Gp method: hasclose energetics of the prediction of cleavage

Summary

The parameter G_p allows to define a criterion of starting validates in the field of cleavage (fracture brittle intragranular in the presence of plasticity). The bases of this approach are first of all pointed out: tally energy behavior of the structures, modeling of the crack by a notch, principle of minimization of energy and definition of the criterion. One discusses the link between the comprehensive approach and the local approach before specifying then essential components with the implementation of G_p in *Code_Aster*.

Contents

1 Introduction

In this chapter one starts by introducing the limits of the classical approach in breaking process (§1.1), then the objective and the interest of the approach are pointed out G_p (§1.2). One approaches finally the bases of this method with (§1.4).

1.1 Limits of the classical approach

In breaking process, the classical parameter allowing to define a criterion of starting of an already existing crack[1] is the parameter G, rate of elastic refund of energy. The parameter G is valid in linear elasticity like in non-linear elasticity. I.e. the physical framework and assumptions are well defined, and that the formula of computation $G\theta$ is in coherence with this framework. On the other hand, relevance of the criterion of rupture $G=Gc$ with the experimental results is another question, not tackled in this document. In elastoplasticity, the parameter G is in general not valid. However, if the loading remains radial and monotonous, then G is valid. This assumption implies that the equations of plasticity are equivalent to the equations of Hencky (which are those of non-linear elasticity [**R4.20.01**]). However, the cases of loadings strictly proportional are rather rare (even do not exist when they are structures presenting of the geometrical defects such as cracks).

1.2 Objective and interest of Gp

The objective of the method G_p is to define a valid parameter of starting in incremental plasticity. This theory supposes a decoupling of plasticity and cracking (case of the rupture by cleavage). Thus the use of G_p to predict cleavage will allow to reduce conservatisms of the classical approach. Sdefinition has called on the free energy like with some geometrical parameters necessary to the definition of the crack, which is modelled by a notch. This parameter G_p has the unit of a density of energy surface and is calculated in the adjacent zone with the face of notch.

The interest of the method is triple: G_p is a deterministic criterion of starting, in other words it makes it possible to evaluate if there is starting or not crack. This parameter is coherent with G rubber band [BOG MANGANESE 13, LOR 14], and it is valid within an elastoplastic framework in loading nonproportional and discharge.

Note:

- G_p is not a criterion of stability or instability of the defect.
- G_p does not predict length of propagation.

1.3 History of the development of the Gp approach

The development of this approach, who find its origins at the beginning of the years 2000, is exit of work of Lorentz, Wadier and Debryune [LOR00]. The crack is then modelled by a cut of the plan but the idea to introduce a notch is already mentioned there in prospect and will quickly be adopted. Lmodeling of the crack by U hasnotch is not presented in 2003 (CR) in [BOG MANGANESE 03d] and in 2004 (articles) in [LOR 04] and [BOG MANGANESE 04].

The approach was presented many times at the time of international conferences by being initially of its interest for the tank in Dbe situations in load/discharge [BOG MANGANESE 00], [BOG MANGANESE 01a], [BOG MANGANESE 01b], then by showing interpretations of experimental results illustrating the effect “small defect” [BOG MANGANESE 03a] [BOG MANGANESE 03c] then the effect of hot preloading [BOG MANGANESE 03b] [BOG MANGANESE 05] [BOG MANGANESE 09].

1 The direction of the term *starting* is here different from the terminology used in fatigue, where *starting* mean creation of a crack starting from a healthy material. In rupture, one should rather say criterion of “ *restarting* ”, but one usually employs the term of*starting*.

A synthesis pointing out the bases of the Gp approach as well as the principal results of validation is carried out in [BOG MANGANESE 13]. It is to date the only article published in an international newspaper, dedicated to the Gp approach. So he looks like reference as for the Gp approach.

It should be noted that attempt S to predict the stop of crack thanks to an approach of the type Gp (G-delta) were realized in the past [BOG MANGANESE 07a], but this way is not continued any more at the present time .

