Estimate of the lifetime in fatigue to large many cycles and in fatigue oligocyclic

Summary:

Most industrial structures are subjected to variable efforts in the time which, repeated a large number of times can lead to their rupture by tiredness. One presents in this note the principal features of the orders POST_FATIGUE [U4.83.01] and/or CALC_FATIGUE [U4.83.02] and/or CALC_CHAMP [U4.81.04] which makes it possible to estimate the limit of endurance and the office plurality of damage of a part.

The various methods available are:

- linear office plurality: methods based on uniaxial tests (methods of Wöhler, Manson-Whetstone sheath and Taheri).
  These methods have as a common point to determine a value of damage starting from the evolution in the course of the time of a component **scalar** characterizing, for the calculation of the damage, the amplitude of constraints or structural deformations.
  With this intention, it is necessary to extract by a method of counting of cycles, the elementary cycles of loading undergone by the structure, to determine the elementary damage associated with each cycle and to determine the total damage by a rule of linear office plurality;
- nonlinear office plurality: method of Lemaître and method of Lemaître-Sermage
  These methods make it possible to calculate the damage D at every moment t, starting from the data of the tensor of the constraints $\sigma(t)$ and of the cumulated plastic deformation $p(t)$;
- limit of endurance: criteria of Crossland and Dang Van Papadopoulos
  These criteria apply to uniaxial or multiaxial loadings in constraints **periodicals**. They provide a value of criterion indicating if there is tiredness or not. The definite equivalent constraints for these criteria can also be used for to calculate the office plurality of damage.

Contents

Contents

1 Introduction

The industrial experiment shows that the ruptures of or structure machine components under normal functioning are generally due to tiredness. Its masked progressive character very often leads to a brutal rupture.

One understands by tiredness the consecutive modification of the properties of materials to the application of cycles of efforts, cycles whose repetition can lead to the rupture of the parts made up with these materials [bib1].

Various methods are available for the evaluation of the damage. The second part of this document is devoted to the presentation of oldest which is methods based on uniaxial tests: method of Wöhler, method of Manson-Whetstone sheath and more recently methods suggested by S. Taheri (EDF-R&D/AMA).

These methods have as a common point to determine a value of damage starting from the evolution during the characterizing time of a scalar component, for the calculation of the damage, the state of constraints or structural deformations.

The evaluation of the damage is based on the use of curves of tiredness of the material (Wöhler or Manson-Whetstone sheath), associating a variation of constraint of amplitude given to a number of acceptable cycles.

To use these curves starting from a real uniaxial loading, it is necessary to treat the history of the constraints or the deformations by identifying elementary cycles (cf [§2.2]).

The difficulty in defining a cycle for a complex signal explains the profusion of the methods of counting appeared in the literature [bib2].

Two methods among most usually used were introduced into *Code_Aster* :

- counting of the extents in cascade or method RAINFLOW,
- rule RCC_M.

One adds to it a third method which we will call method of “natural” counting and which respects the order of application of the cycles of loading.

For each elementary cycle, one evaluates an elementary damage using methods founded on the curves of Wöhler, Manson-Whetstone sheath or both simultaneously.

For the method of Wöhler (cf [§2.3]) the user can correct the constraint to be integrated in the curve of Wöhler by:

- a concentration factor of constraints K_T, to take account of the geometry of the part,
- an elastoplastic coefficient of concentration K_e ,
- a correction of Goodman or To stack in the diagram of Haigh to take account of the average constraint of the cycle.

In addition, one proposes to define the curve of Wöhler in three different forms, a point by point discretized form and two analytical forms.

The method of Manson-Whetstone sheath (cf [§2.4]) applies to loadings in deformations. The curve of Manson-Whetstone sheath is defined in a single form, forms discretized point by point.

The methods of Taheri (cf [§2.5]) also apply to loadings in deformations and require the data of the curve of Manson-Whetstone sheath and possibly of the curve of Wöhler. Their characteristic is to take account about application of the elementary cycles of loading with the structure, contrary to the two other methods.

Note:

Three methods of extraction of the elementary cycles are available: method of Rainflow, rule of RCC_M and "natural" counting.

The first two methods do not take account about application of the cycles what is of no importance for the calculation of the damage by the methods of Wöhler or Manson-Whetstone sheath.

For the calculation of the damage by the methods of Taheri, it is necessary to use the method of extraction of the cycles by "natural" counting [§2.2.3] which respects the order of application of the cycles.

For the whole of these methods calculation of the total damage undergone by the structure is determined by a method of office plurality, the rule To mine.

The third part of this document presents the methods of Lemaître and Lemaître-Sermage which are "analytical" methods making it possible to calculate the damage D (in each moment t) starting from the data of the tensor of constraints $\sigma(t)$ and of the cumulated plastic deformation $p(t)$. These two methods apply to loadings in unspecified constraints (uniaxial or multiaxial).

A linear rule of office plurality can be used to determine the total damage undergone by the structure.

Lastly, the criteria of Crossland and Dang Van Papadopoulos are introduced in fourth and last part of this document. They apply to unspecified loadings (uniaxial or multiaxial) in constraints and periodicals. They provide a value of criterion indicating if there is tiredness or not.

From the value of the criterion, one can specify a scalar component characterizing the state of the structure for calculation of the damage and determine a value of damage by using the curve of Wöhler of material.

2 Methods of Wöhler, Manson-Whetstone sheath and Taheri

2.1 Extraction of the peaks

The user provides to *Code_Aster* a function which defines the history (scalar) loading in a given point. For that, it has the keyword `HISTORY`.

On this history of the loading, which can be complex, a first operation of extraction of the peaks is carried out. This operation consists in reducing the history of loading to the only fundamental peaks.

Note:

> *In fatigue, one names loading in a point given the value of the answer of the structure in this point.*
> *In the use of the curves of Wöhler, it is about constraint in this point.*
> *In the use of the curves of Manson-Whetstone sheath, it is about deformation in this point.*
> *The history of loading is thus the evolution in the course of the time of a constraint, or a deformation.*

If the function remains increasing or decreasing on more than two consecutive points, one removes the intermediate points to keep only the two extreme points.

One also removes history of the loading the points for which the variation of the value of the constraint or the deformation is lower than a certain level chosen by the user. That amounts applying a filter to the history of the loading. The value of the level of the filter is introduced by the user under the keyword `DELTA_OSCI`.

For illustration let us consider the following history of loading:

N° not	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
Moment	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	
Loading	4.	7.	2.	10.	9.6	9.8	5.	9.	3.	4.	2.	2.4	2.2	12.	

N° not	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Moment	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.
Loading	5.	11.	1.	4.	3.	10.	6.	8.	12.	4.	8.	1.	9.	4.	6.

The extraction of the peaks of this history of loading, with a value of delta of 0.9 conduit to destroy all the oscillations of amplitude lower than 0.9. What leads to the following history of loading:

N° not	1	2	3	4	7	8	9	10	11	14	15	16	17
Moment	0.	1.	2.	3.	6.	7.	8.	9.	10.	13.	14.	15.	16.
Loading	4.	7.	2.	10.	5.	9.	3.	4.	2.	12.	5.	11.	1.

N° not	18	19	20	21	23	24	25	26	27	28	29		
Moment	17.	18.	19.	20.	22.	23.	24.	25.	26.	27.	28.		
Loading	4.	3.	10.	6.	12.	4.	8.	1.	9.	4.	6.		

Note:

> *Let us note* ch *the value of the loading;* ch *can be a constraint or a deformation.*

History of loading was removed:

- the point 5 because $\Delta ch = |ch(5) - ch(4)| < 0.9$,
- the point 6 because $\Delta ch = |ch(6) - ch(4)| < 0.9$,
- the point 12 because $\Delta ch = |ch(12) - ch(11)| < 0.9$,
- the point 13 because $\Delta ch = |ch(13) - ch(11)| < 0.9$.

In the same way the point is removed 22 because the history of loading is increasing between the points $21{,}22$ and 23 . Thus only the extreme points are kept.

2.2 Methods of counting of cycles

During their life, the industrial structures are generally subjected to complex loadings whose levels of requests are variable.

The methods of counting of cycles make it possible to extract from the history of loading, of the elementary cycles according to various criteria.

Code_Aster propose three distinct methods including two nonstatistical methods among the methods most usually used.

2.2.1 Method RAINFLOW

The method of counting of the extents in cascade more often called method of RAINFLOW, defines cycles which physically correspond to loops of hysteresis in the stress-strains plan. In the literature, several alternatives of this method are counted.

The algorithm implemented in *Code_Aster* essentially that is proposed by recommendation AFNOR A 03-406 of November 1993 [bib3] (with characteristics which is specified during the presentation of the detail of the algorithm) and breaks up into three stages:

- One **first stage** who consists in rearranging the history of the loading $\sigma(t)$ or $\varepsilon(t)$ so that the loading begin with the maximum value, in absolute value, of the loading.

Note:

> *In recommendation AFNOR A 03-406, it is not mentioned a rearrangement of the history of loading. This rearrangement is however carried out in software POSTDAM [bib2] and included in Code_Aster.*

- **second stage** consist in extracting the elementary cycles from the history of loading thus rearranged.

  The method consists in being based on four successive points of the history of loading $(ch(i),\ i = 1,\ \text{Nbpoint})$.

  One notes:

$$X = |ch(i+1) - ch(i)| \text{ et } Y = |ch(i+2) - ch(i+1)|$$
$$\text{et } Z = |ch(i+3) - ch(i+2)|.$$

As long as Y is strictly higher than X or with Z, one traverses the history of the loading while moving of a point towards the line (what amounts incrementing the value of i).

As soon as Y is lower or equal to X and inferior or equal to Z, it is considered that one met an elementary cycle which is defined by the two points $(i+1)$ and $(i+2)$. The amplitude of the cycle is given by $\Delta ch = |ch(i+1)-ch(i+2)|$.

When the cycle is extracted one removes the two points of the history of loading and one continues the algorithm.

- **third stage** consist in treating the residue, i.e. the remaining history of loading after the stage of extraction of the cycles.

  With this intention, one adds the same residue with his continuation realising possibly some care on the level of connection following the values of the extrema considered thus that value of the first and the last slope of the residue.

  The last point of the residue the first point of the cycle succeeds. So the points considered can not seem extrema more. If that occurs, it is advisable to eliminate them. Eight different cases are encountered. To treat them explicitly, let us call R_1 and R_2 the first two points of the residue and R_{n-1} and R_n its last two points.

