Extrapolation of measurements on a digital model in dynamics

Summary:

A approach of extrapolation of experimental results of measurement in dynamics (displacements, speeds, accelerations, strains, stresses) on a digital model is presented. Based on a representation of the structure on a basis of projection chosen beforehand, it consists of the resolution of the opposite problem defined by the identification of the generalized coordinates relating to the base of projection. The resolution suggested uses a minimization, within the meaning of least squares, by using the decomposition LU or the decomposition in singular values (SVD), of a functional calculus possibly regularized via the addition of a criterion of proximity of a solution known a priori. In the case of a temporal identification, an explicit formulation of information a priori is proposed.

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1 Problems

One wishes to estimate numerically, the behavior in any point of a structure starting from the mesures taken in some points of the structure. Taking into account the costs and constraints of accessibility, experimental measurements are generally of number limited and located into cubes places which inevitably are not requested. Thus, in dynamics, the knowledge of the zones of stress concentration and the local values of constraints is crucial to check the mechanical resistance of the material. One is then brought to extrapolate results of measurement located, on the whole of the digital grid of the structure.
The approach of extrapolation suggested is based on a representation of the structure on a basis of judiciously selected projection (clean modes, static answer,…). It consists of the determination of the generalized coordinates relative to this base of projection. The resolution suggested uses a minimization, within the meaning of least squares, by decomposition out of LU or singular values (SVD), of a functional calculus possibly regularized via the addition of a criterion of proximity of a solution known a priori. In the case of a temporal identification, an explicit formulation of information a priori is proposed.

2 Notations

$q, \dot{q}, \ddot{q}$: vectors of displacements, speeds and accelerations in the physical reference mark

$\eta, \dot{\eta}, \ddot{\eta}$: vectors of displacements, speeds and accelerations generalized

Φ : formed matrix by the basic vectors of projection (displacements)

Φ_ϵ : formed matrix by the basic vectors of projection (deformations)

Φ_σ : formed matrix by the basic vectors of projection (forced)

$\bar{\Phi}$: matrix of the basic vectors (displacements), restricted with the measured degrees of freedom

$\bar{\Phi}_\epsilon$: matrix of the basic vectors (deformations), restricted with the measured degrees of freedom

$\bar{\Phi}_\sigma$: matrix of the basic vectors (forced), restricted with the measured degrees of freedom

I : matrix identity

N_{num} : many basic vectors of projection, N_{exp} : many degrees of freedom measured

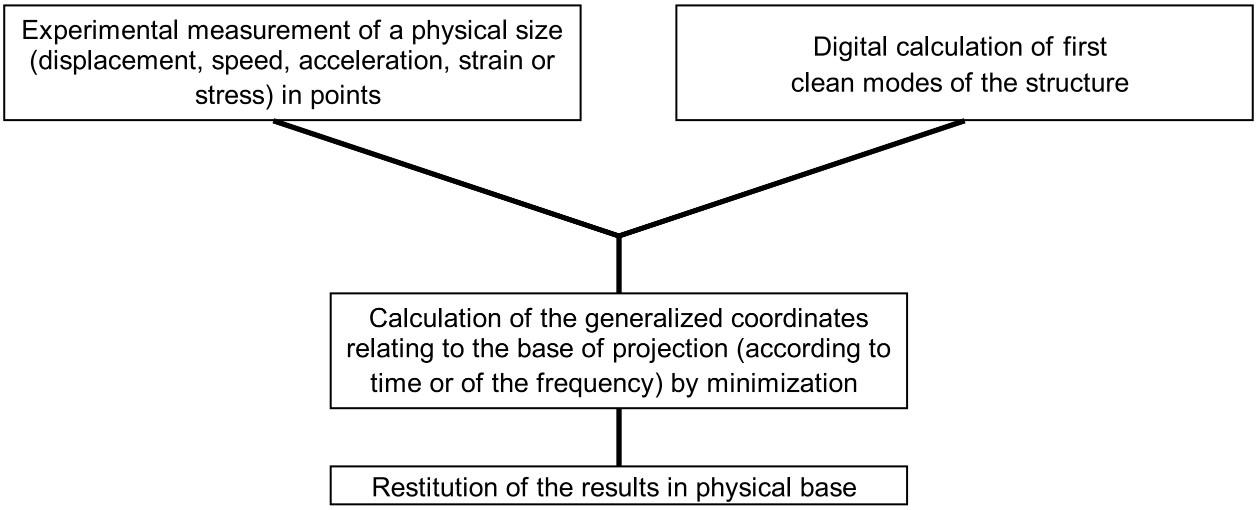
t : variable time, ω : variable pulsation

TF : transform of Fourier, TF^{-1} : opposite transform of Fourier

With$^+$: pseudo-opposite of matrix A

3 Approach of extrapolation

The approach which one wishes to set up in order to extrapolate of the vibratory results of measurement on a digital model breaks up into 4 stages [bib1]:



This approach is based on the concept of projection of a field in a space of lower size corresponding to the space of the digital model and then extrapolation on the space of the digital model. The fact of projecting the field in a space of reduced size generates necessarily a loss of information during extrapolation. One thus sees here the importance of the choice of the base of projection. This base can be made of modal answer and/or static answer. It is supposed here that the digital model is linear.