Since the departure in inactivity of the main actor of method (Y. Wadier), a new generation of engineers of EDF R & D took again the torch and continuous to promote the Gp approach attached to the scientific community [GEN 16] [JUL 17] . Recently, the Gp approach was extended to the configurations for which the zone around the point of the defect is in compression [HAB 17] .

1.4 Bases of the Gp approach

Method G_p places itself in the context of a total energy formulation of brittle fracture. The approach consists in determining if a propagation of defect (preserving the geometry of the bottom of defect and with constant mechanical fields) makes it possible to decrease the total energy of the structure (principle of minimization). For that purpose, the way of cracking is supposed to be known *a priori* . Under this assumption, the energy framework proposed by Frankfurt and Marigo [FRA 93, FRA 98] can be put into practice to predict the brutal starting of cracks. However, this framework shows certain characteristics which limit of it range [LOR 08], in particular of the undesirable scale effects and the incompatibility of the theory with loadings of imposed the forces type. This is why two types of modifications were brought to solve these difficulties, while preserving the form of introduced energies. On the one hand, the crack is replaced by a notch whose ray becomes a parameter of the model. In addition, one examines whether the absence of propagation is a minimum (total) with fixed mechanical fields, therefore one carries out a total minimization but in only one direction. The energy nature of the formulation suggested authorizes the introduction of plastic mechanisms.

2 Modeling of the defects in the form of notch

One places itself in modeling 2D. Within the framework of the approach G_p , the real crack located in the field Ω is not modelled by a surface of discontinuity of the fields of displacements, but by a “notch” out of U *(H1)*, of thickness $D=2R$, with R the ray of the bottom of notch (see [Figure 2-a]). It is supposed that the notch can be propagated without changing form *(H2)* and without junction *(H3)*, in mode I. Thus, Lbe wayS of cracking are preset and prone to an evolution crack continues (i.e. pas de fissuresS “in dotted lines” along the way).

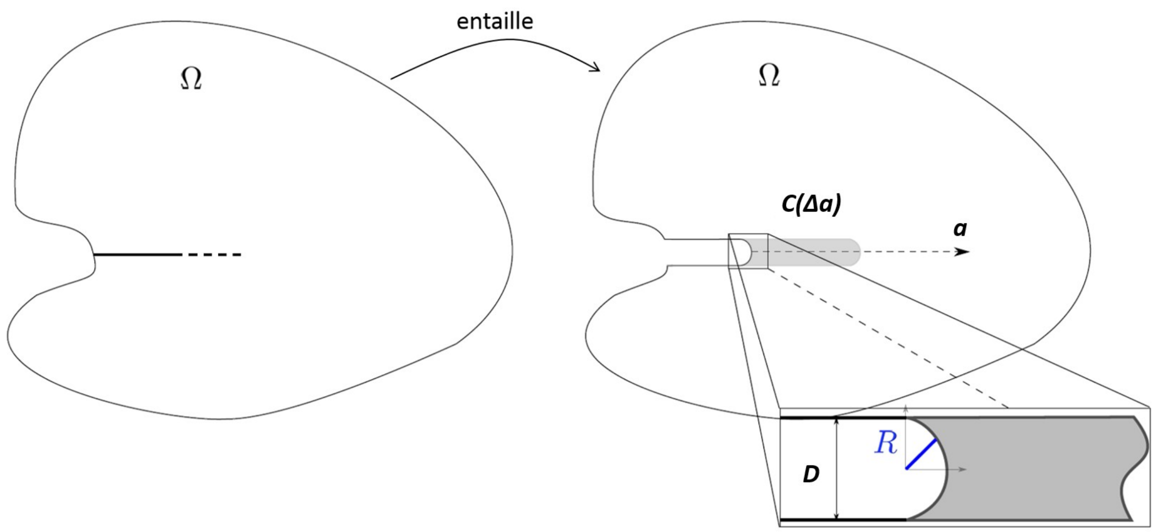

Figure 2-a – Modeling DU defect by a notch and representation of the zone of virtual propagation $C(\Delta a)$ in gray.

