}\\\text{irréversible}}} \quad (9)$$

$J^{ème}$ principal component of the total deviatoric deformation is governed by the equations:

$$\eta_r^d \cdot \dot{\varepsilon}_r^{fd,j} + k_r^d \cdot \varepsilon_r^{fd,j} = h \cdot \sigma^{d,j} \quad (10)$$

And:

$$\eta_i^d \cdot \dot{\varepsilon}_i^{fd,j} = h \cdot \sigma^{d,j} \quad (11)$$

with:

- k_r^d the modulus of rigidity associated with clean creep deviatoric reversible,
- η_r^d the viscosity of the stage of Kelvin Voigt associated with clean creep deviatoric reversible,
- η_i^d the nonlinear deviatoric viscosity of the fluid of Maxwell.

3.1.3 Description of not viscous linearity

The non-linearity of viscosity is interpreted according to [bib6] like the result of a spherical consolidation of the sample ([bib7]) and of a tangle or blocking of displacements of the layers HSC, components of the mortar. A coefficient of “consolidation” is thus introduced according to the same idea to control the evolution of viscosities. This additional coefficient intervenes on the laws of evolutions of the bodies of Maxwell (spherical and deviatoric). It depends directly on the standard of the tensor of the cumulated irreversible differed deformations. This extension of the assumptions posed by [bib6] allows a taking into account of non-linearity for any type of ways (with or without spherical loading). The explicit formulation of the bodies of Maxwell is the following one:

$$\eta_i^s = \eta_{i,0}^s \cdot \exp\left(\frac{\|\varepsilon_m^{fi}\|}{\kappa}\right) \text{ and } \eta_i^d = \eta_{i,0}^d \cdot \exp\left(\frac{\|\varepsilon_m^{fi}\|}{\kappa}\right) \quad (12)$$

with:

- $\eta_{i,0}^s$ the initial viscosity of the fluid of bearing Maxwell on the spherical part
- $\eta_{i,0}^d$ the initial viscosity of the fluid of bearing Maxwell on the deviatoric part
- κ deformation characteristic related to an amplified viscosity of a factor $\exp(1)$.
- $\|\varepsilon_m^{fi}\|$ Irreversible equivalent deformation, i.e. the standard of the complete tensor (spherical and deviatoric) of deformations of irreversible creep, maximum value attack during the loading.

The construction of $\|\varepsilon_m^{fi}\|$ follows following logic: $\|\varepsilon_m^{fi}\| = max\left(\|\varepsilon_m^{fi}\|, \sqrt{\underline{\underline{\varepsilon}}^{fi} : \underline{\underline{\varepsilon}}^{fi}}\right)$.

3.1.4 Restriction of number parameters of the model

The equivalence of the rheological chains deviatoric and spherical makes it possible to obtain, by respecting the following expression, an apparent Poisson's ratio of constant creep:

$$\frac{\eta_{i,0}^s}{\eta_{i,0}^d} = \frac{\eta_r^s}{\eta_r^d} = \frac{k_r^s}{k_r^d} = \frac{(1+\nu)}{(1-2\cdot\nu)} = \beta \quad (13)$$

For the use of the model `BETON_BURGER` on uniaxial tests of creep, one seldom has the radial deformations of the samples making difficult the identification of the whole of the parameters of the model. A first approximation consists in assuming the relation (13) and limit thus the number of parameters to be determined with four.

3.1.5 Thermo-activation of the deformations of clean creep

The experimental results of clean creep test in temperature show that the deformation of creep thermo-is activated. For temperatures T lower with 55°C, it obeys a law of the Arrhenius type:

$$\varepsilon^f(T)=\varepsilon^f(T_0)\cdot\exp\left(\frac{-E_{ac}}{R}\cdot\left(\frac{1}{T}-\frac{1}{T_0}\right)\right) \quad (14)$$

with:

- T_0 the temperature of reference
- E_{ac} the energy of activation
- R the constant of perfect gases (8.3144621 $J.K^{-1}.mol^{-1}$)

