Rate of refund of energy in thermoelasticity non-linear

Summary:

One presents the calculation of the rate of refund of energy by the method theta in 2D or 3D for a non-linear thermoelastic problem. The relation of nonlinear elastic behavior is described in [R5.03.20].

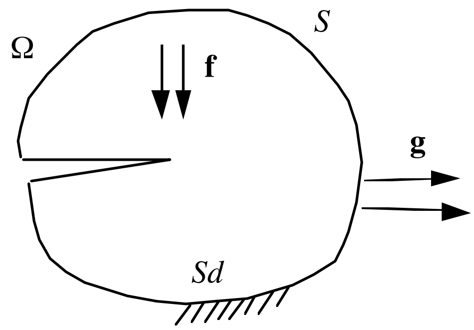
Contents

1 Calculation of the rate of refund of energy by the method theta in nonlinear thermoelasticity

1.1 Relation of behavior

One considers a fissured solid occupying the field Ω space R^2 or R^3 . That is to say:

- u the field of displacement,
- T the field of temperature,
- f the field of voluminal forces applied to Ω ,
- g the field of surface forces applied to a part S of $\partial\Omega$,
- U the field of displacements imposed on a part S_d of $\partial\Omega$.



The behavior of the solid is supposed to be elastic non-linear such as the relation of behavior coincides with the elastoplastic law of Hencky-Von Put (isotropic work hardening) in the case of a loading which induces a radial and monotonous evolution in any point. This model is selected in the orders `CALC_G` via the keyword `RELATION=' ELAS_VMIS_LINE'` or `'ELAS_VMIS_TRAC'` or `'ELAS_VMIS_PUIS'` under the keyword factor `BEHAVIOR` [R5.03.20].

One indicates by:

- ε the tensor of deformations,
- $\boldsymbol{\varepsilon}^\circ$ the tensor of the initial deformations,
- $\boldsymbol{\sigma}$ the tensor of the constraints,
- $\boldsymbol{\sigma}^\circ$ the tensor of the initial constraints,
- $\boldsymbol{\Psi}(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^\circ, \boldsymbol{\sigma}^\circ, T)$ density of free energy.

ε is connected to the field of displacement u by:

$$\boldsymbol{\varepsilon}(u)=\frac{1}{2}(\mathbf{u}_{\mathbf{i},\mathbf{j}}+\mathbf{u}_{\mathbf{j},\mathbf{i}})$$

Density of free energy $\boldsymbol{\Psi}(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^\circ, \boldsymbol{\sigma}^\circ, T)$ is a convex and differentiable, known function for a given state [R5.03.20 éq 3]. The relation of behavior of material is written in the form:

$$\boldsymbol{\sigma}_{\mathbf{ij}}=\frac{\partial\boldsymbol{\Psi}}{\partial\boldsymbol{\varepsilon}_{\mathbf{ij}}}(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}^\circ, \boldsymbol{\sigma}^\circ, T)$$

It derives from the potential free energy. For this hyperelastic relation of behavior, one can give a direction to the rate of refund of energy within the framework of the comprehensive approach in breaking process. It is not the case for a plastic relation of behavior.

1.2 Potential energy and relations of balance

One defines spaces of the fields kinematically acceptable V and V_0.

$$\begin{aligned} V &= \{\mathbf{v}\,\text{admissibles}, \mathbf{v}=\mathbf{U}\,\text{sur}\, S_d\} \\ V_0 &= \{\mathbf{v}\,\text{admissibles}, \mathbf{v}=\mathbf{O}\,\text{sur}\, S_d\} \end{aligned}$$

With the assumptions of the paragraph [§1.1] (with $\boldsymbol{\varepsilon}° = \boldsymbol{\sigma}° = 0$), the relations of balance in weak formulation are:

$$\begin{cases} \mathbf{u} \in V \\ \int_\Omega \boldsymbol{\sigma}_{\mathbf{ij}} \mathbf{v}_{\mathbf{i,j}}\, d\Omega = \int_\Omega \mathbf{f}_\mathbf{i} \mathbf{v}_\mathbf{i}\, d\Omega + \int_\Omega \mathbf{g}_\mathbf{i} \mathbf{v}_\mathbf{i}\, d\Gamma, \quad \forall \mathbf{v} \in V_0 \end{cases}$$