4 Relations between the physical sizes and the generalized sizes

It is supposed that the intrinsic behavior of the structure is represented in a space generated by N_{num} basic vectors of projection. The transformation of Rayleigh-Ritz establishes the relation between the degrees of freedom of the structure in the physical reference mark and its generalized coordinates:

$$\mathbf{q}=\mathbf{\Phi}\,\boldsymbol{\eta}$$

In this formulation, the matrix of the basic vectors contains all space information; the generalized coordinates, as for them, depend:

- time, in the case of a calculation of temporal answer: $\mathbf{q}(M,t)=\mathbf{\Phi}(M)\boldsymbol{\eta}(t)$
- pulsation, in the case of a harmonic calculation of answer: $\mathbf{q}(M,\omega)=\mathbf{\Phi}(M)\boldsymbol{\eta}(\omega)$

One can thus deduce from them very simply the following relations:

	Harmonic answer	Temporal answer
Displacement	$\mathbf{q}(M,\omega)=\mathbf{\Phi}(M)\boldsymbol{\eta}(\omega)$	$\mathbf{q}(M,t)=\mathbf{\Phi}(M)\boldsymbol{\eta}(t)$
Speed	$\dot{\mathbf{q}}(M,w)=j\boldsymbol{\omega}\,\Phi(M)\boldsymbol{\eta}(\omega)$	$\dot{\mathbf{q}}(M,t)=\mathbf{\Phi}(M)\dot{\boldsymbol{\eta}}(t)$
Acceleration	$\ddot{\mathbf{q}}(M,w)=-\omega^2\mathbf{\Phi}(M)\boldsymbol{\eta}(\omega)$	$\ddot{\mathbf{q}}(M,t)=\mathbf{\Phi}(M)\ddot{\boldsymbol{\eta}}(t)$
Deformations	$\boldsymbol{\epsilon}(M,\omega)=\mathbf{\Phi}_{\epsilon}(M)\boldsymbol{\eta}(\omega)$	$\boldsymbol{\epsilon}(M,t)=\mathbf{\Phi}_{\epsilon}(M)\boldsymbol{\eta}(t)$
Constraints	$\boldsymbol{\sigma}(M,\omega)=\mathbf{\Phi}_{\sigma}(M)\boldsymbol{\eta}(\omega)$	$\boldsymbol{\sigma}(M,t)=\mathbf{\Phi}_{\sigma}(M)\boldsymbol{\eta}(t)$

All these formulations thus present an equivalent form: in the continuation of the document, we will treat primarily the case of temporal displacement, but the got results are applicable to all the other sizes: speed, acceleration, strain and stress.
In the same way, the relations established according to time are applicable in the spectral field:

$$TF\left(\mathbf{q}(M,t)\right)=\mathbf{\Phi}(M)\,TF\left(\boldsymbol{\eta}(t)\right)=\mathbf{\Phi}(M)\boldsymbol{\eta}(\omega)$$
$$TF^{-1}\left(\mathbf{q}(M,\omega)\right)=\mathbf{\Phi}(M)\,TF^{-1}\left(\boldsymbol{\eta}(\omega)\right)=\mathbf{\Phi}(M)\boldsymbol{\eta}(t)$$

5 Calculation of the generalized coordinates

5.1 Formulation of the problem

The calculation of the generalized coordinates η be carried out on the matrix of displacements (respectively speeds, accelerations, strains, stresses) restricted to the measured degrees of freedom, by resolution of the matric system:

$$q_{\mathbf{exp}}=\bar{\Phi}\,\eta$$

Dimensions of the matrix $\bar{\mathbf{\Phi}}$ “to reverse” are (N_{exp}, N_{num}).
It is seen here that the calculation of the generalized coordinates is carried out in a restricted space: the dimension of the space generated by the basic vectors is lower corresponding to digital model, one exploits only information with the measured degrees of freedom.

5.2 Determination of a quasi-solution

For the resolution of the opposite problem, 3 cases can arise:

- $N_{exp}=N_{num}$: the number of measured degrees of freedom is equal to the number of basic vectors of projection which one wishes to identify the generalized coordinates.
  In this case, there exists a single solution with the problem of inversion: $\boldsymbol{\eta}=\bar{\mathbf{\Phi}}^{-1}q_{\mathbf{exp}}$
- $N_{exp}>N_{num}$: the number of measured degrees of freedom is higher than the number of basic vectors of projection of the digital model which one wishes to identify the coordinates generalize.
  In this case, there does not exist exact solution with the problem of inversion. A quasi-solution can however be defined, which minimizes the distance: $\left|q_{\mathbf{exp}}-\bar{\mathbf{\Phi}}\boldsymbol{\eta}\right|$. The formula $\eta=\left[\bar{\mathbf{\Phi}}^T\bar{\mathbf{\Phi}}\right]^+\bar{\mathbf{\Phi}}^T q_{\mathbf{exp}}$ then provides the solution (single) within the meaning of least squares. In this expression, the matrix $\left[\bar{\mathbf{\Phi}}^T\bar{\mathbf{\Phi}}\right]^+\bar{\mathbf{\Phi}}^T$ indicate the opposite matrix generalized of $\bar{\mathbf{\Phi}}$. The calculation of pseudo-opposite can be carried out by using the decomposition LU or the decomposition in singular values (SVD).

