Multiaxial criteria of starting in fatigue

Summary:

In this note we propose a formulation of the criteria of MATAKE, DANG VAN and FATEMI-SOCIE and criteria in formula adapted to the framework of the office plurality of damage under multiaxial loading **periodical** and **not periodical**. These criteria are usable in the order `CALC_FATIGEU`.

The first part of this document is devoted to the criteria of MATAKE and DANG VAN adapted to the periodic multiaxial loadings. In this part after having approached the concepts of endurance and office plurality of damage and the general form of the criteria of tiredness, we describe the two models of DANG VAN and MATAKE (Plan criticizes) designed to carry out calculations of office plurality of damage under multiaxial loading. One details there the definition of the various plans of shearing associated with the points of Gauss or the nodes, as well as the definition of an amplitude of loading through the circle circumscribed with the way of the loading in the plan of shearing. Finally criteria available in *Code_Aster* are presented.

In the second part we propose a formulation of the criteria of MATAKE, DANG VAN and FATEMI-SOCIE within the framework of the office plurality of damage under nonperiodic multiaxial loading. To define a cycle in the variable case amplitude, we reduce the history of the loading to a unidimensional function of time by projecting the point of the vector shearing on an axis, and we use a method of counting of cycles. Here we choose method RAINFLOW. The criteria of MATAKE, DANG VAN and FATEMI-SOCIE adapted to the office plurality of damage under nonperiodic loading are established in *Code_Aster*.

Besides the well established criteria, one lays out in Code_Aster **criteria in formula** allowing the user of to build new preset criteria according to the sizes. This kind of criterion is detailed in third part of this document.

Lastly, the option `VMIS_TRESCA` allows to calculate the maximum variation, in the course of time, of a tensor of constraint according to the criteria of Von Mises and Tresca.

Contents

1 Introduction

The models of endurance in fatigue multiaxial under periodic loading are of the models of the following type:

$$VAR_{amplitude}+a\times VAR_{moyenne}<b \ ,$$

where b is the threshold of endurance in simple shearing, and *has* a positive constant without dimension. $VAR_{amplitude}$ is a certain definition of the amplitude (half of the variation) of the cycle of loading and $VAR_{moyenne}$ is a variable in connection with the constraint (or sometimes deformation) or the constraints (or sometimes the deformations) average. The models are characterized by definitions different from $VAR_{amplitude}$ and $VAR_{moyenne}$.
To pass from the endurance to the office plurality of the damage, one introduces an equivalent constraint definite by:

$$\sigma_{eq}=VAR_{amplitude}+a\times VAR_{moyenne} \ .$$

This equivalent constraint gives us a unit damage on the curve of tiredness. Like the second member of the inequation b corresponds to the threshold in shearing, one needs a curve of tiredness in shearing. But the curves of tiredness in shearing are rare since difficult to obtain, one thus tries to use the curves of tiredness in traction alternate compression. For that it is necessary to about multiply the equivalent constraint by a corrective coefficient $\sqrt{3}$.

The macroscopic models of MATAKE (critical plan) and macro microphone of DANG-VAN are described. It is shown that under certain assumptions the model of DANG-VAN is similar to the macroscopic model of MATAKE. The only difference lies in the variable $VAR_{moyenne}$: DANG-VAN uses the hydrostatic pressure, while MATAKE employs the normal constraint as regards maximum amplitude of shearing.

After having defined the plan of shearing, we express the shear stress in this plan. The plans of shearing are then explored according to a method described in the reference [bib4] which consists in cutting out the surface of a sphere into pieces of equal sizes.

The normal vectors being known we then determine for each plan the points which are most distant from/to each other. Among those we find the two points which are most distant one from the other. That being made we use, if necessary, the method of the circle passing by three points in order to obtain the circle circumscribed with the way of loading.

In the first part of this document we present the models of endurance in fatigue multiaxial under loading periodicals, as well as the concept of office plurality of damage. The passage of the endurance to the office plurality of damage is also approached.

In the second part the criteria of MATAKE and DANG-VAN are then presented under the aspects limiting of endurance and office plurality of damage under periodic loading.

The third part is devoted to the definition of the plan of shearing, the expression of shear stresses in this plan and finally, in the manner of exploring the plans of shearing.

The fourth part is dedicated to the determination of the circle circumscribed with the way of shearing in the plan of the same name. Finally we describe the criteria and the sizes which are introduced into *Code_Aster*.

After having extended the models of MATAKE and DANG-VAN to the office plurality of damage under periodic loading, we present the adaptation of these models to the office plurality of damage under nonperiodic loading. Thus, the fifth part is devoted to the definition of the elementary equivalent constraint. We describe also the criterion of modified FATEMI-SOCIE.

The sixth part is reserved in the manner of selecting the axis (or the two axes) on which is project the history of the cission.

The seventh part is dedicated to the projection itself of the point of the vector cission on this axis or these two axes. Lastly, concerning the criteria of MATAKE and DANG-VAN formulated in office plurality of damage under nonperiodic loading, we describe the sizes which are introduced into *Code_Aster.*

2 Preliminaries

In this part we treat the concepts of limit of endurance and office plurality of damage. We also present the general form of the criteria of tiredness.

2.1 Limit of endurance and office plurality of damage, uniaxial case

In the uniaxial case, the rigorous definition of the threshold of endurance is the half-amplitude (half of the variation) of loading defined in constraint below which the lifetime is infinite. However, as in practice the lifetime can never be infinite, one defines limits of endurance in 10^7, 10^8, etc cycles of loading. There exists another way of seeing the things: since in practice the infinite lifetime does not exist, one uses the concept of office plurality of damage. The approach by the office plurality of damage consists in defining a limit of many cycles beyond which the cumulated damage is equal to one. Thus limit with 10^7 wants to say that afterwards 10^7 cycles the cumulated damage is equal to 1.

2.2 Criterion of tiredness, multiaxial case

In the literature a certain number of criteria were proposed to define it **threshold of endurance** under multiaxial cyclic loading. The general form of these criteria is:

$$VAR_{amplitude} + a \times VAR_{moyenne} < b \quad \text{éq 2.2-1}$$

where b is the threshold of endurance in simple shearing, a is a positive constant without dimension. $VAR_{amplitude}$ is a certain definition of the half-amplitude (half of the variation) of the cycle and $VAR_{moyenne}$ is a variable in connection with the constraint (or sometimes deformation) or the constraints (or sometimes the deformations) average. Various models are characterized by definitions different from $VAR_{amplitude}$ and $VAR_{moyenne}$.

To pass from the endurance to **office plurality of the damage**, one can define a constraint (or a deformation) equivalent:

$$\sigma_{eq} = VAR_{amplitude} + a \times VAR_{moyenne} \quad \text{éq 2.2-2}$$

This equivalent constraint gives us a unit damage on the curve of tiredness. As the second member of the inequation [éq 2.2-1] corresponds to the threshold in shearing, one needs a curve of tiredness in shearing. But the curves of tiredness in shearing are rare since difficult to obtain, one thus tries to use the curves of tiredness in traction alternate compression. For that it is necessary to be coherent at least on the level of the threshold of endurance i.e. to multiply σ_{eq} by a constant about $\sqrt{3}$ to be able to use the curve of tiredness in traction. The value $\sqrt{3}$ is the exact value for a criterion of the type Put, in experiments this coefficient is smaller than $\sqrt{3}$.

2.3 Definition of an amplitude of loading in the multiaxial case

In *Code_Aster*, there exist two definitions of amplitude of loading in the multiaxial case:

A : ray (half diameter) of the sphere circumscribed with the way of the loading;
B : half of the maximum of the distance between two unspecified points of the way.

It is clear that in the case of a loading being defined on a sphere, A and B give the same amplitude. On the other hand, if one takes a way (two-dimensional) in the form of an equilateral triangle of with dimensions l, the definition A we gives $l/\sqrt{3}$, while the definition B we gives $l/2$. To work within a conservative framework we take as definition of the amplitude (half-variation) of a way of loading the ray of the sphere (or rings for the case 2D) circumscribed.

2.4 Definition of the plan of shearing

In a point M of a continuous medium we express the tensor of the constraints $\boldsymbol{\sigma}$ in an orthonormal reference mark $(O, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$. With the unit normal $\boldsymbol{n}$ components (n_x, n_y, n_z) in the orthonormal reference mark, we associate the vector forced $\boldsymbol{F}=\boldsymbol{\sigma}.\boldsymbol{n}$ components (F_x, F_y, F_z). This vector $\boldsymbol{F}$ can break up into a normal vector with $\boldsymbol{n}$ and a scalar carried by $\boldsymbol{n}$, that is to say:

$$\boldsymbol{F}=N\,\boldsymbol{n}+\boldsymbol{\tau} \qquad \text{éq 2.4-1}$$

where N represent the normal constraint and the vector τ the shear stress. In the reference mark $(O, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$, components of the vector τ are noted: (τ_x, τ_y, τ_z). The vector τ results directly from [éq 2.4-1] and the normal constraint:

$$N=\boldsymbol{F}\,.\,\boldsymbol{n} \quad \text{from where} \quad \boldsymbol{\tau}=\boldsymbol{F}-\boldsymbol{F}\,.\,\boldsymbol{n}\;\boldsymbol{n}\,. \qquad \text{éq 2.4-2}$$

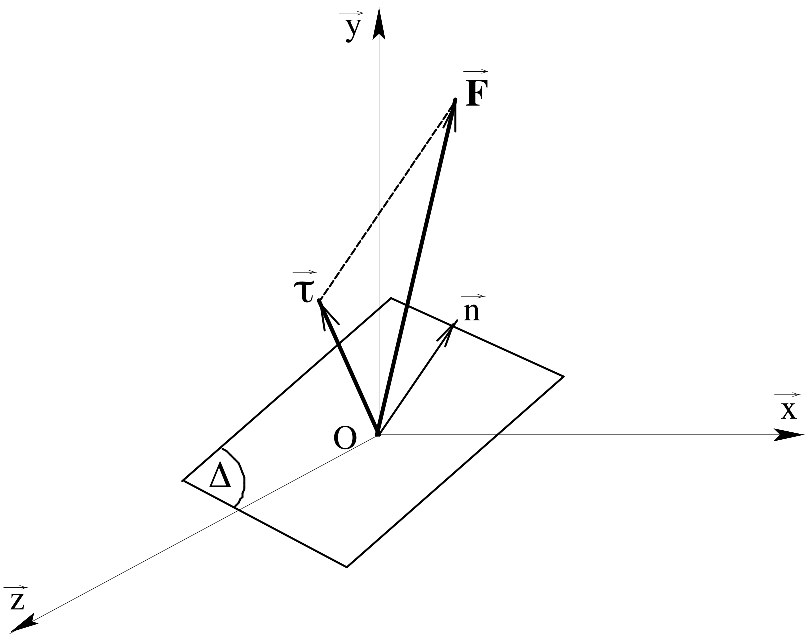


Figure 2.4-a: Representation of the vectors of constraint $\boldsymbol{F}$ and of shear stress $\boldsymbol{\tau}$

3 Criteria of MATAKE (critical plan) and DANG VAN

Here we clarify the criterion of MATAKE and DANG VAN at the same time from the limiting point of view of endurance and the point of view of the office plurality of damage.

3.1 Criterion of MATAKE

In this kind of criterion deformation and stress fields is calculated under the assumption of elasticity, confer reference [bib1]. As it was known as in chapter 2, in the multiaxial case the criterion of endurance is generally written in the form:

$$VAR_{amplitude} + a \times VAR_{moyenne} < b \quad \text{éq 3.1-1}$$

Amplitude of loading : In the case of the criterion of MATAKE at each point of the structure (or not of Gauss for a calculation with the finite elements) to calculate $VAR_{amplitude}$ one proceeds in the following way:

[1] for each plan of normal $\boldsymbol{n}$ one calculates the amplitude of shearing by determining the circle circumscribed with the way of shearing in this plan;

[2] the normal is sought $\boldsymbol{n}*$ for which the amplitude is maximum. This amplitude is indicated by $\Delta\tau(\boldsymbol{n}*)$.

Average constraint : For the calculation of $VAR_{moyenne}$ one proceeds in the following way:

[1] as regards normal $\boldsymbol{n}*$ one calculates on a cycle the maximum normal constraint indicated by $N_{\max}(\boldsymbol{n}*)$.

The criterion of endurance is written:

$$\frac{\Delta\tau(\boldsymbol{n}*)}{2} + a\,N_{\max}(\boldsymbol{n}*) \leq b \ ,$$

where a and b are two positive constants and b represent the limit of endurance in simple shearing.

Identification of the constants : to determine the constants a and b two simple tests should be used. Two possibilities exist:

A pure shear test plus an alternate tensile test compression. In this case the constants are given by: $b=\tau_0 \quad a=\left(\tau_0-\frac{d_0}{2}\right)/\frac{d_0}{2}$, where τ_0 represent the limit of endurance in alternated pure shearing and d_0 limit of endurance in alternate pure traction and compression.

Two tensile tests compression, alternated and the other not. The constants are given by:

$$a=\frac{(\Delta\sigma_2-\Delta\sigma_1)}{(\Delta\sigma_1-\Delta\sigma_2)-2\,\sigma_m},$$

$$b=\frac{\sigma_m}{(\Delta\sigma_2-\Delta\sigma_1)+2\,\sigma_m}\times\frac{\Delta\sigma_1}{2}$$

where $\Delta\sigma_1$ is the amplitude of loading for the alternate case and $\Delta\sigma_2$ for the case where the average constraint is nonworthless.

3.2 Criterion of DANG VAN

It is supposed that the material remains overall elastic while it is plasticized locally. The interesting physical assumption of the model is that the material adapts locally (it becomes rubber band after being last by plasticity) below the limit of endurance, which corresponds to nonthe initiation of crack. Above the limit of endurance there is locally accommodation plastic thus initiation of crack.
The basic assumptions of the microphone-macro interaction, Flax-Taylor, make it possible to write:

$$\sigma_{ij}^{Loc}(t)=\sigma_{ij}(t)+\rho_{ij}(t)$$
$$\rho_{ij}(t)=-2\,\mu\,\varepsilon_{ij}^{p}(t)$$

One indicates the local constraint by $\sigma_{ij}^{Loc}(t)$, the total constraint by $\sigma_{ij}(t)$, the local residual stress by $\rho_{ij}(t)$ and by $\varepsilon_{ij}^{p}(t)$ local plastic deformation. As soon as there is adaptation the local plastic deformation becomes constant and thus the local residual stress also.

Criterion of plasticity:
In a point of the continuous medium (where there is a distribution of the crystallographic directions random of the grains), one supposes that there is only one grain which is plasticized and this, following only one system of slip. This system of slip will be that which will be most favorably directed, i.e., the grain in which the greatest scission will occur (the projection of the vector shearing on a given direction). The slip is done in the plans of normal $\boldsymbol{n}=(n_1, n_2, n_3)$ and the direction of slip is defined by the vector $\boldsymbol{m}=(m_1, m_2, m_3)$. Two vectors $\boldsymbol{n}$ and $\boldsymbol{m}$ are orthogonal.
The law of **Schmid** known as that so that there is no irreversible slip (plastic deformation) it is necessary that the scission, a certain threshold does not exceed, that is to say:

$$\forall\,\boldsymbol{m}\quad\forall\,\boldsymbol{n}\quad|\tau^{Loc}(\boldsymbol{n},\boldsymbol{m},t)|-\tau_y^{Loc}(t)\leq 0 \qquad \text{éq 3.2-1}$$

where

$$\tau^{Loc}(t)=a_{ij}\sigma_{ij}^{loc}\quad\text{et}\quad a_{ij}=\frac{1}{2}(m_i n_j+n_i m_j)$$

The drawing of [Figure 3.2-a] the watch that the maximum value of τ^{Loc} , indicated by $\tau_{\max}^{Loc}$, is obtained by the orthogonal projection of $F^{Loc}=\sigma_{ij}^{Loc}\,n_j$ as regards normal $\boldsymbol{n}$. The relation [éq 3.2-1] must in particular be checked if one replaces τ^{Loc} by its raising $\tau_{\max}^{Loc}$, this one is written then:

$$\forall\,\boldsymbol{n}\quad|\tau_{\max}^{Loc}(\boldsymbol{n},t)|-\tau_y^{Loc}(t)\leq 0 \qquad \text{éq 3.2-2}$$

where $\tau_y^{Loc}(t)$ is the threshold of the microscopic or local scission. $\tau_y^{Loc}(t)$ depends on the variables of work hardening.

