Data processing method of the rod-ties for the calculation of reinforcement

Summary:

This document presents the macro-order of postprocessing CALC_BT, an alternative of assistance per computer for the application of the Method Rod-Ties which is used for to calculate the reinforcement of an element of structure with of concrete-armed by the method of the rod-ties. This one consists in finding a lattice registered in the structure which one wants to calculate the reinforcement on the basis of a calculation on homogeneous material (concrete). The rods of this lattice are out of concrete, the ties out of steel.

The originality of the routine is that to automate the research of the lattice, operation which traditionally is carried out with the hand.

Contents

1 Introduction

In the field of civil engineering, and more specifically in the field of the Reinforced concrete structures (BA), for the design of solid elements and little requested in inflection such as corbels, beams and plates with changes precipice in their section, the beam-veils or the veils flags, the Method Rod-Tie (MBT) can be privileged compared to the classical algorithms of calculation of reinforcement which are based on a flexional resistance to the various limiting states (for example Wood-Armor or Capra-Maury [Capra et al., 1978]).

The MBT remains an alternative adapted for the design of structures out of concrete presenting an elastic or plastic behavior whose framework of application is well defined in the codes of structural design out of concrete like Eurocodes (§5.6.4 and 6.5 of [In 1992-1 - 1]). Nevertheless, this method presents the major drawback to require an important investment in human resources or capacity of calculation for, respectively, its manual application or an automatic approach by optimization of topology.

The order `CALC_BT` present a method of implementation semi-automatic of the MBT, which is coherent with [In 1992-1 - 1] which recalls that model adapted rod-ties can be defined from isostatic of constraint and distributions of constraints obtained pursuant to the theory of linear elasticity. For this reason the macro-order exploits for the rods and the ties the directions of the principal constraints, σ_I and σ_{III} , obtained starting from a preliminary calculation on a model finite elements of the studied structural element.

2 Description of the method Rod-Ties

2.1 Introduction and motivation

In the zones of discontinuities (see §2.2) structural elements, i.e subjected to concentrated loadings or having changes abrupt of geometry, the conventional methods of analysis in inflection seem more not to be sufficient [Thompson, 2002]. These zones are generally reinforced by using rules of good practices based on the experiment or empirical directives.

The method Rod-Ties (BT) is a procedure of rational design for the calculation of reinforcement of local areas of the reinforced concrete structures (called zones D for discontinuity); the procedure is based on simple formulations and mechanical assumptions to be easily applied in the design.

In a general way, the BT method relates to the idealization of zone of discontinuity by the means of an internal lattice able to represent the distribution and the advance of the forces inside the structure. The lattice is composed of:

- Rods (elements in compression) which models the compression zones of the concrete,
- Ties (elements in traction) which models traction in the steel reinforcement,
- Nodes which represent the zones of interconnection of the elements or the zones anchoring in the concrete.

The rods as well as the ties are linear elements which carry only thrust loads. This mechanism of lattice must correctly balance the loads applied to the system. The failure of the system is thus dictated by the rupture of one or more of its elements or also defined by excessive compressive stresses in the rods or the nodes. Ideally, only the first mode of failure should occur.

2.2 Areas of discontinuity

The basic principles of the theory of the curvilinear structures imply that a linear distribution of the deformations occurs through the depth of an element, namely that: the plane sections remain plane. The element is thus dominated by the behavior in inflection, and the design can proceed section by section; this kind of elements are usually referred like areas B. For the design of the elements in inflection, the stresses compressive are classically supposed on a rectangular block of constraints, while tensile stresses are supposed to be taken again by the longitudinal reinforcements.

In a contrary way, the areas D ("D" meaning discontinuity) occur in the proximity of loads or geometrical discontinuities. The loads applied, the reactions of support and the changes geometrical precipice are discontinuities which "disturb" the distribution of the constraints within the member near as of places where they act. The thick beams, the openings and the corbels are examples of geometrical discontinuities which correspond to the existence of areas D.

A characteristic of the areas D, relates to the fact that the efforts through the depth of member present a non-linear profile, therefore, the assumptions subjacent with the design methods in inflection are not validated. According to the principle of Saint-Coming, an analysis of the elastic constraints indicates that the constraints due to the axial loads and with the inflection, approach a linear distribution at a distance roughly equal to the depth of the member, H, far from discontinuity. In other words, a non-linear distribution of the constraints exists according to the depth of a member starting from the point where discontinuity is introduced [Schlaich and al., 1987].

