Models of Bordet and Rice and Tracey

Summary

Lbe bases ofS models of local approach of the rupture who allowing to model brittle fracture (model of Bordet) and ductile starting (model of Rice and Tracey) are recalled.

In the case DU brittle model of fracture (Bordet), one describes how is calculated the probability of rupture of a structure starting from knowledge of the mechanical fields requesting it. While placing oneself in the case general of a nonmonotonous way of thermomechanical loading and by supposing that certain parameters of the models do not depend on the temperature, one establishes the general expression of the cumulated probabilities of rupture.

Then, one presents the model leading to the law of growth of the cavities of Rice and Tracey as well as the ductile criterion of starting being reported to it. Lastly, indications concerning the implementation of these models in *Code_hasster* are summarized.

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1 Introduction

One is interested here in a metal structure requested thermomécaniquement. One seeks to determine criteria of rupture of this structure, representative of the two mechanisms met on certain steels:

- at low temperature, certain metallic materials (as the steel of tank) can behave like fragile materials while breaking brutally by cleavage,
- at higher temperature, appears the ductile tear.

In opposition to the comprehensive approach, itS models of Bordet and Rice-Tracey presented here are based on the knowledge of the mechanical fields in the zones most requested to obtain a local criterion of rupture representative of the concerned physical mechanisms (instability of the microscopic cracks of cleavage or increase then coalescence in porosity).
The bases of model of Beremin which also makes it possible to model brittle fracture are given in documentation [R7.02.04].
Advices of use of the models Beremin and Bordet are given in documentation [U2.05.08].

2 modèle of Bordet

One presents here quickly the model of Bordet. For more details, one will be able to refer to [3], [4] or [5].
The model of Bordet is based on the same bases as that of Beremin. It defines a probability of rupture per local cleavage. However, in the model of Beremin, one supposes the creation of microscopic cracks at the time of the attack of the threshold of plasticity, and these microscopic cracks remain potentially active throughout the loading which is followed from there. However, in steels, the total rupture is mainly related to microscopic cracks lately created. It is thus advisable to take into account the level of plastic deformation reaches at every moment. This is already possible in the model of Beremin *via* the option of plastic correction defined in Doc. [U4.81.22].
In the model of Bordet, this is taken into account by considering that the probability of rupture per cleavage is the product of the probability of nucleation and propagation at the same moment.

2.1 Probability of local rupture per cleavage of the model of BordeT

The local criterion is defined here like the event statistics simultaneously to meet the conditions of nucleation and propagation of the microscopic cracks. In the model, the nucleation and the propagation of the microscopic cracks are regarded as events independent; nucleation indicates the rupture of a carbide leading to the formation of a microscopic crack, whereas the propagation is defined like the local instability of cleavage only guided by the local constraints.
The probability of local rupture per cleavage is written then:

$$P_{cliv}=P_{nucl}P_{propa} \quad (1)$$

ON understands by plastic deformation ε_p equivalent plastic deformation defined by:

$$\varepsilon_p=\sqrt{\frac{2}{3}\boldsymbol{\varepsilon}_p:\boldsymbol{\varepsilon}_p} \quad (2)$$

This definition is valid only for the elastoplastic laws of behavior with criterion of von Mises. In *Code_hasster*, only the laws of behavior of this type are thus adapted to this model.

2.1.1 Local probability of nucleation

With the definition given higher, the probability of nucleation of a microscopic crack during an increment of plastic deformation $d\varepsilon_p$ is proportional to the yield stress $\sigma_{ys}(T,\dot{\varepsilon}_p)$ at the temperature T and the speed of plastic deformation $\dot{\varepsilon}_p$:

$$P_{nucl}\propto N_{unc}(\varepsilon_p)\sigma_{ys}(T,\dot{\varepsilon}_p)d\varepsilon_p \quad (3)$$

This remains true as long as the number of microcracked carbides remains weak in front of the number of healthy carbides $N_{unc}(\varepsilon_p)$. The rate of microfissuring of carbides is constant for a given plastic deformation; the number of healthy sites varies exponentially with ε_p . While calling $\sigma_{ys,0}$ yield stress at a temperature and a speed of plastic deformation of reference and $\varepsilon_{p,0}$ a plastic deformation of reference, probability P_{nucl} can express itself as follows:

$$P_{nucl}\propto\frac{\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}}\exp\left(\frac{-\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}}\frac{\varepsilon_p}{\varepsilon_{p,0}}\right) \quad (4)$$

If the plastic deformation is small, therefore if the carbide cracking is limited enough, i.e. $\frac{\sigma_{ys,0}\varepsilon_{p,0}}{\sigma_{ys}(T,\dot{\varepsilon}_p)}\gg\varepsilon_p$, then the probability of nucleation is reduced to $P_{nucl}\propto\frac{\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}}$.