One then defines a zone of virtual propagation of the notch called $C(\Delta a)$, who corresponds to a zone *damaged*, of thickness D and length Δa. The zone $C(\Delta a)$ illustrated by [Figure 2-b] can be defined as the union of a rectangle R_1 and of a half-disc C_2, to which the half-disc is cut off C_1 :

$$C(\Delta a)=(R_1 \cup C_2)-C_1 \quad (1)$$

Lbe entities R_1, C_1 and C_2 are defined below, being given a notch along the axis X and coordinates (X_0, Y_0) center of the circular bottom of the notch [BAR 12]:

$$R_1=(X,Y) \text{ tels que } \begin{cases} 0 \leqslant X-X_0 \leqslant \Delta a \\ -R \leqslant Y-Y_0 \leqslant R \end{cases} \quad (2)$$

$$C_1=(X,Y) \text{ tels que } \begin{cases} (X-X_0)^2+(Y-Y_0)^2 \leqslant R^2 \\ X-X_0 \geqslant 0 \end{cases} \quad (3)$$

$$C_2=(X,Y) \text{ tels que } \begin{cases} (X-X_0-\Delta a)^2+(Y-Y_0)^2 \leqslant R^2 \\ X-X_0-\Delta a \geqslant 0 \end{cases} \quad (4)$$

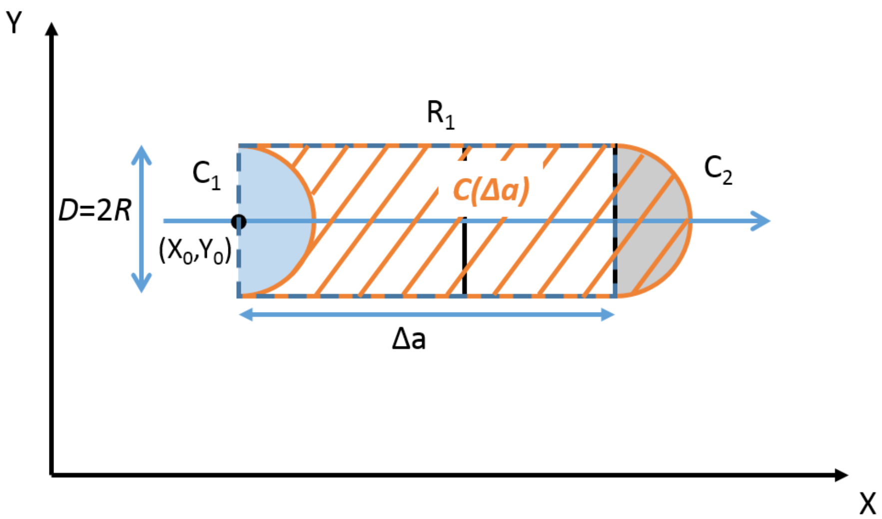

Figure 2-b – Definition geometrical of the notch: the zone $C(\Delta a)$ in hatched, the rectangle R_1 in dotted line, and half-discs C_1 and C_2.

Note: the integral on $C(\Delta a)$ of a constant w independent of space is written simply:

$$\int_{C(\Delta a)} w \, d\Omega = w \times \Delta a \times D \quad (5)$$

EN any rigour, we are not any more in the context of the breaking process but in that of the mechanics of the damage, by considering a model of brutal damage *(assumption H4)* total *(assumption H5)*. The brutal term, in opposition to progressive, means that only two states of damage are possible for material: healthy or damaged. The total term, in opposition to partial, indicates that the material damaged to a worthless residual rigidity.

A field of damage bivalué χ is thus defined on the field Ω, with value in $[0,1]$. By definition, $\chi=0$ corresponds to healthy material, and $\chi=1$ with damaged material. A model of damage makes it possible to fix a critical stress. For recall, one fixes the thickness of the zone on which the damage is propagated: one considers only the evolutions of the damage which correspond to the growth of the notch $C(\Delta a)$, where Δa is the length on which the notch is propagated, and D its thickness. One thus controls the energy dissipated at the time of the projection of the damaged zone. One notes $\Omega \setminus C(\Delta a)$ the unit made up of the difference between the field Ω and virtual propagation of the notch zones it $C(\Delta a)$. Then one a:

$$\chi(\Delta a)(\boldsymbol{x})=\begin{cases}1 & \text{si} \quad \boldsymbol{x}\in C(\Delta a)\\ 0 & \text{si} \quad \boldsymbol{x}\in\Omega\setminus C(\Delta a)\end{cases} \tag{6}$$

Note:
One could have distinguished the thickness from the preexistent notch, of that of the damaged zone $C(\Delta a)$. This would have led to a more complex model (with more parameters). By preoccupation with a simplicity one thus does not make this distinction.