To satisfy this relation and while keeping a constant characteristic time, the stiffness of the springs and the viscosity of the shock absorbers are modified in the following way:

$$k^{rs}(T)=k^{rs}(T_0)\cdot\exp\left(\frac{E_{ac}}{R}\cdot\left(\frac{1}{T}-\frac{1}{T_0}\right)\right) \quad (15)$$

$$k^{rd}(T)=k^{rd}(T_0)\cdot\exp\left(\frac{E_{ac}}{R}\cdot\left(\frac{1}{T}-\frac{1}{T_0}\right)\right) \quad (16)$$

$$\eta^{rs}(T)=\eta^{rs}(T_0)\cdot\exp\left(\frac{E_{ac}}{R}\cdot\left(\frac{1}{T}-\frac{1}{T_0}\right)\right) \quad (17)$$

$$\eta^{rd}(T)=\eta^{rd}(T_0)\cdot\exp\left(\frac{E_{ac}}{R}\cdot\left(\frac{1}{T}-\frac{1}{T_0}\right)\right) \quad (18)$$

$$\eta_0^{is}(T)=\eta_0^{is}(T_0)\cdot\exp\left(\frac{E_{ac}}{R}\cdot\left(\frac{1}{T}-\frac{1}{T_0}\right)\right) \quad (19)$$

$$\eta_0^{id}(T)=\eta_0^{id}(T_0)\cdot\exp\left(\frac{E_{ac}}{R}\cdot\left(\frac{1}{T}-\frac{1}{T_0}\right)\right) \quad (20)$$

$$\kappa(T)=\frac{\kappa(T_0)}{\exp\left(\frac{E_{ac}}{R}\cdot\left(\frac{1}{T}-\frac{1}{T_0}\right)\right)} \quad (21)$$

To determine the energy of activation E_{ac} , it is highly advised to employ experimental data of creep tests in temperature. Without these data, the value by default of the energy of activation is 17.5kJ.mol $^{-1}$.

The field of temperature is thus essential in precondition of mechanical calculation. If nevertheless the field of temperature is not provided, thermo-activation is not taken into account in the model, i.e. $k^{rs}(T)=k^{rs}(T_0)$, $k^{rd}(T)=k^{rd}(T_0)$, $\eta^{rs}(T)=\eta^{rs}(T_0)$, $\eta^{rd}(T)=\eta^{rd}(T_0)$, $\eta_0^{is}(T)=\eta_0^{is}(T_0)$, $\eta_0^{id}(T)=\eta_0^{id}(T_0)$, $\kappa(T)=\kappa(T_0)$.

3.2 Creep of desiccation

One supposes to be able to break up the creep of desiccation $\Delta\varepsilon^{fdess}$ in two parts called intrinsic and structural [bib4]:

$$\Delta\varepsilon^{fdess}=\Delta\varepsilon_{int}^{fdess}+\Delta\varepsilon_{struct}^{fdess} \tag{22}$$

It is agreed that the structural deformation is not a component of deformation in oneself, therefore in this document the only component of the creep of desiccation relates to the intrinsic part:

$$\Delta\varepsilon^{fdess}=\Delta\varepsilon_{int}^{fdess} \tag{23}$$

Bazant and al. [bib10] suggest that the drying and the application of a loading in compression simultaneously are responsible for the microphone-diffusion of the molecules between the macro-pores and the micropores. The microphone-diffusion of the water molecules would support the rupture of the connections between the particles of freezing inducing the deformation of creep of desiccation. It is one of the physicochemical phenomena most complicated to model resulting from a coupling between the constraint, clean creep and drying. They propose the following equation (equation of a shock absorber) to take into account the creep of desiccation (intrinsic) hasU level elementary:

$$\dot{\varepsilon}^{fdess}=\frac{|\dot{h}|\sigma}{\eta^{fd}} \tag{24}$$

with:

- ε^{fdess} deformation of the creep of desiccation,
- η^{fd} a parameter material (in $[Pa\cdot sec]$ in the S.I),
- h the relative humidity which evolves in time, fact of the case of evolution.

4 Description model under MFront

The behavior is defined in the file BETON_BURGER.mfront.