2.3 Basic principles

A design of BT model adheres to two principles:

1) the model out of resulting lattice must be in balance with the system of external forces

2) the concrete element must have a capacity of sufficient deformation to guarantee the supposed distribution of the forces [Schlaich and al., 1987]; the good length of anchoring of reinforcement is an implicit requirement in order to guarantee ductility necessary.

In a complementary way, the compressive stresses developed in the concrete should not exceed the resistance of the concrete, and the traction developed with the reinforcements must be lower than the resistant force of the steel section. If all the requirements mentioned are satisfied, the application of the BT Method should lead to a preserving design [Williams and al., 2012].

2.4 Application

The following paragraphs describe the manual application of the BT method.

2.4.1 *Zoning*

The first stage consists in determining whether the BT method is a good alternative to solve the problem. On the principle of Saint-Coming, the structure can be divided into areas B and D. the process of design of the MBT must be used to conceive the sections which are qualified as areas D.

2.4.2 Definition of the boundary conditions

The second phase consists in determining the loading cases which the structure must support. If the structure is made up at the same time of areas B and areas D, only the areas D will be conceived using the BT method. In this way, each area D must be treated as an isolated element whose boundary conditions nodal come from their interaction with the adjacent elements and from the total reaction of the supports of the structure in the case of load considered (figure 2.4.2-a): the forces acting in extreme cases of the area D concretize the boundary conditions for the design of the BT model.

The internal efforts and the moments with the interfaces between areas B and D can be supposed punctually concentrated and must be applied to the limit of the area D (figure 2.4.2-has). As one can deduce some, a total elastic analysis of the structure must be carried out in order to determine the reactions of support and the loads of interface for various areas D.

By considering that the elements of the BT model cannot resist certain types of loads (for example, moments and distributed loads), certain modifications can be necessary to produce a system of equivalent loading.

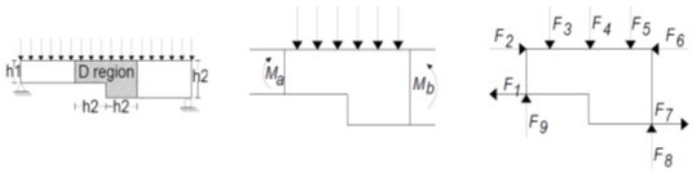

Figure 2.4.2-a: Local zone of a beam to variable thickness. Total element (left), Limits of the area D (center) and isolated model (right).

2.4.3 Definition of the model

On the basis of system of equivalent concentrated loadings derived from the preceding stage, the engineer projects the rods and the ties as right elements of a node to another with an aim of developing a stable structure. This process can possibly include the interaction of all the initial nodes and can moreover include secondary nodes according to the criteria of the originator. For this stage, a model of continuous finite elements, solved in linear elasticity, is usually used in order to visualize the direction of constrained principal in the structure and to align the elements of the BT model according to the stress trajectories [Schlaich and al., 1987].

2.4.3.1 Analysis of the BT model and section of the elements

The section of each element of the BT model must be sufficient to resist in full safety the force which forwards there, without exceeding the elastic limit of steel (for the ties) or limiting resistance in the concrete (for the rods). As for a conventionally reinforced structure, the minimal section, A_{st} , of an element subjected to an axial load can be given using the following equation:

$$A_{st}=\frac{F_u}{\varphi R}$$

In this expression:

F_u corresponds to the greatest force applied in the element

R is the resistance of material considered (σ_y for steel and σ_c for the concrete)

φ a safety factor indicated in the suitable regulation

2.4.4 Checking of the nodes

Because of the level of the constraint which must be balanced in a small volume of concrete, the nodes of the structure lattice are the areas most strongly requested of a BT model. In particular, the nodes are only one simplified idealization of a more complex reality and the definition of their geometry also rests on the criteria of the originator. Ideally, the nodes can be conceived so that the constraint on all the faces is the same one. If the constraint is the same one, the report of the surface of the side face is proportional to the force applied. In this case, the node is called hydrostatic node: the principal constraints are equal on all the sides and stresses shear disappear.