3 Principle of minimization of energy

3.1 Energy formulation within the elastic framework

An energy formulation characterizes the answer of a structure as a a minimum of energy compared to the unit of the variables which one chose to describe the mechanical state of the structure [LOR 08]. In the elastic case, it is a question of minimizing the potential energy compared to the field of displacement. In order to take into account the dissipative phenomena, the approach G_p be based on work of Frankfurt and Marigo [FRA 93, FRA 98] dedicated to a model of brutal damage *partial*. Unlike the elastic case, energy depends on two fields, the field of displacement $\boldsymbol{u}$ and a field of damage χ with value in $[0,1]$, 0 corresponding to healthy material of voluminal free energy φ_s and 1 with damaged material of voluminal free energy φ_d :

$$E_{tot}(\boldsymbol{u},\chi)=\int_{\Omega} \varphi(\varepsilon(\boldsymbol{u}),\chi)+w_c\chi \quad ; \quad \varphi(\varepsilon(\boldsymbol{u}),\chi)=\chi\varphi_d(\varepsilon)+(1-\chi)\varphi_s(\varepsilon) \tag{7}$$

Où one restricts oneself with imposed displacements (external potential of the efforts W_{ext} is null). The additional term $w_c\chi$ measurement energy required to pass from the healthy state in a damaged state. After discretization in time, Frankfurt and Marigo apply whereas the fields of displacement $\boldsymbol{u}$ and of damage χ carry out a minimum of incremental potential energy:

$$(\boldsymbol{u},\chi)=\underset{\boldsymbol{u},\chi}{argmin}\, E_{tot}(\boldsymbol{u},\chi) \tag{8}$$

One considers thereafter the model of brutal damage presented by Frankfurt and Marigo, with this close residual rigidity is worthless for a state damaged (model of brutal damage *total*). Then, one retains theory of Frankfurt and Marigo that with displacement given, the evolution of the damage is controlled by the minimization of total energy compared to χ. In this case, a total damage or external forces is not problematic any more. Thus, Dyears the elastic framework, the evolution of the damage is obtained by the minimization of energy potential incremental total following:

$$E_{tot}(\boldsymbol{u},\chi)=\int_{\Omega}\left[(1-\chi)\Phi_{el}+\chi w_C\right]d\,\Omega-W_{ext}(\boldsymbol{u}) \tag{9}$$

where Φ_{el} is the density of free energy, $\Phi_{el}=\int_{\Omega}\frac{1}{2}[\sigma:A^{-1}:\sigma]d\Omega$ with A the tangent matrix of the behavior, and w_c L'voluminal energy dissipated in the process D' damage in each material point. In this new approach, one can take loadings of the type forces imposed: potential of the effort outsides W_{ext} is not necessarily null.

3.2 Formal amendment of energy

One of the limitations of the model of damage suggested is that it does not distinguish traction from compression, because of form of the density of free energy, so that the restitution of an energy of compression contributes as much to the propagation of the defect that an energy of traction [HAB 16]. One selected artificially to eliminate the compression zones from the integral *(assumption H6)*. For that we consider a dissymmetry traction/compression in the formulation of energy considered, while basing ourselves on the method developed by Badel in [BAD 01]. Thus, while placing oneself in the clean reference mark of the deformations, one will adopt following elastic energy:

$$\Phi_t^{el}(\boldsymbol{\varepsilon}^{el})=\frac{\lambda}{2}tr(\boldsymbol{\varepsilon}^{el})^2 H(tr(\boldsymbol{\varepsilon}^{el}))+\mu\sum_i(\varepsilon_i^{el})^2 H(\varepsilon_i^{el}) \quad (10)$$