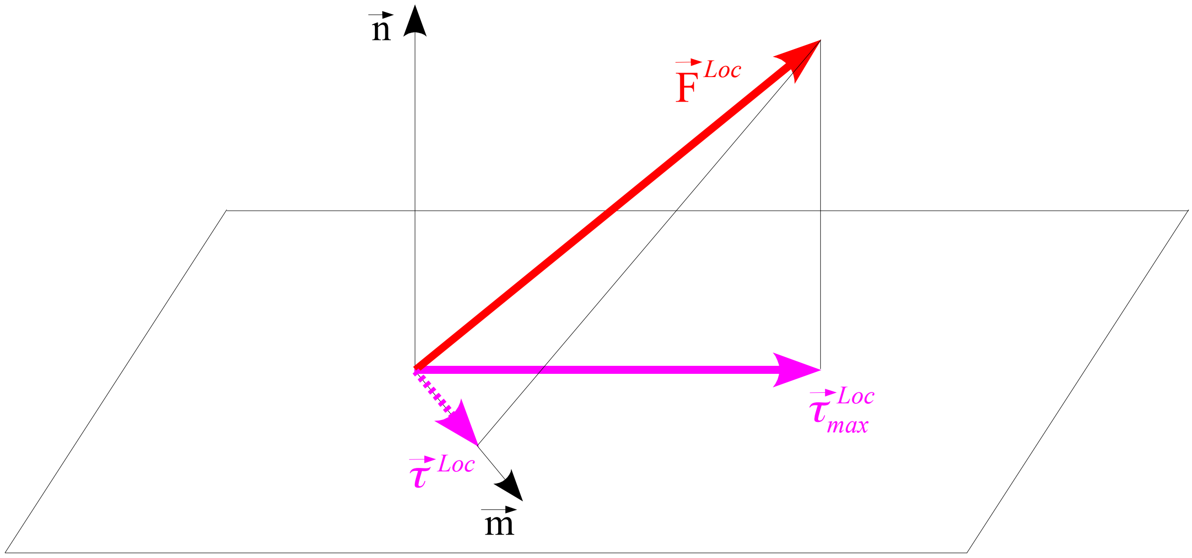


Figure 3.2-a: Projection of F^{Loc} as regards normal $\boldsymbol{n}$

One chooses a microscopic work hardening of the linear isotropic type. That makes it possible to show the existence of a field of adaptation [bib2], [bib3].
At the state adapted by analogy with the formula:

$$\sigma_{ij}^{Loc}(t)=\sigma_{ij}(t)+\rho_{ij}^{*}$$

one has, if one places oneself in the plan $(\boldsymbol{n},\boldsymbol{m})$ in such a way that the scission is maximum, the following formula:

$$\tau_{\max}^{Loc}(\boldsymbol{n},t)=\tau(n,t)+\tau^{*}(n)$$

where $\tau(\boldsymbol{n},t)$ is the vector macroscopic shearing defined in [the Figure 3.2-b] and where $\tau^{*}(\boldsymbol{n})$ is the microscopic vector residual shearing (independent of time since we are in an adapted state).

Criterion of tiredness
Introduction of the maximum pressure: DANG VAN uses instead of the normal constraint on a plan, as that is done in the model MATAKE, the maximum hydrostatic pressure on a cycle. The criterion is thus written:

$$\underset{\boldsymbol{n},t}{MAX}\left(\left|\tau_{\max}^{Loc}(\boldsymbol{n},t)\right|+a\ P_{\max}^{Loc}\right)\leq b$$

As the hydrostatic pressures local and total are identical the criterion becomes:

$$\underset{\boldsymbol{n},t}{MAX}\left(\left|\tau_{\max}^{Loc}(\boldsymbol{n},t)\right|+a\ P_{\max}\right)\leq b$$

For a positive maximum pressure we have:

$$\underset{\boldsymbol{n},t}{MAX}\left(\left|\tau_{\max}^{Loc}(\boldsymbol{n},t)\right|\right)+a\ P_{\max}\leq b$$

For one **always negative pressure** one can take $P_{\max}=0$ to remain conservative.

Assumption on $\tau^*(\boldsymbol{n})$

In the radial case where the direction of maximum shearing is defined in advance one can calculate in an exact way $\tau^*(\boldsymbol{n})$. In the case general DANG VAN proposes the following method to do a calculation simplified of $\tau^*(\boldsymbol{n})$. One gives for a plan n the macroscopic way of the vector shearing defined previously. The vector residual shearing taking into account the preceding assumption is defined by MO, where M is the center of the circle circumscribed with the way of the end of the vector shearing in the plan of shearing.

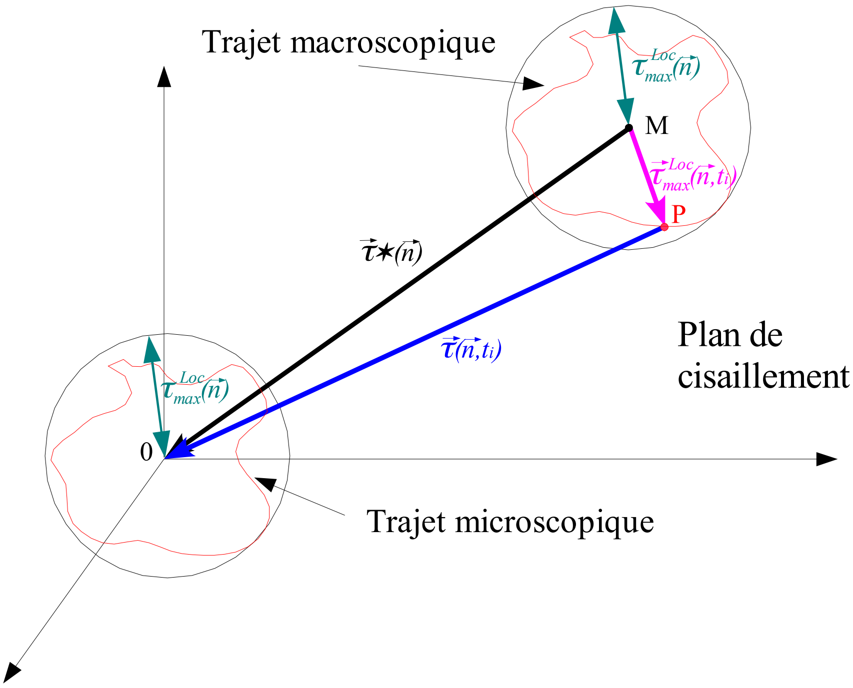


Figure 3.2-b: Ways macro microphone/in the plan of shearing

Final formulation: taking into account the two formulas :

$$\tau_{\max}^{Loc}(\boldsymbol{n},t)=\tau(\boldsymbol{n},t)+\tau^*(\boldsymbol{n}) \quad \text{and} \quad \underset{\boldsymbol{n},t}{MAX}\left(\left|\tau_{\max}^{Loc}(\boldsymbol{n},t)\right|\right)+a\ P_{\max}\leq b$$

one finds oneself with

$$\underset{\boldsymbol{n},t}{MAX}\left(\left|\boldsymbol{MP}\right|\right)+a\ P_{\max}\leq b$$

where P is a point running of the way of shearing in the plan of normal $\boldsymbol{n}$.

Identification of the constants : to determine the constants a and b two simple tests should be used. Two possibilities exist:

- **A pure shear test plus a tensile test alternate compression**. In this case the constants are given by: $b=\tau_0$ $a=(\tau_0-d_0/2)/(d_0/3)$.
- **Two tensile tests compression, alternated and the other not**. The constants are given by:

$$a=\frac{3}{2}\times\frac{\left(\Delta\sigma_2-\Delta\sigma_1\right)}{\left(\Delta\sigma_1-\Delta\sigma_2\right)-2\ \sigma_m} \qquad b=\frac{\sigma_m}{\left(\Delta\sigma_2-\Delta\sigma_1\right)+2\ \sigma_m}\times\frac{\Delta\sigma_1}{2}\ .$$

with $\Delta\sigma_1$ the amplitude of loading for the alternate case and $\Delta\sigma_2$ for the case where the average constraint is nonworthless.

3.3 MATAKE and DANG VAN modified for the office plurality of damage

The models of MATAKE and DANG VAN were proposed initially for the study of the limit of endurance. As the infinite lifetime does not exist one uses limits of endurance with, 10^6, 10^7, 10^n cycles of loading. Thus the initial criteria of MATAKE and DANG VAN are presented like criteria of going beyond a threshold and do not give an office plurality of damage. The use, in particular of the criterion of DANG VAN, in automotive industries is suitable since the sought objective is nonthe going beyond a threshold of endurance contrary to the problems of EDF where one wishes to follow the damage.
Thus we use for the office plurality of damage an equivalent constraint of MATAKE or DANG VAN defined by:

$$\text{MATAKE} \qquad \sigma_{\text{eq}} = \frac{\Delta\tau}{2}(n^*) + a\,N_{\max}(n^*).$$

$$\text{DANG VAN} \qquad \sigma_{\text{eq}} = \text{MAX}_{\boldsymbol{n},t}(|\boldsymbol{MP}|) + aP_{\max}$$

The taking into account of the surface treatment east summarized with the taking into account of the harmful effect of the pre - work hardening over the lifetime in controlled deformation [bib5]. In the models of MATAKE and DANG VAN the effect of pre-work hardening is taken into account by multiplying the half-amplitude of shear stress by a corrective coefficient higher than the unit, noted c_p :

$$\text{MATAKE} \qquad \sigma_{\text{eq}} = c_p \frac{\Delta\tau}{2}(n^*) + a\,N_{\max}(n^*),$$

$$\text{DANG VAN} \qquad \sigma_{\text{eq}} = c_p \text{MAX}_{\boldsymbol{n},t}(|\boldsymbol{MP}|) + aP_{\max}$$

These equivalent constraints are to be used on a curve of tiredness in shearing. For the use on a curve of tiredness in traction compression it is necessary to multiply these equivalent constraints by a corrective coefficient, noted here α :

$$\text{MATAKE} \qquad \sigma_{\text{eq}} = \alpha\left(c_p \frac{\Delta\tau}{2}(n^*) + a\,N_{\max}(n^*)\right),$$

$$\text{DANG VAN} \qquad \sigma_{\text{eq}} = \alpha\left(c^p \text{MAX}_{\boldsymbol{n},t}(|\boldsymbol{MP}|) + aP_{\max}\right)$$

4 Calculation of the plan of maximum shearing

We use here the definition of the plan of shearing introduced in the paragraph [§2.4]. Practically, for us the point M continuous medium will be a point of Gauss.

4.1 Expression of shear stresses in the plan Δ

For reasons of symmetry we vary the unit normal $\boldsymbol{n}$ according to a half-sphere using the angles γ and φ , cf [Figure 4.1-a].
In the reference mark $(O, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$, the unit normal vector $\boldsymbol{n}$ is defined by:

$$n_x = \sin\gamma \ \cos\varphi \quad n_y = \sin\gamma \ \sin\varphi \quad n_z = \cos\gamma \ . \qquad \text{éq 4.1-1}$$

We introduce a new reference mark $(O, \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{n})$ where $\boldsymbol{n}$ is perpendicular to the plan of shearing Δ and where $\boldsymbol{u}$ and $\boldsymbol{v}$ are in this plan, cf [Figure 4.1-a]. In the reference mark $(O, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ unit vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are respectively defined by:

$$u_x = -\sin\varphi \quad u_y = \cos\varphi \quad u_z = 0 \ , \qquad \text{éq 4.1-2}$$

$$v_x = -\cos\gamma \ \cos\varphi \quad v_y = -\cos\gamma \ \sin\varphi \quad v_z = \sin\gamma \ . \qquad \text{éq 4.1-3}$$

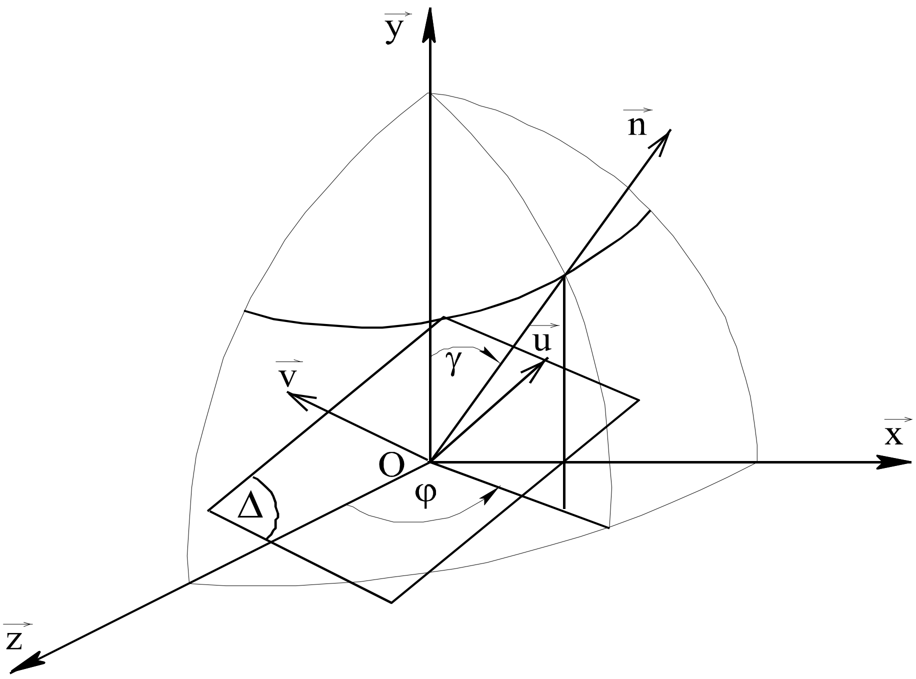


Figure 4.1-a: Location of the normal $\boldsymbol{n}$ with a plan by the angles γ and φ

In the plan Δ , components τ_u and τ_v vector $\boldsymbol{\tau}$ representing the shear stress are obtained by the relations:

$$\tau_u = \boldsymbol{u} \cdot \boldsymbol{\tau} = u_x \tau_x + u_y \tau_y + u_z \tau_z \ , \quad \text{éq 4.1-4}$$

$$\tau_v = \boldsymbol{v} \cdot \boldsymbol{\tau} = v_x \tau_x + v_y \tau_y + v_z \tau_z \ . \quad \text{éq 4.1-5}$$

On [Figure 4.1-b], we represented shear stresses in the plan Δ as well as the reference mark $(O, \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{n})$.