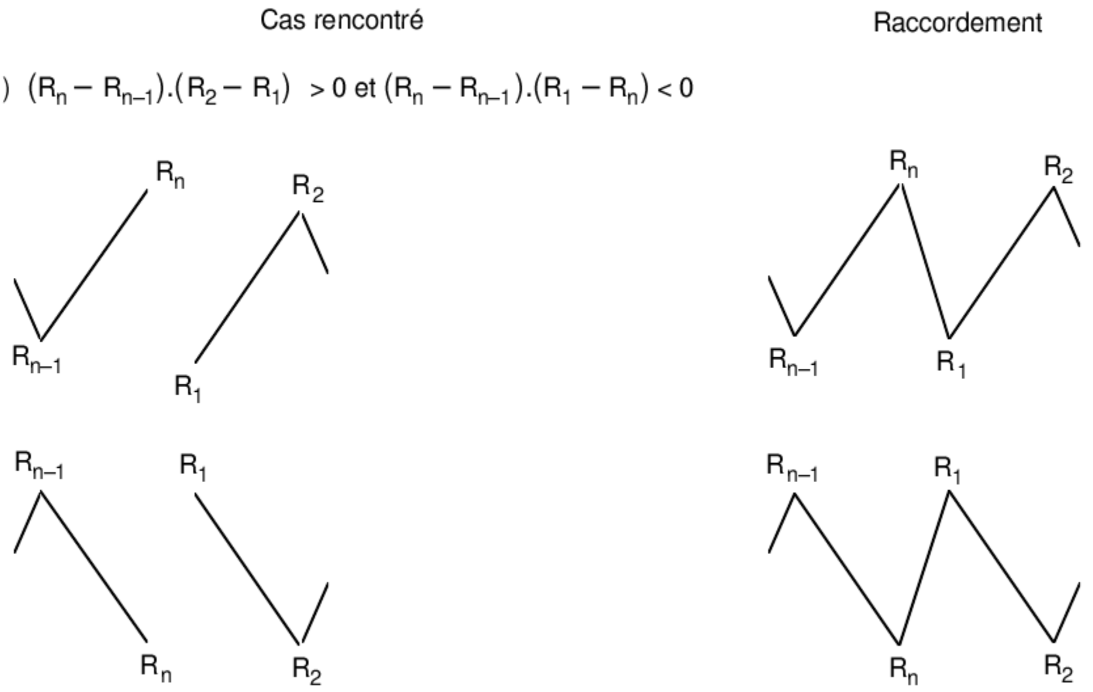
Note:

Recommendation AFNOR A 03-406 fact also state of possible a pre - treatment of the signal, which would consist of a filtering of the signal (suppression of the parasites) and of a quantification of the history of loading.

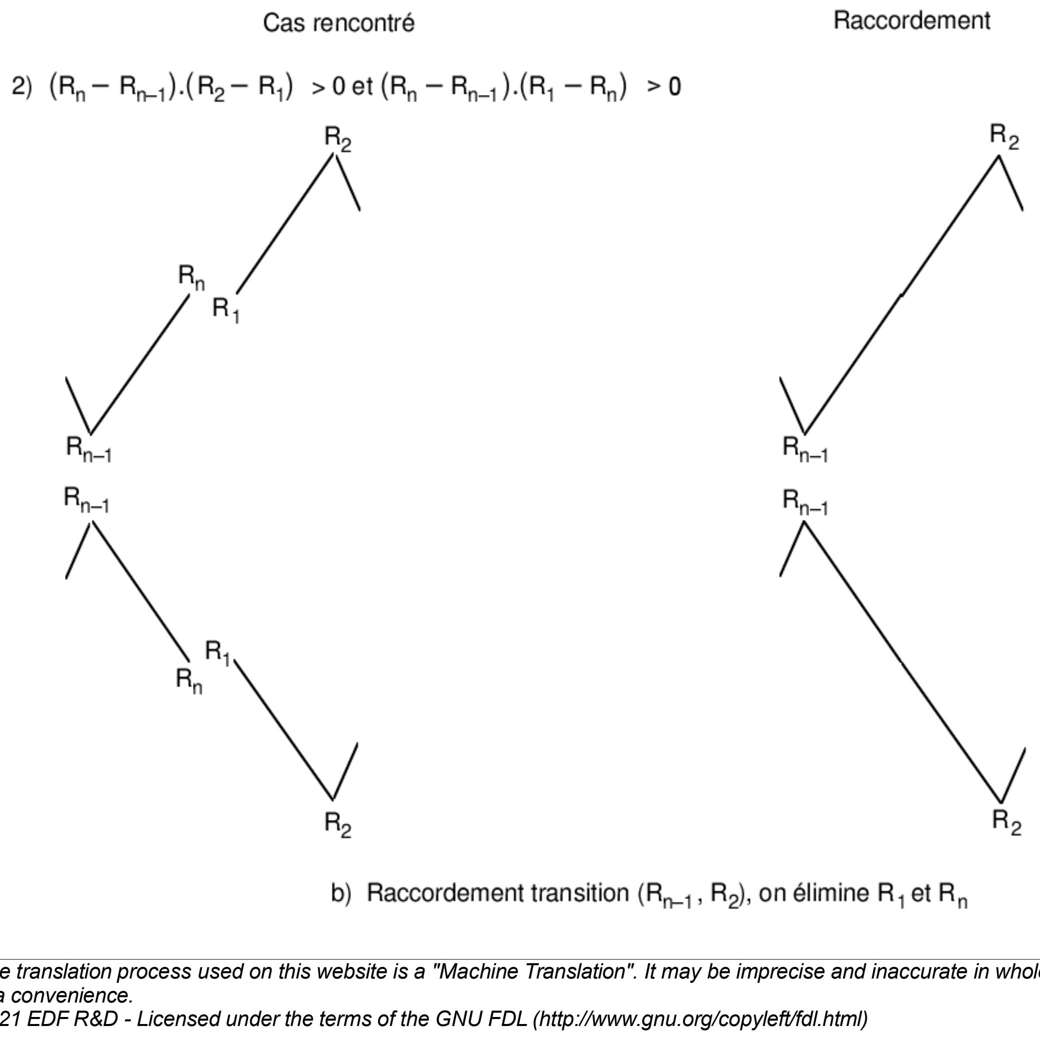
The filtering of the signal is possible, at the request of the user (see [§2.1]. Extraction of the peaks).

The quantification of the signal can be useful for the speed of the analysis of the results of the analysis of tiredness. Practically, the quantification of the signal consists to cut out the maximum extent of the signal in classes of intervals of constant width called not, and to bring back to a value representative of a given class (its median value in general) all the values located in this class. This possibility of preprocessing of the signal as for it, is not available in Code_Aster.

In the special case where the history of loading is constant (for example, average loading applied), Code_Aster will count the whole history of loading like a cycle of worthless amplitude.



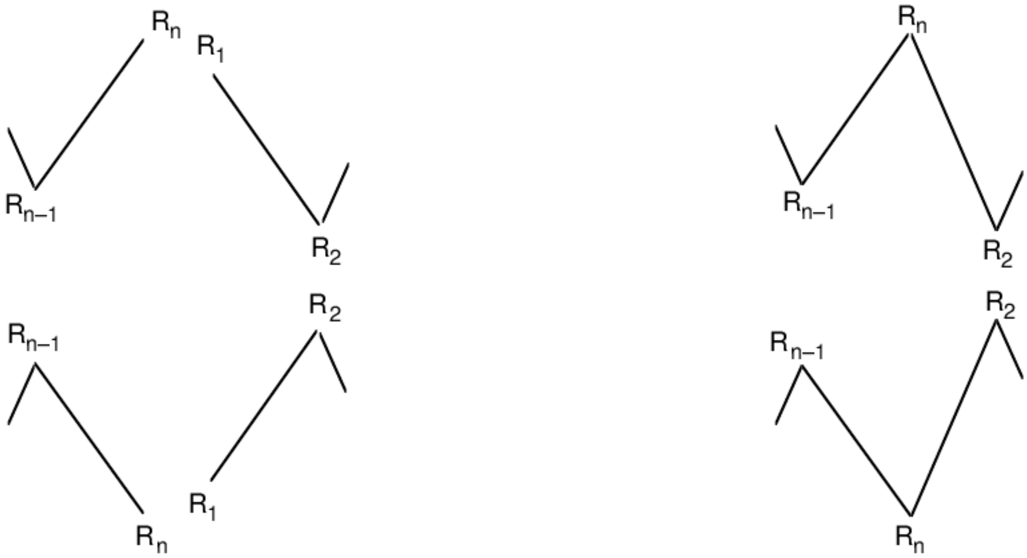
a) Raccordement sans problème : transition (R_n, R_1)



b) Raccordement transition (R_{n-1}, R_2), on élimine R_1 et R_n

Cas rencontré Raccordement

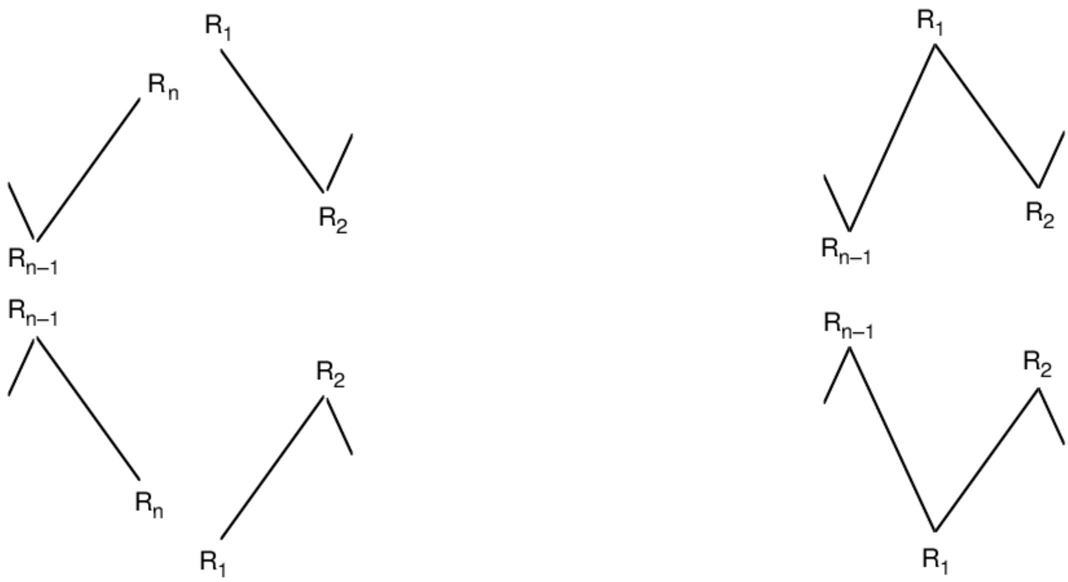
3) $(R_n - R_{n-1}).(R_2 - R_1) < 0$ et $(R_n - R_{n-1}).(R_1 - R_n) < 0$



c) Raccordement transition (R_n, R_2), on élimine R_1

Cas rencontré Raccordement

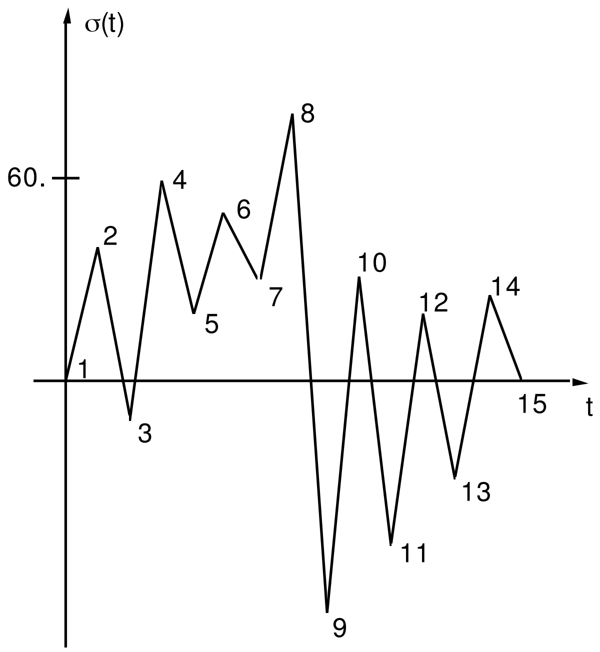
4) $(R_n - R_{n-1}).(R_2 - R_1) < 0$ et $(R_n - R_{n-1}).(R_1 - R_n) > 0$



d) Raccordement transition (R_{n-1}, R_1), on élimine R_n

In order to illustrate the method and to clarify the points which would remain obscure, the following history of loading is considered (which for the example is considered of type forced):

N° not	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Moment	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
Loading	0.	40.	– 10.	60.	20.	50.	30.	80.	– 70.	30.	– 50.	20.	– 30.	25.	0.



