

Computation of T-stress by extrapolation of displacement field

Abstract:

The computation of T-stress IS implemented for linear, homogeneous and elastic-isotropic materials with planes ace mesh (FEM only). The accuracy of T-stress results obtained by extrapolation of the displacement field method IS clearly improved yew HTE meshing elements are quadratic. It is also highly recommended to uses "Barsoum" elements type in the ace tip (elements where the mid-side Nodes are located At the quarter of edges).

1 Theory

Based on one isotropic-linear-elasticity theory, when a crack in an elastic body is subjected to external forces (see Fig.1), the stress field in the vicinity of the crack tip edge can be expressed by the Williams' expansion. The stress near the crack tip edge can be written according to equations 12, where σ_{ij} is the stress tensor, r represents the distance of element from the crack tip, and θ is the angle of element with respect to the crack front axes located at the crack tip [1].

$$\sigma_{rr} = \frac{K_1}{\sqrt{(2\pi r)}} \left(\frac{5}{4} \cos\left(\frac{\theta}{2}\right) - \frac{1}{4} \cos\left(\frac{3\theta}{2}\right) \right) + \frac{K_2}{\sqrt{(2\pi r)}} \left(-\frac{5}{4} \sin\left(\frac{\theta}{2}\right) + \frac{3}{4} \sin\left(\frac{3\theta}{2}\right) \right) + T \cos^2(\theta)$$

Equation 1

$$\sigma_{\theta\theta} = \frac{K_1}{\sqrt{(2\pi r)}} \left(\frac{3}{4} \cos\left(\frac{\theta}{2}\right) + \frac{1}{4} \cos\left(\frac{3\theta}{2}\right) \right) + \frac{K_2}{\sqrt{(2\pi r)}} \left(-\frac{3}{4} \sin\left(\frac{\theta}{2}\right) + \frac{3}{4} \sin\left(\frac{3\theta}{2}\right) \right) + T \sin^2(\theta)$$

Equation 2

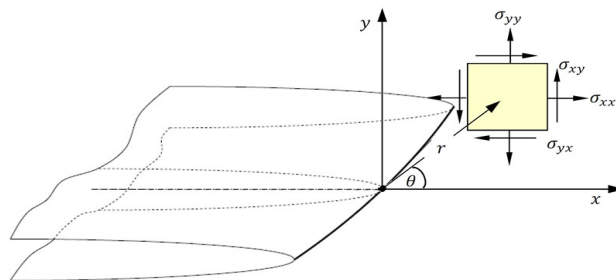


Fig.1 Stress in the Cartesian coordinate system ahead of the crack tip in a crack of infinite length

The non-singular term is known as the T-stress, and it characterizes the crack tip constraint; this is particularly useful in the assessment of the integrity of structures containing defects. A robust method for obtaining T-stress is via the displacement field along the crack faces. This method is based on the asymptotic development of the displacement field in the crack tip. For $\theta = -\pi$ however $\theta = +\pi$ the singular term of σ_{rr} vanishes and hence near the crack tip, T-stress can be represented by:

$$T = E' \frac{[U_{\theta=\pi}(r)] - [U_{\theta=\pi}(0)]}{r}$$

where $[U]$ is the displacement step between the crack lips and hence $[U_{\theta=\pi}(0)]$ indicates the crack tip displacement; details of the method are given elsewhere [2]. E' is defined for plane stress and plane strain conditions as:

$$E' = \begin{cases} E & \text{Plane stress} \\ \frac{E}{1-\nu^2} & \text{Plane strain} \end{cases}$$

Note:

- It is also possible to compute T-stress according to the stress fields, but the accuracy of the vector forces on the lips of the crack are less accurate than displacements (resulting from the stress singularity in the crack tip).

- The to use must take care to refine the mesh in the vicinity of ace tip so that enough points are available for extrapolation.
- The method used young stag is HTeoretically less accurate than the integration integral method for computation of T-stress since the integral is taken over has domain of elements surrounding the ace and errors in room solution parameters cuts less effect one the evaluated quantities. However, I N this method No spec ia L ace tip elements are required and the deteRmination of node sets for the calculation of J-integral T-stress is simpler than in the approhasCH. Thus it makes possible to obtain easily relatively applicable been worth of T-stress for any of type mesh strategy. Different The comparison of the methods of calculating is always useful to estimate the accuracy of the results obtained.