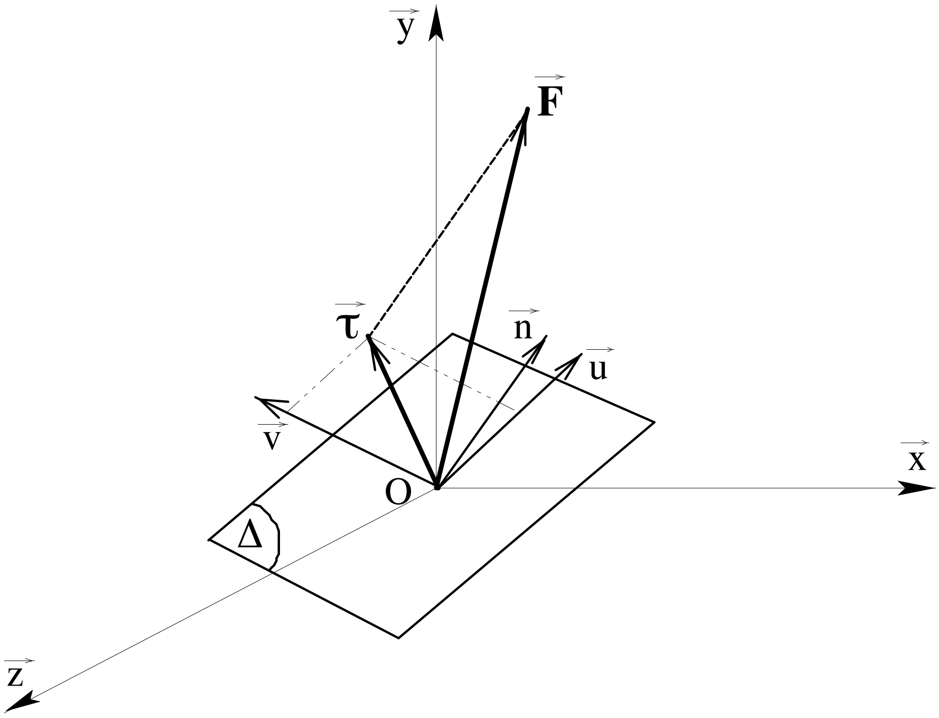


Figure 4.1-b: Representation of the vector stress shear $\boldsymbol{\tau}$ in the plan Δ

Now our problems are to determine for each point of Gauss or each node of a grid the plan of normal $\boldsymbol{n}$ such as $|\boldsymbol{\tau}|$ that is to say maximum. With this intention we vary the unit normal $\boldsymbol{n}$.

4.2 Exploration of the plans of shearing

The method that we present here is resulting from the reference [bib4]. Its principle is the following. As indicated in the paragraph [§4.1], for reasons of symmetry we vary the unit normal $\boldsymbol{n}$ according to a half-sphere using the angles γ and φ , cf [Figure 4.1-a]. The question which comes immediately is which must be the step of variation of the angles γ and φ . Indeed, it is necessary to find an optimum between the smoothness of exploration and a reasonable computing time insofar as it is necessary to make this operation at each point of Gauss of the grid. The author of the reference [bib4] proposes to divide the surface of the half sphere into facets of equal surfaces to the center of which the unit normal $\boldsymbol{n}$ is positioned, cf [Figure 4.2-a]. In practice surfaces are not strictly equal but of the same order of magnitude.

The step value of variation of γ , $\Delta\gamma$ is worth 10 degrees. The angle φ vary according to a step $\Delta\varphi$ who is function of the angle γ . More γ is weak or close to 180 degrees and more $\Delta\varphi$ must be large to preserve a surface of about constant facet. It is in the vicinity of $\gamma = 90°$ that $\Delta\varphi$ is smallest. [Table 4.2-1] the cutting summarizes which was retained.

With this method the number of facet thus the number of normal vectors to explore is equal to 209 for a half sphere.

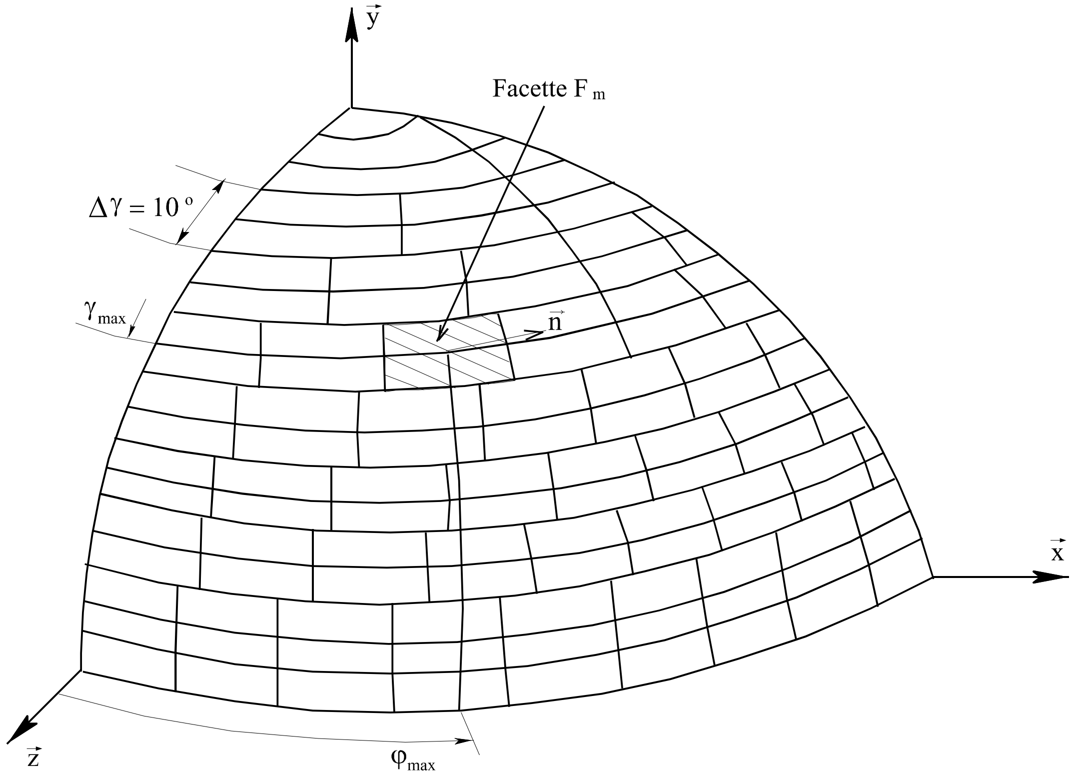


Figure 4.2-a: Division of the surface of the half sphere in facets

γ °	$0°$	$10°$	$20°$	$30°$	$40°$	$50°$	$60°$
$\Delta\Phi$ °	$180°$	$60°$	$30°$	$20°$	$15°$	$12,857$	$11,25$
Many facets	1	3	6	9	12	14	16

γ °	$70°$	$80°$	$90°$	$100°$	$110°$	$120°$	$130°$
$\Delta\Phi$ °	$10,588$	$10°$	$10°$	$10°$	$10,588$	$11,25$	$12,857$
Many facets	17	18	18	18	17	16	14

γ °	$140°$	$150°$	$160°$	$170°$	$180°$
$\Delta\Phi$ °	$15°$	$20°$	$30°$	$60°$	$180°$
Many facets	12	9	6	3	1

Table 4.2-1: Number of facet according to γ and of $\Delta\Phi$

In order to determine the normal vector $\boldsymbol{n}$ who will give the plan of maximum shearing with a good precision, the author recommends to resort to four additional successive refinings. The first consists in exploring eight new facets around the initial normal vector, like illustrates it [Figure 4.2-b].

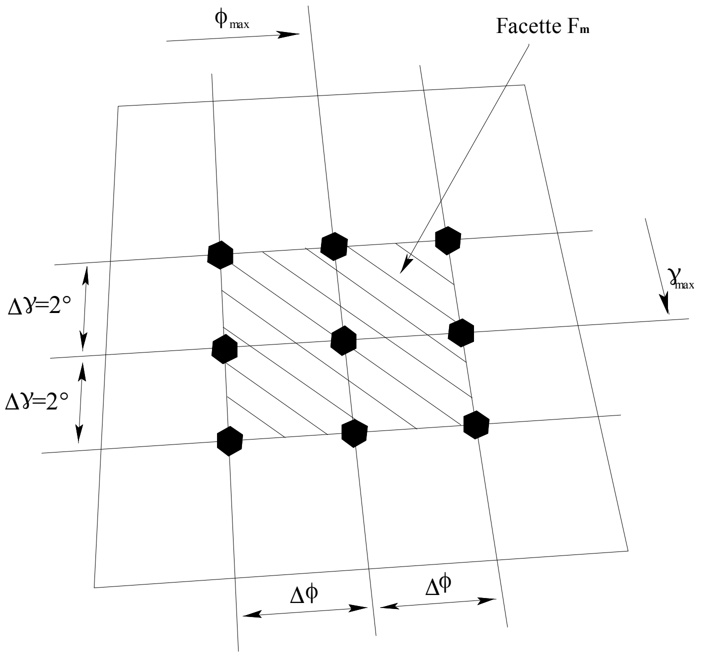


Figure 4.2-b: Representation of the eight additional facets around $\boldsymbol{n}$

In this case $\Delta\gamma$ is equal to two degrees and for $\gamma\in]0°, 180°[$, $\Delta\phi=\Delta\gamma/\sin\gamma$. For the last three refinings, $\Delta\gamma$ is equal to 1,0.5 and 0.25 degrees, respectively.

Typical case. If the facet F_m is perpendicular to $\boldsymbol{y}$, one considers the six facets all around it located at $\gamma=5°$ and respectively definite by $\Phi=0°$, $\Phi=60°$, $\Phi=120°$, $\Phi=180°$, $\Phi=240°$ and $\Phi=300°$, cf [Figure 4.2-c].

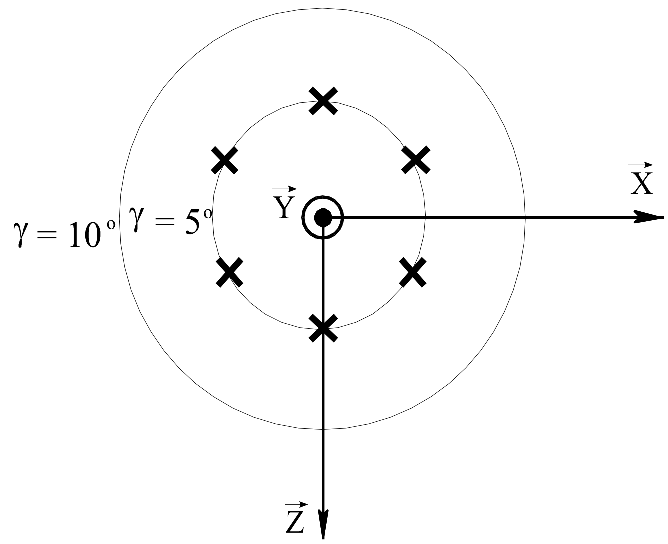


Figure 4.2-c: Localization of the explored facets when F_m is normal with $\boldsymbol{y}$

For each point of Gauss or each node we explore the 209 normal vectors $\boldsymbol{n}$. With each normal vector a history of shearing concretized by a certain number of points located in the plan with shearing is associated Δ axes u and v . Now it is a question of finding the circle circumscribed at the points belonging to the plan of shearing so as to deduce to it half amplitude from it from shearing.

5 Calculation of the half-amplitude of shearing

The problems are thus to find the circle circumscribed at a certain number of points located in a plan. The half-amplitude of shearing will be equal to the radius of the circumscribed circle.

5.1 General presentation of the calculation of the circumscribed circle

The method that we use is an exact method which breaks up into four stages.

Stage 1
We frame the points and we determine the coordinates of the four corners of the framework in the reference mark $(0, u, v)$, and coordinates of the center of the framework O cf [Figure 5.1-a] and [Figure 5.1-c]. **In the typical case** where the framework is summarized with a horizontal line or vertical it half length of the line is equal to the half-amplitude of shearing.

Stage 2
The objective of the second phase is to select the two most distant points. In order not to examine the distance between all the possible pairs of points, we build four sectors, cf [Figure 5.1-a] and [Figure 5.1-c]. These sectors are at the four corners of the framework and are delimited on the one hand, by the contour of the framework and on the other hand, by an arc of a circle whose center is the opposite corner and the ray the large side of the framework which in fact undervalues the distance between the two most distant points. Finally, we evaluate the distances between the points of the four sectors two to two:

distances between the points of sector 1 and the points of sector 2;
distances between the points of sector 1 and the points of sector 3;
distances between the points of sector 1 and the points of sector 4;
distances between the points of sector 2 and the points of sector 3;
distances between the points of sector 2 and the points of sector 4;
distances between the points of sector 3 and the points of sector 4.

In the typical case where the report on the small side of the framework on large on the east side strictly lower than $\sqrt{3/4}$ we do not evaluate the distances between the points belonging to sectors 1 and the 2 nor distances between the points of sectors 3 and 4, case of the example of [Figure 5.1-a].

Stage 3
In the third stage we build the fields 1 and 2 in which we will seek the points which are apart from the initial circumscribed circle, cf Stage 4. The purpose of the constitution of fields 1 and 2 is to reduce the number of points to be explored at the time of stage 4. The principles of constructions of these two fields are the following.

- From the points mediums on the two large sides of the framework (Omi_1 and Omi_2 , cf [Figure 5.1-b] and [Figure 5.1-d]) we trace an arc of a circle whose ray corresponds to undervaluing value of the half amplitude of shearing and is equal to the half length on the large side of the framework.
- Center of the framework O we trace four arcs of a circle whose ray is also undervaluing it of the value of the half amplitude of shearing.

If O_i the center of the circle circumscribes initial has a component according to the axis u who places it between Omi_1 and O , then if there exists a point of which the distance to O_i is higher than R_i the ray of the circle circumscribes initial, it can be only in field 1, cf [Figure 5.1-b].