The method of RAINFLOW thus leads, on this example, (see [§Annexe1], for detail of the stages of the algorithm) with the determination of 7 elementary cycles defined by the maximum value and the minimal value of the loading, for each cycle.

Cycle 1: `VALMAX = 20.` `VALMIN = – 30.`
Cycle 2: `VALMAX = 25.` `VALMIN = 0.`
Cycle 3: `VALMAX = 30.` `VALMIN = – 50.`
Cycle 4: `VALMAX = 40.` `VALMIN = – 10.`
Cycle 5: `VALMAX = 50.` `VALMIN = 30.`
Cycle 6: `VALMAX = 60.` `VALMIN = 20.`
Cycle 7: `VALMAX = 80.` `VALMIN = – 70.`

Note:

- *The calculation of the damage which does not take account about appearance of the elementary cycles of loading, it is without consequence to rearrange the history of the loading.*
- *For the methods of Taheri, the order of application of the elementary cycles of loading is taken into account, also is necessary it to be very vigilant with the use of such a method of counting of cycles. He is advised, for the calculation of the damage by the methods of Taheri, to use the method of "natural" counting known as [§2.2.3].*

2.2.2 Method RCC_M

This method consists in forming the elementary cycles of request while starting with those which cause the greatest variations.

Thus for a history of comprising loading N points, one determines $N/2$ elementary cycles if N is even and $N/2+1$ if N is odd.

The algorithm breaks up into two stages. The first stage consists in ordering the history of loading of smallest to the greatest value of the constraint, or the deformation.

The second stage consists, as for it, to form the elementary cycles with the greatest variation of the value of the constraint, or the deformation.

On the history of loading $ch(t)$ rearranged, the elementary cycles are defined by:

$$\begin{cases} \text{VALMAX} = ch_{N+1-i} & \text{pour } i = 1, \ N/2 \\ \text{VALMIN} = ch_i & \end{cases}$$

If N one is odd determines a definite additional cycle by:

$$\begin{cases} \text{VALMAX} = ch_{N/2+1} & \text{si } ch_{N/2+1} > ch_m \\ \text{VALMIN} = -ch_{N/2+1} + 2 * ch_m & \end{cases}$$

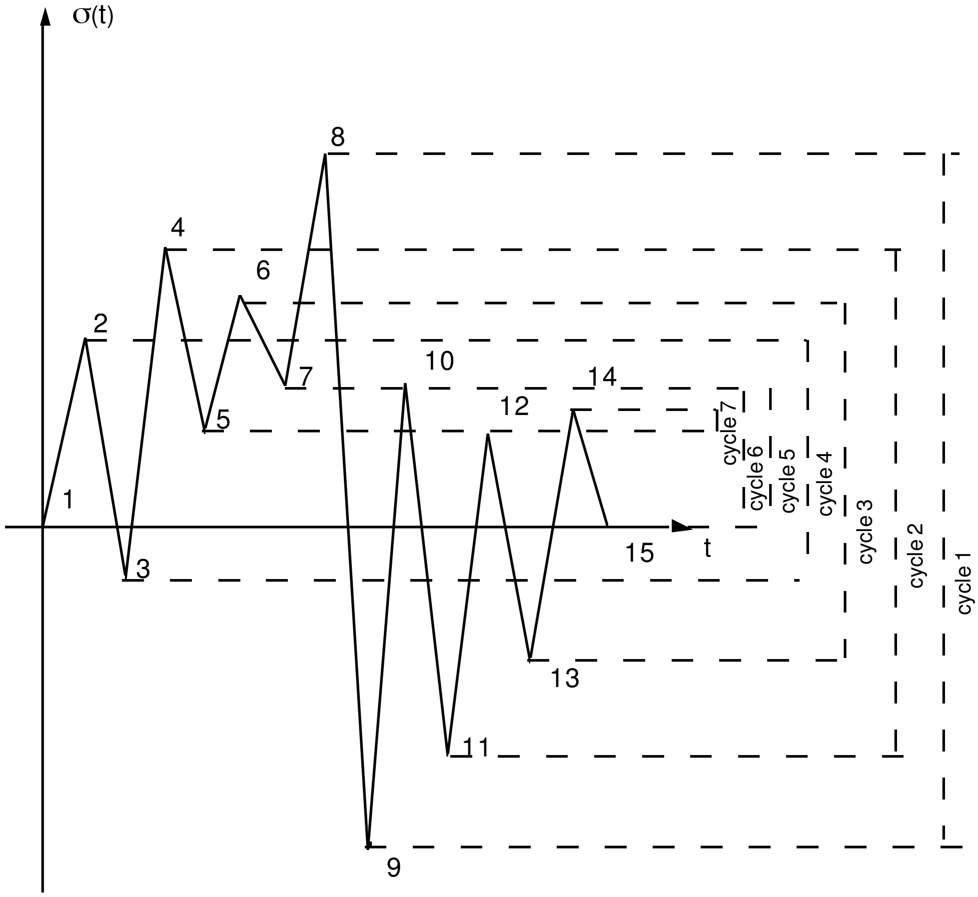
and

$$\begin{cases} \text{VALMAX} = ch_{N/2+1} & \text{sinon} \\ \text{VALMIN} = -ch_{N/2+1} + 2 * ch_m & \end{cases}$$

where $ch_m =$ average constraint or average deformation of the loading = $\frac{1}{N}\sum_{1}^{N} ch_i$.

To illustrate method RCC_M let us consider the same example as that used for the method RAINFLOW (of which the loading was considered of type forced).

N° not	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Moment	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
Loading	0.	40.	– 10.	60.	20.	50.	30.	80.	– 70.	30.	– 50.	20.	– 30.	25.	0.



The first stage which consists in ordering the history of the loading, of smallest with the greatest value of the loading, conduit to following storage:

N° not	9	11	13	3	1	15	5	12	14	7	10	2	6	4	8
Loading	– 70.	– 50.	– 30.	– 10.	0.	0.	20.	20.	25.	30.	30.	40.	50.	60.	80.

The history of loading being composed of 15 points, method RCC_M determines 8 elementary cycles:

Cycle 1:	`VALMAX = 80.`	and	`VALMIN = - 70.`	
Cycle 2:	`VALMAX = 60.`	and	`VALMIN = - 50.`	
Cycle 3:	`VALMAX = 50.`	and	`VALMIN = - 30.`	
Cycle 4:	`VALMAX = 40.`	and	`VALMIN = - 10.`	
Cycle 5:	`VALMAX = 30.`	and	`VALMIN = 0.`	
Cycle 6:	`VALMAX = 30.`	and	`VALMIN = 0.`	
Cycle 7:	`VALMAX = 25.`	and	`VALMIN = 20.`	
Cycle 8:	`VALMAX = 20.`	and	`VALMIN = 6.`	because $\left(\sigma_m = \frac{1}{N}\sum_1^N \sigma_i = 6.\right)$

Note:

This method of counting of cycles does not take absolutely account about appearance of the cycles, and systematically orders the elementary cycles by decreasing amplitude. This method must be used with vigilance for the calculation of the damage by the methods of Taheri whose characteristic is to take account about application of the cycles of loading. For the calculation of the damage by the methods of Taheri, it is strongly advised to use the method of “natural” counting of cycles known as [§2.2.3].

2.2.3 Method "naturalness"

This method consists in generating the cycles in the order of their appearance in the history of loading.

Thus for a history of loading of $N+1$ points, one determines $N/2$ elementary cycles if N par and $N/2+1$ elementary cycles if N odd.

The method consists in being based on three successive points of the history of loading.

One notes $X=|ch(i+1)-ch(i)|$ and $Y=|ch(i+2)-ch(i+1)|$.

If $X \geq Y$ it is considered that one met an elementary cycle which is defined by the two points (i) and $(i+1)$.

The amplitude of the cycle is given by $\Delta ch=|ch(i+1)-ch(i)|$.

If $X<Y$ it is considered that one met an elementary cycle which is defined by the two points $(i+1)$ and $(i+2)$.

The amplitude of the cycle is given by $\Delta ch=|ch(i+2)-ch(i+1)|$.

When the cycle is extracted the two points are removed (i) and $(i+1)$ history of loading and one continues the algorithm.

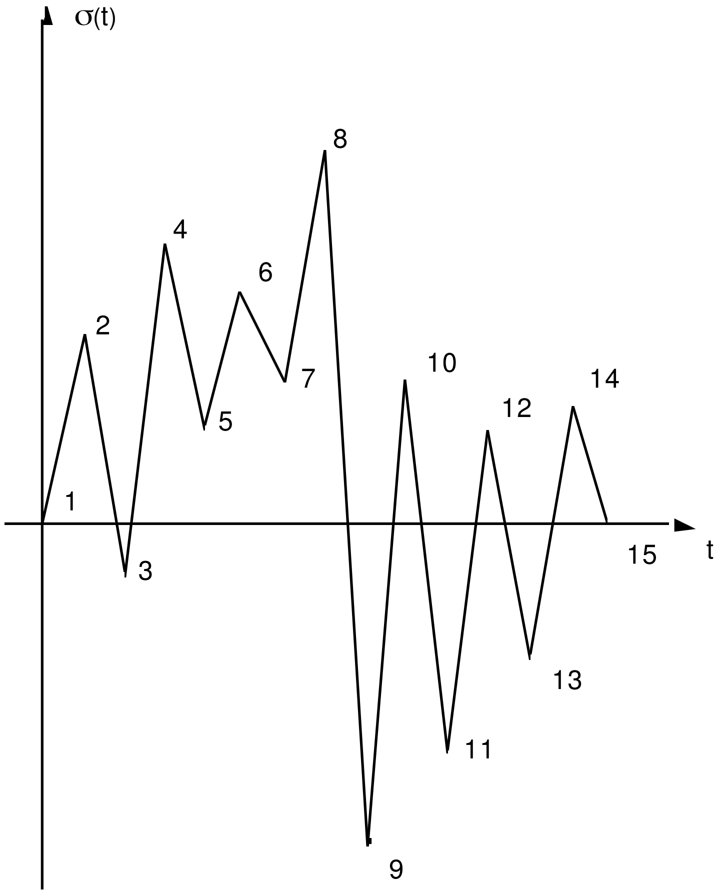
If the number of points $(N+1)$ history of loading is odd, the algorithm described previously makes it possible to discuss all items.

If the number of points $(N+1)$ history of loading is even, it remains to discuss the two remaining items.

It is considered that these two points form a cycle defines by the two points N and $(N+1)$. The amplitude of the cycle is given by $\Delta ch=|ch(N+1)-ch(N)|$.

To illustrate this method let us consider the same example as that used for methods RAINFLOW and RCC_M.

N° not	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Moment	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
Loading	0.	40.	– 10.	60.	20.	50.	30.	80.	– 70.	30.	– 50.	20.	– 30.	25.	0.



The history of loading being composed of 15 points, the method "naturalness" determines 7 elementary cycles:

Cycle 1:	VALMAX = 40.	and	VALMIN = - 10.
Cycle 2:	VALMAX = 60.	and	VALMIN = - 10.
Cycle 3:	VALMAX = 50.	and	VALMIN = 20.
Cycle 4:	VALMAX = 80.	and	VALMIN = - 70.
Cycle 5:	VALMAX = 30.	and	VALMIN = - 70.
Cycle 6:	VALMAX = 30.	and	VALMIN = - 50.
Cycle 7:	VALMAX = 25.	and	VALMIN = - 30.