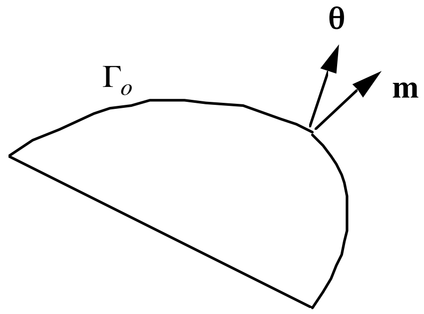
They are obtained by minimizing the total potential energy of the system:

$$\mathbf{W}(\mathbf{v}) = \int_\Omega \boldsymbol{\Psi}(\boldsymbol{\varepsilon}(\mathbf{v}), T)\, d\Omega = \int_\Omega \mathbf{f}_\mathbf{i} \mathbf{v}_\mathbf{i}\, d\Omega + \int_\Omega \mathbf{g}_\mathbf{i} \mathbf{v}_\mathbf{i}\, d\Gamma$$

The demonstration is identical to that in linear elasticity [R7.02.01 §1.2].

1.3 Lagrangian expression of the rate of refund of energy

That is to say $\boldsymbol{m}$ the unit normal with Γ_0 located in the tangent plan at $\partial\Omega$ in Ω.



That is to say the field $\boldsymbol{\theta}$ such as:

$$\boldsymbol{\theta} \in \Theta = \{\boldsymbol{\mu}\ \text{tels que}\ \boldsymbol{\mu}\,.\,\mathbf{n} = 0\ \text{sur}\ \Omega\}$$

while noting $\mathbf{n}$ the normal with $\partial\Omega$.

The rate of refund of energy G is solution of the variational equation:

$$\int_{\Gamma_o} G\boldsymbol{\theta}\cdot\mathbf{m}\,\mathrm{dS}=G(\boldsymbol{\theta}),\ \forall\,\boldsymbol{\theta}\in\Theta$$

where $G(\boldsymbol{\theta})$ is defined by:

$$\begin{aligned}
G(\boldsymbol{\theta}) &= \int_\Omega \boldsymbol{\sigma}_{ij}\mathbf{u}_{i,p}\boldsymbol{\theta}_{p,j}-\boldsymbol{\Psi}\,\boldsymbol{\theta}_{k,k}-\frac{\partial\boldsymbol{\Psi}}{\partial T}\mathbf{T}_{,k}\boldsymbol{\theta}_k\,d\,\Omega\\
&+\int_\Omega\left(\boldsymbol{\sigma}_{ij}-\frac{1}{2}\boldsymbol{\sigma}^\circ_{ij}\right)\boldsymbol{\varepsilon}^\circ_{ij,k}\boldsymbol{\theta}_k-\left(\boldsymbol{\varepsilon}_{ij}-\boldsymbol{\varepsilon}^{th}_{ij}-\frac{1}{2}\boldsymbol{\varepsilon}^\circ_{ij}\right)\boldsymbol{\sigma}_{ij,k}\boldsymbol{\theta}_k\,d\,\Omega\\
&+\int_\Omega \mathbf{f}_i\mathbf{u}_i\boldsymbol{\theta}_{k,k}+\mathbf{f}_{i,k}\boldsymbol{\theta}_k\mathbf{u}_i\,d\,\Omega\\
&+\int_S \mathbf{g}_{i,k}\boldsymbol{\theta}_k\mathbf{u}_i+\mathbf{g}_i\mathbf{u}_i\left(\boldsymbol{\theta}_{k,k}-\frac{\partial\boldsymbol{\theta}}{\partial\mathbf{n}_k}\mathbf{n}_k\right)d\,\Gamma\\
&-\int_{S_d}\boldsymbol{\sigma}_{ij}\mathbf{n}_j\mathbf{U}_{i,k}\boldsymbol{\theta}_k\,d\,\Gamma
\end{aligned}$$

The demonstration is identical to that of the calculation of G in linear elasticity [R7.02.01]. The expression is the same one, postprocessing is thus identical.