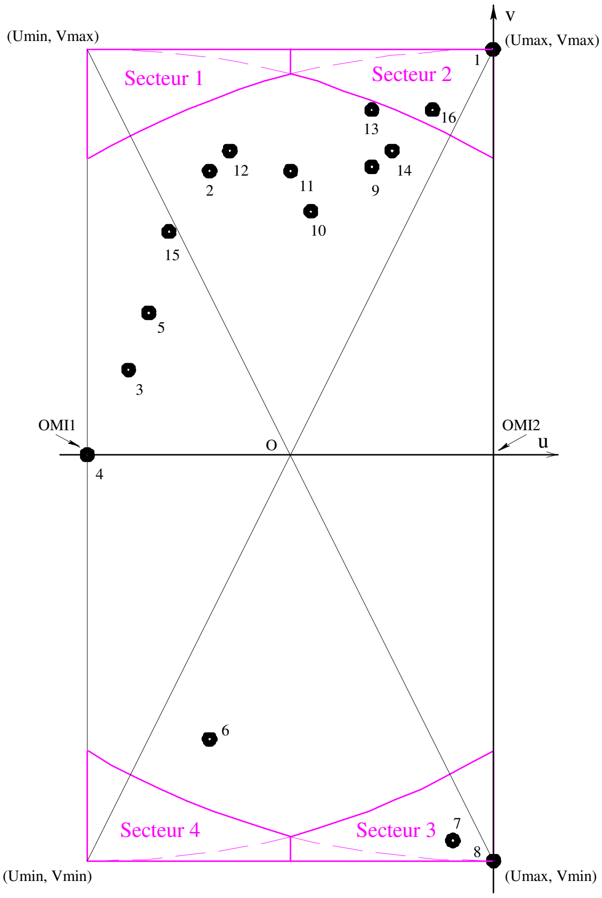


Figure 5.1-a: Exemple1, localization sectors

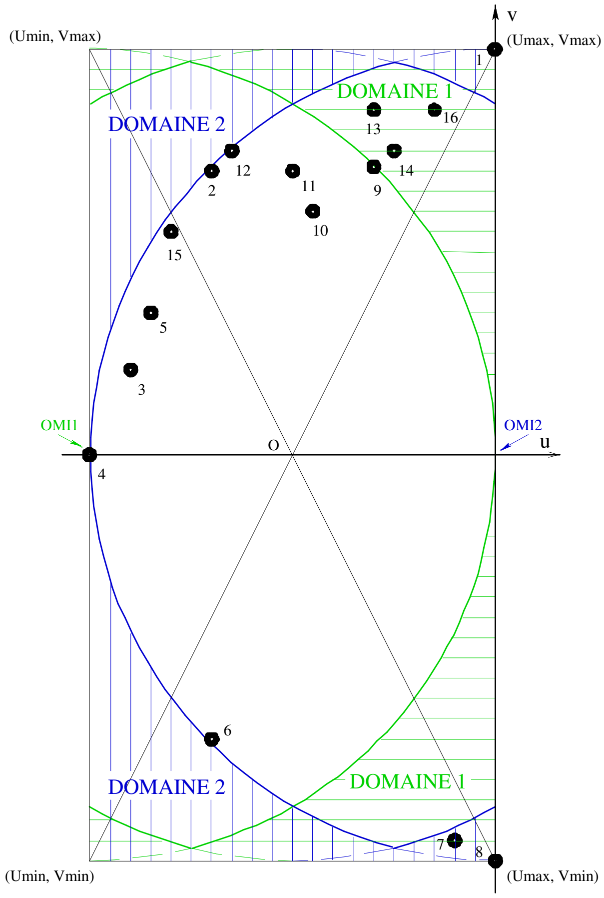


Figure 5.1-b: Exemple1, localization fields

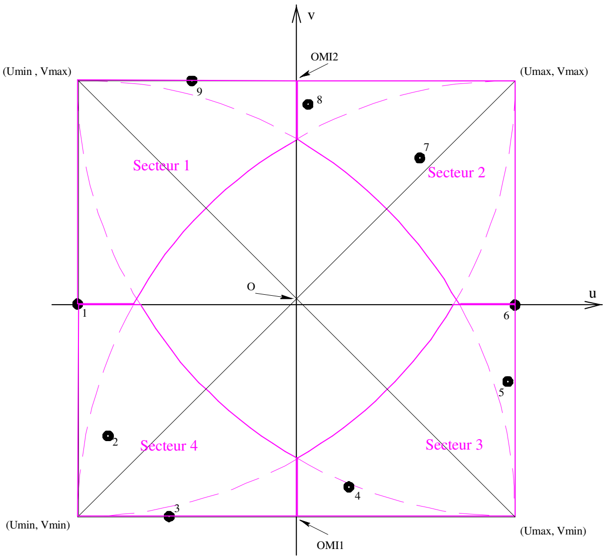


Figure 5.1-c: Exemple2, localization sectors

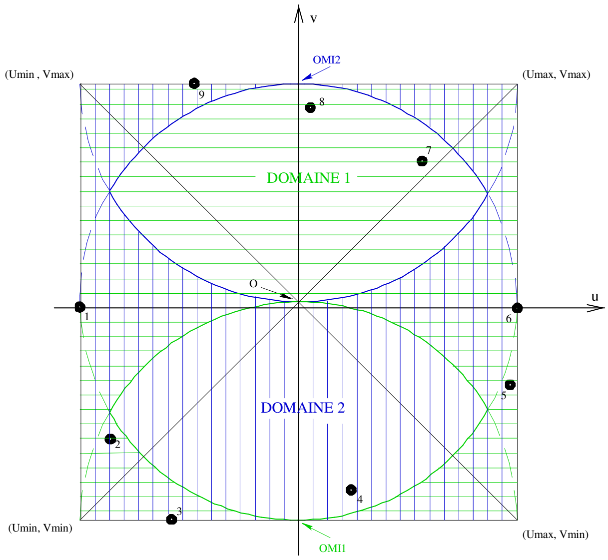


Figure 5.1-d: Exemple2, localization fields

Stage 4

The goal of the fourth stage is to find the circle circumscribed by the method of the circle passing by three points, cf [§5.2]. With this intention, we calculate the point medium O_1 associated with the two most distant points which we note P_1 and P_2, we deduce the value from it from a first noted ray R_1. According to the position of O_1 compared to the main roads of the framework passing in its center, we seek either in field 1, or in field 2, if it has there a point located at a distance higher than the half outdistances measured between the two most distant points P_1 and P_2. Let us note P_3 such a point. If there is no point such as P_3 then it half amplitude of shearing is equal to R_1, cf [Figure 5.1-c]. On the other hand, if P_3 exist we seek the coordinates of the point located at equal distance from P_1, P_2 and P_3; we note this point O_2. We obtain a new ray thus, R_2 thus new a half amplitude of shearing. Again, according to the position of O_2 compared to the main roads of the framework passing in its center, we seek either in field 1, or in field 2, if it has there a point located at a distance higher than R_2 of O_2. Let us note P_4 such a point. If there is no point such as P_4 then it half amplitude of shearing is equal to R_2. On the other hand, if P_4 exist we seek the smallest circle circumscribed at the four points: P_1, P_2, P_3 and P_4 by using the method of the circle successively passing by three points, cf [§5.2]. That provides us a new center O_3 and a new ray R_3. As previously, according to the position of O_3 compared to the main roads of the framework passing in its center, we seek either in field 1, or in field 2, if it has there a point located at a distance higher than R_3 of O_3. Let us note P_5 such a point. If there is no point such as P_5 then it half amplitude of shearing is equal to R_3. On the other hand if a point such as P_5 exist we have five points, if we want to use the preceding method, where it has only four points concerned there, it is necessary to eliminate one from the five points. That cannot be the last: P_5, therefore we preserve preceding iteration the three points which made it possible to determine O_3 and R_3, i.e. the smallest circumscribed circle. Let us suppose that P_1 that is to say thus eliminated. We thus seek the smallest circle circumscribed at the four points: P_2, P_3, P_4 and P_5 by using the method of the circle successively passing by three points, cf [§5.2]. That provides us a new center O_4 and a new ray R_4. According to the position of O_4 compared to the main roads of the framework passing in its center, we seek either in field 1, or in field 2, if it has there a point located at a distance higher than R_4 of O_4. If it is not the case it half amplitude of shearing is equal to R_4 and the circumscribed circle has as a center O_4, cf [Figure 5.1-f]. Contrary, if such a point exists we remake an iteration identical to the preceding one.

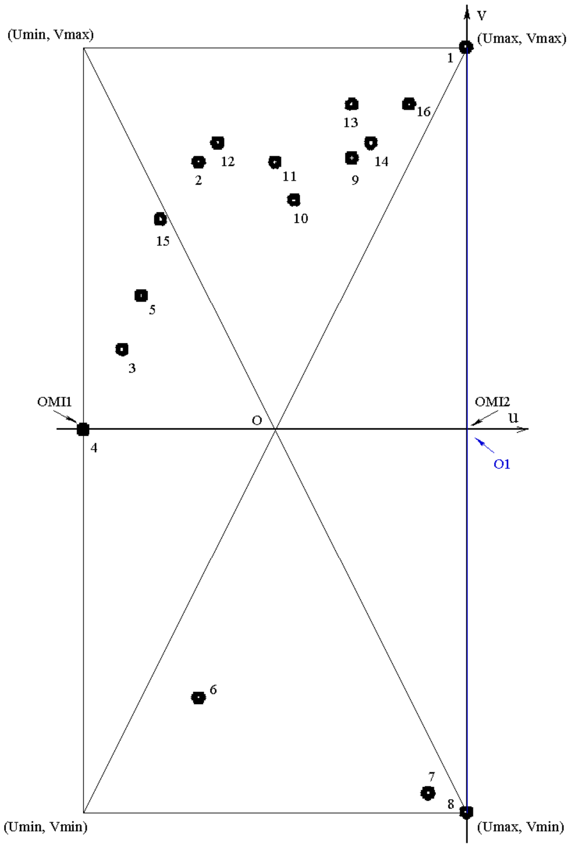


Figure 5.1-e: Exemple1, research of the circumscribed circle

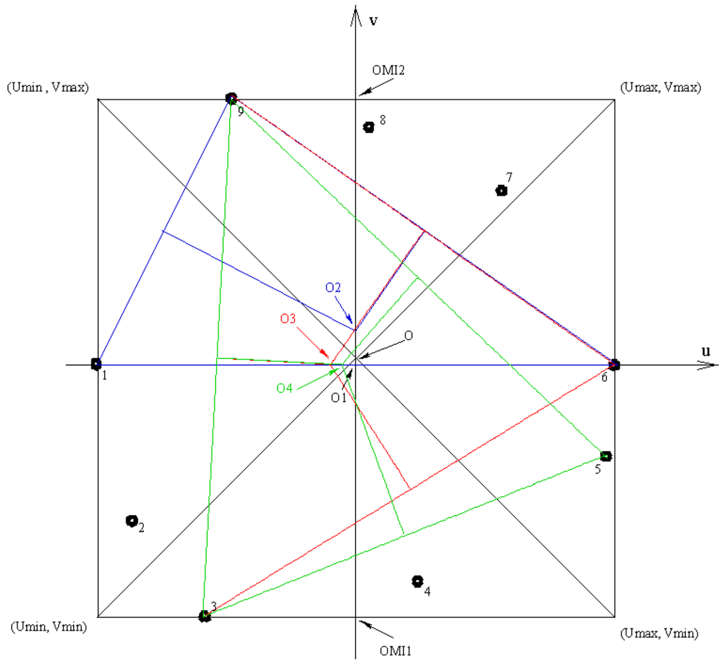


Figure 5.1-f: Exemple2, research of the circumscribed circle

5.2 Description of the method of the circle passing by three points

In this paragraph we will treat the case general, then the typical cases.

5.2.1 Case general

To determine the circumscribed circle at three points P_0, P_1 and P_2, cf [Figure 5.2.1-a], we proceed in three stages.

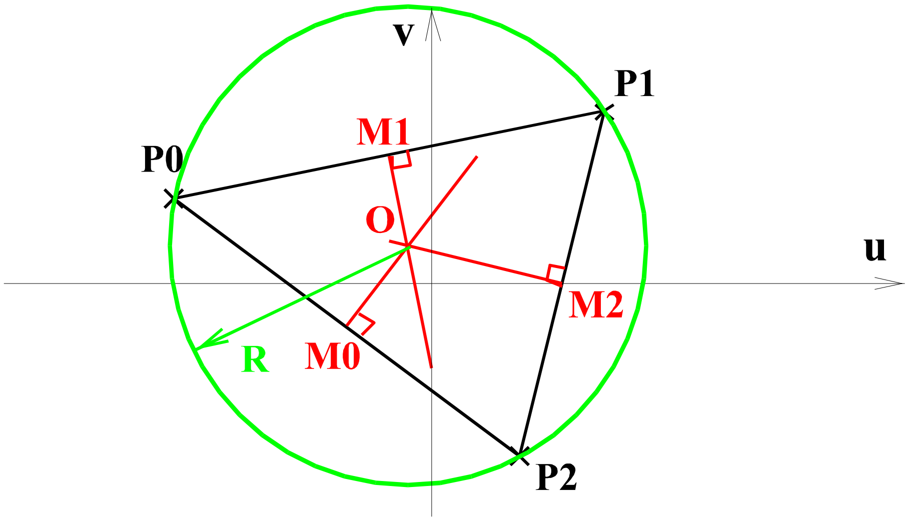


Figure 5.2.1-a: Determination of the circle passing by three points

Stage 1
We calculate the coordinates of the three points mediums: M_0, M_1 and M_2, cf [Figure 5.2.1-a].

Stage 2
We determine the normals passing by the three points mediums: M_0, M_1 and M_2, cf [Figure 5.2.1-a]. These normals are of the right-hand sides of the type $v = a\,u + b$ where a and b are constants which it is possible to calculate with the coordinates of the points P_0, P_1, P_2, M_0, M_1 and M_2. Let us describe, now, the manner of determining these normals.

1) Normal with the segment $P_0 \; P_1$ passing by M_1
We determine the punctual coordinates $M_1^{'}$ by rotation of 90° of the segment $P_0 \; M_1$:

$$\begin{aligned} U_{M1'} &= U_{M1} + \left(V_{M1} - V_{P0}\right) \\ V_{M1'} &= V_{M1} + \left(U_{P0} - U_{M1}\right) \end{aligned} \qquad \text{éq 5.2.1-1}$$

where U_k and V_k with $k = M1', M1, P0$ the components represent u and v points $M_1^{'}$, M_1 and P_0. We deduce the constants from them a_0 and b_0 line representing the normal with the segment $P_0 \; P_1$ passing by M_1:

$$a_0=(V_{M1'}-V_{M1})/(U_{M1'}-U_{M1})$$
$$b_0=(U_{M1'}\,V_{M1}-U_{M1}\,V_{M1'})/(U_{M1'}-U_{M1}) \quad \text{éq 5.2.1-2}$$

In the typical case where $(U_{M1'}-U_{M1})=0$, we force a_0 and b_0 with zero and we obtain the coordinates of the center O circle circumscribed at the points P_0 , P_1 and P_2 by a specific method described in the paragraph [§5.2.2].

2) Normal with the segment P_0 P_2 passing by M_0

We determine the punctual coordinates $M0'$ by rotation of 90° of the segment P_0 M_0 :

$$U_{M0'}=U_{M0}+(V_{M0}-V_{P0})$$
$$V_{M0'}=V_{M0}+(U_{P0}-U_{M0}) \quad \text{éq 5.2.1-3}$$

where U_k and V_k with $k=M0',M0,P0$ the components represent u and v points $M0'$, M_0 and P_0 . We deduce the constants from them a_1 and b_1 line representing the normal with the segment P_0 P_2 passing by M_0 :

$$a_1=(V_{M0'}-V_{M0})/(U_{M0'}-U_{M0})$$
$$b_1=(U_{M0'}\,V_{M0}-U_{M0}\,V_{M0'})/(U_{M0'}-U_{M0}) \quad \text{éq 5.2.1-4}$$

In the typical case where $(U_{M0'}-U_{M0})=0$, we force a_1 and b_1 with zero and we obtain the coordinates of the center O circle circumscribed at the points P_0 , P_1 and P_2 by a specific method described in the paragraph [§5.2.2].

3) Normal with the segment P_1 P_2 passing by M_2

We determine the punctual coordinates M_2' by rotation of 90° of the segment P_1 M_2 :

$$U_{M2'}=U_{M2}+(V_{M2}-V_{P1})$$
$$V_{M2'}=V_{M2}+(U_{P1}-U_{M2}) \quad \text{éq 5.2.1-5}$$

where U_k and V_k with $k=M2',M2,P1$ the components represent u and v points M_2' , M_2 and P_1 . We deduce the constants from them a_2 and b_2 line representing the normal with the segment P_1 P_2 passing by M_2 :

$$a_2=(V_{M2'}-V_{M2})/(U_{M2'}-U_{M2})$$
$$b_2=(U_{M2'}\,V_{M2}-U_{M2}\,V_{M2'})/(U_{M2'}-U_{M2}) \quad \text{éq 5.2.1-6}$$

In the typical case where $(U_{M2'}-U_{M2})=0$, we force a_2 and b_2 with zero and we obtain the coordinates of the center O circle circumscribed at the points P_0 , P_1 and P_2 by a specific method described in the paragraph [§5.2.2].