Note:

This method is that which it is strongly recommended to use in the case of the calculation of the damage by the methods of Taheri.

2.3 Calculation of the damage: method of Wöhler

The number of cycles to the rupture is determined by interpolation of the curve of Wöhler of material for a level of alternate constraint given (to each elementary cycle corresponds a level of amplitude of constraint $\Delta\sigma = |\sigma_{max} - \sigma_{min}|$ and an alternate constraint $S_{alt} = 1/2\,\Delta\sigma$).

The damage of an elementary cycle is equal contrary to many cycles to the rupture $D = 1/N$.

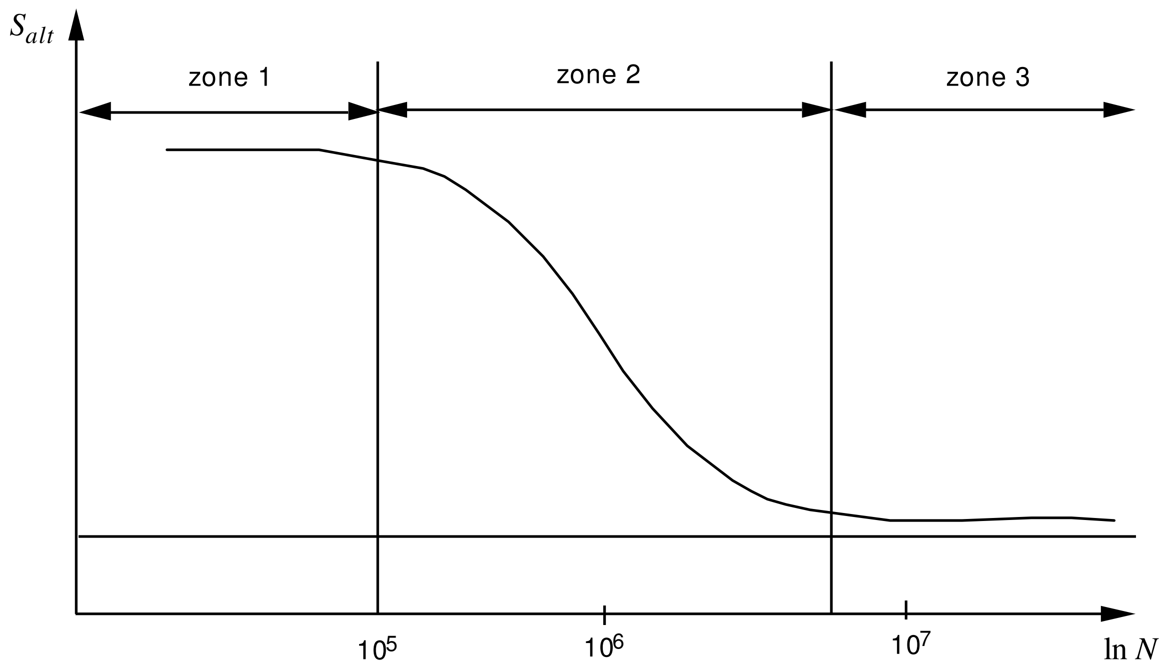
In the case of a uniaxial homogeneous test with an alternate constraint pure (or symmetrical), the number of cycles to the rupture is given starting from a diagram of endurance, still called curve of Wöhler or curve $S-N$.

In the case of geometrical defects or of elementary cycles of nonworthless average constraint, of the corrections of the curve of Wöhler are necessary before the determination amongst cycles for the rupture and thus to the elementary damage.

2.3.1 Diagram of endurance

The diagram of endurance, also called curve of Wöhler or curve $S-N$ (curve constraint-number of cycles to the rupture) is obtained in experiments by subjecting test-tubes to cycles of periodic efforts (generally sinusoidal) of normal amplitude σ and of constant frequencies, and by noting the number of cycles N with the end of which the rupture occurs.

The curve of Wöhler is thus defined for a given material and is presented in the form:



N : Nombre de cycle à la rupture

where S_{alt} = the alternate constraint of the cycle = $\frac{1}{2}|\sigma_{max}-\sigma_{min}|$

One distinguishes three zones on this curve:

- a zone of oligocyclic fatigue, under strong constraint, where the rupture occurs after very a small number of alternations,
- zone of tiredness or of endurance limited, where the rupture is reached after a number of cycles which grows when the constraint decrease,
- a zone of unlimited endurance or security zone, under low constraint, for which the rupture does not occur before a number given of cycles superior to the lifetime under consideration for the part.

There exist many expressions of the diagram of endurance:

- Oldest is that of Wöhler:

$$\ln(N) = a - bS_{alt}$$ éq 2.3.1-1

where N is the number of cycles to the rupture,
S_{alt} the alternate constraint applied,
a and b two characteristics of material.

This analytical expression does not give an account well, of a branch horizontal or asymptotic of the curve SN supplements, but it gives a representation often very good of average part of the curve.

- By 1910, Basquin proposes the formula:

$$\ln(N) = a - b\ln(S_{alt})$$ éq 2.3.1-2

to take account of the curve of the curve of Wöhler which connects the branch downward to the horizontal branch.

D = damage of an elementary cycle = $1/N = A\,S_{alt}^{\beta}$ where $A = e^{-a}$ and $\beta = b$.

- Another analytical shape of the curve of Wöhler is proposed in POSTDAM to take account of the curve out of the singular zone:

$$S_{alt} = 1/2(E_C/E)\Delta\sigma$$ éq 2.3.1-3

$$\begin{aligned} \textit{où}\quad & E_C = \text{Module d'Young associé à la courbe de fatigue du matériau,} \\ & E = \text{Module d'Young utilisé pour déterminer les contraintes}. \\ & X = \mathrm{LOG}_{10}(S_{alt}) \\ & N = 10^{a0 + a1X + a2X^2 + a3X^3} \\ & D = \begin{cases} 1/N & \text{si } S_{alt} \geq S_l \quad \text{où } S_l \text{est la limite d'endurance du matériau} \\ 0. & \text{sinon} \end{cases} \end{aligned}$$

Note:

If one takes $a2 = a3 = 0$ et $E_C/E = 1$ *one finds the formula of Basquin.*

The user can introduce the curve of Wöhler into the operator `DEFI_MATERIAU` [U4.43.01] in three distinct forms:

- **a point by point discretized form** (keyword `WOHLER` under the keyword factor `TIREDNESS` in `DEFI_MATERIAU`).

  The curve of Wöhler is in this case a function which gives the number of cycles to the rupture N according to the alternate constraint S_{alt} and for which the user chooses the mode of interpolation:

- 'LOG' ----> interpolation logarithmic curve on the number of cycles to the rupture and on the alternate constraint (formula of Basquin per pieces),
- 'LIN' ----> linear interpolation on the number of cycles to the rupture and on the alternate constraint (this interpolation is disadvised because the curve of Wöhler is absolutely not linear in this reference mark).
- 'LIN', 'LOG' interpolation in logarithmic curve on the number of cycles to the rupture and into linear on the alternating load, which leads to the expression given by Wöhler.

The user must also choose the type of prolongation of the function on the right and on the left (if it is necessary to interpolate the function in an unauthorized point by the definition of the function there is program stop by fatal error).

- **an analytical form of Basquin** (keywords A_BASQUIN and BETA_BASQUIN under the keyword factor TIREDNESS in DEFI_MATERIAU)

  $D=A S_{alt}^{\beta}$ They are the constants A and β used in this formula which is to be introduced by the user (in accordance with code POSTDAM).

- an analytical form except singular zone

$$S_{alt}=\text{contrainte alternée}=1/2\left(E_C/E\right)\Delta\sigma$$
$$X=\text{LOG}_{10}\left(S_{alt}\right)$$
$$N=10^{a0+a1\,X+a2\,X^2+a3\,X^3}$$
$$D=\begin{cases}1/N & \text{si } S_{alt}\geq S_l \quad \text{où } S_l \text{ est la limite d'endurance du matériau}\\ 0. & \text{sinon}\end{cases}$$

The user must introduce:

E_C = Young modulus associated with the curve with tiredness with the material (keyword E_REFE under the keyword factor TIREDNESS in DEFI_MATERIAU)
E = Young modulus used to determine the constraints (keyword E under the keyword factor ELAS in DEFI_MATERIAU),

constants of material $\text{a0}, \text{a1}, \text{a2} \,\text{et}\, \text{a3}$ (keywords A0, A1, A2 and A3 under the keyword factor TIREDNESS in DEFI_MATERIAU)
and S_l limit of endurance of the material (keyword SL under the keyword factor TIREDNESS in DEFI_MATERIAU).

Note:

This expression of the damage is available in the same form in software POSTDAM.

2.3.2 Influence of the geometrical parameters on the endurance

2.3.2.1 Coefficient of stress concentration

According to the geometry of the part, it can be necessary to balance the value of the pressure applied by the coefficient of stress concentration K_T. K_T is a coefficient function of the geometry of the part, geometry of the defect and type of loading.
This coefficient is given by the user under the keyword K_T keyword factor COEF_MULT.

It is used to apply to the history of the loading, a homothety of report K_T, which amounts multiplying all the values of the history of loading by the coefficient K_T.

(The calculation of the damage will be done on a history of loading $\sigma(t) = K_T \times \sigma(t)$).

2.3.2.2 Elastoplastic coefficient of concentration

It can also be necessary to balance the value of the pressure applied by the elastoplastic coefficient of concentration K_e.

The elastoplastic coefficient of concentration K_e (aimed to the articles B3234.3 and B3234.5 of RCC_M [bib4]) is defined as being the relationship between the amplitude of real deformation and the amplitude of fictitious deformation determined by the elastic analysis.

An acceptable value of the coefficient K_e can be given by [bib4]:

$$\begin{cases} K_e = 1 & \text{si} \quad \Delta\sigma < 3S_m \\ K_e = 1 + (1-n)(\Delta\sigma/3S_m - 1)/(n(m-1)) & \text{si} \quad 3S_m < \Delta\sigma < 3mS_m \\ K_e = 1/n & \text{si} \quad 3mS_m < \Delta\sigma \end{cases}$$

where S_m is the acceptable maximum constraint,

and n and m two constants depending on material.

The elastoplastic factor K_e is a report of homothety of the loading. This factor depend on the amplitude of the loading. It is applied, cycle by cycle to the values of the maximum and minimal constraint of each cycle.

Data S_m, n and m are introduced under the keywords `SM_KE_RCCM`, `N_KE_RCCM` and `M_KE_RCCM` under the keyword factor `TIREDNESS` in `DEFI_MATERIAU`.

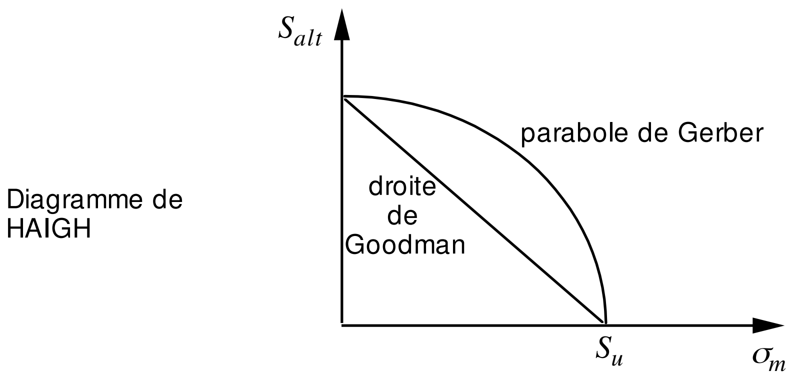
The user asks for the taking into account of the elastoplastic concentration factor while indicating `CORR_KE` : '`RCCM`'in `POST_FATIGUE` [U4.83.01].