Usually, a classification of the type of nodes is made according to the nature of the elements which converge towards a zone of connection. According to the sign of the convergent forces, the node can be called CCC, CCT, CTT or TTT (C for compressive forces and T for the forces of tension). If more than three elements convergent in a node, it is recommended to amalgamate groups of forces in order to reduce their number. However, that is not always possible and other possible types of nodal combinations are also accepted (e.g. CCTT, CCCC)

3 Method of assistance to the calculation of the BT models in 2D

3.1 Description of the algorithm

The algorithm is divided into five sub-routines including the analysis and the postprocessing of a model. The sub-routines of the algorithm general make it possible to achieve various tasks:

1. Introduction of the continuous model (modeling planes)
2. Construction of the basic structure starting from the extrema local of the principal constraints minor and major and the direction of minor principal constraints
3. Definition of the boundary conditions
4. Optimization of the basic structure
   A. Optimization of the sections according to method FSD "Fully Stressed Design"
   B. Topological optimization + FSD
5. Acquisition of the model Rod-Ties (BT)

3.2 Introduction of the continuous model

To be launched, the algorithm requires a field of results of the type `evol_elas` or `evol_noli` , geometry (recoverable in the structure result), the list of the materials (structure of given of type `to subdue`) and boundary conditions imposed on the nodes. The boundary conditions must be applied by considering the recommendations of the § 2.3 and § 2.4.
Since the BT model is defined by one-way elements assembled in a two-dimensional reference mark, the slopes of the rods and ties are related to the direction of the plane constraints.

3.3 Construction of the basic structure

The basic structure is built on the basis of result of the elastic linear analysis acquired in the preceding stage. Results, such as the field of direction and the levels of constraint, are used as predictors of the geometrical characteristics of the final model.

The first stage of this sub-routine consists in delimiting the zones of the structure where elements of type rod or tie can be placed. The division of the geometry is established according to the following principles:

1. Resulting division must take account of the coordinates of the potential structural nodes (maximum buildings of stress fields).

2. Each subdivision must be sufficiently large to be able to contain a rod within its limits.

3. At the same time, the subdivisions must be sufficiently small to present a clearly dominating direction of the constraint minor σ_{III} (by considering only the results of model EF initial located inside each division).

For this purpose, maximum and minimal values local of σ_I and σ_{III} are required in the geometry. To carry out this task, a grid, of interpolation is superimposed on the initial grid (Figure 3.3-a). The grid presents a constant step regulated in direction X, `PAS_X`, and a step in direction Y, `PAS_Y`. Like recommendation, these two values must be lower or equal to the smallest dimension of elements in the model.

The algorithm determines the derivative partial to give the slope of a discrete function $f=\sigma(x,y)$ in an unspecified point (a,b) in the directions parallel with the axes of coordinates. According to the definition of the derivative partial, the derivative $\partial f/\partial x$ is obtained while fixing the value y with a constant b and by differentiating the function $f(x,b)$ at the point $x=a$. Same manner, the partial derivative compared to is obtained while fixing $x=a$ and by differentiating the function $f(a,y)$ in. $y=b$ The evolution of the value of the derivative is used to find the peaks and the valleys of the function. The first derived one is calculated in form *backward* and *forward* what makes it possible to distinguish the maxima of the minima without calculating the derivative second [Levy, 2010].

The positions of the maxima of constraint recommend the presence of the nodes in the BT model and influence the size and the distribution of the internal elements of the structure. In the same way, the position of these maxima is used to define zones where direction of the major principal constraint σ_I present a dominating direction. In the intention to define these zones, a division of Voronoï is applied at this stage.

The division of Voronoï is a cutting of a space bi- or dimensional sorting in cells starting from a discrete whole of points called "germs" [Aurenhammer, 2013]. Each cell locks up only one germ, and forms the whole of the points of the plan closer to this germ than of all the others. The cell represents to some extent the "zone of influence" of the germ.

By using the division of Voronoï and while being based on the position of the peaks of constraint like the germs, a division of the geometry in a finished number of cells is carried out. For the maxima of constraint found at the edge of the geometry, the associated germs will be the intersections of the "skin" of the geometry with a circle of infinitesimal radius whose center is located at coordinated peak of constraint.

The preceding stage is intended to directly delimit the length and the distribution of the elements in compression. In addition, the slope of each one of its elements is supposed to be parallel with the directions of the principal constraint (classical assumption in the development of the BT method).

By considering that the rods must approach the distribution and the direction of the minor constraint, σ_{III} , the slope of an element crossing a cell of Voronoï is proposed like the angular average of the inclination associated with the field of direction of the minor constraint with the elements resting inside this cell.