Parser/DSL	Implicit
Algorithm	NewtonRaphson_NumericalJacobian
	@Epsilon 1.E-11
Internal variables (@StateVariable)	real ESPHR real ESPHI real ELIM Stensor EDEVR Stensor EDEVI Stensor Edess
Auxiliary internal variables (@AuxiliaryStateVariable)	Stensor EF
Variables of orders (@ExternalStateVariable)	real HYGR real HYDR real SECH
Modelings	'3D' 'AXIS' 'D_PLAN'
Deformations	'SMALL' 'PETIT_REAC' 'GDEF_LOG'

5 Description of the internal variables

The following table gives the correspondence between the number of the internal variables accessible by *Code_hasster* and their description in the case of a modeling 2D (D_PLAN or AXIS):

Recall: Lbe four first internal variables (in the case 2D) under MFront are always the elastic strain.

Number of the variable	Description
5	Reversible spherical deformation
6	Irreversible spherical deformation
7	Equivalent deformation of clean creep irreversible max
8	Reversible deviatoric deformation, component 11
9	Reversible deviatoric deformation, component 22
10	Reversible deviatoric deformation, component 33
11	Reversible deviatoric deformation, component 12
12	Irreversible deviatoric deformation, component 11
13	Irreversible deviatoric deformation, component 22
14	Irreversible deviatoric deformation, component 33
15	Irreversible deviatoric deformation, component 12
16	Creep of desiccation, component 11
17	Creep of desiccation, component 22
18	Creep of desiccation, component 33
19	Creep of desiccation, component 12
20	Clean deformation creep, component 11
21	Deformation clean creep, component 22
22	Deformation clean creep, component 33
23	Clean deformation creep, component 12

The following table gives the correspondence between the number of the internal variables accessible by *Code_hasster* and their description in the case of a modeling 3D :

Recall: Lbe six first internal variables (in the case 3D) under MFront are always the elastic strain.

Number of the variable	Description
7	Reversible spherical deformation
8	Irreversible spherical deformation
9	Equivalent deformation of clean creep irreversible max
10	Reversible deviatoric deformation, component 11
11	Reversible deviatoric deformation, component 22
12	Reversible deviatoric deformation, component 33
13	Reversible deviatoric deformation, component 12
14	Reversible deviatoric deformation, component 13
15	Reversible deviatoric deformation, component 23
16	Irreversible deviatoric deformation, component 11
17	Irreversible deviatoric deformation, component 22
18	Irreversible deviatoric deformation, component 33
19	Irreversible deviatoric deformation, component 12
20	Irreversible deviatoric deformation, component 13
21	Irreversible deviatoric deformation, component 23
22	Creep of desiccation, component 11
23	Creep of desiccation, component 22
25	Creep of desiccation, component 33
26	Creep of desiccation, component 12
26	Creep of desiccation, component 13

27	Creep of desiccation, component 23
28	Clean deformation creep, component 11
29	Clean deformation creep, component 22
30	Clean deformation creep, component 33
31	Clean deformation creep, component 12
32	Clean deformation creep, component 13
33	Clean deformation creep, component 23

6 Notations

$\underline{\underline{\varepsilon}}$ tensor of the total deflections

$\underline{\underline{\varepsilon}}^f$ tensor of the deformations of clean creep

$\underline{\underline{\varepsilon}}^e$ tensor of the elastic strain

$\varepsilon^{fs}\underline{\underline{1}}$ spherical part of the tensor of the deformations of clean creep

$\varepsilon_r^{fs}\underline{\underline{1}}$ reversible spherical part of the tensor of the deformations of clean creep

$\varepsilon_i^{fs}\underline{\underline{1}}$ irreversible spherical part of the tensor of the deformations of clean creep

$\underline{\underline{\varepsilon}}^{fd}$ deviatoric part of the tensor of the deformations of clean creep

$\underline{\underline{\varepsilon}}_r^{fd}$ reversible deviatoric part of the tensor of the deformations of clean creep

$\underline{\underline{\varepsilon}}_i^{fd}$ irreversible deviatoric part of the tensor of the deformations of clean creep

$\underline{\varepsilon}^{fi}$ complete tensor of the deformations of irreversible creep