2.3.3 Influence of the average constraint

If the part is not subjected to pure or symmetrical alternate constraints, i.e. if the average constraint of the cycle is not worthless, resistance to the dynamic stresses of the material (its limit of endurance) decreases.

One thus balances the curve of Wöhler to calculate the number of effective cycles to the rupture using various diagrams.

The diagram of Haigh makes it possible to determine the evolution of the limit of endurance according to the average constraint σ_m and of the alternate constraint S_{alt}.



Starting from a cycle (S_{alt}, σ_m) identified in the signal one calculates the value of the corrected alternate constraint S'_{alt}.

Si l'on utilise la droite de Goodman $S'_{alt} = \frac{S_{alt}}{1-\frac{\sigma_m}{S_u}}$

Si l'on utilise la parabole de Gerber $S'_{alt} = \frac{S_{alt}}{1-\left(\frac{\sigma_m}{S_u}\right)^2}$

If the line of Goodman is used: $S'_{alt} = \frac{S_{alt}}{1-\frac{\sigma_m}{S_u}}$

If one uses the parabola To stack: $S'_{alt} = \frac{S_{alt}}{1-\left(\frac{\sigma_m}{S_u}\right)^2}$

It is noticed that this last does not differentiate the average constraint in traction and compression.

where S_u is the limit with the rupture of material.

The influence of the average constraint is taken into account only on request of the user (keyword `CORR_HAIG`).

Notice :

If the curve of Wöhler is defined by the analytical form except singular zone [éq 2.3.1 - 3], of the extents of variation of constraints being below the limit of endurance can find itself higher than this one. To avoid that, the limit of endurance is corrected S_l by taking a limit of corrected endurance [bib5]:

$$S'_l = \frac{S_l}{1-\frac{\sigma_m}{S_u}} \quad \text{for the line of Goodman}$$

$$S'_l = \frac{S_l}{1-\left(\frac{\sigma_m}{S_u}\right)^2} \quad \text{for the parabola To stack}$$

2.4 Calculation of the damage: method of Manson-Whetstone sheath

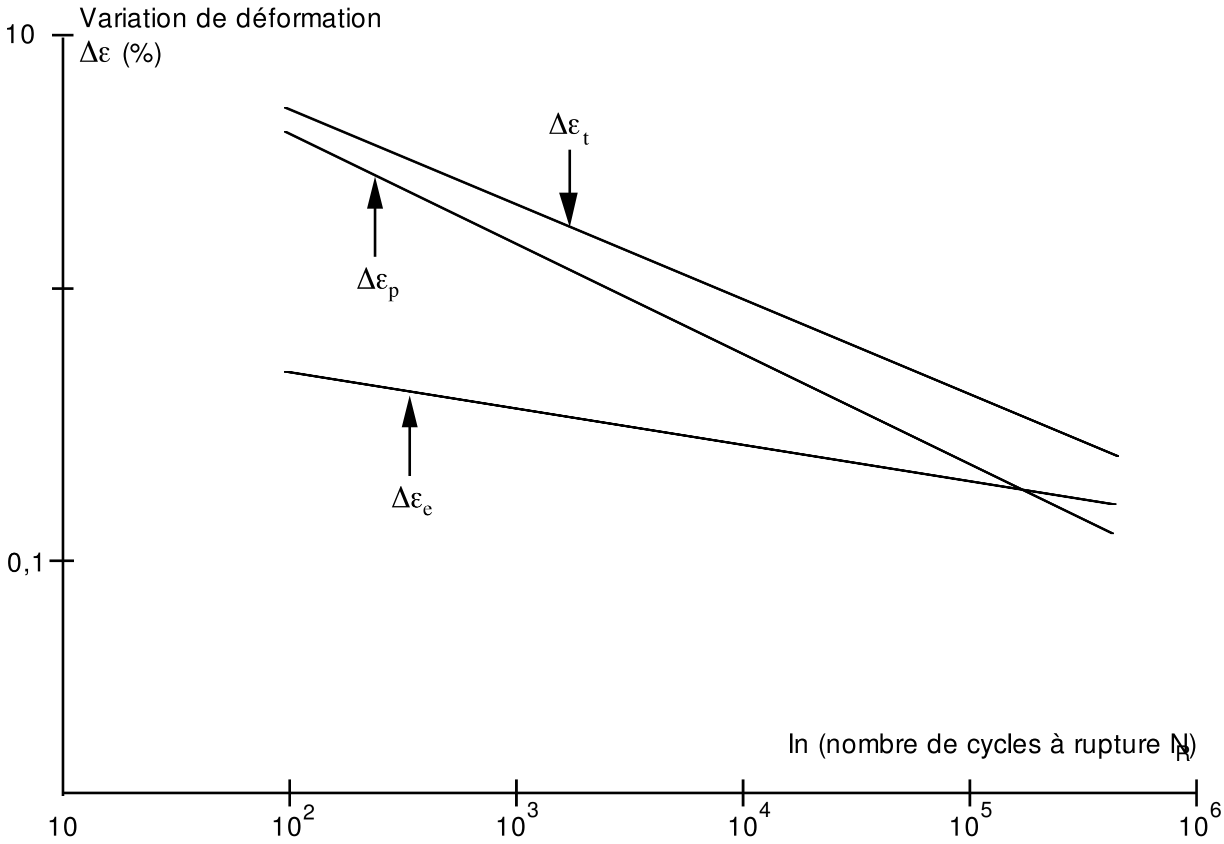
The scope of application of the method of Manson-Whetstone sheath [bib1] is the oligocyclic plastic tiredness, which as its name indicates it shows two fundamental characteristics:

- it is plastic, i.e. a significant plastic deformation occurs with each cycle,
- it is oligocyclic, i.e. the materials have an endurance finished with this kind of request.

To describe the behavior of materials in fatigue oligocyclic plastic, one uses tests with alternate imposed deformation.

In the case, of a uniaxial homogeneous test with an alternated deformation, the number of cycles to the rupture is given starting from a diagram of resistance, which connects the variation of deformation to the number of cycles involving the rupture.

In the diagram of resistance, one separates the deflections total, elastic and plastic. These diagrams are still known under the name of Whetstone sheath-Manson which proposed them in 1950.



Relations $\frac{\Delta \varepsilon_e}{2} - \ln(N)$ and $\frac{\Delta \varepsilon_p}{2} - \ln(N)$ are lines. The relation $\frac{\Delta \varepsilon_t}{2} - \ln(N)$ have, as for it, a curve towards the positive deformations.

It was shown that a relation power connected the plastic deformation $(\Delta\varepsilon_p)$ and elastic strain $(\Delta\varepsilon_e)$ with the number of cycles to the rupture, which leads to the following relations:

$$\Delta \varepsilon_p = A \; N^{-a}$$
$$\Delta \varepsilon_e = B \; N^{-b}$$
$$\Delta \varepsilon_t = A \; N^{-a} + B \; N^{-b}$$

where a and b are two characteristics of material (in general a is close to $0{,}5$ and b neighbor of $0{,}12$); A and B , two constants of material.

The user can introduce the curve of Manson-Whetstone sheath in a single mathematical form: point by point discretized form. It is a function which gives the number of cycles to the rupture N according to the amplitude of deformation $\left(\Delta \varepsilon_{t/2}\right)$.

As for the curve of Wöhler, the user can choose the mode of interpolation on the number of cycles to the rupture and on the amplitude of deformation.

The type of prolongation of the function on the right and on the left is also with the choice of the user.

The damage of an elementary cycle is equal contrary to many cycles to the rupture $D=1/N$.

2.5 Calculation of the damage: method of Taheri

The methods of calculating of the damage proposed by Taheri [bib12] are two: they will be named respectively Taheri-Manson and Taheri-mixed. These methods apply to loadings characterized by a scalar component of standard deformation.

These methods have as a characteristic to take account about application of the elementary cycles of loading with the structure. For this reason, it is advisable to be vigilant with the choice of the method of counting of the cycles. It is strongly advised to use the method of "natural" counting known as method [§2.2.3].

2.5.1 Taheri-Manson method

Are N cycles elementary of half-amplitude $\frac{\Delta\varepsilon_1}{2}, \cdots \frac{\Delta\varepsilon_n}{2}$.

The value of the elementary damage of the first cycle is determined by interpolation on the curve of Manson-Whetstone sheath of material.

The calculation of the elementary damage of the following cycles is carried out by the algorithm:

- if $\frac{\Delta\varepsilon_{i+1}}{2} \geq \frac{\Delta\varepsilon_i}{2}$

  the value of the elementary damage of the cycle $(i+1)$ is determined by interpolation on the curve of Manson-Whetstone sheath of material.

- if $\frac{\Delta\varepsilon_{i+1}}{2} < \frac{\Delta\varepsilon_i}{2}$

  one determines:

$$\frac{\Delta\sigma_{i+1}}{2} = F_{NAPPE}\left(\frac{\Delta\varepsilon_{i+1}}{2},\ \underset{j<i}{\mathrm{Max}}\left(\frac{\Delta\varepsilon_j}{2}\right)\right)$$

  then

$$\frac{\Delta\varepsilon^*_{i+1}}{2} = F_{FONC}\left(\frac{\Delta\sigma_{i+1}}{2}\right).$$

F_{NAPPE} is the cyclic curve of cyclic work hardening with préécrouissage of material.

F_{FONC} is the cyclic curve of work hardening of material.

The value of the damage of the cycle $(i+1)$ is determined by interpolation of $\frac{\Delta\varepsilon^*_{i+1}}{2}$ on the curve of Manson-Whetstone sheath of material.

Notice :

If all the cycles applied are arranged by ascending value of the amplitude of deformation, this method is identical to the method of Manson-Whetstone sheath.

2.5.2 Taheri-Mixed method

Are N cycles elementary, of half-amplitude $\frac{\Delta\varepsilon_1}{2}, \cdots \frac{\Delta\varepsilon_n}{2}$.

The value of the elementary damage of the first cycle is determined by interpolation on the curve of Manson-Whetstone sheath of material.

The calculation of the elementary damage of the following cycles is carried out by the algorithm:

- if $\frac{\Delta\varepsilon_{i+1}}{2} \geq \frac{\Delta\varepsilon_i}{2}$

  the value of the elementary damage of the cycle $(i+1)$ is determined by interpolation on the curve of Manson-Whetstone sheath of material.

- if $\frac{\Delta\varepsilon_{i+1}}{2} < \frac{\Delta\varepsilon_i}{2}$

  one determines:

$$\frac{\Delta\sigma_{i+1}}{2} = F_{NAPPE}\left(\frac{\Delta\varepsilon_{i+1}}{2},\ \operatorname*{Max}_{j<i}\left(\frac{\Delta\varepsilon_j}{2}\right)\right)$$

  where F_{NAPPE} is the cyclic curve of cyclic work hardening with préécrouissage of material.

  The value of the damage of the cycle $(i+1)$ is obtained by interpolation of $\frac{\Delta\sigma_{i+1}}{2}$ on the curve of Wöhler of material.

Notice :

If all the cycles applied to the structure are arranged by ascending value of the amplitude of deformation, this method is identical to the method of Manson-Whetstone sheath.