Où λ and μ the coefficients of Lamé indicate which characterize the tensor of rigidity. Eigenvalues of the tensor of deformation rubber band are noted ε_i^{el}. H is the function of Heaviside such as:

$$H(x)=\begin{cases}1 & \text{si } x\geqslant 0\\ 0 & \text{si } x<0\end{cases} \quad (11)$$

3.3 Ext.ension with the elastoplastic framework

In the case of an elastoplastic behavior, the definition of energy incremental total E_{tot} is wide by Lorentz *et al.* [LOR 00], with the help of the definition of new total potentials free energy and dissipation . One restricts oneself here with the framework of isotropic materials . The state of a material point is described by its deformation $\boldsymbol{\varepsilon}$, its damage χ , but also by its plastic deformation $\boldsymbol{\varepsilon}^p$ and of the internal variables α characterizing work hardening. One supposE:

- that **plastic dissipation is uncoupled from that related to the damage** (quasi-fragile materials), *(assumption H7)* . Plastic potential of dissipation D_{pl} thus depends on only the variable plastic interns.
- that the energy blocked by work hardening in dislocations E_{bl} is not restorable by the mechanism of cracking *(phenomenologic assumption H8)* . One considers thus the following expression incremental potential energy :

$$E_{tot}(\boldsymbol{u},\boldsymbol{\varepsilon}^p,\alpha,\chi)=\int_{\Omega}\left[(1-\chi)\Phi_t^{el}+\chi w_C\right]d\Omega+E_{bl}(\alpha)+D_{pl}(\Delta\boldsymbol{\varepsilon}^p,\Delta\alpha)-W_{ext}(\boldsymbol{u}) \quad (12)$$

where Δ indicate the variation of a size during the increment considered.

4 Definition of a criterion of starting in elastoplasticity

Under the terms of the results relating to the energy formulations developed by Frankfurt and Marigo, the solution of the elastoplastic problem with damage minimizes the potential energy (12) *(assumption H9)* . The integration of the equations of behavior is written like the minimum of potential energy compared to the internal variables $(\boldsymbol{\varepsilon}^p,\alpha)$, the equilibrium equations are expressed like the minimum of potential energy compared to the field of displacement $\boldsymbol{u}$. Lastly, the evolution of the damage is controlled by the minimum of potential energy compared to χ .

The question of starting can be formulated in the following way [LOR 08] : in a given state $\left(\boldsymbol{u},\boldsymbol{\varepsilon}^p,\alpha\right)$ correspondent with a quasi-static evolution without propagation of the notch, the solution without propagation $\Delta a=0$ it is always licit in comparison with the minimization of (12) ? If so, then one applies that there is not starting *(assumption H10)*. In the contrary case, i.e. if the optimum is not reached in $\Delta a=0$, one applies thatthere is propagation.

One considers thus that**there is not starting of defect** as long as the elastoplastic solution without evolution of the damage is a solution of the problem, i.e.:

$$\begin{cases} (\boldsymbol{u},\boldsymbol{\varepsilon}^p,\alpha)=\underset{\boldsymbol{u},\boldsymbol{\varepsilon}^p,\alpha}{argmin}\,E_{tot}\left(\boldsymbol{u},\boldsymbol{\varepsilon}^p,\alpha,\chi=0\right) \\ \text{et} \\ \forall\,\Delta a>0 \quad E_{tot}(\boldsymbol{u},\boldsymbol{\varepsilon}^p,\alpha,\chi=0)\leqslant E_{tot}(\boldsymbol{u},\boldsymbol{\varepsilon}^p,\alpha,\chi(\Delta a)) \end{cases} \tag{13}$$

Thus, by expressing the second-row forward thanks to the equation (12), one obtains after simplification that $\forall\,\Delta a>0$, there is not starting of the defect as long as :

$$\int_{\Omega}\Phi_t^{el}(\varepsilon(\boldsymbol{u})-\boldsymbol{\varepsilon}^p)\,d\,\Omega\leqslant\int_{\Omega}\left[\left(1-\chi(\Delta a)\right)\Phi_t^{el}(\varepsilon(\boldsymbol{u})-\boldsymbol{\varepsilon}^p)+\chi(\Delta a)\,w_C\right]d\,\Omega \tag{14}$$