1.4 Establishment of G in nonlinear thermoelasticity in *Code_Aster*

The types of elements and loadings, the environment necessary are the same ones as for the establishment of G in linear thermoelasticity [R7.02.01 §2.4].

For the calculation of the various terms of $G(\boldsymbol{\theta})$, in a given state, one recovers the density of free energy $\boldsymbol{\Psi}(\boldsymbol{\varepsilon},T)$, deformations $\boldsymbol{\varepsilon}$ and constraints $\boldsymbol{\sigma}$, calculated for the linear relation of behavior not - (routine NMELNL).

It is supposed that $\boldsymbol{\varepsilon}^\circ=\boldsymbol{\sigma}^\circ=0$ (identical term in linear or non-linear thermoelasticity). The density of free energy is written then [R5.03.20 §1.5]:

- in linear thermoelasticity:

$$\boldsymbol{\Psi}(\boldsymbol{\varepsilon},T)=\frac{1}{2}K\left(\boldsymbol{\varepsilon}_{kk}-3\alpha\left(T-T_{réf}\right)\right)^2+\frac{2\mu}{3}\boldsymbol{\varepsilon}^2_{eq}$$

with

$$\boldsymbol{\varepsilon}^2_{eq}=\frac{3}{2}\boldsymbol{\varepsilon}^D_{ij}\boldsymbol{\varepsilon}^D_{ij}=\frac{3}{2}\left(\boldsymbol{\varepsilon}_{ij}-\frac{1}{3}\boldsymbol{\varepsilon}_{kk}\boldsymbol{\delta}_{ij}\right)\left(\boldsymbol{\varepsilon}_{ij}-\frac{1}{3}\boldsymbol{\varepsilon}_{kk}\boldsymbol{\delta}_{ij}\right)$$

$$\boldsymbol{\varepsilon}^2_{eq}=\frac{3}{2}\left(\boldsymbol{\varepsilon}_{ij}\boldsymbol{\varepsilon}_{ij}-\frac{1}{3}\boldsymbol{\varepsilon}^2_{kk}\right)$$

- in non-linear thermoelasticity $\left(2\mu\,\boldsymbol{\varepsilon}_{eq}\geq\boldsymbol{\sigma}_y\right)$:

$$\boldsymbol{\Psi}(\boldsymbol{\varepsilon},T)=\frac{1}{2}K\left(\boldsymbol{\varepsilon}_{kk}-3\alpha\left(T-T_{réf}\right)\right)^2+\frac{1}{6\mu}\mathbf{R}\left(p\left(\boldsymbol{\varepsilon}_{eq}\right)\right)^2+\int_0^{p(\boldsymbol{\varepsilon}_{eq})}R(s)\,ds$$

with $\mathbf{R}\left(p\left(\boldsymbol{\varepsilon}_{eq}\right)\right)$: function of work hardening.

For a linear isotropic work hardening (`RELATION= 'ELAS_VMIS_LINE'`) one a:

$$\mathbf{R}(p)=\boldsymbol{\sigma}_y+p\frac{E\,E_T}{E-E_T}=\boldsymbol{\sigma}_y+a\,p$$

$$p=\frac{\boldsymbol{\sigma}_{eq}-\boldsymbol{\sigma}_y}{3\mu+a}\quad \text{avec}\; a=\frac{E\,E_T}{E-E_T}$$

$$A(p)=\int_0^p \mathbf{R}(s)\,ds=\ \boldsymbol{\sigma}_{yp}+\frac{1}{2}a\,p^2=\frac{1}{2}\boldsymbol{\sigma}_y\,p+\frac{p}{2}(\boldsymbol{\sigma}_y+a\,p)$$

$$A(p)=\frac{p}{2}(\boldsymbol{\sigma}_y+\mathbf{R}(p))$$

Postprocessing is then identical to the problem in linear elasticity except for the thermal term:

$$\text{THER}=-\frac{\partial\boldsymbol{\Psi}}{\partial T}\mathbf{T}_{,\mathbf{k}}\boldsymbol{\theta}_{\mathbf{k}}$$

If coefficients of Lamé $\lambda(T)$ and $\mu(T)$ are independent of the temperature, this term is null. In the contrary case, it is necessary to calculate $\frac{\partial\boldsymbol{\Psi}}{\partial T}(\boldsymbol{\varepsilon},T)$ at a given moment.