The damage of an elementary cycle is equal contrary to many cycles to the rupture $D = 1/N$.

2.6 Calculation of the total damage

The simplest approach and most known to determine the total damage of a part subjected to n_i cycles of alternate constraint S_{alt} or of alternate deformation E_{alt} is the linear rule of the damage suggested by Mining:

$$Di=\frac{n_i}{N_i}$$

Under operation, the structures are subjected to various loadings of different amplitudes. Undergone tiredness is due to the accumulation of the elementary damage and the total damage is calculated using the rule of office plurality To mine [bib6]:

$$D_{total}=\sum_i \frac{n_i}{N_i}$$

In the case of Wöhler and Manson-Whetstone sheath, this law supposes that the damage increases linearly with the number of imposed cycles and that it is independent of the level of loading and about application of the levels of loading (whereas in experiments, it is shown that the order of application of the loading is a significant factor for the lifetime of material).

The calculation of the total damage is required by the user with the keyword `OFFICE PLURALITY`.

The methods suggested by Taheri take account about application of the loading, in the calculation of the elementary damage associated with each cycle.

2.7 Conclusion

For the methods based on uniaxial tests, the calculation of the total damage undergone by a part subjected to a history of loading breaks up into several stages:

- extraction of the peaks of the history of loading, to lead to a simpler history,
- extraction of the elementary cycles of the history of loading by a method of counting of cycles,
- calculation of the elementary damage associated with each elementary cycle resulting from the real history of the loading,
  - possibly (and for the method of Wöhler), correction of the loading by a coefficient of stress concentration K_T,
  - possibly (and for the method of Wöhler), correction of the loading by an elastoplastic coefficient of concentration K_e,
  - possibly (and for the method of Wöhler), correction of Haigh to take account of the nonworthless value of the average constraint,
- calculation of the total damage, by a linear rule of office plurality.

3 Calculation of the damage of generalized Lemaître

This law of damage relates to the study of the starting of a macroscopic crack, using a post - processor of mechanics of damage based on a unified formulation of the laws of evolution of the damage. This one uses, on the one hand, of the laws of evolution of the damage specific to the various mechanisms considered, and, on the other hand, a model plus general based on a micromechanical analysis of the phenomenon of starting.

This law offers a single formalism which supposes that the various damage mechanisms all are controlled by the plastic deformations, elastic deformation energy and process of instability.

3.1 The law of Lemaître generalized

The law of Lemaître generalized consists of an enrichment of the method of calculating of damage of Lemaitre [bib7] by the introduction of a law in power (model of Lemaître-Sermage). She is written [bib14]:

$$\begin{cases} \dot{D} = \left[\frac{Y}{S}\right]^s \dot{p} & \text{si } p > p_D \\ D = 0 & \text{sinon} \end{cases} \qquad \text{éq 3.1-1}$$

with:

$$Y = \frac{\sigma_{eq}^2}{2\,E(1-D)^2} R_\nu \quad \text{et} \quad R_\nu = \frac{2}{3}(1+\nu) + 3(1-2\,\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2 .$$

Y is the rate of refund of density of elastic deformation energy.

R_ν is the function of triaxiality.

$\frac{\sigma_H}{\sigma_{eq}}$ is the rate of triaxiality.

$\sigma_{eq} = \sqrt{\frac{3}{2}\,\sigma_{ij}^D\,\sigma_{ij}^D}$ is the equivalent constraint of von Mises.

$\sigma_{ij}^D = \sigma_{ij} - \frac{1}{3}\,\sigma_{kk}\,S_{ij}$ is the diverter of the constraint.

p_D is the threshold of damage.

S and s are characteristics material.

$p(t)$ is the cumulated plastic deformation.

This law thus makes it possible to calculate the damage $D(t)$ starting from the data of the tensor of the constraints $\sigma(t)$ and of the cumulated plastic deformation $p(t)$.

The equation [éq 3.1-1] can be written:

$$\begin{cases} (1-D)^{2s} dD = \left[\frac{C}{S}\right]^s dp & \text{si } p > p_D \\ D = 0 & \text{sinon} \end{cases}$$ éq 3.1-2

with:

$$C = \frac{\sigma_{eq}^2}{2\ E} R_\nu$$

The integration of the equations [éq 3.1-2] enters t_i and t_{i+1} conduit with:

$$\begin{cases} \int_{D(t_i)}^{D(t_{i+1})} (1-D)^{2s} dD = \int_{p(t_i)}^{p(t_{i+1})} \left[\frac{C}{S}\right]^s dp & \text{si } p > p_D \\ D = 0 & \text{sinon} \end{cases}$$ éq 3.1-3

It is noticed that there exists a primitive for the term on the left of the equation [éq 3.1-3] but not for that on the right. A digital diagram of integration is thus used to calculate the integral of it. The equation [éq 3.1-3] can be written:

$$\begin{cases} \frac{[1-D(t_i)]^{2s+1} - [1-D(t_{i+1})]^{2s+1}}{2s+1} = \frac{1}{2}\left(\left[\frac{C(t_i)}{S(t_i)}\right]^s + \left[\frac{C(t_{i+1})}{S(t_{i+1})}\right]^s\right)[p(t_{i+1}) - p(t_i)] & \text{si } p > p_D \\ D(t_{i+1}) = 0 & \text{sinon} \end{cases}$$ éq 3.1-4

It is supposed that $D(t_o) = 0$. The value of the damage $D(t_i),\ i = 0, n$ for each moment t_i can be given starting from the equation [éq 3.1-4]. In *Code_Aster*, this size is named DOM_LEM.

The final damage with the rupture $D_r = D(t_r)$ is thus associated with the time-to-failure t_r .

Remarks :

- *It is considered that the characteristics material E (Young modulus), ν (Poisson's ratio) and S (parameter material) depend on the temperature T .*
- *The value of the Young modulus and the value of the Poisson's ratio are defined in DEFI_MATERIAU [U4.43.01] under the keyword factor ELAS_FO .*
- *Values of S , p_D and of s are defined in DEFI_MATERIAU under the keyword factor DOMMA_LEMAITRE and operands S , ESPS_SEUIL and EXP_S . Parameters S and p_D can depend on the temperature TEMP .*
- *The law of Lemaître is obtained by assigning the value $s=1$*

3.2 Identification of the parameters of the law of Lemaître generalized

It is noticed that the equation [éq 3.1-4] is valid for tiredness and creep. For tiredness, p is the cumulated plastic deformation. For creep, p is the instantaneous plastic deformation if one neglects the elastic strain.

It is noted that the parameters materials S and s depend strongly not only on temperature T but also of the constraint σ (via the expression of C). The process of determination of S and s starting from the fatigue tests was presented in [bib14].

This part aims at presenting a simple method to identify the parameters materials S and s starting from the uniaxial tests of creep.

Initially, the temperature is fixed T. One will carry two creep tests to two levels of constraint σ_1 and σ_2 sufficient close so that parameters S and s can be regarded as constant between these two levels of constraint. One indicates p_{r1} and p_{r1} plastic deformations due to the rupture, measured starting from the tests of creep associated with σ_1 and σ_2, respectively.

Like the constraint σ is maintained constant during the creep test and that $D(t_o)=0$, $D_r=D(t_r)=1$, the equation [éq 3.1-4] can be written for the levels of constraint σ_1 and σ_2 like:

$$\frac{S^s}{2s+1}=[C(\sigma_1)]^s.p_{r1}$$ éq 3.2-1

$$\frac{S^s}{2s+1}=[C(\sigma_2)]^s.p_{r2}$$ éq 3.2-2

Parameters S and s are the solution of the system of equations [éq 3.2-1] and [éq 3.2-2]. From these equations, the parameter s is given like:

$$s=\frac{\log\frac{p_{r2}}{p_{r1}}}{\log\frac{C(\sigma_1)}{C(\sigma_2)}}$$ éq 3.2-3

Then, S can be given is starting from the equation [éq 3.2-1], that is to say starting from the equation [éq 3.2-2].

For $\sigma_1<\sigma_2$, starting from the equation [éq 3.1-7], the value of s is positive (physically acceptable) if and only if $p_{r1}>p_{r2}$. In the contrary case, there does not exist positive solution for the system of equations [éq 3.2-1] and [éq 3.2-2], i.e., the model of damage of Lemaitre is not applicable in this case.

It is noted that this simple method makes it possible to identify the parameters S and s at various temperatures and various levels of pressure applied starting from the experimental/digital curves of creep.

4 Criteria of Crossland and Dang Van Papadopoulos

The criteria [bib9] and [bib13] allow for metal structures subjected to constraints forced following one a large number of cycles to distinguish the loadings damaging from the others.

One can classify the criteria in two categories according to nature of their approach:

- macroscopic approach: criterion of Crossland,
- microscopic approach: criterion of Dang Van Papadopoulos.

The criteria of Crossland and Dang Van Papadopoulos apply to uniaxial or multiaxial loadings periodic.

The goal of these criteria is not to determine a value of damage, but a value of criterion R_{crit} such as:

$$\begin{cases} R_{crit} \leq 0 \quad \text{pas de dommage} \\ R_{crit} > 0 \quad \text{dommage possible (fatigue).} \end{cases}$$

4.1 Criterion of Crossland

The criterion of Crossland is empirical and is written only starting from macroscopic variables.

In fact, starting from trial runs, one could note that the amplitude of cission as well as the hydrostatic pressure played a fundamental role in the mechanisms of tiredness of the structures.

This is why, Crossland applied the criterion:

$$R_{crit} = \tau_a + a\ P_{\max} - b$$

where

$$\tau_a = \frac{1}{2} \underset{0 \leq t_0 \leq T}{\text{Max}} \ \underset{0 \leq t_1 \leq T}{\text{Max}} \ \| \sigma^D_{(t_1)} - \sigma^D_{(t_0)} \| = \text{amplitude of cission}$$

with σ^D diverter of the tensor of the constraints.

$$P_{\max} = \underset{0 \leq t \leq T}{\text{Max}} \left(\frac{1}{3}\ trace\ \sigma \right) = \text{maximum hydrostatic pressure.}$$

$$a = \left(\tau_0 - \frac{d_0}{\sqrt{3}} \right) / \left(\frac{d_0}{\sqrt{3}} \right) \ \text{ et } \ b = \tau_0$$

with:

$\tau_0 =$ limit of endurance in alternated pure shearing,

$d_0 =$ limit of endurance in alternate pure traction and compression.

4.2 Criterion of Dang Van Papadopoulos

It appeared that the crack initiation of tiredness is a microscopic phenomenon occurring on a scale about the grain. This is why, of the criteria of tiredness, starting from local microscopic variables were applied.

The implemented criterion [bib8], [bib9] and [bib10] in *Code_Aster* is the criterion of Dang Van Papadopoulos, who is written in the form:

$$R_{crit}=k^{*}+a\,P_{max}-b$$

where:

$k^{*}=\frac{R}{\sqrt{2}}$ if $R=\underset{0\le t\le T}{Max}\sqrt{\left(\sigma^{D}(t)-C^{*}\right):\left(\sigma^{D}(t)-C^{*}\right)}$

$k^{*}=R$ if $R=\underset{0\le t\le T}{Max}\sqrt{J_{2}(t)}=\underset{0\le t\le T}{Max}\sqrt{\frac{1}{2}\left(\sigma^{D}(t)-C^{*}\right):\left(\sigma^{D}(t)-C^{*}\right)}$

with:

1) R , the ray of the smallest sphere circumscribed with the way of loading within the space of diverters of the constraints;
2) $J_{2}(t)$, the second invariant of the diverters of the constraints;
3) $C^{*}=\underset{C}{Min}\ \underset{t}{Max}\sqrt{\left(\sigma^{D}(t)-C\right):\left(\sigma^{D}(t)-C\right)}$, the center of the hypersphère.