Stage 3

In the case general, we deduce from the constants a_0 , b_0 , a_1 , b_1 , a_2 and b_2 coordinates of the center O circle circumscribed at the points P_0 , P_1 and P_2 from three manner different. Let us note O_0 , O_1 and O_2 the same center O , obtained in three different ways, and U_k and V_k , where $k=O_0, O_1, O_2$, the components represent u and v points O_0 , O_1 and O_2 :

$$\begin{aligned} U_{O_0} &= (b_1 - b_0)/(a_0 - a_1) \\ V_{O_0} &= (a_0\ b_1 - a_1\ b_0)/(a_0 - a_1) \end{aligned} \quad \text{éq 5.2.1-7}$$

$$\begin{aligned} U_{O_1} &= (b_2 - b_0)/(a_0 - a_2) \\ V_{O_1} &= (a_0\ b_2 - a_2\ b_0)/(a_0 - a_2) \end{aligned} \quad \text{éq 5.2.1-8}$$

$$\begin{aligned} U_{O_2} &= (b_2 - b_1)/(a_1 - a_2) \\ V_{O_2} &= (a_1\ b_2 - a_2\ b_1)/(a_1 - a_2) \end{aligned} \quad \text{éq 5.2.1-9}$$

After having checked that equalities: $U_{O_0} \equiv U_{O_1} \equiv U_{O_2}$ and $V_{O_0} \equiv V_{O_1} \equiv V_{O_2}$ we are satisfied determine the radius of the circle circumscribed by calculating the distance enters O and one of the three points P_0 , P_1 or P_2 .

5.2.2 Typical cases

In this paragraph we treat the three typical cases of stage 2 of the paragraph [§5.2.1].

Typical case where $(U_{M1'} - U_{M1}) = 0$

In this case we obtain the components immediately u and v center O by:

$$\begin{aligned} U_O &= U_{M1} \\ V_O &= (a_1\ b_2 - a_2\ b_1)/(a_1 - a_2) \end{aligned} \quad \text{éq 5.2.2-1}$$

Typical case where $(U_{M0'} - U_{M0}) = 0$

Here components u and v center O are given by:

$$\begin{aligned} U_0 &= U_{M0} \\ V_O &= (a_0\ b_2 - a_2\ b_0)/(a_0 - a_2) \end{aligned} \quad \text{éq 5.2.2-2}$$

Typical case where $(U_{M2'} - U_{M2}) = 0$

In this last case, them u and v center O are given by:

$$\begin{aligned} U_0 &= U_{M2} \\ V_O &= (a_0\ b_1 - a_1\ b_0)/(a_0 - a_1) \end{aligned} \quad \text{éq 5.2.2-3}$$

The value of the ray of the circumscribed circle is obtained same manner as in the case general; i.e., while calculating the distance enters O and one of the three points P_0 , P_1 or P_2 .

5.3 Criteria with critical plans

In this paragraph we give the list of the criteria with critical plans, cf [bib3], which are programmed as well as a brief description.
Notation:

$\boldsymbol{n}*$	: normal with the plan in which the amplitude of shearing is maximum;
$\Delta\tau(\boldsymbol{n})$	: amplitude of shearing in a plan of normal $\vec{n}$;
$N_{\max}(\boldsymbol{n})$	: maximum normal constraint as regards normal $\vec{n}$ during the cycle;
τ_0	: limit of endurance in alternate pure shearing;
d_0	: limit of endurance in alternate pure traction and compression;
$N_m(\boldsymbol{n})$	: average normal constraint as regards normal $\vec{n}$ during the cycle;
$\varepsilon_{\max}(\boldsymbol{n})$	: maximum normal deformation as regards normal $\vec{n}$ during the cycle;
$\varepsilon_m(\boldsymbol{n})$	: average normal deformation as regards normal $\vec{n}$ during the cycle;
P	: hydrostatic pressure;
c_p	: harmful effect of pre-work hardening in controlled deformation, $c_p \geq 1$.

Criterion of MATAKE

$$\frac{\Delta\tau(\boldsymbol{n}*)}{2}+a\ N_{\max}(\boldsymbol{n}*)\leq b \qquad \text{éq 5.3-1}$$

where a and b are two constant data by the user, they depend on characteristic materials and are worth:

$$a=\left(\tau_0-\frac{d_0}{2}\right)/\frac{d_0}{2} \qquad b=\tau_0 .$$

Moreover, we define an equivalent constraint within the meaning of MATAKE, noted $\sigma_{eq}(n*)$:

$$\sigma_{eq}(\boldsymbol{n}*)=\left(c_p\frac{\Delta\tau(\boldsymbol{n}*)}{2}+a\ N_{\max}(\boldsymbol{n}*)\right)\frac{f}{t},$$

where f/t represent the report of the limits of endurance in alternate inflection and torsion.

Criterion of DANG VAN

$$\frac{\Delta\tau(\boldsymbol{n}*)}{2}+a\ P\leq b \qquad \text{éq 5.3-2}$$

where a and b are two constant data by the user, they depend on characteristic materials and are worth:

$$a=\frac{3}{2}\times\frac{(\Delta\sigma_2-\Delta\sigma_1)}{(\Delta\sigma_1-\Delta\sigma_2)-2\ \sigma_m} \qquad b=\frac{\sigma_m}{(\Delta\sigma_2-\Delta\sigma_1)+2\ \sigma_m}\times\frac{\Delta\sigma_1}{2} .$$

Moreover, we define an equivalent constraint within the meaning of DANG VAN, noted $\sigma_{eq}(\boldsymbol{n}*)$:

$$\sigma_{eq}(\boldsymbol{n}*)=\left(c_p\frac{\Delta\tau(\boldsymbol{n}*)}{2}+a\ P\right)\frac{c}{t} ,$$

where c/t represent the report of the limits of endurance in alternate shearing and traction.

5.4 Many cycles to the rupture and damage

From $\sigma_{eq}(\boldsymbol{n}*)$ and from a curve of Wöhler we deduct the number of cycles to the rupture: $N(\boldsymbol{n}*)$, then the damage corresponding to a cycle: $D(\boldsymbol{n}*)=1/N(\boldsymbol{n}*)$.

6 Criteria with variable amplitude

The criteria with variable amplitude are implemented when the loading is not periodic. When the loading is not periodic it is necessary to break up the way of loading undergone by the structure into elementary under-cycles using a method of counting of cycles. If the loading is nonradial there is no tested multiaxial method of counting. Consequently we choose, as in the literature, to use the method of counting RAINFLOW [bib7] which needs as starter for a scalar. This is why we reduce to a dimension the scission, which is the orthogonal projection of the vector forced on a plan, by projecting the point of the vector scission on one or two axes. Another important difference with the criteria with critical plan is that it is not the amplitude of shearing which makes it possible to select the critical plan but the office plurality of damage which results from the elementary under-cycles.

The method of projection that we use is clarified in chapters 7 and 8. In the continuation we describe the way in which we made evolve the criteria of MATAKE and DANG VAN to adapt them to the cases where the loading is not periodical.

6.1 Criterion of modified MATAKE

In the context of the office plurality of damage and a periodic loading, the criterion of MATAKE [bib6], is written in the following way:

$$\sigma_{eq}=c_p\frac{\Delta\tau\left(\vec{n}^*\right)}{2}+a\ N_{\max}\left(\vec{n}^*\right) \qquad \text{éq 6.1-1}$$

where σ_{eq} represent the constraint equivalent within the meaning of the criterion of MATAKE and with:

$\vec{n}^*$	normal with the plan for which the amplitude of shearing is maximum;
$\Delta\tau\left(\vec{n}^*\right)/2$	maximum half-amplitude of shearing;
a	constant which perhaps defined by an alternate pure traction and shear test - alternate compression or test in alternate traction and compression and nonalternate traction and compression;
$N_{\max}\left(\vec{n}^*\right)$	maximum normal constraint as regards normal $\vec{n}^*$ during the cycle;
c_p	harmful effect of pre-work hardening in controlled deformation $c_p\geq 1$.

To calculate the cumulated damage if the loading is not periodical the first stage consists in determining the scission (vector shearing) in a plan of normal $\vec{n}$ at every moment of the loading. The technique which is used with this intention is described in the reference [bib6]. In the second stage we start by reducing the history of the scission to a unidimensional function of time by projecting the point of the vector scission on one or two axes defined in the plan of normal $\vec{n}$ considered, cf chapter 7 and 8. Thus the evolution of the projected scission is summarized with the relation: $\tau_p=f(t)$ what makes it possible to use the method of counting RAINFLOW. On the figure [Figure 6.1-a] we show the values reached by the end of the vector shearing in a plan of normal $\vec{n}$ before projection on an axis or two axes and the figure [Figure 6.1-b] these same values after projection on an axis. At this stage we should introduce the concept of elementary equivalent constraint σ_{eq}^i. Practically this concept has the same meaning as the concept of equivalent constraint defined by the relation [éq 6.1-1], but it applies to the elementary under-cycles resulting from the method of counting RAINFLOW. Thus starting from the projected scission τ_p we calculate elementary equivalent constraints σ_{eq}^i.

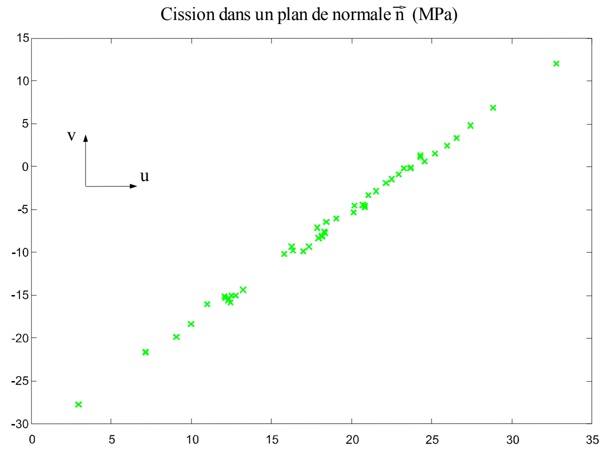


Figure 6.1-a: Points of the vector cission before projection

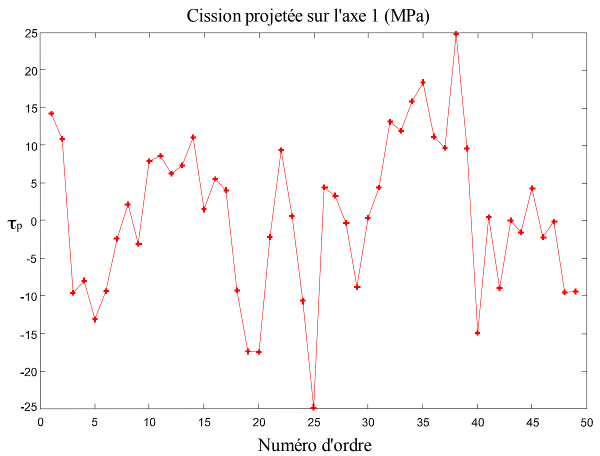


Figure 6.1-b: Points of the vector cission after projection on an axis

Method RAINFLOW breaks up $\tau_p = f(t)$ in periodic elementary under-cycles and breeze the history of the loading, as we show it on the figure [Figure 6.1-c]. Thus, for a normal $\vec{n}$ data method RAINFLOW provides for each elementary under-cycle two values, points high and low, of the point of the vector cission $\tau^i_{p_1}(\vec{n})$ and $\tau^i_{p_2}(\vec{n})$ associated with two values of maximum normal constraint $N^i_1(\vec{n})$ and $N^i_2(\vec{n})$.

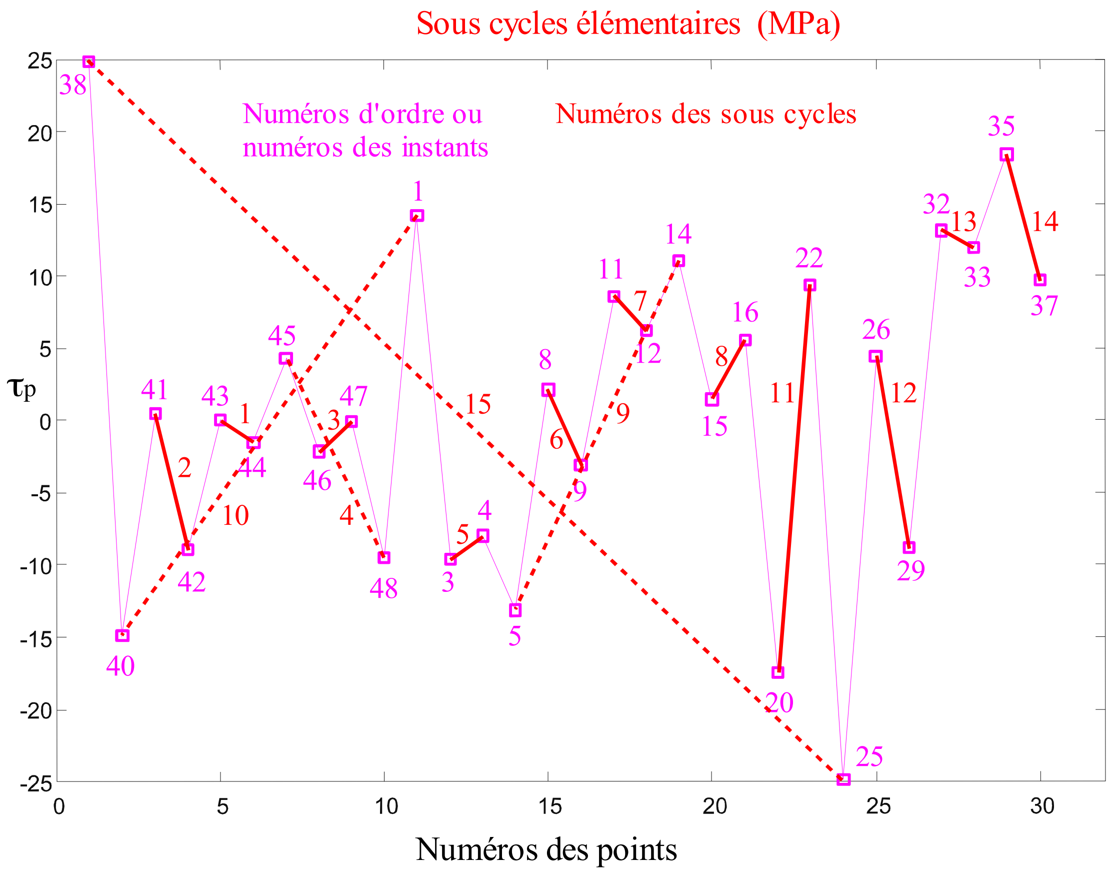


Figure 6.1-c: Fifteen elementary under-cycles after treatment by method RAINFLOW

For the criterion of MATAKE we define the elementary equivalent constraint in the following way:

$$\sigma^{i}_{eq}(\vec{n})=c_p\frac{Max\left(\tau^{i}_{p_1}(\vec{n}),\tau^{i}_{p_2}(\vec{n})\right)-Min\left(\tau^{i}_{p_1}(\vec{n}),\tau^{i}_{p_2}(\vec{n})\right)}{2}+a\ Max\left(N^{i}_{1}(\vec{n}),N^{i}_{2}(\vec{n}),0\right) \quad \text{éq 6.1-2}$$

For the office plurality of damage, this elementary equivalent constraint is to be used with a curve of tiredness in shearing. If a curve of tiredness in traction compression is used it is necessary to multiply [éq 6.1-2] by a corrective coefficient which corresponds to the report of the limits of endurance in inflection and alternate torsion and that we note α :

$$\sigma^{i}_{eq}(\vec{n})=\alpha\left(c_p\frac{Max\left(\tau^{i}_{p_1}(\vec{n}),\tau^{i}_{p_2}(\vec{n})\right)-Min\left(\tau^{i}_{p_1}(\vec{n}),\tau^{i}_{p_2}(\vec{n})\right)}{2}+a\ Max\left(N^{i}_{1}(\vec{n}),N^{i}_{2}(\vec{n}),0\right)\right)$$

éq 6.1-3

From $\sigma_{eq}^{i}(\vec{n})$ and from a curve of Wöhler we deduct the number of cycles to the rupture $N^{i}(\vec{n})$ and elementary damage $D^{i}(\vec{n})=1/N^{i}(\vec{n})$ correspondent with an elementary under-cycle. We use a linear office plurality of damage. That is to say k the number of elementary under-cycles, for a normal $\vec{n}$ fixed, the cumulated damage is equal to:

$$D(\vec{n})=\sum_{i=1}^{k} D^{i}(\vec{n}) \quad \text{éq 6.1-4}$$

To determine the normal vector $\vec{n}^{*}$ corresponding to the maximum cumulated damage it is enough to vary $\vec{n}$ and to calculate [éq 6.1-4]. The normal vector $\vec{n}^{*}$ corresponding to the maximum cumulated damage is then given by:

$$D(\vec{n}^{*})=\underset{\vec{n}}{Max}\left(D(\vec{n})\right)$$

6.2 Criterion of modified DANG VAN

Within the framework of the damage and a periodic loading, the criterion of DANG VAN is written:

$$\sigma_{eq}(\vec{n}^{*})=c_{p}\frac{\Delta\tau(\vec{n}^{*})}{2}+a\ P$$

where σ_{eq} represent the constraint equivalent within the meaning of the criterion of DANG VAN and with:

$\vec{n}^{*}$	normal with the plan for which the amplitude of shearing is maximum;
$\Delta\tau\left(\vec{n}^{*}\right)/2$	maximum half-amplitude of shearing;
a	constant which perhaps defined by an alternate pure traction and shear test - alternate compression or test in alternate traction and compression and nonalternate traction and compression;
P	maximum hydrostatic pressure during the cycle;
c_{p}	harmful effect of pre-work hardening in controlled deformation $c_{p}\geq 1$.