For a linear isotropic work hardening, one a:

$$\frac{\partial\boldsymbol{\Psi}}{\partial T}(\boldsymbol{\varepsilon},T)=\left[\frac{1}{2}\frac{dK(T)}{dT}(\boldsymbol{\varepsilon}_{kk}-3\alpha(T-T_{réf}))-3\mathrm{K}\left(\alpha+\frac{d\alpha(T)}{dT}(T-T_{réf})\right)\right](\boldsymbol{\varepsilon}_{kk}-3\alpha T-T_{réf})$$
$$+\frac{R(p)}{6\mu^2}\left[2\mu\frac{dR(P)}{dT}-\frac{d\mu(T)}{dT}R(p)\right]+\frac{dA(p)}{dT}$$

with

$$\frac{dR(p)}{dT}=\frac{d\,\boldsymbol{\sigma}_y(T)}{dT}+\frac{d\,a(T)}{dT}\,p+a\frac{d\,p(T)}{dT}$$

$$\frac{d\,a(T)}{dT}=\frac{1}{(E-E_T)^2}\left(\frac{d\,E_T(T)}{dT}E^2-\frac{d\,E(T)}{dT}E_T^2\right)$$

$$\frac{dp(T)}{dT}=\frac{1}{(3\mu+a)^2}\left[(\boldsymbol{\sigma}_y-\boldsymbol{\sigma}_{eq})\left(3\frac{d\mu(T)}{dT}+\frac{dA(T)}{dT}\right)-(3\mu+a)\frac{d\,\boldsymbol{\sigma}_y(T)}{dT}\right]$$

$$\frac{\mathrm{dA}(p)}{dT}=\frac{1}{2}\frac{dp(T)}{dT}(\boldsymbol{\sigma}_y+R_p)+\frac{1}{2}p\left(\frac{d\,\boldsymbol{\sigma}_y(T)}{dT}+\frac{\mathrm{d\,Rp}(T)}{dT}\right)$$

1.5 Warning

Caution! By definition, in the case general:

$$\Psi(\boldsymbol{\varepsilon}, T) \neq \boldsymbol{\sigma} : \boldsymbol{\varepsilon}$$

Although it is possible to carry out a followed elastoplastic calculation by the calculation of G in nonlinear elasticity, it should well be known that does not have any thermodynamic direction and that it is normal that the result depends on the field $\boldsymbol{\theta}$.

2 Calculation of the rate of refund of energy by the method theta in great transformations

One extends the relation of behavior of [§1] to great displacements and great rotations, insofar as it derives from a potential (hyperelastic law). This functionality is started by the keyword `DEFORMATION=' GROT_GDEP'` in the order `CALC_G`.

2.1 Relation of behavior

One indicates by:

- $\mathbf{E}$ the tensor of deformations of Green-Lagrange,
- $\mathbf{S}$ the tensor of the constraints of Piola-Lagrange called still second tensor of Piola - Kirchoff,
- $\Psi(\mathbf{E})$ density of energy internal.

The behavior of the solid is supposed to be hyperelastic, namely that:

- $\mathbf{E}$ is connected to the field of displacement $\mathbf{u}$ measured compared to the configuration of reference Ω_0 by:

$$\mathbf{E_{ij}}(\mathbf{u}) = \frac{1}{2}(\mathbf{u_{i,j}} + \mathbf{u_{j,i}} + \mathbf{u_{k,i}}\mathbf{u_{k,j}})$$

- $\mathbf{S}$ is connected to the tensor of the constraints of cauchy T by:

$$\mathbf{S_{ij}} = \det(\mathbf{F})\mathbf{F_{ik}^{-1}}\mathbf{T_{kl}}\mathbf{F_{jl}^{-1}}$$