It was observed [Mendoza-Chávez, 2018], that the size and the form of the grid (especially in the vicinity of the boundary conditions) are two factors being able to bring results which, point of considering digital, represent maxima of constraint at very weak distances one of the other. A distance lower than the value of the minimal size of the elements can generate digital problems with respect to the generation of a division of Voronoï containing of blank cells. In order to avoid the appearance of the very close maxima, a distance from tolerance of proximity of the maxima of constraint, `TOLE_BASE`, can be defined. In this way, two (or several) maxima of concentration found at a distance equal or lower than the value of tolerance will be amalgamated with the position of the average of cordonnées of the nodes concerned. If one of the maxima is with the coordinates of one of the boundary conditions, the close nodes are amalgamated to the maximum associated with the limiting condition.

Once the slope of the rods was calculated in each cell, an iterative algorithm is implemented for the generation of the rods. Starting from the coordinates of the germs corresponding to the maximum local value of the minor constraint σ_{III} , the algorithm takes an initial node, N_i , and a line projects, according to the slope calculated for the current cell. The projected line is delimited by the limits of the zone in progress, generating a final node N_f . The following iteration takes the last final node like a new initial node and proceeds in the same manner to calculate another final node. The procedure continues for all the germs until the generated "branches" reach the limits of the geometry. Consequently, the algorithm of generation of rods produces branches which start with the coordinates of the concentrated loadings and the stress concentrations of compression, and are diffused in all the field of design. Even if two or several branches could cross the same zones, until this stage, they do not share necessarily common nodes. Consequently, a procedure of nodal fusion with a tolerance of fusion given, `TOLE_BT`, is put in work in this state.

As in the case of the rods, the ties are supposed to approach the distribution of the principal elastic constraints σ_I . Moreover, these elements must also be adapted to the way of already existing rods.

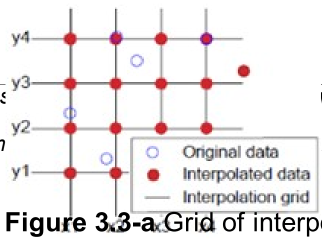

Figure 3.3-a Grid of interpolation of data [Mendoza-Chávez, 2018]

On the basis of the principle that the local extrema of the constraint provide a reliable indicator of the position of the nodes of the BT model, the distribution of the secondary elements is obtained by considering each point of major concentration stress σ_I like a node of birth of elements. This node of birth is attached by a straight line to each one of existing nodes in the way of rods.

3.4 Definition of the boundary conditions

The loads and supports are assigned with the nodes of the BT model closest to the coordinates of the points of application of the boundary conditions of the continuous model. On the basis of the position of the nodes contained in the groups associated with the boundary conditions with the continuous model, the node nearest to the model lattice is found to give identical conditions: imposed displacements and forces applied to the nodes will be transported continuous model towards the model lattice with the same intensity and direction. The process generates conditions equivalent to the representation with elements plans.

3.5 Definition of the boundary conditions

The first three sub-routines aim to the construction of the basic structure. The structure thus created is characterized by the representation of the potential nodes and a large number of elements. At this stage, the quantity of elements present in the structure does not make it possible to identify realizable a BT model clearly. Moreover, the elements bar do not have elementary characteristics, particularly their section.

In order to highlight a realizable BT model starting from the basic structure, an iterative diagram of optimization allowing for choice of material of each element (`STEEL` or `CONCRETE`) and, more important still, to evaluate the section of each element was implemented.

The selected criterion of optimization is based on the technique *Fully Stressed Design* (FSD), a criterion of simple and intuitive optimization based on the following statement: “For an optimal design, each member of the structure must be subjected to a maximum constraint in at least one of the conditions of load of design” [Ganzreli, 2013]:

$$a_{m,i}=\frac{F_{m,i}}{\sigma_{m,R}}$$

where

$a_{m,i}$ is the section of the element m with the iteration i .

$F_{m,i}$ is the thrust load in the element m with the iteration i .

$\sigma_{m,R}$ is the working stress of material associated with the element m (`SIGMA_C` for the concrete and `SIGMA_Y` for steel)

Within each iteration, the elements increase, reduce or preserve the value of their section according to the supported constraint: when the request exceeds the working stress of the element, the section is increased. In a similar way, the section is reduced if the pressure applied is lower.