Thereafter, we will omit the argument of elastic energy adopted to relieve the expression. By developing on the one hand the integral of left and right-hand side on the fields $C\left(\Delta a\right)$ and $\Omega\setminus C\left(\Delta a\right)$, and by using the definition of the field of damage (6) in addition, it comes after simplification :

$$\int_{\Omega\setminus C(\Delta a)}\Phi_t^{el}\,d\,\Omega+\int_{C(\Delta a)}\Phi_t^{el}\,d\,\Omega\leqslant\int_{\Omega\setminus C(\Delta a)}\Phi_t^{el}\,d\,\Omega+\int_{C(\Delta a)}w_C\,d\,\Omega \tag{15}$$

However, according to (5) :

$$\int_{C(\Delta a)}w_C\,d\,\Omega=w_c\times\Delta a\times D \tag{16}$$

The inequality (13) express yourself then like below :

$$\forall\,\Delta a>0 \qquad \frac{\int_{C(\Delta a)}\Phi_t^{el}\,d\,\Omega}{\Delta a}\leqslant w_c\times D \tag{17}$$

One DéfiniT the term of left $\widetilde{G}_p(\Delta a)$ such as:

$$\widetilde{G}_p(\Delta a)=\frac{\int_{C(\Delta a)}\Phi_t^{el}\,d\,\Omega}{\Delta a} \qquad (18$$

And the term of right-hand side is defined $G_{pc}=w_c\times D$ like the energy of cracking of material. Lcondition of not has starting is written then :

$$\forall\,\Delta a>0\,,\quad \widetilde{G}_p(\Delta a)\leqslant G_{pc} \tag{19}$$

The parameter then is defined G_p like the maximum value of the function $\widetilde{G}_p(\Delta a)$:

$$G_p = \max_{\Delta a} \tilde{G}_p(\Delta a) \tag{20}$$

The criterion of nonrestarting is written $G_p \leqslant G_{pc}$. The criterion ofstarting can then be written $G_p > G_{pc}$. This criterion imply the knowledge of two parameters material, w_c and D. For practical reasons, one prefers to choose D and G_{pc}. These parameters thus require an identification (see the paragraph §6.2).

The parameter G_p is calculated like the maximum (compared to Δa, length of propagation virtual) integral on the field $C(\Delta a)$ free energy modified Φ_t^{el}, divided by Δa. It is thus a density of average elastic energy (kJ/m^2). It is not here a rate of refund of energy because the energy considered for minimization is that of the moment running, and included not the rebalancing of the fields after propagation.
Following the example DU K_J $(MPa.m^{1/2})$, definite from J in a way similar to the relation of Irwin $J = \frac{(1-\nu^2)}{E} K_J^2$ - valid in 2D plane deformation and 3D -, one can also define a parameter K_{G_P} $(MPa.m^{1/2})$ from G_p using this same relation:

$$G_p = \frac{(1-\nu^2)}{E} K_{G_P}^2 \tag{21}$$

5 Link between comprehensive approach and local

Lorentz [LOR 08] and Wadier [BOG MANGANESE 07B] position the energy approach G_p compared to the comprehensive approaches and local of the rupture. More recent than the comprehensive approach in breaking process, the local approach aims at predicting the ruin of a structure while being pressed on the microscopic mechanism of cleavage. She gives an account of the dispersion of the results by an intrinsically probabilistic modeling. The model of Beremin, for example, defines the probability of rupture and the constraint of Weibull σ_w at time t by:

$$P_R(t) = 1 - \exp\left[-\left(\frac{\max_{\tau \le t} \sigma_w(\tau)}{\sigma_c}\right)^m\right] \quad ; \quad \sigma_w(\tau) = \left[\frac{1}{V_0} \int_{\Omega_p(\tau)} \sigma_I(\tau)^m \, d\Omega\right]^{\frac{1}{m}} \tag{22}$$