2.1.2 Local probability of propagation

As in the model of Beremin [R4.02.04] , the density of size of the carbide microscopic cracks is supposed to be distributed according to a law power reverses (with α and β parameters material independent of the temperature and speeds of deformation and l size of the microscopic crack):

$$f(l)=\frac{\alpha}{l^{\beta}} \quad (5)$$

The cracking of a carbide will not be propagated in ferrite (because of inertia alone of propagation, by dynamic effect) that in the condition of the presence of a sufficiently high local constraint. A higher limit of ferritic size of microscopic crack $l_{max}=l(\sigma_{th})$ nucleate is thus introduced; it makes it possible to define the minimum of local constraint necessary to the propagation of the largest possible ferritic microscopic crack. The probability of propagation is thus written:

$$P_{propa}(\sigma_I)=\int_{l_c(\sigma_I)}^{l_{max}} f(l)\,dl \quad (6)$$

With vec σ_I the maximum principal constraint, σ_{th} the threshold in lower part of which the propagation cannot take place and l_c ferritic critical size of microscopic crack, obeying the relation of Griffith for one elliptic microscopic crack:

$$l_c(\sigma)=\frac{\pi E \gamma_p}{2(1-\nu^2)\sigma^2} \quad (7)$$

Withvec E the Young modulus, ν the Poisson's ratio and γ_p surface density of energy. One obtains finally:

$$\begin{cases} P_{propa}(\sigma_I) = 0 & \sigma_I<\sigma_{th} \\ P_{propa}(\sigma_I) = \left(\left(\frac{\sigma_I}{\sigma_u}\right)^m-\left(\frac{\sigma_{th}}{\sigma_u}\right)^m\right) & \sigma_I\geqslant\sigma_{th} \end{cases} \quad (8)$$

With m parameter independent of the temperature, following the example of α and β , and σ_u who can depend on it (if the Young modulus depends on it).
As for the model of Beremin, the effects of orientation of the microscopic cracks compared to the direction of the maximum principal constraint are taken into account only *via* the parameter σ_u .

2.1.3 Local probability of cleavage

As specified previously, the local probability of cleavage is supposed of being the product of the probability of nucleation and the probability of propagation; for that one considers that during an increment of plastic deformation infinitesimal, the active constraint is constant. From where, if $\sigma_I\geqslant\sigma_{th}$:

$$\begin{aligned} P_{cliv}(\sigma_I, d\varepsilon_p) &\propto P_{nucl}(d\varepsilon_p).P_{propa}(\sigma_I) \\ &\propto \frac{\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}}\exp\left(\frac{-\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}}\frac{\varepsilon_p}{\varepsilon_{p,0}}\right)\left(\left(\frac{\sigma_I(\varepsilon_p)}{\sigma_u}\right)^m-\left(\frac{\sigma_{th}}{\sigma_u}\right)^m\right)d\varepsilon_p \end{aligned} \quad (9)$$

This equation does not state that the microscopic cracks nucleate remain active during the increment of plastic deformation, but although the condition of propagation for each one of these microscopic cracks is determined by the value of the local stress field at the time of creation.
The probability that a ferritic microscopic crack is created and is propagated on an interval of plastic deformation $[0,\varepsilon_{p,u}]$ is then:

$$P_{cliv}(\sigma_I,\varepsilon_{p,u})\propto\int_0^{\varepsilon_{p,u}}\frac{\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}}\exp\left(\frac{-\sigma_{ys}(T,\dot{\varepsilon}_p)}{\sigma_{ys,0}}\frac{\varepsilon_p}{\varepsilon_{p,0}}\right)\left(\left(\frac{\sigma_I(\varepsilon_p)}{\sigma_u}\right)^m-\left(\frac{\sigma_{th}}{\sigma_u}\right)^m\right)d\varepsilon_p \quad (10)$$

This probability is reduced well to 0 if the constraint remains lower than σ_{th} during all the way of loading.