$\underline{\varepsilon}^{fdess}$ tensor of the deformations of creep of desiccation

$\underline{\underline{\sigma}}$ tensor of the total constraints

$\sigma^s\underline{\underline{1}}$ spherical part of the tensor of the constraints

$\underline{\underline{\sigma}}^d$ deviatoric part of the tensor of the constraints

h internal relative moisture

k_r^s rigidity connects associated with spherical Kelvin Voigt

k_r^d rigidity connects associated with deviatoric Kelvin Voigt

η_i^s viscosity connects associated with the spherical unrecoverable deformations

η_r^s viscosity connects associated with spherical Kelvin Voigt

η_i^d viscosity connects associated with the deviatoric unrecoverable deformations

η_r^d viscosity associated with deviatoric Kelvin Voigt

η^{fd} viscosity associated with creep with desiccation

$x, \underline{x}, \underline{\underline{x}}$ indicate respectively a scalar, a vector and a tensor of order 2.

$x_n, x_{n+1}, \Delta x_n$ indicate the value of the quantity respectively X at time t_n, at time t_{n+1} and variation of x during the interval $[t_n; t_{n+1}]$.

7 Bibliography

1) BENBOUDJEMA F.: Modeling of the deformations differed from the concrete under biaxial requests. Application to the buildings engines of nuclear power plants, Memory of D.E.A. Advanced materials – Engineering of the Structures and the Envelopes, 38 p. (+ additional) (1999).

2) BENBOUDJEMA F., MEFTAH F., HEINFLING G., POPE Y.: Digital and analytical study of the spherical part of the clean model of creep UMLV for the concrete, notes technical HT 2/25/040 /A, 56 p (2002).

3) BENBOUDJEMA F., MEFTAH F., TORRENTI J.M., POPE Y.: Algorithm of the clean model of creep and desiccation UMLV coupled to an elastic model, notes technical HT - 2/25/050 /A, 68 p (2002).

4) GRANGER L.: Behavior differed from the concrete in the enclosures of nuclear power plant: analysis and modeling, Doctorate of the ENPC (1995).

5) Relation of behavior of Granger for the clean creep of the concrete, Documentation *Code_Aster* [R7.01.01], 16 p (2001).

6) SADDLER A., BUFFO-LACARRIERE L.: Towards a modeling simple and unified clean creep, withdrawal and creep in desiccation of the concrete, EJECE, volume 13(10), pages 1161-1182, 2009.

7) ACKER P.: On the origins of the shrinking and creep of the concrete. French review of Génie Civil, vol.7, n°6, p.761-776.

8) MANDEL, J: Course of Mechanics of the Continuous Mediums. Volume II, Mechanics of the solids. Gauthier-Villard Editor, 837 pages, 1966.

9) BENBOUDJEMA F.: Contribution to the analysis of the deformations differed in cementing materials and of its effects in the works of Génie Civi *, Memory of obtaining the habitlation to direct research, ENS Cachan.* 2012

10) BAZANT, Z.P., CHERN, J.C.: Concrete creep variable At humidity: constitutive law and mechanism. *Materials and Structures* (RILEM, Paris), 18, Jan., p. 1-20 (1985).

8 Features and checking

This document relates to the law of behavior BETON_BURGER (keyword BEHAVIOR of STAT_NON_LINE) and its associated material BETON_BURGER (order DEFI_MATERIAU).

This law of behavior is checked by the cases following tests:

SSNV163	Clean calculation of creep	[V6.04.163]
SSNV174	Taking into account of the withdrawal in the models BETON_UMLV and BETON_BURGER	[V6.04.174]
SSNV180	Taking into account of thermal dilation and the creep of desiccation in the models BETON_UMLV and BETON_BURGER	[V6.04.180]
SSNV181	Checking of the good taking into account of shearing in the models BETON_UMLV and BETON_BURGER	[V6.04.181]
SSNV235 C, D	Influence of the temperature in the evolution of creep	[V6.04.235]
COMP003	Test of behaviors specific to the concretes. Simulation in a material point	V6.07.103
COMP011	Thermomechanical validation of the laws for the concrete	V6.07.111