with $\sigma_I(t)$ and $\Omega_p(t)$ the maximum principal constraint and the field in the course of plasticization at time respectively. V_0 is a volume of reference, σ_c the critical stress and m an exhibitor without dimension. Thus, the expression of the probability of rupture is based in particular on the assumption of the weak link, i.e the ruin is associated with the starting of the microscopic defect more penalizing. In preoccupations with a comparison between the two models, one can define a probability of rupture of the approach G_p by the formula:

$$P_R(t) = 1 - \exp\left[-a\left(\frac{G_p}{G_{p0}}\right)^{m/2}\right] \tag{23}$$

where a and m are constants identified starting from the experimental results, and G_{p0} a constant such as P_r that is to say equal to 5 % when G_p is equal to the value identified for a probability of rupture of 5 %.

The comparison enters the two approaches revealed a certain number of links. The sizes intervening in the criterion of initiation (elastic energy or constraint) are similar. These sizes are realised on a zone located in the vicinity of the bottom of notch. Lastly, ON can be condUit to take into account the hydrostatic constraint [BOG MANGANESE 07b].

The model of Beremin allows a natural taking into account of the distinction traction/compression thanks to the introduction of the maximum principal constraint rather than that of elastic energy. Contrary, approach G_p offer a simpler transition with the former approaches from the industrial field. It makes it possible to establish the link with the classical comprehensive approach because it is founded like it on energy principles: $G_p = G$ notch in elasticity [LOR 14].

6 Setting in work in *Code_Aster*

Mechanical calculation is carried out under the assumption of a thermoelastoplastic behavior associated with a criterion of Von Mises with isotropic or kinematic work hardening linear (`VMIS_ISOT_TRAC`, `VMIS_ISOT_LINE`, `VMIS_CINE_LINE`). We will specify of what the calculation consists of G_p and how the identification of the parameters materials is carried out $R = D/2$ (ray of the notch) and G_{pc} (rupture limit).
The method for calculation and identification with Code_Aster is presented in [U2.05.08]. The simple documentation of use is in [U4.82.31].

6.1 Calculation of Gp

The calculation of G_p, realized using the macro order `CALC_GP`, is based on the use of `POST_ELEM` who allows the calculation of elastic energy on a group of meshs. Modelings (finite elements, small deformations, etc) and loadings usable are those of the order `POST_ELEM`, keyword `ENER_ELTR`. More precisely it acts, for each moment envisaged in the list of the moments of calculation, to carry out the two following stages:

1 Tout initially to calculate the quantity $\tilde{G}_p(\Delta a)$ for ascending values of Δa by:

$$\tilde{G}_p(\Delta a) = \frac{\int_\Omega \Phi_t^{el} \, d\Omega}{\Delta a}$$

In 2D it is thus necessary to identify the elements of the zone $C(\Delta a)$ by a group of meshs defined in the level of the grid as presented in [Figure 2-a], or by a geometrical zone of points of Gauss, then to calculate elastic energy on this zone then to divide it by Δa.
To identify the elements of the zone $C(\Delta a)$ one will operate as follows: the elements of the first chip will set up a first group of meshs, the elements of chips 1 and 2 will set up a second group, the elements of the chips $1, 2, 3, \ldots, i$ will constitute one i ème group, etc It is necessary to envisage a sufficiently large number of chips to be able to find the maximum of $\tilde{G}_p(\Delta a)$, which is generally at a distance from approximately $3R$ bottom of notch.

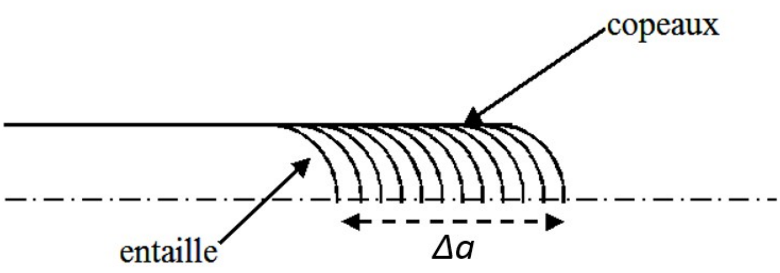

Figure 6.1-1 - Definition of the chips in the grid

2 Ensuite it is a question of identifying the maximum of this function:

$$G_p = \max_{\Delta a} \tilde{G}_p(\Delta a)$$

One will find examples and advices of use in the document [**U2.05.01**], in the tests ssnp131 (see [V6.03.131] and ssnv218 (see [V6.04.218]).