When the loading is not periodical, we calculate the damage by the same process as that used for the criterion of MATAKE. The only difference lies in the definition of the elementary equivalent constraint:

$$\sigma_{eq}^{i}(\vec{n})=\frac{Max\left(\tau_{p_1}^{i}(\vec{n}),\tau_{p_2}^{i}(\vec{n})\right)-Min\left(\tau_{p_1}^{i}(\vec{n}),\tau_{p_2}^{i}(\vec{n})\right)}{2}+a\ Max\left(P_{1}^{i}(\vec{n}),P_{2}^{i}(\vec{n}),0\right) \quad \text{éq 6.2-1}$$

where P_{1}^{i} and P_{2}^{i} the two values of the hydrostatic pressure represent attached to each under - elementary cycle. This elementary equivalent constraint is to be used with a curve of tiredness in shearing. If one must employ a curve of tiredness in traction compression it is necessary to multiply [éq 6.2-1] by the corrective coefficient α :

$$\sigma_{eq}^{i}(\vec{n})=\alpha\left(c_{p}\frac{Max\left(\tau_{p_1}^{i}(\vec{n}),\tau_{p_2}^{i}(\vec{n})\right)-Min\left(\tau_{p_1}^{i}(\vec{n}),\tau_{p_2}^{i}(\vec{n})\right)}{2}+a\ Max\left(P_{1}^{i}(\vec{n}),P_{2}^{i}(\vec{n}),0\right)\right)$$

After having defined the criteria of MATAKE and DANG-VAN within the framework of the office plurality of damage and a nonperiodic loading, it remains us to specify the technique of projection which we propose.

6.3 Criterion of modified FATEMI-SOCIE

6.3.1 Description

The criterion of FATEMI and SOCIE is a criterion of type critical plan [9], [10]. Initially formulated for periodic loadings, we propose a version adapted to the nonperiodic loadings of it.
In this criterion the parameter a is defined as follows: $a=k/\sigma_y$ where σ_y is the elastic limit and k a coefficient which depends on material. We will reconsider the way of calculating k. This criterion mixes shearing in deformation and the maximum normal constraint. We propose to define an equivalent deformation "elementary" in the following way:

$$\varepsilon_{eq}^{i}(\vec{n})=\alpha\left(c_p\ \frac{Max\left(\gamma_{p_1}^{i}(\vec{n}),\gamma_{p_2}^{i}(\vec{n})\right)-Min\left(\gamma_{p_1}^{i}(\vec{n}),\gamma_{p_2}^{i}(\vec{n})\right)}{2}\left[1+a\ Max\left(N_1^{i}(\vec{n}),\ N_2^{i}(\vec{n}),\ 0\right)\right]\right)$$

where $\gamma_{p_1}^{i}$ and $\gamma_{p_2}^{i}$ the extreme shearing strains of the under-cycle number represent i.

Except a definition different from the criterion, the approach used to calculate the damage is identical to the two preceding criteria. Lastly, it is also the maximum damage which makes it possible to select the critical plan.

It will be noted that the shearing strains used in the criterion of FATEMI and SOCIE are distortions γ_{ij} ($i\neq j$). If one uses the shearing strains of the tensorial type ϵ_{ij} ($i\neq j$), they should be multiplied by a factor 2 because $\gamma_{ij}=2\epsilon_{ij}$.

6.3.2 Identification of the coefficient *K*

The author proposes to identify the coefficient k starting from tests in pure traction and compression and pure alternate torsion on a thin tube [9], [10]. In order not to introduce skew, the two kinds of tests must be realized on the same type of test-tube. Before presenting the formula which defines k we introduce the following notations:

- ν : Poisson's ratio, (is generally worth $0{,}3$ for our materials);
- ν_p : coefficient of incompressibility of the plastic deformations (is worth $0{,}5$ in [9] and [10]);
- E : Modulus of elasticity of Young;
- G : Elastic modulus of rigidity;
- N_f : Many cycles to the rupture.

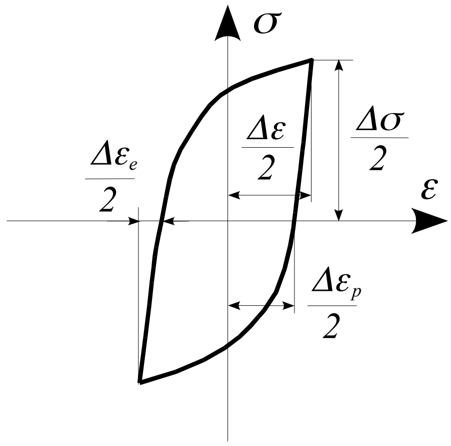
Contrary to the two preceding criteria this one can treat the cases where there remains elastoplastic zones in the structure. The coefficient k is defined by the relation following:

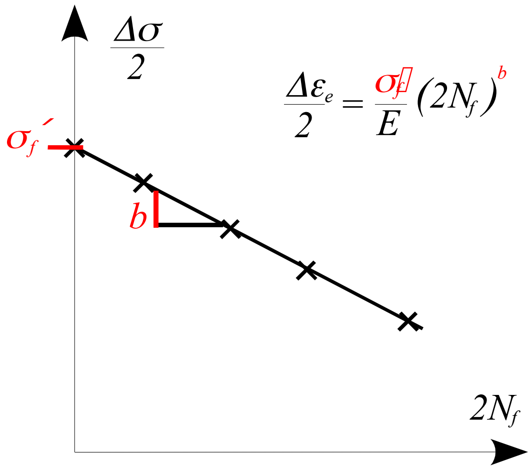
$$k=\left[\frac{\dfrac{\tau'_f}{G}\left(2N_f\right)^{b_o}+\gamma'_f\left(2N_f\right)^{c_o}}{(1+\nu)\dfrac{\sigma'_f}{E}\left(2N_f\right)^{b}+\left(1+\nu_p\right)\varepsilon'_f\left(2N_f\right)^{c}}-1\right]\frac{k'(0,002)^{n'}}{\sigma'_f\left(2N_f\right)^{b}}\ ,$$

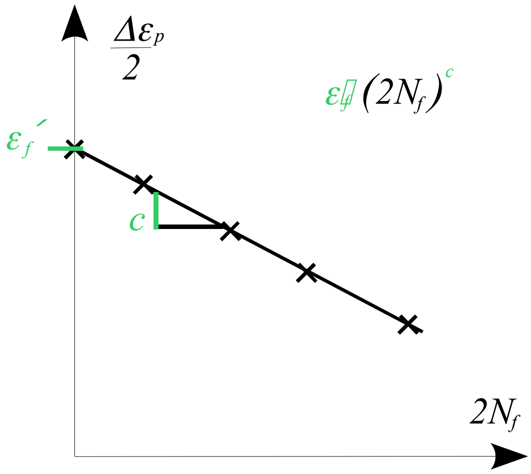
where terms: τ'_f , b_o , γ'_f , c_o , σ'_f , b , ε'_f , c , k' and n' are defined by the means of tests.

Tests in pure traction and compression

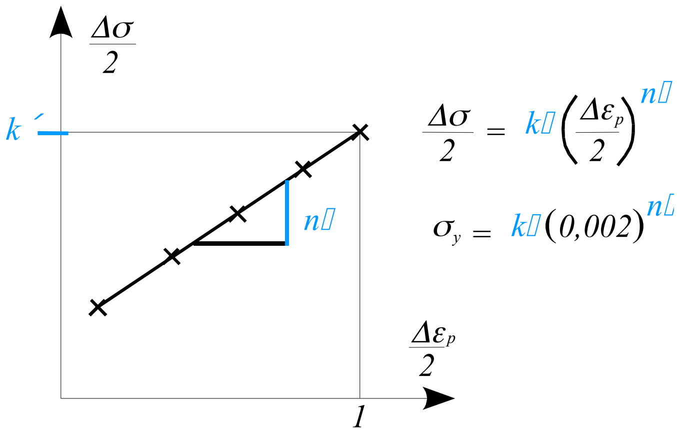
The tests in pure traction and compression make it possible to identify the coefficients: σ'_f , b , ε'_f , c , k' and n' .







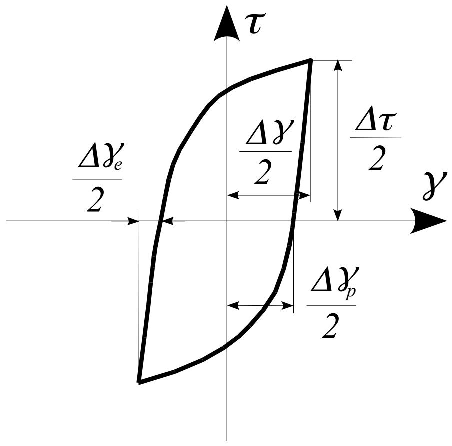
$$\varepsilon_a=\frac{\Delta\varepsilon}{2}=\frac{\sigma'_f}{E}(2N_f)^b+\varepsilon'_f(2N_f)^c$$

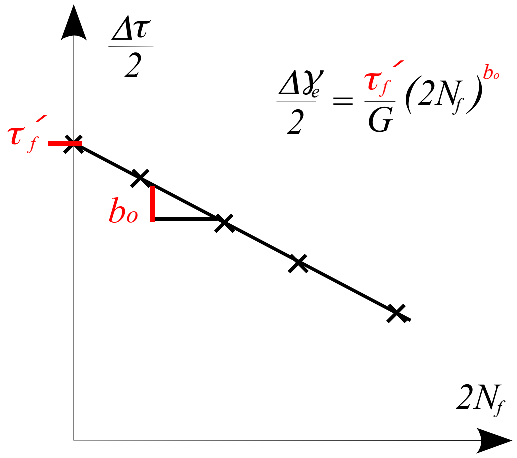


The curves opposite use scales Log-Log.

Tests in pure alternate torsion

The tests in pure alternate torsion make it possible to identify the coefficients:
τ'_f , b_o , γ'_f , c_o .





$$\gamma_a = \frac{\Delta\gamma}{2} = \frac{\tau'_f}{G}(2N_f)^{b_o} + \gamma'_f (2N_f)^{c_o}$$

$$\gamma_a = \frac{\Delta\gamma}{2} = \frac{\tau'_f}{G}(2n_f)^{b_0} + \gamma'_f (2N_f)^{c_0}$$

7 Choice of the axes of projection

With regard to the projection of the end of the vector cission we propose two options:

- a projection on an axis,
- a projection on two axes.

The axis of option 1 is in the same way given that the first axis of option 2. The second axis of option 2 is orthogonal with the first axis of this option.

7.1 Projection on an axis

We place ourselves in a plan of normal $\vec{n}$ data where each point represents the position of the point of the vector shearing at one moment, for more details to see the reference [bib6]. In this plan we build the smallest framework which contains all the points at every moment representing the end of the vector cission. The two diagonals of the framework enable us to define two axes: axis 1 corresponds to the segment $\overline{AC}$, and centers it 2 corresponds to the segment $\overline{DB}$, cf [Figure 7.1].

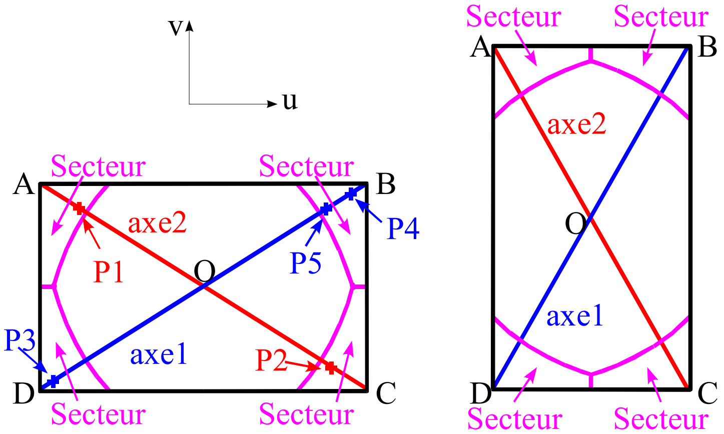


Figure 7.1-a **Figure 7.1-b**

Figure 7.1: Definition of the axes of projection

We choose a priori the axis of projection among axes 1 and 2 because the diagonal of the framework is larger than the large side of the framework what has as a virtue to dilate a little the projected points. In addition the loadings which interest us are of thermal origin with the result that the points representing the evolution of the point of the vector cission, in the plans of normal $\vec{n}$, are generally aligned on an axis, as we show it on the figure [Figure 6.1-a].

Sectors 1,2,3 and 4 are built same manner as in the reference [bib6]. Only the points which are in these sectors are projected orthogonally on axes 1 and 2.

We define the axis of projection as being the axis on which the distance between two projected points is largest.

For example, on [Figure 7.1-a] the axis of projection is axis 1 since the length of the segment $\overline{P_3 P_4}$ is higher than the length of the segment $\overline{P_1 P_2}$. This definition of the axis of projection makes it possible to make sure that the axis of projection retained will make it possible to give an account of the largest amplitude of shearing projected.

According to the presence or absence of points in sectors 1,2,3 and 4 the determination of the axis of projection can be immediate, it is then not necessary to implement the procedure of selection described above. For more details the reader will be able to refer to appendix 1.

A second axis is necessary to distinguish the case where the points representing the point of the vector cission are aligned on an axis of the case where these points describe a circle.

7.2 Construction of the second axis

The second axis of projection is orthogonal with the initial axis of projection and it passes by the point O.

Since we know the coordinates of the points A, B, C and D, to characterize the second axis completely it is enough to determine the punctual coordinates M such as:

$\vec{DB} \cdot \vec{OM}=0$ if the initial axis is axis 1,
$\vec{AC} \cdot \vec{OM}=0$ if the initial axis is axis 2.

8 Projection of shearing

In this chapter we describe the process of projection on the initial axis, or first axis, and the second axis. We point out that projection on these two axes is orthogonal.

8.1 Case where axis 1 is the initial axis

This case is represented on [Figure 8.1-a]. We place in the reference mark $(O,\vec{u},\vec{v},\vec{n})$. Definitions of $\vec{u}$, $\vec{v}$ and $\vec{n}$ are given in the reference [bib6]. In the plan $(\vec{u},\vec{v})$ of normal $\vec{n}$ points A, B, C, D and O have respectively, for coordinates $(U_{\min},V_{\max})$, $(U_{\max},V_{\max})$, $(U_{\max},V_{\min})$, $(U_{\min},V_{\min})$ and (U_O,V_O).