$\mathbf{F}$ being the gradient of the transformation which makes pass from the configuration of reference Ω_0 with the current configuration Ω, connected to displacement by:

$$\mathbf{F_{ij}} = (\boldsymbol{\delta}_{\mathbf{ij}} + \mathbf{u_{i,j}})$$

The relation of behavior of a material hyperelastic is written in the form:

$$\mathbf{S}_{\mathbf{ij}} = \frac{\partial \mathbf{\Psi}}{\partial \mathbf{E}_{\mathbf{ij}}} = \frac{\partial \mathbf{\Psi}}{\partial \mathbf{E}_{\mathbf{jj}}} = \mathbf{S}_{\mathbf{ji}}$$

This relation describes a non-linear elastic behavior, similar to that of [§1.1]. She gives the opportunity of dealing with the problems of breaking process without integrating plasticity into it. And in the case of a monotonous radial loading, it makes it possible to obtain strains and stresses of the structure similar to those which one would obtain if the material presented an isotropic work hardening. The material hyperelastic has a reversible mechanical behavior, i.e. any cycle of loading does not generate any dissipation.

This model is selected in the order `CALC_G` [U4.82.03] via the keyword:

`RELATION: 'ELAS'`
for an elastic relation "linear", i.e. the relation between the strains and the stresses considered is linear,

`RELATION: 'ELAS_VMIS_LINE'` or `'ELAS_VMIS_TRAC'` or `'ELAS_VMIS_PUIS'`
for a "nonlinear" relation of elastic behavior (law of HENCKY-VON PUT at isotropic work hardening).

Such a relation of behavior makes it possible in any rigour to take into account great deformations and great rotations. However, one confines oneself with small deformations to ensure the existence of a solution and to be identical to an elastoplastic behavior under a monotonous radial loading [R5.03.20 §2.1].

2.2 Potential energy and relations of balance

The loading considered is reduced to a nonfollowing surface density $\mathbf{R}$ applied to a part Γ_0 edge of Ω_0 (assumption of the dead loads [R5.03.20 §2.2]).

One defines a space of the fields kinematically acceptable V:

$$V = \{ \mathbf{v}\,\text{admissibles}, \mathbf{v} = 0\,\text{sur}\,\Gamma_0 \}$$

The relations of balance in weak formulation are:

$$\int_{\Omega_0} \mathbf{F}_{\mathbf{ik}} \mathbf{S}_{\mathbf{kj}} \mathbf{v}_{\mathbf{i},\mathbf{j}}\, d\,\Omega_0 = \int_{\Gamma} \mathbf{R}_{\mathbf{i}} \mathbf{v}_{\mathbf{i}}\, d\,\Gamma$$

They can be obtained by minimizing the total potential energy of the system:

$$W(\mathbf{v}) = \int_{\Omega_0} \mathbf{\Psi}(\mathbf{E}(\mathbf{v}))\, d\,\Omega - \int_{\Gamma} \mathbf{R}_{\mathbf{i}} \mathbf{v}_{\mathbf{i}}\, d\,\Gamma$$

Indeed, if this functional calculus is minimal for the field of displacement $\mathbf{u}$, then:

$$
\begin{aligned}
\delta W &= \int_{\Omega_0} \frac{\partial \Psi}{\partial \mathbf{E_{ij}}} \delta \mathbf{E_{ij}} d\Omega - \int_{\Gamma} \mathbf{R_i} \delta \mathbf{v_i} d\Gamma = 0, \qquad \forall \delta \mathbf{v} \in V \\
&= \int_{\Omega_0} \mathbf{S_{ij}} \frac{1}{2} (\delta \mathbf{v_{i,j}} + \delta \mathbf{v_{j,i}} + \delta \mathbf{v_{p,i}} \mathbf{u_{p,j}} + \mathbf{u_{p,i}} \delta \mathbf{v_{p,j}}) d\Omega - \int_{\Gamma} \mathbf{R_i} \delta \mathbf{v_i} d\Gamma \\
&= \int_{\Omega_0} \mathbf{S_{ij}} (\boldsymbol{\delta_{ip}} + \mathbf{u_{p,i}}) \delta \mathbf{v_{p,i}} d\Omega - \int_{\Gamma} \mathbf{R_i} \delta \mathbf{v_i} d\Gamma \\
&= \int_{\Omega_0} \mathbf{F_{pi}} \mathbf{S_{ij}} \delta \mathbf{v_{p,j}} d\Omega - \int_{\Gamma} \mathbf{R_i} \delta \mathbf{v_i} d\Gamma \\
&= \int_{\Omega_0} \mathbf{F_{ik}} \mathbf{S_{kj}} \delta \mathbf{v_{i,j}} d\Omega - \int_{\Gamma} \mathbf{R_i} \delta \mathbf{v_i} d\Gamma = 0
\end{aligned}
$$