2.2 Total probability of rupture per cleavage of Bordet

The preceding paragraph made it possible to determine the local probability of rupture per cleavage. While following the principle of weak link, the total probability of rupture per cleavage on n_c potential sites of initiation is written:

$$P_{Bordet}=1-\exp\left(-\sum_{i=1}^{n_c} P_{cliv}\left(\sigma_{I,i},\varepsilon_{p,u,i}\right)\right) \quad (11)$$

This equation can be expressed according to the volume of the process zones by introducing an infinitesimal volume dV on which deformations and constraints are constant (in the digital case, the point of Gauss). In order to simply compare the probabilities of Beremin and Bordet on an example given, one can define a constraint of Bordet of the same type as that of Weibull:

$$\sigma_{Bordet}=\left\{\int_{V_p}\left(\int_0^{\varepsilon_{p,u}}\frac{\sigma_{ys}\left(T,\dot{\varepsilon}_p\right)}{\sigma_{ys,0}}\left(\left(\sigma_I\left(\varepsilon_p,dV\right)\right)^m-\left(\sigma_{th}\right)^m\right)\exp\left(\frac{-\sigma_{ys}\left(T,\dot{\varepsilon}_p\right)}{\sigma_{ys,0}}\frac{\varepsilon_p}{\varepsilon_{p,0}}\right)d\,\varepsilon_p\right)\frac{dv}{V_0}\right\}^{\frac{1}{m}} \quad (12)$$

and the probability of total rupture per cleavage is written with a distribution of Weibull:

$$P_{Bordet}=1-\exp\left(-\left(\frac{\sigma_{Bordet}}{\sigma_u}\right)^m\right) \quad (13)$$

If the number of microcracked carbides is weak in front of the number of healthy carbides, the term into exponential is close to 1 and the constraint of Bordet is some simplified.

2.3 Discussion

2.3.1 Bordet or Beremin?

The model of Bordet is slightly more complex and end that the model of Beremin.
One of its advantages is to consider the maximum principal constraint at every moment, and not the maximum principal constraint during the loading; consequently nothing prevents the probability of Bordet of decreasing, contrary to that of Beremin.

Moreover, the model of Bordet gives an account owing to the fact that an area with a lower constraint but a more important plastic deformation can be more critical than a zone in which the constraints are higher but the lower level of plastic deformation.

However, the model of Bordet requires the knowledge of additional parameters material, as one will describe it in the paragraph hereafter.

2.3.2 Parameters material

The model of usual Beremin requires the knowledge of three parameters materials: two parameters of form of the law of Weibull, m and σ_u , as well as the ground volume of the plastic zone V_0 ; only the parameter σ_u depends on the temperature. With these three parameters, it is possible to add the plastic deformation threshold making it possible to define the plastic zone on which integration is carried out.

The first three parameters are formally preserved by the model of Bordet.

Note: they are only ***formally*** preserved, they can be different and require a calibration of the same type as that done for Beremin and presented in [R7.02.09].

Other parameters are added to it.

The threshold of plasticity $\sigma_{ys}(T, \dot{\varepsilon}_p)$ is one function *has minimum* temperature and potentially of the speed of plastic deformation and its value of reference $\sigma_{ys,0}$.
The critical stress below which the propagation of the ferritic microscopic cracks cannot take place, σ_{th} , independent of the conditions of loading.
In the full version only (with the exponential term taken into account), a plastic deformation of reference $\varepsilon_{p,0}$ the method of identification is not given and who seems rather delicate. It will be noted that the author him even (cf [8], [9]) seems to use for certain studies and validation the version of the model in which this parameter does not intervene (for the user of Code_hasster, it is enough to specify `PROBA_NUCL=' NON'`)

2.4 Setting in work in *Code_hasster*

The probability and constraint the calculus of Bordet is carried out by the operator `POST_BORDET`. It Nécessite to have carried out an elastoplastic thermomechanical calculation *via* the operator `STAT_NON_LINE`. Advices of use of this model are given in documentation [U2.05.08].
In the currently established version, the temperature of the medium on which calculation is carried out must be uniform (but can evolve in the course of time); this limitation does not seem crippling insofar as the plastic zone at a peak of crack is in general rather small.
It is possible to carry out calculation with or without the term into exponential; if this term is required, the parameter material $\varepsilon_{p,0}$ must be well informed.
In all the cases, one at every moment calculates the sizes such as the maximum principal constraint and the equivalent plastic deformation. The value of the parameters is determined $\sigma_u(T)$ and $\sigma_{ys}(T, \dot{\varepsilon}_p)$ according to the data material provided by the user, then one calculates the total constraint of Bordet by summation on the points of gauss of the elements of the group of meshs indicated by the user, then finally the total probability of rupture of Bordet over every moment until that requested. Calculation carried out is written with final as follows:

$$\sigma_{Bordet} = \left\{ \sum_{o=1}^{n_{inst}} \sum_{el=1}^{n_{elem}} \sum_{pg=1}^{n_{pg}} \left(\begin{array}{c} \dfrac{\sigma_{ys}\left(T(o), \dot{\bar{\varepsilon}}_{p,el,pg}(o)\right)}{\sigma_{ys,0}} \\ Max\left(\bar{\sigma}_{1,el,pg}^{m}(o) - \sigma_{th}^{m}, 0\right) \\ \exp\left(\dfrac{-\sigma_{ys}\left(T(o), \dot{\bar{\varepsilon}}_{p,el,pg}(o)\right)}{\sigma_{ys,0}} \dfrac{\bar{\varepsilon}_{p,el,pg}(o)}{\varepsilon_{p,0}} \right) \\ \Delta\varepsilon_{p,el,pg}(o) \dfrac{V_{el,pg}}{V_0} \end{array} \right) \right\}^{1/m} \quad (14)$$

Withvec n_{inst} the number of moments over which is calculated, n_{elem} the number of elements contained in the group of meshs asked by the user and n_{pg} the number of points of gauss of each one of these elements. DE more, for any scalar a , $\bar{a}(o) = \dfrac{a(o) + a(o-1)}{2}$, $\dot{a}(o) = \dfrac{a(o) - a(o-1)}{t(o) - t(o-1)}$ and $\Delta a(o) = a(o) - a(o-1)$.

The result is a table containing the global values of the constraint of Bordet and the probability of rupture of Bordet.

So that calculation is correct, it is necessary that the user informs the keyword `COEF_MULT` as recommended in the user's documentation of `POST_BORDET`.

3 modèle of Rice and Tracey

One is interested now in the case of ductile starting. By considering an element of initially healthy volume, ductile this element tears it results from the following elementary mechanisms:

- nucleation of cavities caused by the decoherence of inclusions present in material,
- growth then coalescence of these cavities.

3.1 Cavity isolated in an infinite plastic rigid matrix

In an analytical approach of comprehension, Rice and Tracey studied the behavior of a cavity, initially spherical (surface S_v), isolated in an infinite isotropic medium (volume V), of behavior of vone Settings rigid elastic plastic (limit σ_0), incompressible, subjected ad infinitum at a speed of deformation $\dot{\varepsilon}^{\infty}$ unspecified (noted constraint σ^{∞} ad infinitum). They show that the field rate of travel, solution of the posed mechanical problem, minimizes the functional calculus:

$$Q(\dot{u})=\int_V \left[s_{ij}(\dot{\varepsilon})-s_{ij}^{\infty}\right]\dot{\varepsilon}_{ij}\,dV-\sigma_{ij}^{\infty}\int_{S_v} n_i\dot{u}_j\,dS \quad (15)$$

3.2 Approximate law of the growth of the cavities

While managing to minimize this functional calculus in various situations, Rice and Tracey then showed the dominating influence of the rate of triaxiality $\frac{\sigma_m^{\infty}}{\sigma_{eq}^{\infty}}$ (with σ_m^{∞} trace and σ_{eq}^{∞} equivalent of von Mises of the constraint imposed on the element of volume considered) on the growth rate of the cavities.

They display even a law of growth of the cavities, certainly approached, but very near to the results of the preceding model. Thus, in each principal direction (K) associated at the speed of deformation $\dot{\varepsilon}^{\infty}$, the rate of elongation of a cavity rises with:

$$\dot{R}_K=\left\{\chi\,\dot{\varepsilon}_K^{\infty}+\dot{\varepsilon}_{eq}^{\infty}D\right\}R_K \quad (16)$$

For a principal direction K, the parameter R_K is ray of the cavity, $\dot{\varepsilon}_K^{\infty}$ principal value the speed of ad infinitum imposed deformation and $\dot{\varepsilon}_{eq}^{\infty}$ the value equivalentE of von Mises speed of ad infinitum imposed deformation. This relation in which coefficients χ and D depend on the situation considered:

- $\chi=\frac{5}{3}$ for a linear matrix of work hardening or a perfectly plastic matrix with low level of triaxiality or $\chi=2$ in the case of a perfectly plastic matrix atstrong rate of triaxiality,
- $D=\alpha\exp\left(\frac{3\sigma_m^{\infty}}{2\sigma_0}\right)$ for a perfectly plastic matrix or $D=\frac{\sigma_m^{\infty}}{4\sigma_{eq}^{\infty}}$ for a linear matrix of work hardening.

  $\alpha=0,283$ is the value given by Rice and Tracey whereas more precise calculations (cf [bib4]) showed that this coefficient is higher ($\alpha=1,28$).

Mudry then proposed to apply these theoretical results to the case of the steel of tank, *i.e.* :

- intermediate behavior enters the extreme cases of behavior studied by Rice and Tracey with a reasonable work hardening not no one but,
- fissured structures (high rate of triaxiality).