Note:

It is the definition of R who uses $J_{2}(t)$ who is programmed.

$$P_{max}=\text{ maximum hydrostatic pressure }=\underset{0\le t\le T}{\text{Max}}\left(\frac{1}{3}\ trace\ \sigma\right)$$

$$a=\left(\tau_{0}-\frac{d_{0}}{\sqrt{3}}\right)/\left(\frac{d_{0}}{3}\right)\ \text{ and }\ b=\tau_{0}$$

with:

$\tau_{0}=$ limit of endurance in alternated pure shearing,

$d_{0}=$ limit of endurance in alternate pure traction and compression.

The basic idea of Papadopoulos is to write that the grain obeys a criterion of plasticity of the type von Mises instead of the criterion of plasticity of the Tresca type used by Dang Van.

Papadopoulos conducted a campaign of comparisons between the results provided by its criterion and of the experimental results, which shows that the predictions of the criterion of Papadopoulos are excellent for the loadings closely connected; they are a little less precise for the ways nonclosely connected.

In its thesis [bib10] Papadopoulos shows that the criterion of Crossland and the criterion of Dang Van Papadopoulos give the same results for radial loadings.

The algorithm employed for the calculation of the ray of the smallest sphere circumscribed with the way of loading within the space of diverters of constraints, is that proposed in [bib11]. It is about a recurring algorithm which rests on the second invariant of the diverters of the constraints.

Let us note S_i the value of the diverter of the constraints at the moment t_i, O_n the center of the hypersphère to the iteration n, R_n the ray of the hypersphère to the iteration n and x the "isotropic parameter of work hardening" of the algorithm.

- Phase of initialization of the algorithm:

$$O_1 = \frac{1}{N}\sum_{i=1}^{N} S_i$$
$$R_1 = 0.$$

- Iteration of the stage n at the stage $n+1$:

  one supposes O_n and R_n known. One calculates then:

$$D = \|S_{i+1} - O_n\|$$
$$P = D - R_n$$

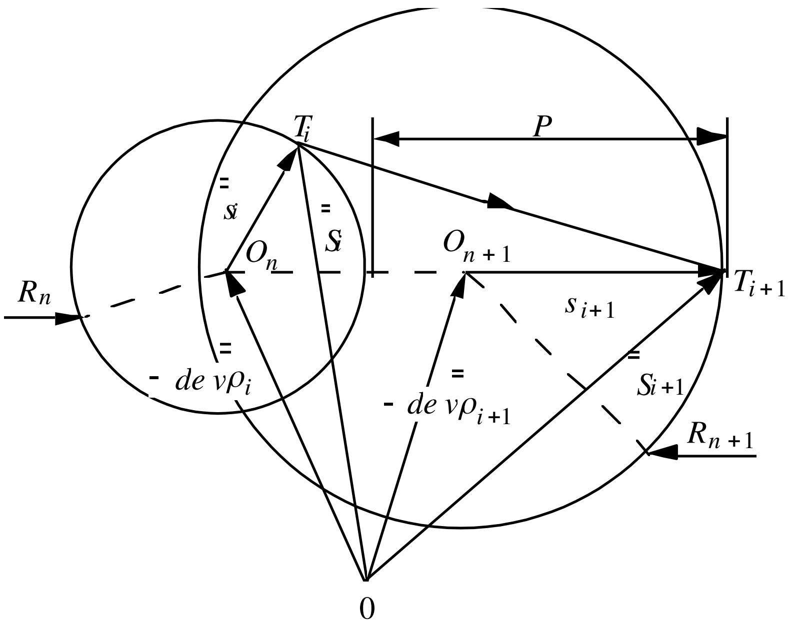
  - If $P > 0$

$$R_{n+1} = R_n + x \;.\; P$$
$$O_{n+1} = S_{i+1} + R_{n+1}\frac{O_n - S_i}{\|O_n - S_{i+1}\|}$$

  - If $P < 0$

$$R_{n+1} = R_n$$
$$O_{n+1} = O_n$$

The algorithm ends when all the points S_i are in the hypersphère of center O_n and of ray R_n .



4.3 Calculation of a value of damage

These two criteria applicable to multiaxial periodic loadings make it possible to say if there is damage or not:

$$\begin{cases} R_{crit} \le 0 & \text{pas de dommage} \\ R_{crit} > 0 & \text{dommage possible (fatigue)}. \end{cases}$$

These criteria do not provide a value of damage. It can however be interesting to calculate a value of damage by using the curves of Wöhler of material. With this intention, should be defined an equivalent constraint σ^*, value to be interpolated on the curve of Wöhler.

The curves of Wöhler can be built starting from shear tests in which case the limit of endurance is τ_0, but are more generally built starting from tensile tests - compression for which the limit of endurance is $d_0 (d_0 < \tau_0)$.

So that there is coherence between the criterion and the curve of Wöhler it is necessary that:

$$\left.\begin{cases} \sigma^* \le \tau_0 & \text{pas de dommage} \\ \sigma^* > \tau_0 & \text{dommage} \end{cases}\right\} \text{pour une courbe de Wöhler définie en cisaillement,}$$
$$\left.\begin{cases} \sigma^* \le d_0 & \text{pas de dommage} \\ \sigma^* > d_0 & \text{dommage} \end{cases}\right\} \text{pour une courbe de Wöhler définie en traction-compression}.$$

It thus seems possible to us to take:

$$\sigma^* = R_{crit} + \tau_0 \text{ pour une courbe de Wöhler en cisaillement (ce qui est assez rare),}$$
$$\sigma^* = (R_{crit} + \tau_0) * (d_0 / \tau_0) \text{ pour une courbe de Wöhler en traction-compression}.$$

In a general way, the user can take $\sigma^* = (R_{crit} + \tau_0) * corr$ where $corr$ is a coefficient of correction introduced by the user.

By default, this coefficient $corr$ is taken equal to (d_0 / τ_0) (case of the curve of Wöhler introduced in traction and compression).

Notice :

In the literature, one does not find presentation of a approach of use of a criterion to calculate a value of damage. It is known however that certain industrialists use such a approach, but without knowing the adopted form of it.

The approach implemented in Code_Aster is proposed by department AMA.

5 Conclusion

In this note the various methods of calculating of the damage available are exposed is in the operator POST_FATIGUE maybe in the operator CALC_FATIGUE, that is to say in the two orders simultaneously.

One can classify these methods in two big classes:

- estimate of the damage to great numbers of cycles,
- estimate of the damage in fatigue oligocyclic plastic.

In the first class of problems, one finds the method of Wöhler, based on uniaxial tests, and which applies to loadings in constraint. One also finds in this class, the criterion of Crossland, which is an empirical criterion being based on macroscopic sizes and the criterion of Dang Van Papadopoulos who is based on microscopic phenomena.
The two criteria are addressed to loadings in constraints which can be uniaxial or multiaxial but periodic.

In the second class of problems, one finds the method of Manson-Whetstone sheath and the methods of Taheri, which apply to loading in deformations.

The whole of the methods based on uniaxial tests (method of Wöhler, method of Manson - Whetstone sheath and methods of Taheri) are available in the two operators POST_FATIGUE and CALC_FATIGUE.

The criteria, as for them, are only available in POST_FATIGUE.

6 Bibliography

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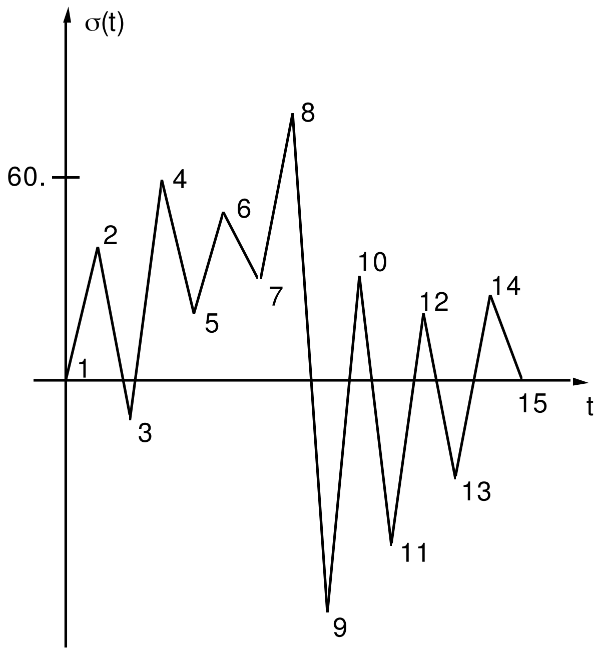
7 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
6	*A.M DONORE, F. MEISSONNIER EDF-R&D/AMA*	initial text
7.4	*A.M DONORE, F. MEISSONNIER EDF-R&D/AMA*	

Annexe 1

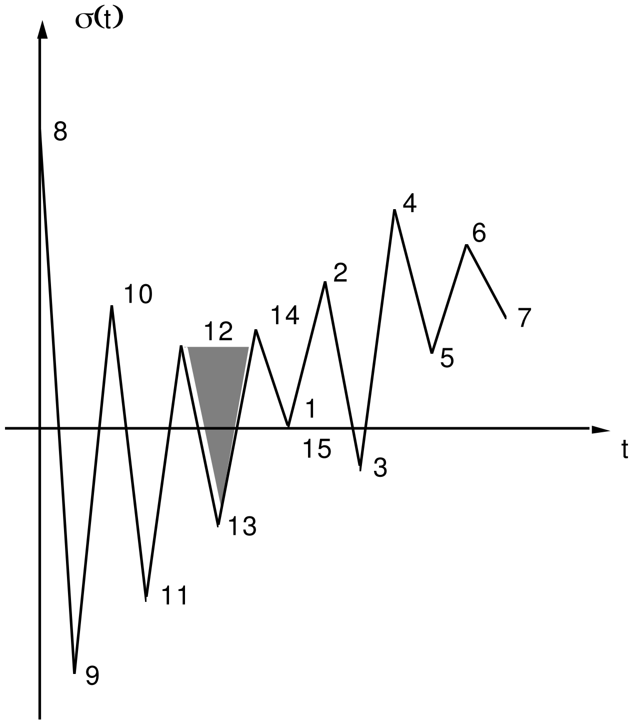
The following history of loading is considered (which for the example is considered of type forced):

N° not	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Moment	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
Loading	0.	40.	– 10.	60.	20.	50.	30.	80.	– 70.	30.	– 50.	20.	– 30.	25.	0.



The stage of rearrangement of the history of loading leads to the following loading:

N° not	8	9	10	11	12	13	14	15	2	3	4	5	6	7
Loading	80.	– 70.	30.	– 50.	20.	– 30.	25.	0.	40.	– 10.	60.	20.	50.	30.