We thus find the equilibrium equations and the relation of behavior while having posed:

$$\mathbf{S}_{ij} = \frac{\partial \Psi}{\partial \mathbf{E}_{ij}}$$

2.3 Lagrangian expression of the rate of refund of energy in non-linear thermoelasticity and great transformations

By definition, the rate of refund of energy G is defined by the opposite of the derivative of the potential energy in balance compared to the field Ω [bib1]. It is calculated by the method theta, which is a Lagrangian method of derivation of the potential energy [bib4] and [bib2]. Transformations are considered $F^{\eta}: \mathbf{M} \rightarrow \mathbf{M} + \eta \boldsymbol{\theta}(\mathbf{M})$ field in Ω_0 a field Ω_{η} who correspond to propagations of the crack. With these families of configuration of reference thus defined Ω_{η} correspond of the families of deformed configurations where the crack was propagated. The rate of refund of energy G is then the opposite of the derivative of the potential energy $W(\mathbf{u}(\eta))$ with balance compared to the initial evolution of the bottom of crack: η

$$G = -\left(\frac{\mathrm{d}\, W(\mathbf{u}(\eta))}{\mathrm{d}\eta}\right)_{\eta=0}$$

One notes as in [feeding-bottle 4] par. Lagrangian derivation in a virtual propagation of crack speed $\boldsymbol{\theta}$. That is to say $\boldsymbol{\varphi}(\eta, \mathbf{M})$ an unspecified field (η positive reality and $\mathbf{M}$ belonging to the field Ω_0), we will note:

$$\bar{\boldsymbol{\varphi}}(\eta, \mathbf{M}) = \bar{\boldsymbol{\varphi}}(\eta, F^{\eta} \mathbf{M}) \,\mathrm{et}\, \dot{\boldsymbol{\varphi}} = \left(\frac{\partial \bar{\boldsymbol{\varphi}}}{\partial \eta}\right)_{\eta=0}$$

Potential energy definite on Ω_η is brought back on Ω_0, $\mathbf{R}$ is supposed to be independent of η, derivation compared to the parameter of propagation η is then easy and the rate of refund of energy in this propagation is solution of the variational equation:

$$\int_{\Gamma_o} G\,\boldsymbol{\theta}\cdot\mathbf{m}\,\mathrm{dS}=G(\boldsymbol{\theta}) \qquad ,\forall\,\boldsymbol{\theta}\in\boldsymbol{\Theta}$$

with:

$$-G(\boldsymbol{\theta})=\int_{\Omega_o}\left(\overbrace{\boldsymbol{\Psi}(\mathbf{E},T)}^{\cdot}\right)+\boldsymbol{\Psi}(\mathbf{E},T)\,\boldsymbol{\theta}_{\mathbf{k,k}}\,\mathrm{d}\,\Omega-\int_\Gamma \mathbf{R_i}\,\dot{\mathbf{u}}_\mathbf{i}+\mathbf{R_{i,k}}\,\boldsymbol{\theta}_\mathbf{k}\,\mathbf{u_i}+\mathbf{R_i}\,\mathbf{u_i}\left(\boldsymbol{\theta}_{\mathbf{k,k}}-\frac{\partial\boldsymbol{\theta}}{\partial\mathbf{n_k}}\mathbf{n_k}\right)\mathrm{d}\,\Gamma$$

However:

$$\left(\overbrace{\boldsymbol{\Psi}(\mathbf{E},T)}^{\cdot}\right)=\frac{\partial\boldsymbol{\Psi}}{\partial\mathbf{E}_{ij}}\dot{\mathbf{E}}_{ij}+\frac{\partial\boldsymbol{\Psi}}{\partial T}\dot{T}$$

Thereafter, we will consider only the term $\frac{\partial\boldsymbol{\Psi}}{\partial\mathbf{E}_{ij}}\dot{\mathbf{E}}_{ij}$, the thermal term being treated in the same way that into small displacement [R7.02.01].