In order to avoid the problems of stability in the matrix of rigidity caused by the elements with worthless section, a minimal section, `SECTION_MIN`, can be adopted, so that the optimal solution must accept that certain members are not completely solicited.

A minimal section different from zero product, generally, “secondary” elements of which the only goal is often to guarantee the condition of not-singularity on the matrix of rigidity and to avoid the internal mechanisms of the structure. These elements can simply be been unaware of at the last stage of optimization, [Ohsaki, 2002].

Initialization (Stage t_0) : The initial value of the sections is proposed as being unit. Taking into account the fact that the selection of the initial sections can affect the performance of the algorithm, and even the solution of optimization, the user can choose a different initial value.

The initial model considers that all the elements are out of steel. This consideration is made only once.

During the first iteration, the algorithm associates materials according to the sign of the force forwarding in each element; the elements subjected to tractive efforts are associated with the characteristics of steel and the elements undergoing of the compressive forces, or with the worthless efforts, are associated with the characteristics of the concrete.

Selection of cross section (Stage t_1): The section of each element is applied as a variable of design with a condition of nonworthless lower limit [Li, 1990].

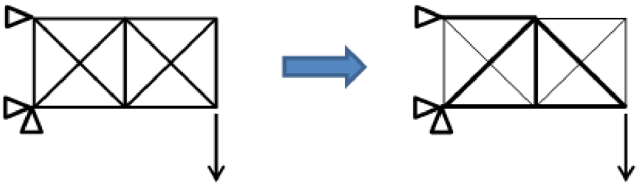

Figure 3.5-a: Optimization of sections. Reduction of section of the elements

Generally, taking into account the fact that the elements resistant to compression have a low slenderness ratio, the effects of buckling are neglected. With each iteration, balance is checked by the relation $Ku=P$ and the sections evolve in correspondence with the report of axial stress to the current iteration.

Elimination of the secondary elements topological Optimization (Stage t_2) : As the stage T_1 advance, the section of certain elements of the structure decreases until reaching the limit of the not-worthless lower limit. By considering that the individual deformation energy of these elements is weak (compared to the elements of more important section in the model), their contribution with the total rigidity of the structure is negligible. These elements can be qualified "secondary" elements.

Figure 3.5-b: Topological optimization. Elimination of elements

In order to remove the "secondary" elements without having repercussions on the stability of the BT lattice (with being known, to avoid a bad conditioning of the matrix of total rigidity), a sub-routine of selection and elimination of elements is installation. Assembly of the matrix of total rigidity, K_G , is divided into two stages: at the time of the first stage, one buckles on the assembly of the principal elements; at the time of the second phase, one buckles on the secondary elements. Inside the second loop, an evaluation of the conditioning of the matrix is made during the addition of an element or a group of elements of weak section. If the matrix of rigidity is well conditioned, the algorithm erases the elements not added according to the pseudonym codes according to:

```
FOR all principal elements:
    Assemble K_G = ∑ k_ei

WHILE cond (K_G) < 1/(os precision):
    Add a secondary element K_G + k_ei
```

End: The end of the algorithm can occur in two different ways:

1) Evolution of convergence to the current iteration, C , reached the threshold prescribed RESI_RELA_SECTION

$$C=\sqrt{\sum \left(\frac{a_{m,i-1}-a_{m,i}}{a_{m,i}}\right)^2}$$

where $a_{m,i}$ is the section of the element m with the iteration i .

2) The iteration count reached the maximum, NMAX_ITER, prescribed.

3.6 Diagrams of optimization

By adopting the same method as in the reference [Mendoza-Chávez, 2018] the reduction of volume of the model is carried out using a hybrid diagram of optimization which comprises an optimization of the sections (stage T_1) and a topological optimization (stage T_2). The user has the possibility of launching the algorithm of construction of an optimized BT model, by using one of two different diagrams:

SECTION : Optimization of the sections

REPORT : Optimization of the sections + topological optimization

LE iterative process is represented with the following pseudo code :

```
T 0 :
Initial sections
Considered material (steel)
Solve for displacements
IF sigma <= 0
       Assign material steel

T 1  :
Initial sections
Updated material
WHILE I <= #Iterations and C <= C*:
       Solve for displacements
       IF ABS (sigma) > sigma limit:
             Increase section
       ELSE IF ABS (sigma) < sigma limit:
             Decrease section

T 2 :
Yew C <= C 2 :
Topology optimization
Principal FOR all elements:
      Assemble

 WHILE cond () < 1 (bone precision):
      Add has secondary element
```