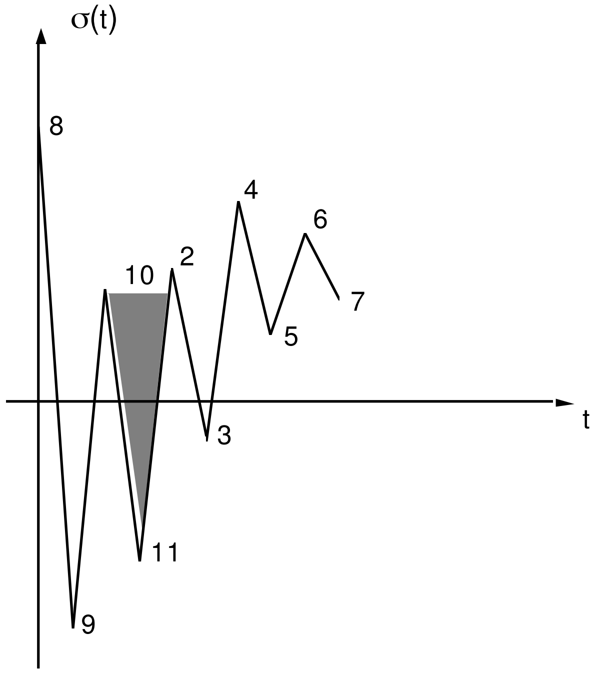


The second stage consists in extracting the elementary cycles. The first extracted cycle is the cycle defined by items 12 and 13 since $|\sigma(12)-\sigma(13)|$ is lower than $|\sigma(14)-\sigma(13)|$ and $|\sigma(12)-\sigma(13)|$ is lower than $|\sigma(12)-\sigma(11)|$.

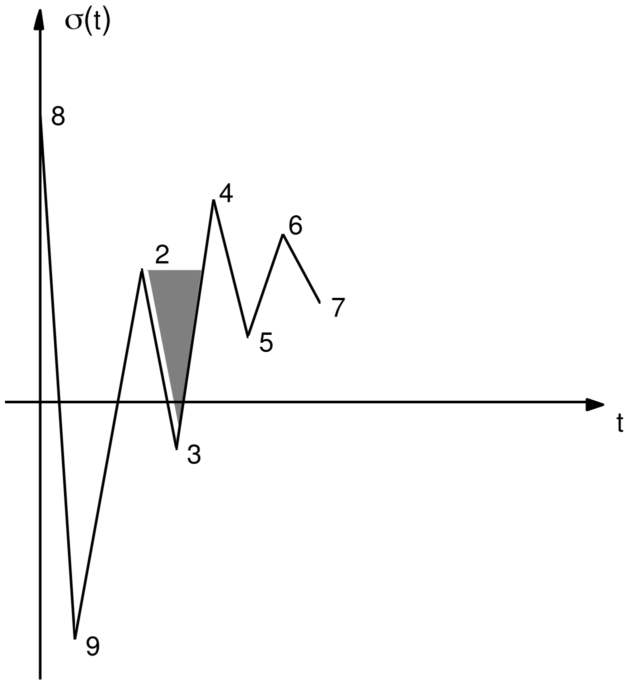
Cycle 1: $\mathrm{VALMAX}=20$ and $\mathrm{VALMIN}=-30$.

The cycle having been extracted one removes these two points of the history of the loading, and one starts again on the remaining history.

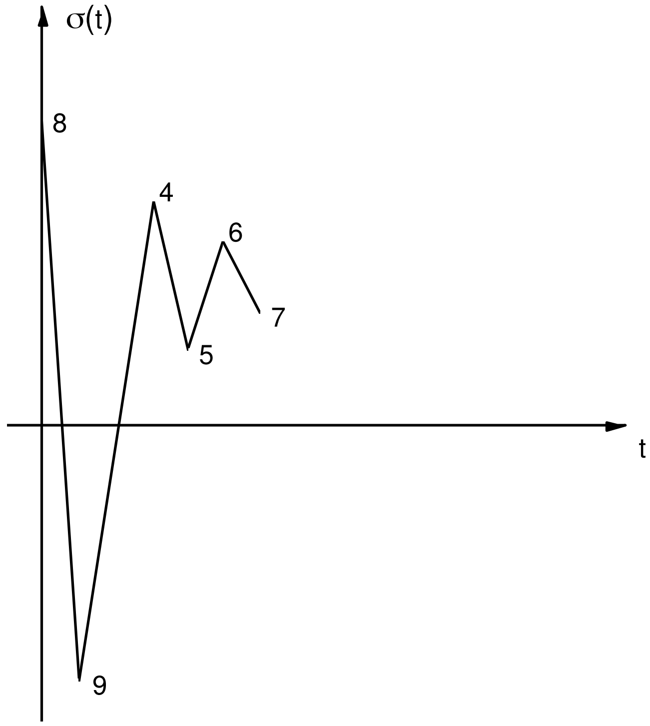
The following cycle extract is the cycle defined by items 14 and 15.
Cycle 2: $\mathrm{VALMAX}=25$ and $\mathrm{VALMIN}=0$.
The remaining history, after suppression of these two points is:



One extracts then the cycle defined by items 10 and 11.
Cycle 3: $\mathrm{VALMAX}=30$ and $\mathrm{VALMIN}=-50$.
One sets out again on the following history of loading:

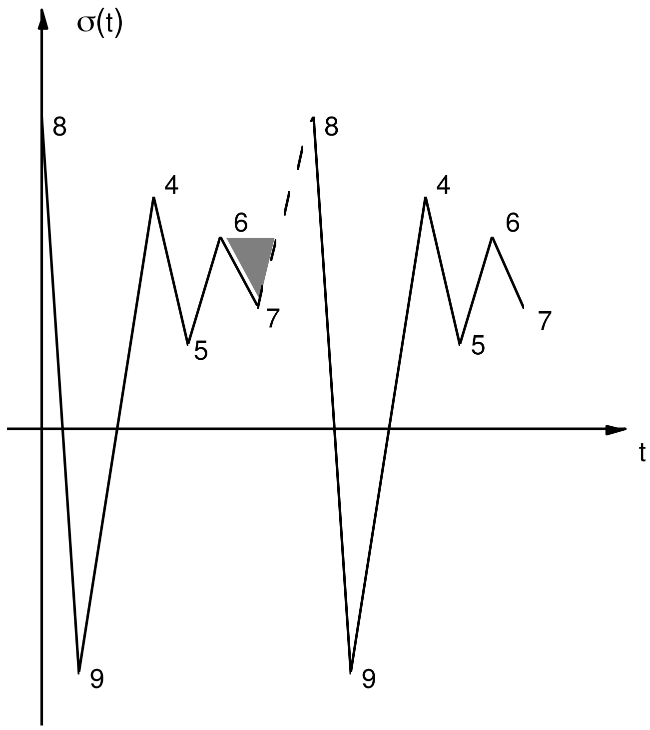


The following cycle extract is defined by items 2 and 3.
Cycle 4: $\mathrm{VALMAX}=40$ and $\mathrm{VALMIN}=-10$.
The remaining history of loading is (it is the residue of the history of the loading):



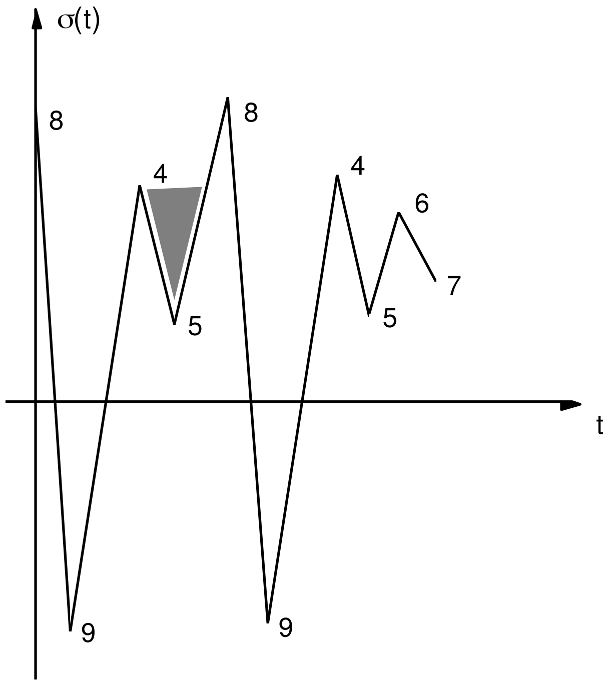
One cannot extract any more from cycles, because all the history of the loading was traversed.

One thus passes at the third stage, which consists in treating the residue:

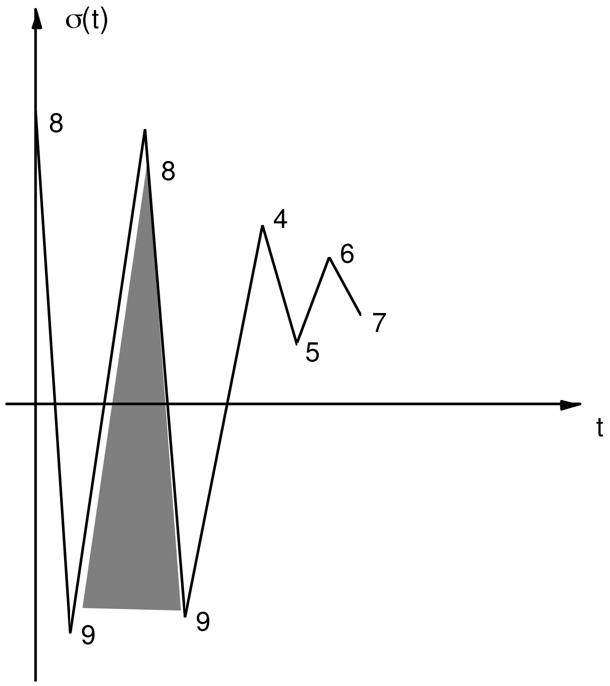


One adds the same residue with his continuation, and one starts again the second phase on this loading.

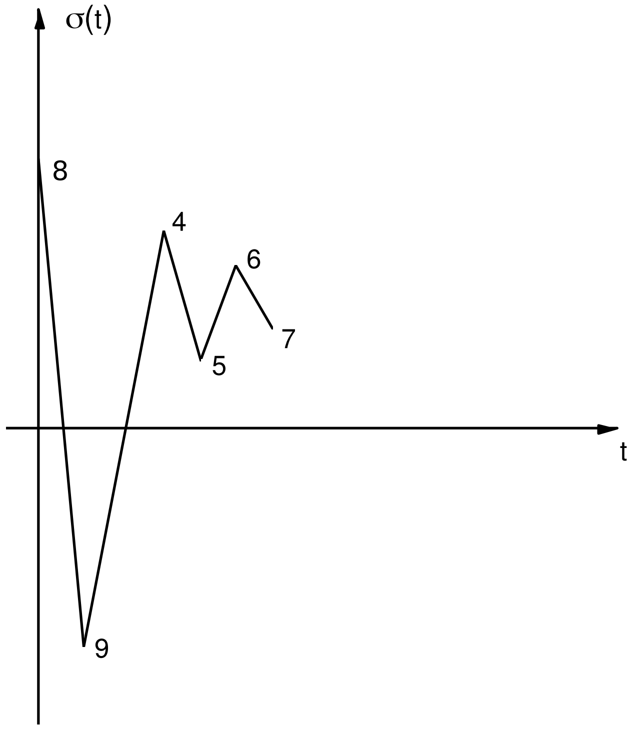
The following cycle extract is defined by items 6 and 7.
Cycle 5: $VALMAX=50$ and $VALMIN=30$.
The remaining history of loading is:



The following cycle extract is defined by items 4 and 5.
Cycle 6: VALMAX = 60. and VALMIN = 20.
The remaining history of loading is:



The last extracted cycle is a cycle defined by items 8 and 9.
Cycle 7: $\mathrm{VALMAX}=80$ and $\mathrm{VALMIN}=-70$.



It is noticed well that when one applies counting RAINFLOW to the unit made up of the two residues, one obtains in the end counting again the initial residue.