And according to proposal 2 of [bib4]:

$$\begin{aligned}\dot{\mathbf{E}}_{ij}=&\frac{1}{2}\left(\dot{\mathbf{u}}_{\mathbf{i,j}}+\dot{\mathbf{u}}_{\mathbf{j,i}}+\dot{\mathbf{u}}_{\mathbf{k,i}}\,\mathbf{u}_{\mathbf{k,j}}+\mathbf{u}_{\mathbf{k,i}}\,\dot{\mathbf{u}}_{\mathbf{k,j}}\right)\\&-\frac{1}{2}\left(\mathbf{u}_{\mathbf{i,p}}\,\mathbf{q}_{\mathbf{p,j}}+\mathbf{u}_{\mathbf{j,p}}\,\mathbf{q}_{\mathbf{p,i}}+\mathbf{u}_{\mathbf{k,p}}\,\mathbf{q}_{\mathbf{p,i}}\,\mathbf{u}_{\mathbf{k,j}}+\mathbf{u}_{\mathbf{k,i}}\,\mathbf{u}_{\mathbf{k,p}}\,\mathbf{q}_{\mathbf{p,j}}\right)\end{aligned}$$

One can eliminate $\dot{\mathbf{u}}$ expression of G as in small deformations by noticing that $\dot{u}$ is kinematically acceptable (cf [bib3] for the problems of regularity) and by using the equilibrium equation:

$$\begin{aligned}&\int_{\Omega_o}\left(\overbrace{\boldsymbol{\Psi}(\mathbf{E})}^{\cdot}\right)\mathrm{d}\,\Omega-\int_\Gamma \mathbf{R_i}\,\dot{\mathbf{u}}_\mathbf{i}\,\mathrm{d}\,\Gamma=\\&\int_{\Omega_o}\frac{\partial\boldsymbol{\Psi}}{\partial\mathbf{E}_{ij}}\frac{1}{2}\left(\dot{\mathbf{u}}_{i,j}+\dot{\mathbf{u}}_{j,i}+\dot{\mathbf{u}}_{k,i}\,\mathbf{u}_{\mathbf{k,j}}+\mathbf{u}_{\mathbf{k,i}}\,\dot{\mathbf{u}}_{k,j}\right)d\,\Omega\\&-\int_\Gamma \mathbf{R_i}\,\dot{\mathbf{u}}_i\,d\,\Gamma-\int_{\Omega_o}\frac{\partial\boldsymbol{\Psi}}{\partial\mathbf{E}_{ij}}\frac{1}{2}\left(\mathbf{u}_{\mathbf{i,p}}\boldsymbol{\theta}_{\mathbf{p,j}}+\mathbf{u}_{\mathbf{j,p}}\boldsymbol{\theta}_{\mathbf{p,i}}+\mathbf{u}_{\mathbf{k,p}}\boldsymbol{\theta}_{\mathbf{p,i}}\mathbf{u}_{\mathbf{k,j}}+\mathbf{u}_{\mathbf{k,i}}\mathbf{u}_{\mathbf{k,p}}\boldsymbol{\theta}_{\mathbf{p,j}}\right)d\,\Omega\\&=-\int_{\Omega_o}\mathbf{S}_{ij}\left(\mathbf{u}_{\mathbf{i,p}}\boldsymbol{\theta}_{\mathbf{p,j}}+\mathbf{u}_{\mathbf{k,i}}\mathbf{u}_{\mathbf{k,p}}\boldsymbol{\theta}_{\mathbf{p,j}}\right)d\,\Omega\\&=-\int_{\Omega_o}\mathbf{S}_{ij}\left(\mathbf{d}_{ki}+\mathbf{u}_{\mathbf{k,i}}\right)\mathbf{u}_{\mathbf{k,p}}\boldsymbol{\theta}_{\mathbf{p,j}}\,d\,\Omega\\&=-\int_{\Omega_o}\mathbf{S}_{ij}\mathbf{F}_{ki}\mathbf{u}_{\mathbf{k,p}}\mathbf{q}_{\mathbf{p,j}}\,d\,\Omega\\&=-\int_{\Omega_o}\mathbf{F}_{ik}\mathbf{S}_{kj}\mathbf{u}_{\mathbf{i,p}}\mathbf{q}_{\mathbf{p,j}}\,d\,\Omega\end{aligned}$$