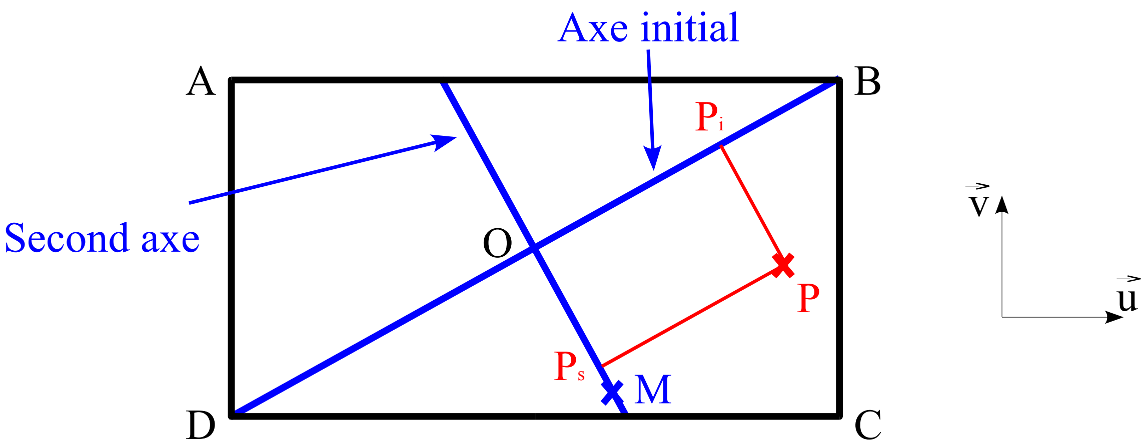


Figure 8.1-a: Projection if axis 1 is the initial axis

8.1.1 Determination of the second axis

Here to determine the second axis we solve the equation:

$$\vec{DB} \cdot \vec{OM}=0 \quad \text{éq 8.1.1-1}$$

where coordinates U_M, V_M point M are the unknown factors.
The equation [éq 8.1.1-1] is also written in the following form:

$$(U_{\max}-U_{\min})(U_M-U_O)+(V_{\max}-V_{\min})(V_M-V_O)=0$$

what leads to:

$$V_M = V_O - \frac{(U_{\max} - U_{\min})}{(V_{\max} - V_{\min})}(U_M - U_O)$$

By giving a value of U_M different from U_O we obtain immediately V_M .

8.1.2 Projection of an unspecified point on the initial axis

Starting from a point P unspecified known, the first stage consists in calculating the punctual coordinates P such as:

$$\vec{DB}.\vec{PP'} = 0$$

While proceeding like previously, we obtain the relation:

$$V_{P'} = V_P \frac{(U_{\max} - U_{\min})}{(V_{\max} - V_{\min})}(U_{P'} - U_P)$$

where $V_{P'}$ result from a value from $U_{P'}$ different from U_P.

In the plan (u,v) the initial axis and the segment $\bar{PP}'$ are lines closely connected respectively described by $v = a_i u + b_i$ and $v = a_P u + b_P$, therefore to know the coordinates of the point project on the initial axis P_p we solve the equation:

$$a_i u + b_i = a_P u + b_P$$

where

$$a_i = \frac{(V_{\max} - V_{\min})}{(U_{\max} - U_{\min})}, \qquad b_i = \frac{(U_{\max} V_{\min} - U_{\min} V_{\max}}{(U_{\max} - U_{\min})},$$

$$a_P = \frac{(V_{P'} - V_P)}{(U_{P'} - U_P)}, \qquad b_P = \frac{(U_{P'} V_P - U_P V_{P'})}{(U_{P'} - U_P)}.$$

One obtains:

$$U_{P_i} = \frac{b_p - b_i}{a_i - a_p}$$

$$V_{P_i} = \frac{a_i b_P - a_P b_i}{a_i - a_P}$$

The projection of an unspecified point on the second axis is described in appendix 2.

8.2 Case where axis 2 is the initial axis

This case is represented on [Figure 8.2-a]. As previously, in the plan $(\vec{u}, \vec{v})$ points A , B , C , D and O have respectively, for coordinates $(U_{\min}, V_{\max})$, $(U_{\max}, V_{\max})$, $(U_{\max}, V_{\min})$, (U_{min}, V_{min}) and $(U_{0,} V_0)$.

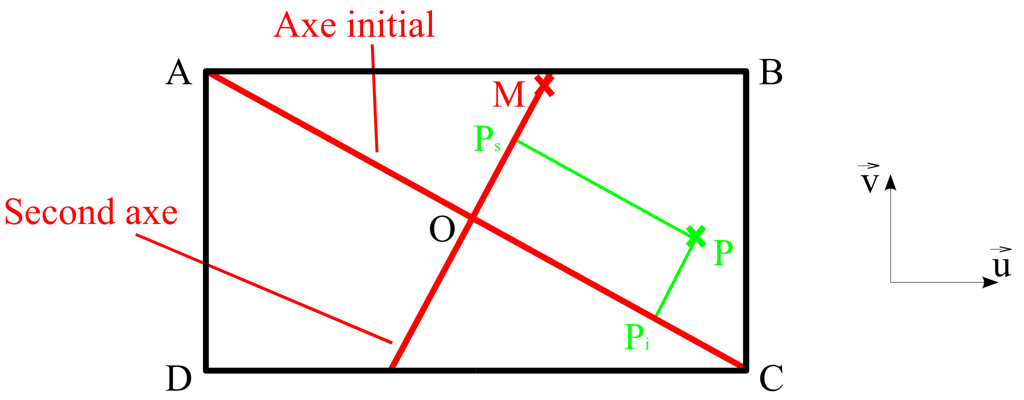


Figure 8.2-a: Projection if axis 2 is the initial axis

8.2.1 Determination of the second axis

Here to determine the second axis we solve the equation:

$$\overrightarrow{AC} \, . \, \overrightarrow{OM} = 0 \qquad \text{éq 8.2.1-1}$$

where coordinates (U_M , V_M) point M are the unknown factors.

The equation [éq 8.2.1-1] is also written in the following form:

$$(U_{\max} - U_{\min})(U_M - V_O)(V_{\max} - V_{\min})(V_M - V_O) = 0$$

what leads to:

$$V_M = V_O + \frac{(U_{\max} - U_{\min})}{(v_{\max} - V_{\min})}(U_M - U_O)$$

By giving a value of U_M different from U_O we obtain immediately V_M .

8.2.2 Projection of an unspecified point on the initial axis

Starting from a point P unspecified known, the first stage consists in calculating the punctual coordinates P' such as:

$$\vec{AC} \, . \, \vec{PP}' = 0$$

While proceeding like previously, we obtain the relation:

$$V_{P'} = V_P \frac{(U_{\max} - U_{\min})}{(V_{\max} - V_{\min})}(U_{P'} - U_P)$$

where for a value of $U_{P'}$ different from U_P we calculate $V_{P'}$.

In the plan (u , v) the initial axis and the segment $P\bar{P}'$ are lines closely connected respectively described by $v = a_i u + b_i$ and $v = a_p u + b_p$, therefore to know the coordinates of the point project on the initial axis P_p we solve the equation:

$$a_i u + b_i = a_p u + b_p$$

where

$$a_i=-\frac{(V_{max}-V_{min})}{(U_{max}-U_{min})}\ , \qquad b_i=\frac{(U_{max}\ V_{max}-U_{min}\ V_{min})}{(U_{max}-U_{min})}\ ,$$

$$a_p=\frac{(V_{P'}-V_P)}{(U_{P'}-U_P)}\ , \qquad b_p=\frac{(U_{P'}V_p-U_P V_{P'})}{(U_{P'}-U_P)}\ .$$

One obtains:

$$U_{P_i}=\frac{b_P-b_i}{a_i-a_P}\ , \qquad V_{P_i}=\frac{a_i\ b_P-a_P\ b_i}{a_i-a_P}\ .$$

The projection of an unspecified point on the second axis is described in appendix 2.

8.3 Definition of the module and orientation of the axis of projection

We propose to define the sign of the module of the point project compared to the initial axis. That is to say the reference mark $(O\ ,\ \vec{u}\ ,\ \vec{v}\ ,\ \vec{n})$ in which the scission evolves. In this reference mark if the component U_{P_i} point project is higher or equal to the zero sign of the module is positive, if not it is negative. In short the module and the sign of the module of the point project are in the following way defined:

$$P_{mod}=\sqrt{\overline{OP_i}^2+\overline{OP_s}^2} \quad si \quad U_{P_i}\geq 0\ ,$$
$$P_{mod}=-\sqrt{\overline{OP_i}^2+\overline{OP_s}^2} \quad si \quad U_{P_i}<0\ .$$

The definition of the module differentiates the loadings closely connected from the circular loadings. In accordance with the experiment a circular loading will be regarded as being more damaging that a loading refines [bib1].

9 Criteria in formula

9.1 For the periodic loading

For the periodic loading, the calculation of the damage is carried out only on the first complete cycle. The first part of the monotonic history of the loading corresponding to the loading is not taken into account because this one aims to impose a loading average not no one. For the elastic behavior, calculation is carried out between the maximum value and the minimal value of the cycle considered. For the elastoplastic behavior, calculation is carried out between the first discharge and the second discharge.

The list of sizes available is in the following table:

TYPE_CHARGE = 'PERIODIC', CRITERION = 'FORMULE_CRITERE'
The sizes available are: 'DTAUMA' : half-amplitude of shearing in maximum constraint ($\Delta\tau(\mathbf{n}^*)/2$) 'PHYDRM' : hydrostatic pressure (P) 'NORMAX' : normal maximum constraint as regards normal ($N_{max}(\mathbf{n}^*)$) 'NORMOY' :average normal constraint on the critical level ($N_{moy}(\mathbf{n}^*)$) 'EPNMAX' :maximum normal deformation on the critical level ($\varepsilon_{Nmax}(\mathbf{n}^*)$) 'EPNMOY' :average normal deformation on the critical level ($\varepsilon_{Nmoy}(\mathbf{n}^*)$)

`'DEPSPE'` :half-amplitude of the equivalent plastic deformation ($\Delta\varepsilon_{eq}^{p}/2$)

$$\Delta\varepsilon_{eq}^{p}/2=\frac{1}{2}max_{t1}\,max_{t2}\sqrt{\frac{2}{3}(\boldsymbol{\epsilon}^{p}(t_1)-\boldsymbol{\epsilon}^{p}(t_2)):(\boldsymbol{\epsilon}^{p}(t_1)-\boldsymbol{\epsilon}^{p}(t_2))}$$

`'EPSPR1'` :half-amplitude of the first principal deformation (with the taking into account of the sign)

$$\frac{\epsilon_{max}^{1}-\epsilon_{min}^{1}}{2}$$

`'SIGNM1'` :maximum normal constraint on the level associated with ε_1

$$max_t(\boldsymbol{\sigma}(t).\boldsymbol{n}_1(t).\boldsymbol{n}_1(t))$$

where $\boldsymbol{n}_1(t)$ is the normal vector of the plan associated with ε_1 .

`'DENDIS'` :density of dissipated energy (W_{cy})

$$W_{cy}=\int_{cylce}\boldsymbol{\sigma}:\dot{\boldsymbol{\epsilon}}^{p}dt$$

where $\dot{\boldsymbol{\epsilon}}^{p}$ represent the rate of the plastic deformation.

`'DENDIE'` :density of energy of the elastic distortions (W_e)

$$W_e=\int_{cylce}\langle\boldsymbol{s}:\dot{\boldsymbol{e}}^{e}\rangle dt$$

where $\boldsymbol{s}$ represent the deviatoric part of the constraint $\boldsymbol{\sigma}$, $\boldsymbol{e}^{e}$ represent the deviatoric part of the constraint $\boldsymbol{\epsilon}^{e}$ and $\langle x\langle$ give x if $x\geq 0$ and 0 give if $x<0$.

`'APHYDR'` :half-amplitude of the hydrostatic pressure (P_a)

$$P_a=\frac{P_{max}-P_{min}}{2}$$

`'MPHYDR'` :average hydrostatic pressure (P_m)

$$P_m=\frac{P_{max}-P_{min}}{2}$$

`'DSIGEQ'` :half-amplitude of the equivalent constraint ($\Delta\sigma_{eq}/2$)

$$\frac{\Delta\sigma_{eq}}{2}=\frac{1}{2}max_{t1}\,max_{t2}\sqrt{\frac{3}{2}(\boldsymbol{s}(t_1)-\boldsymbol{s}(t_2)):(\boldsymbol{s}(t_1)-\boldsymbol{s}(t_2))}$$

`'SIGPR1'` :half-amplitude of the first principal constraint (with the catch in sign)

$$\frac{\sigma_{max}^{1}-\sigma_{min}^{1}}{2}$$

`'EPSNM1'` :normal maximum deformation on the level associated with σ_1

$$max_t(\boldsymbol{\epsilon}(t).\boldsymbol{n}_1(t).\boldsymbol{n}_1(t))$$

where $\boldsymbol{n}_1(t)$ is the normal vector of the plan associated with σ_1 .

`'INVA2S'` :half-amplitude of the second invariant of the deformation ($J_2(\Delta\epsilon)$)

$$J_2(\Delta\epsilon)=\frac{1}{2}max_{t1}\,max_{t2}\sqrt{\frac{2}{3}(\boldsymbol{e}(t_1)-\boldsymbol{e}(t_2)):(\boldsymbol{e}(t_1)-\boldsymbol{e}(t_2))}$$

`'DSITRE'` :half-amplitude of the Tresca half-constraint ($(\sigma_{max}^{Tresca}-\sigma_{min}^{Tresca})/4$)
`'DEPTRE'` : half-amplitude of the Tresca half-deformation ($(\epsilon_{max}^{Tresca}-\epsilon_{min}^{Tresca})/4$)
`'EPSPAC'` : plastic deformation accumulated p
`'RAYSPH'` : the ray of the smallest sphere circumscribed with the way of loading within the space of diverters of the constraints R . See the document [R7.04.01] for the definition of this parameter.
`'AMPCIS'` : amplitude of cission ($\tau_a = \frac{1}{2} \underset{0 \leq t_0 \leq T}{\text{Max}} \underset{0 \leq t_1 \leq T}{\text{Max}} \|\sigma_{(t_1)}^D - \sigma_{(t_0)}^D\|$)
`'DEPSEE'` : half-amplitude of the equivalent elastic strain ($\Delta \varepsilon_e^p/2$)

There exist sizes depending on the orientation of the plan which master keys with through a point of material. For these sizes, one defines criteria of the critical type of plan. The critical plan is the plan which makes the value maximum of a formula criticizes maximum.

`'DTAUCR'` : half-amplitude of constraint shearing as regards normal **N** ($\Delta \tau(\mathbf{n})/2$)
`'DGAMCR'` : half-amplitude of deformation (of engineering) shearing as regards normal **N** ($\Delta \gamma(\mathbf{n})/2$)
`'DSINCR'` : half-amplitude of normal constraint as regards normal **N** ($\Delta N(\mathbf{n})/2$)
`'DEPNCR'` : half-amplitude of normal deformation as regards normal **N** ($\Delta \epsilon_n(\mathbf{n})/2$)
`'MTAUCR'` : maximum constraint shearing as regards normal **N** ($\tau_{max}(\mathbf{n})$)
`'MGAMCR'` : deformation (of engineering) maximum shearing as regards normal **N** ($\gamma_{max}(\mathbf{n})$)
`'MSINCR'` : maximum normal constraint as regards normal **N** ($N_{max}(\mathbf{n})$)
`'MEPNCR'` :maximum normal deformation as regards normal **N** $\varepsilon_{nmax}(\mathbf{n})$

`'DGAMPC'` : half-amplitude of plastic deformation (of engineering) shearing as regards normal **N** ($\Delta \gamma^p/2$)
`'DEPNPC'` :half-amplitude of normal plastic deformation as regards normal **N** ($\Delta \varepsilon_e^p/2$)
`'MGAMPC'` : plastic deformation (of engineering) maximum shearing as regards normal **N** ($\gamma_{max}^p(\mathbf{n})$)
`'MEPNPC'` : maximum normal plastic deformation as regards normal **N** $\varepsilon_{nmax}^p(\mathbf{n})$

It will be noted that there exist two types of shearing strain measurement: distortions of shearing γ_{ij} ($i \neq j$) and shearing strains ϵ_{ij} ($i \neq j$). Let us note that $\gamma_{ij}=2\epsilon_{ij}$. For 'DGAMCR', 'MGAMCR', 'MGAMPC', the distortions of shearing were used γ_{ij} .