He deduced from it the approximate law following, valid for sufficiently high rates of triaxiality (superiors with 0.5):

$$\dot{R} = \alpha\, \dot{\varepsilon}_{eq}^{p^\infty} \exp\left(\frac{3\sigma_m^\infty}{2\sigma_{eq}^\infty}\right) R \quad (17)$$

Dyears which:

- $\dot{\varepsilon}_{eq}^{\infty}$ was substituted by $\dot{\varepsilon}_{eq}^{p^\infty}$ (equivalent (von Mises) of the plastic part the speed of deformation) in order to extend the law of Rice and Tracey to the elastoplastic case,
- elastic limit σ_0 was substituted by σ_{eq}^{∞} in order to take account of the hardening of the matrix around the cavity.

Experimental measurements of growth of porosity for various rates of triaxiality made it possible to validate this expression. These results show that, when the initial proportion of air voids weak rest, the exponential character of the relation between the ray of the cavities and the rate of triaxiality is well confirmed. On the other hand, the coefficient α depends on material considered thus that initial fraction of porosity.

3.3 Ductile criterion of starting

R_0 and $R(t)$ indicating the initial ray of the cavities and with the moment t considered, the ductile criterion of starting adopted here is:

$$\frac{R(t)}{R_0} = \left(\frac{R}{R_0}\right)_c \quad (18)$$

LE first member of this expression result from the integration of the law of growth, in accordance with the indications of the preceding paragraph.

One can object several arguments of principle against the direct use of this law of growth of the cavities of Rice and Tracey like ductile criterion of starting. As follows:

- inclusions, and thus the cavities, are not actually insulated. Worse, they are often gathered in cluster,
- the coalescence of the cavities undoubtedly results from interactions which are also not described in the established model,
- in a fissured structure, the presence of gradients in bottom of crack makes less directly applicable the preceding analysis which relates to an infinite medium subjected to boundary conditions homogeneous.

Nevertheless, by using the preceding criterion, one hopes that this law remains realistic, on average, even in clusters or zones of strong gradients (average on an element of size comparable to that of the model of Beremin). In addition, one makes the assumption that the critical size retained, in general readjusted on geometries given (test-tube CT, for example), represented coalescence, which amounts supposing that coalescence does not depend too much on the nature of the mechanical requests imposed on the element of volume (triaxiality, shearing,…).

Let us notice to finish that the model of Rice and Tracey is only one approached law, valid for important rates of triaxiality (*i.e.* superiors with 0.5).

3.4 Establishment in *Code_hasster*

Let us consider a field Ω_c studied structure which can be the whole of the studied grid, a group of meshS or a mesh. Following an elastoplastic thermomechanical calculation, one knows the evolution of the stress fields, of deformation and plastic deformation in this field and one wishes to determine the space and temporal variations growth rate of the cavities in this field.

With this intention, the keyword is used `RICE_TRACEY` order `POST_ELEM`.

In each point of Gauss of the field Ω_c , one compares the constraints and at every moment calculated speeds of deformation to the quantities applied to the infinite medium considered previously. The law of growth of Rice and Tracey is thus integrated step by step using the following approximate formula:

$$Log\left(\frac{R(t_n)}{R_0}\right)=Log\left(\frac{R(t_{n-1})}{R_0}\right)+0,283\, signe\left(\frac{\sigma_m(t_n)}{\sigma_{eq}(t_n)}\right)Exp\left(1,5.\left|\frac{\sigma_m(t_n)}{\sigma_{eq}(t_n)}\right|\right)\left(\varepsilon_{eq}^p(t_n)-\varepsilon_{eq}^p(t_{n-1})\right) \quad (19)$$

The values of the report are thus at every moment obtained $\frac{R}{R_0}$ in each point of Gauss of the field Ω_c , the sign of the rate of triaxiality allowing the taking into account of evolutions as well in traction as in compression. Two features are then offered in *Code_hasster* : the maximum value and the median value of growth rate.

3.4.1 Research of the maximum value of growth rate

At every moment, one seeks on the whole of the field Ω_c the point of Gauss (and the volume of the associated under-mesh) maximizing $\frac{R}{R_0}$.

3.4.2 Calculation of the median value of growth rate

By squaring on each mesh then moyennation on the field Ω_c aimed, one at every moment deduces the median value from $\frac{R}{R_0}$ on Ω_c .

As in the case of the model of Weibull, an alternative is introduced: preceding temporal integration is then carried out starting from the constraint and of the average plastic deformation by mesh.

4 Bibliography

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