6.2 Identification of the parameters

The energy model is based on the couple of parameters materials (G_{pc}, R) that it is thus a question of determining at each temperature . It is noted that G_{pc} depends actually ray of notch [BOG MANGANESE 13] . We will see that L prediction of the rupture has does not depend on it.

One supposes known S on the one hand the Young modulus E and the critical stress σ_c of a material. In addition, one fact the assumption of knowing tenacity K_{Jc} evaluated in experiments starting from a test of traction on a test-tube CT, for example . The application of the "minimum compared to the damage" at the level of a material point in a state of simple stress tensile gives the relation enters the voluminal energy dissipated by a material point which is damaged w_c and the critical stress [BOG MANGANESE 13] *(H11 assumption)* :

$$w_c = \frac{\sigma_c^2}{2E} \tag{24}$$

One obtains then equation following that it is necessary to solve identify the two parameters :

$$G_{pc}(R) = \frac{\sigma_c^2}{E} R \tag{25}$$

The member of left $G_{pc}(R)$ is a nonlinear function of R . The member of right-hand side is a linear function of R . Thus, to solve (25) it is a question of calculating G_{pc} for various values of R like the figure illustrates it [Figure 6.2-1].

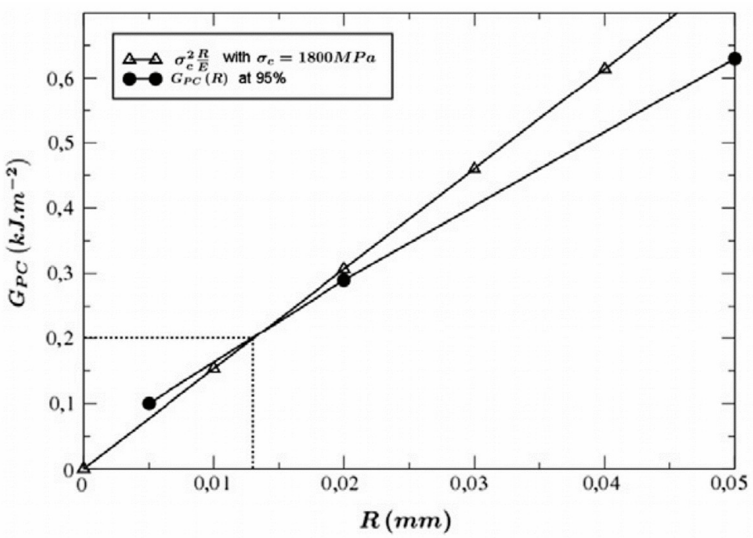

Figure 6.2-1: identification of the couple (G_{pc}, R) [BOG MANGANESE 13]

For each notch of ray R given, the parameter G_{pc} is determined by simulation of a test on test-tube CT in the following way: for each value of the loading, crescent of 0 up to a breaking value, one calculates on the one hand the parameter K_J and in addition the parameter G_p. For the value loading criticizes corresponding to $K_J = K_{Jc}$, one obtains $G_p = G_{pc}$ *(Hypothèse H12)*.

The resolution of equation (25) for various values of R, allows finally to determine and the value of R and the value of G_{pc} at a given temperature. Within the framework of a steel, one finds values ranging between 10 and 100 microns, which is representative of a real crack.

7 Bibliography

The bibliography is voluntarily exhaustive. Paragraph 7.1 lists the academic references: theses articles and congress. Paragraph 7.2 lists certain internal documentations EDF (CR, notes). All are not quoted in body text.

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8 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
10.4	Y. WADIER EDF R & D AMA	Initial text
13	S. JULES EDF R & D AMA	New presentation of the method