Finally, one obtains:

$$G(\boldsymbol{\theta})=\int_{\Omega_0} \mathbf{F}_{ik}\mathbf{S}_{kj}(\mathbf{u}_{i,p}\boldsymbol{\theta}_{p,j})-\boldsymbol{\Psi}(\mathbf{E})\boldsymbol{\theta}_{k,k}\,d\Omega+\int_{\Gamma} \mathbf{R}_{i,k}\boldsymbol{\theta}_k\,\mathbf{u}_i+\mathbf{R}_i\,\mathbf{u}_i(\boldsymbol{\theta}_{k,k}-\frac{\partial\boldsymbol{\theta}}{\partial\mathbf{n}_k}\mathbf{n}_k)\,d\Gamma$$

The expression supplements for the following loadings:

- nonfollowing surface density $\mathbf{R}$ applied to a part Γ edge of Ω_0,
- nonfollowing voluminal density $\mathbf{f}$ applied to the field Ω,

and by taking account of thermics:

$$\begin{aligned} G(\boldsymbol{\theta})= & \int_{\Omega_0} \mathbf{F}_{ik}\mathbf{S}_{kj}(\mathbf{u}_{i,p}\boldsymbol{\theta}_{p,j})-\boldsymbol{\Psi}(\mathbf{E})\boldsymbol{\theta}_{k,k}-\frac{\partial\boldsymbol{\Psi}}{\partial\mathbf{T}}\mathbf{T}_{,k}\boldsymbol{\theta}_k\,d\Omega \\ & +\int_{\Omega_0} \mathbf{f}_i\,\mathbf{u}_i\boldsymbol{\theta}_{k,k}+\mathbf{f}_{i,k}\boldsymbol{\theta}_k\,\mathbf{u}_i\,d\Omega \\ & +\int_{\Gamma} \mathbf{R}_{i,k}\boldsymbol{\theta}_k\,\mathbf{u}_i+\mathbf{R}_i\,\mathbf{u}_i(\boldsymbol{\theta}_{k,k}-\frac{\partial\boldsymbol{\theta}}{\partial\mathbf{n}_k}\mathbf{n}_k)\,d\Gamma \end{aligned}$$

2.4 Establishment in *Code_Aster*

The comparison of the formulas of $G(\boldsymbol{\theta})$ [§1.3] and [§1.4] watch that terms of $G(\boldsymbol{\theta})$ are very close. The introduction of the great transformations requires little modification in postprocessing.

The presence of the keyword `DEFORMATION=' GROT_GDEP'` under the keyword factor `BEHAVIOR` order `CALC_G` indicate that it is necessary to recover the tensor of the constraints of Piola-Lagrange $\boldsymbol{S}$ and the gradient of the transformation $\boldsymbol{F}$ (routines `NMGEOM` and `NMELNL`).

The types of finite elements are the same ones as in linear elasticity [R7.02.01 §2.4]. They are the isoparametric elements 2D and 3D.

The supported loadings are those supported in linear elasticity provided that they are dead loads: typically an imposed force is a dead load while the pressure is a following loading since it depends on the orientation of surface, therefore of the transformation.

2.5 Restriction

With the relation of behavior specified with the §2, there is a formulation of G valid for great deformations for materials very-rubber bands, but… if one wishes a coherence with the actual material which, let us recall it, is elastoplastic, it is imperative to confine itself with small deformations, displacements and rotations being able to be large.

The conditions of loadings proportional and monotonous, essential to ensure the coherence of the model with actual material, lead to important restrictions of the field of with the capable problems being dealt by this method (thermics in particular can lead to local discharges). It cannot thus be a question that of a palliative solution before being able to give a direction to the rate of refund of energy within the framework of plastic behaviors.

3 Bibliography

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4 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
4	E.VISSE EDF-R&D/MMN	Initial text
10,1,1	J.M.Proix, R.Bargellini EDF/R & D /AMA	Change of the keyword GREEN in GROT_GDEP