2 Implementation of extrapolation method

The extrapolation displacement method is implementED in the `POST_T_Q` operator which starts by Computing the displacement field one the total structure. Since THE of definition T-stress are only asymptotically true; extrapolation is restricted to the vicinity of the ace tip limited by has maximum distance $dmax$ aroundD the ace. $dmax$ is has parameter of the `ABSC_CURV_MAXI` operator. In the box of has meshed ace `ABSC_CURV_MAXI` is optional. Yew $dmax$ is not definED in the .comm file, by defect it is automatically assignED in `POST_T_Q` operator ace equal to three times the maximum size of the mesh elements connected to the nodes one the ace face. General The principle of T-stress computation is defined ace follows:

1. Loop one the ace tip nodes
2. ExtracT the ace tip displacement,
3. DefinE the plane normal to ace and the ace tip,
4. Project the displacement field in $\theta=+\pi$ and $\theta=-\pi$,
5. Calculate the difference of the displacement been worth,
6. THE Displacement difference is divideD by r with material multiplicative Factor will obtaine and then extrapolatE Into $r=0$. Yew the solution is perfect, one should obtain has line. Actually, we obtain almost has line with has mesh of the standard "Barsoum".

3 Accuracy of suggested method

The extrapolation displacement method Wace validated one the test box (SSS?) , for which analytical solutionS are known [3]. This test puts, ace detailed in (T₁.odt), considers year elliptic ace in has three-dimensional bodystocking subject to has tensile load. With cube by size of ($h=w=t=16$) , containing has horizOntal circular ace of radius $c=a=1$ is considered (see fig.2). Been worth The of far-field stress, σ , was prescribes ace 1 MPa. Had to symmetry, only one quarter of the elliptical ace is considered. Fine The ace is meshed with quadratic mesh standard in the vicinity of ace tip. The results are compared with reference solution (see Fig.3).

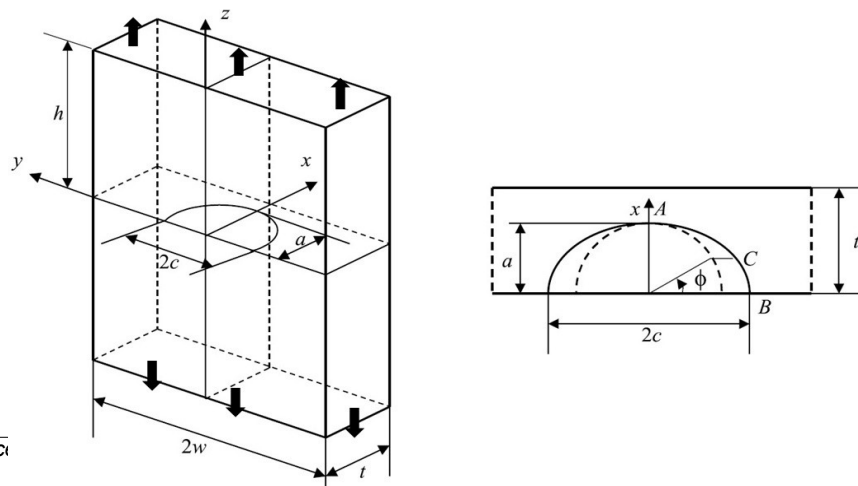


Fig.2 Geometric configuration of the testcase shown in half symmetry [3]

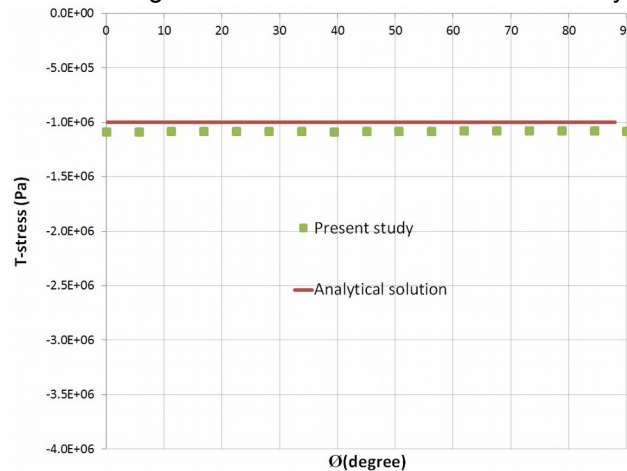


Fig.3 T-stresses compression

4 Conclusion

The results obtained with extrapolation displacement method are in good agreement, with less than 5% error compared to the analytical solution of T-stress. It should be noticed that the asymptotic relation of displacements is valid only for r tending towards 0. Therefore, it is necessary to take care not to choose too broad a domain of extrapolation (distance d_{max} from operator `POST_T_Q` of about 4 to 5 elements).

5 Bibliography

- 1 L. Novotný, Calculation of T stress on 3D specimens with ace. *Procedia Engineering*. 48,489 – 494. 2012,
- 2 S.A. Zahedi, A. Jivkov, Two-parameter fracture characterization of a welded pipe in the presence of residual stress – *Structural Procedia Integrity*. 2,777-784, 2016.
- 3 X. Wang, Elastic T-stress solutions for semi-elliptical surface cracks in finite thickness plates. *Engineering Fractures Mechanics* 70,731-756, 2003.

6 Description of the document versions

Index	Version Aster	Author (S) gold contributor (S), organization	Description of the modifications
With	13.4	A. Zahedi EDF Energy UKC	Initial document