By using at least one of the first six sizes, one implicitly will build the criterion of standard "the critical plan". In this case there, one will find two plans different on which shearing is maximum.

It is noticed that the names of the sizes are identical to those used in the programming. The operators used in the formula must be in conformity with the syntax of Python as indicated in the note [U4.31.05].

It is noted that the equivalent size left for the periodic loading is under the name `'SIG1'` in the result.

9.2 For the loading not-periodical

The list of sizes available is in the following table:

TYPE_CHARGE = 'NON-PERIODIQUE', CRITERION = 'FORMULE_CRITERE'
The sizes available are: `'TAUPR_1'`: shear stress projected of the first top of the under-cycle ($\tau_{p1}(\mathbf{n})$) `'TAUPR_2'`: shear stress projected of the second top of the under-cycle ($\tau_{p2}(\mathbf{n})$)

'SIGN_1' : normal constraint of the first top of the under-cycle ($N_1(\mathbf{n})$)
'SIGN_2' : normal constraint of the second top of the under-cycle ($N_2(\mathbf{n})$)
'PHYDR_1': hydrostatic pressure first top of the under-cycle
'PHYDR_2': hydrostatic pressure second top of the under-cycle
'EPSPR_1': shearing in deformation projected first top under-cycle ($\gamma_{p1}(\mathbf{n})$)
'EPSPR_2': shearing in deformation projected of the second top of the under-cycle ($\gamma^{i}_{p2}(\mathbf{n})$)
'SIPR1_1': first principal constraint first top of the under-cycle ($\sigma_1(1)$)
'SIPR1_2': first principal constraint second top of the under-cycle ($\sigma_1(2)$)
'EPSN1_1': deformation normal on the level associated with $\sigma_1(1)$ first top of the under-cycle
'EPSN1_2': deformation normal on the level associated with $\sigma_1(2)$ second top of the under-cycle
'ETPR1_1': first principal total deflection first top of the under-cycle ($\epsilon_1^{tot}(1)$)
'ETPR1_2': first principal total deflection second top of the under-cycle ($\epsilon_1^{tot}(2)$)
'SITN1_1': constraint Normale on the level associated with $\epsilon_1^{tot}(1)$ first top of the under-cycle
'SITN1_2': constraint Normale on the level associated with $\epsilon_1^{tot}(2)$ second top of the under-cycle
'EPPR1_1': first principal plastic deformation first top of the under-cycle ($\epsilon_1^{p}(1)$)
'EPPR1_2': first principal plastic deformation second top of the under-cycle ($\epsilon_1^{p}(2)$)
'SIPN1_1': constraint Normale on the level associated with $\epsilon_1^{p}(1)$ first top of the under-cycle
'SIPN1_2': constraint Normale on the level associated with $\epsilon_1^{p}(2)$ second top of the under-cycle
'SIGEQ_1': equivalent constraint first top of the under-cycle ($\sigma_{eq}(1)$)
'SIGEQ_2': equivalent constraint second top of the under-cycle ($\sigma_{eq}(2)$)
'ETEQ_1': equivalent total deflection first top of the under-cycle ($\epsilon_{eq}^{tot}(1)$)
'ETEQ_2': equivalent total deflection second top of the under-cycle ($\epsilon_{eq}^{tot}(2)$)

For the loading not-periodical, after having extracted the elementary under-cycles with method RAINFLOW, we calculate an elementary equivalent size by the formula of criterion for any elementary under-cycle. It is noted that under cycle is represented by two states of stress or deformation, noted by the first and the second tops of the under-cycle.

By using it criterion in formula, one implicitly will build the criterion to determine the plan of the maximum damage with a linear office plurality of the damage.

It is noted that the use of the sizes 'TAUPR_1' and 'TAUPR_2' exclude that from 'EPSPR_1' and 'EPSPR_2' because one can project is stress shear, is it shearing in deformation. It is not possible to project all these two parameters simultaneously.

It is noticed that the names of the sizes are identical to those used in the programming. The operators used in the formula must respect the syntax of Python as indicated in the U4.31.05 note.

10 Size and components introduced into *Code_Aster*

10.1 Calculated by CALC_FATIGUE

The computed values are stored at the points of Gauss or the nodes according to the option selected. Size FACY_R (Cyclic Tiredness) was introduced into the catalogue of the sizes. Components of the field of this size, calculated by CALC_FATIGUE [U4.83.02] are described in the following tables.

For the periodic loading and the criteria of the type of plan criticizes of maximum shear stress (with DTAUM1)

DTAUM1 **first** value of the half amplitude max of constraint shearing in the critical plan

VNM1X	component x normal vector with the plan criticizes related to DTAUM1
VNM1Y	component y normal vector with the plan criticizes related to DTAUM1
VNM1Z	component z normal vector with the plan criticizes related to DTAUM1
SINMAX1	normal maximum constraint with the plan criticizes correspondent with DTAUM1
SINMOY1	normal average constraint with the plan criticizes correspondent with DTAUM1
EPNMAX1	normal maximum deformation with the plan criticizes correspondent with DTAUM1
EPNMOY1	average maximum deformation with the plan criticizes correspondent with DTAUM1
SIGEQ1	Constraint equivalent within the meaning of the criterion selected correspondent to DTAUM1
NBRUP1	many cycles before rupture (function of SIGEQ1 and of a curve of Wöhler)
ENDO1	damage associated with NBRUP1 (ENDO1=1/NBRUP1)
DTAUM2	**second** value of the half amplitude max of shearing in the critical plan
VNM2X	component x normal vector with the plan criticizes related to DTAUM2
VNM2Y	component y normal vector with the plan criticizes related to DTAUM2
VNM2Z	component z normal vector with the plan criticizes related to DTAUM2
SINMAX2	normal maximum constraint with the plan criticizes correspondent with DTAUM2
SINMOY2	normal average constraint with the plan criticizes correspondent with DTAUM2
EPNMAX2	normal maximum deformation with the plan criticizes correspondent with DTAUM2
EPNMOY2	average maximum deformation with the plan criticizes correspondent with DTAUM2
SIGEQ2	Constraint equivalent within the meaning of the criterion selected correspondent to DTAUM2
NBRUP2	many cycles before rupture (function of SIGEQ2 and of a curve of Wöhler)
ENDO2	damage associates with NBRUP2 (ENDO2=1/NBRUP2)

Table 5.5-1: Components specific to multiaxial cyclic tiredness for the periodic loading

For the periodic loading and the criteria of the type of plan criticizes with the keyword FORMULE_CRITIQUE

VNM1X	Component x normal vector with the critical plan who maximizes the critical formula
VNM1Y	Component y normal vector with the critical plan who maximizes the critical formula
VNM1Z	Component z normal vector with the critical plan who maximizes the critical formula
SIGEQ1	Constraint equivalent within the meaning of the criterion selected with the critical plan who maximizes the critical formula
NBRUP1	Many cycles before rupture
ENDO1	Damage associated with NBRUP1 (ENDO1=1/NBRUP1)

Table 5.5-2: Components specific to multiaxial cyclic tiredness for the periodic loading and the criteria of the standard plan criticizes with the keyword FORMULE_CRITIQUE

For the loading not-periodical and the criteria of the type of plan criticizes maximum damage

VNM1X	Component x normal vector with the plan criticizes dependent with the damage max
VNM1Y	Component y normal vector with the plan criticizes dependent with the damage max
VNM1Z	Component z normal vector with the plan criticizes dependent with the damage max
ENDO1	Damage associated with the block with loading
VNM2X	Component x normal vector with the plan criticizes dependent with the damage max
VNM2Y	Component y normal vector with the plan criticizes dependent with the damage max
VNM2Z	Component z normal vector with the plan criticizes dependent with the damage max

Table 5.5-3: Components specific to multiaxial cyclic tiredness for the loading not-periodical

For the loading not-periodical, if there exists only one critical plan of the maximum damage, VNM2X, VNM2Y, VNM2Z are identical to VNM1X, VNM1Y, VNM1Z . If several plans exist, one emits an alarm and one leaves the two foregrounds.

10.2 Calculated by POST_FATIGUE

The computed values are stored at the points of Gauss or the nodes according to the option selected. size FACY_R (Cyclic Tiredness) was introduced into the catalogue of the sizes.

Components calculated by POST_FATIGUE [U4.83.01] are described in the following tables.

For the periodic loading and the criteria of the type of plan criticizes of constraint shearing maximum

DTAUM1	**First** value of the half-amplitude max of shearing in the critical plan
VNM1X	Component x normal vector with the plan criticizes related to DTAUM1
VNM1Y	Component y normal vector with the plan criticizes related to DTAUM1
VNM1Z	Component z normal vector with the plan criticizes related to DTAUM1
SINMAX	Normal maximum constraint with the plan criticizes correspondent with DTAUM1
SINMOY	Normal average constraint with the plan criticizes correspondent with DTAUM1
EPNMAX	Normal maximum deformation with the plan criticizes correspondent with DTAUM1
EPNMOY	Average maximum deformation with the plan criticizes correspondent with DTAUM1
SIGEQ	Constraint equivalent within the meaning of the criterion selected correspondent to DTAUM1
NBRUP	Many cycles before rupture (function of SIGEQ1 and of a curve of Wöhler)
TOO BAD	Damage associated with NBRUP1 (ENDO1=1/NBRUP1)
VNM2X	Component x normal vector with the plan criticizes related to DTAUM2
VNM2Y	Component y normal vector with the plan criticizes related to DTAUM2
VNM2Z	Component z normal vector with the plan criticizes related to DTAUM2

Table 5.5-1: Components specific to multiaxial cyclic tiredness for the periodic loading

For the periodic loading and the criteria of the type of plan criticizes with the keyword FORMULE_CRITIQUE

VNM1X	Component x normal vector with the critical plan who maximizes the critical formula
VNM1Y	Component y normal vector with the critical plan who maximizes the critical formula
VNM1Z	Component z normal vector with the critical plan who maximizes the critical formula
SIGEQ1	Constraint equivalent within the meaning of the criterion selected with the critical plan who maximizes the critical formula
NBRUP1	Many cycles before rupture
ENDO1	Damage associated with NBRUP1 (ENDO1=1/NBRUP1)

Table 5.5-2: Components specific to multiaxial cyclic tiredness for the periodic loading and the criteria of the standard plan criticizes with the keyword FORMULE_CRITIQUE

For the loading not-periodical and the criteria of the type of plan criticizes maximum damage

VNM1X	Component x normal vector with the plan criticizes dependent with the damage max
VNM1Y	Component y normal vector with the plan criticizes dependent with the damage max
VNM1Z	Component z normal vector with the plan criticizes dependent with the damage max
TOO BAD	Damage associated with the block with loading
VNM2X	Component x normal vector with the plan criticizes dependent with the damage max
VNM2Y	Component y normal vector with the plan criticizes dependent with the damage max
VNM2Z	Component z normal vector with the plan criticizes dependent with the damage max

Table 5.5-3: Components specific to multiaxial cyclic tiredness for the loading not-periodical

11 Other criteria

11.1 Criterion VMIS_TRESCA

In this part we describe the option VMIS_TRESCA who allows to calculate the maximum variation, in the course of time, of a tensor of constraint according to the criteria of Von Mises and Tresca.

This calculation can be carried out with the nodes or the points of Gauss according to the request of the user.

We give below the algorithm which is programmed in *Code_Aster*.

Notations:

N : Many moments
T_i : Tensor at the moment i
$DIFF_T$: difference between two tensors

$VAVMIS=0.0$

$VATRES=0.0$

For i of 1 with $(N-1)$

For j of $(i+1)$ with N

$DIFF_T=T_i-T_j$

$VMIS=$ Von-put $DIFF_T$

$TRES=$ Tresca of $DIFF_T$

If $VMIS>VAVMIS$ then

$VAVMIS=VMIS$

If $TRES>VATRES$ then

$VATRES=TRES$

11.2 Components of *Code_Aster* used

The computed field by CALC_FATIGUE has as components:

VAVMIS	Maximum amplitude of variation of the criterion of Von Mises
VATRES	Maximum amplitude of variation of the criterion of Tresca

12 Conclusion

In this document we presented the criteria of MATAKE and DANG-VAN adapted to the office plurality of damage under periodic and nonperiodic loading.

When the loading is periodic the criteria of MATAKE and DANG-VAN are tested by the cases tests SSLV135a and SSLV135b. The cases tests SSLV135c and SSLV135d test these two criteria if the loading is not periodical.

The keywords which make it possible to use these two criteria are described in the document [U4.83.02] devoted to the order CALC_FATIGUE. One will be able to also consult the keyword factor CISA_PLAN_CRIT order DEFI_MATERIAU [U4.43.01].

13 Bibliography

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14 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
8,4	J.ANGLE S EDF-R&D/AMA	Initial text

Annexe 1

The various situations are summarized in [A1-1 Table]. In [A1-1 Table], "0" and "1" mean respectively that there are no points and that there is at least a point in the indicated sectors.

Sector 1	Sector 3	Sector 2	Sector 4	Axis of projection
0	0	0	0	Impossible case.
0	0	0	1	Impossible case.
0	0	1	0	Impossible case.
0	0	1	1	Axis 1.
0	1	0	0	Impossible case.
0	1	0	1	Use of the procedure of selection.
0	1	1	0	Use of the procedure of selection.
0	1	1	1	Axis 1.
1	0	0	0	Impossible case.
1	0	0	1	Use of the procedure of selection.
1	0	1	0	Use of the procedure of selection.
1	0	1	1	Axis 1.
1	1	0	0	Axis 2.
1	1	0	1	Axis 2.
1	1	1	0	Axis 2.
1	1	1	1	Use of the procedure of selection.

A1-1 table: Summary of the situations

The impossible cases result from the way in which the framework and the sectors are built. This construction makes impossible the presence of points in no or only one sector.

Annexe 2

The projection of an unspecified point on the second axis is quickly described in this appendix. Starting from a point P unspecified known, we calculate the punctual coordinates P' such as:

$$\overrightarrow{OM}\,.\,\overrightarrow{PP'}=0$$

After simplification it comes the relation:

where a value of $U_{P'}$ different from U_P we gives $V_{P'}$.

In the plan (u, v) the second axis and the segment are lines closely connected respectively described by $v=a_s u+b_s$ and $v=a_P u+b_P$, therefore to know the coordinates of the point project on the second axis P_p we solve the equation:

$$a_s u+b_s=a_P u+b_P$$

where

$$a_s=\frac{(V_M-V_O)}{(U_M-U_O)}\ ,\qquad b_s=\frac{(U_M V_O-U_O V_M)}{(U_M-U_O)}\ ,$$

$$a_P=\frac{(V_{P'}-V_P)}{(U_{P'}-U_P)}\ ,\qquad .$$

One obtains:

$$U_{P_s}=\frac{b_P-b_s}{a_s-a_P}\ ,$$

$$V_{P_s}=\frac{a_s b_P-a_P b_s}{a_s-a_p}\ .$$