3.7 Finalization of the model Rod-Ties

At this stage, the cross sections and the material associated with each element are known. A structure lattice is already defined starting from the reduction of the basic structure but this structure has sometimes "superimposed" elements; elements which share the same nodes initial and final. Even if these elements do not affect the model, their existence can involve mistakes in interpretation because they represent different bodies occupying same space. This is avoided by searching these groups of elements dividing of the nodes and, if necessary, by amalgamating each group in only one element whose surface is equal to the sum of the group. Nevertheless, this fusion is still not possible; the user will have to thus check the existence of concomitant elements in the structure.

Checking of the sections: Based on a design FSD (*Fully Stressed Design*), the section of each element is calculated by taking of account the efforts obtained with the last iteration. Dimensioning guarantees the axial resistance of the elements.

4 Conclusion

The order `CALC_BT` allows to carry out the calculation of reinforcement by using the method Rod-Ties [U4.4208].

The user gives as starter this order the concepts results associated with a linear calculation – rubber band carried out, the geometry and boundary conditions of the model as well as materials to use and of the parameters of desired optimization.

The user can choose between two iterative diagrams of optimization; a diagram of optimization of the sections of the elements (`DIAGRAM = 'SECTION'`) and a second diagram of optimization (`DIAGRAM = 'REPORT'`) what regards also the quantity of the elements as one of the variables to be optimized.

Sizes determined by the order `CALC_BT` are deferred in a table in which one finds the value of the parameters by elements such as the force, the section, the length and it type of element associated (rod or tie).

5 Bibliography

1 [Almeida, 2013] Almeida, V.S., Simonetti, H.L., & Neto, L.O. (2013). Comparative analysis of strut-and-tie models using smooth evolutionary structural optimization. Engineering Structures, 56,1665-1675

2 [Aurenhammer, 2013] Aurenhammer, F., Klein, R., & Lee, D.T. (2013). Voronoi diagrams and Delaunay triangulations. World Scientific Publishing Company.

3 [Capra et al., 1978] Capra, A. and Maury, J. (1978). Automatic calculation of the optimal reinforcement of the plates or reinforced concrete hulls. Annals of the technical Institute of the building and public works, 367.

4 [Ganzereli, 2013] Ganzreli, S. (2013). Direct fully stressed design for displacement constraints. In 10th World Structural Congress one and Multidisciplinary Optimization, pages 19-24.

5 [Levi, 2010] Levy, D. (2010). Introduction to numerical analysis. Department of Mathematics and Center for Scientific Computation and Mathematical Modeling (CSCAMM), University of Maryland.

6 [Li, 1990] Li, X. (1990). Optimum Truss structure design and its engineering application. Computers & Structures, 36(3): 567-573.

7 [Lövgren, 2011] Lövgren, S., & Norberg, E. (2011). Topology Optimization of Vehicle Bodystocking Structure for improved Wrinkle & Handling.

8 [Mendoza-Chávez, 2018] Mendoza-Chávez G. (2018). *Strut-and-Tie models for the design of non-flexural elements: computational aided approach*. University Paris-Is. Doctorate.

9 [EDF R & D, 2018] EDF R & D functional Specifications of the tool for postprocessing by rods and ties 6125-172018-02375 FR Version 0.3

10 [In 1992-1 - 1] European standards - Eurocode 2 Calculation of the concrete structures Left 1-1: General rules and rules for the buildings

11 [Ohsaki, 2002] Ohsaki, Mr. and Swan, C. (2002). Topology and geometry optimization of trusses and frames. Recent advances in optimal structural design.

12 [Schlaich et al., 1987] Schlaich, J., Schäfer, K., & Jennewein, Mr. (1987). Toward has consist design of structural concrete. NCV newspaper, 32(3), 74-150.

13 [Thompson, 2002] Thompson, K. (2002). *The anchorage behavior of headed reinforcement*

14 *in CCT nodes and lap splices.* PhD thesis.

6 Features and checking

The computational tool of models Rod-Ties CALC_BT is checked by the following tests:

ssnp105	Veil of unit size	[V6.03.105]
ssnp106	Beam veils with hopper	[V6.03.106]