- $N_{exp} < N_{num}$: the number of measured degrees of freedom is lower than the number of basic vectors of projection which one wishes to identify the generalized coordinates (what corresponds to the case more running).
  In this case, there exist an infinity of solutions with the problem of inversion and the objective is to determine an acceptable solution by introducing an additional condition (minimal standard of the solution or application of methods known as "of regularization").

5.3 Determination of a regularized opposite solution

5.3.1 Principles of the methods of regularization

The goal of the methods of regularization [bib4], [bib5] is to suggest an approximate and stable solution with respect to the variations of the data input. One does not seek any more to solve the equation of minimization resulting from the formulation: $q_{\mathbf{exp}} = \bar{\mathbf{\Phi}}\,\mathbf{\eta}$, but to determine a solution approximate (or regularized) answering two requirements:

- it satisfies a condition with proximity: one seeks $\mathbf{\eta}_{\delta}$ such as $|q_{\mathbf{exp}} - \mathbf{\Phi}_{num}\mathbf{\eta}_{\delta}| < \delta$,
- she answers additional a condition a priori known as "information".

The methods of regularization thus consist in supplementing the statement of the problem by introducing information a priori to extract, in the family of the solutions which are compatible with the experimental data, that which best corresponds to the problem. This is done by amalgamating in a single criterion a measurement of the fidelity of the solution compared to the experimental data and a measurement of its fidelity to information a priori [bib2].
An approach which can be easily put in work in finished dimension is the regularization by optimization. To bring closer to the method of regularization of Tikhonov [bib3], it consists in considering a solution a priori $\mathbf{\eta}_{priori}$ problem of minimization and to seek the solution of the approximate system nearest to this solution. One then seeks to minimize the following functional calculus:

$$|\mathbf{q}_{\mathbf{exp}} - \bar{\mathbf{\Phi}}\,\mathbf{\eta}|^2 + a\,|\mathbf{\eta} - \mathbf{\eta}_{priori}|^2$$

The parameter a determine the affected weight with information a priori.
The solution of the equation of minimization is given by:

$$\mathbf{\eta} = \left[\bar{\mathbf{\Phi}}^T\bar{\mathbf{\Phi}} + a\,\mathbf{I}\right]^{-1}\left(\bar{\mathbf{\Phi}}^T\mathbf{q}_{\mathbf{exp}} + a\,\mathbf{\eta}_{priori}\right)$$

or, while revealing explicitly the variation compared to the solution a priori:

$$\mathbf{\eta} = \mathbf{\eta}_{priori} + \left[\bar{\mathbf{\Phi}}^T\bar{\mathbf{\Phi}} + a\,\mathbf{I}\right]^{-1}\bar{\mathbf{\Phi}}^T\left(\mathbf{q}_{\mathbf{exp}} - \bar{\mathbf{\Phi}}\,\mathbf{\eta}_{priori}\right)$$

If one poses $\mathbf{\eta}_{priori} = 0$, this formulation consists in seeking the solution known as of "minimal standard" (or Tikhonov of order 0).
The regularizing addition of the term related to the matrix $a\,\mathbf{I}$ has as a role to shift the spectrum of $\bar{\mathbf{\Phi}}^T\bar{\mathbf{\Phi}}$ in order to ensure the stage of matric inversion. This approach of calculation thus makes it possible to implement a procedure of calculation conditioned better, which softens the effects of the noise and which provides a physically acceptable solution.
In addition, the choice of the values of the matrix $a\,\mathbf{I}$ result from a compromise between the stability of the required solution and the confidence which one can grant to the solution a priori.

5.3.2 Choice of information a priori

In the case of the methods of regularization, the choice of information a priori constitutes a stage-key which determines the representativeness of the final results. This choice can be based on a physical

knowledge of the solution or a knowledge of its evolution according to the parameter selected. We provide, in the continuation, an example applied to the determination by minimization of a temporal variable [bib1].
The minimization of a variable according to time can be realized with each step of time independently of the step of previous time. The introduction of information a priori however makes it possible to enrich the functional calculus by supposing a slow evolution by the given variables:

$$\eta_{priori}(t)=\eta(t-dt)$$

This assumption is acceptable only when the step of sampling is sufficiently weak. Indeed, the solution at a given moment is approached by (development of Taylor):

$$\eta(t)=\eta(t-dt)+dt\,\dot{\eta}(t-dt)+o(dt)$$

The maximum frequency of answer of the structure is determined by the pulsation of the mode of the highest nature $\omega_{\mathbf{max}}$ taken in modeling. One thus has:

$$\left|\frac{\eta(t)-\eta(t-dt)}{\eta(t-dt)}\right|<\omega_{\mathbf{max}}dt+|o(dt)|$$

So that term corrective is weak (and thus that information a priori constitutes an approximation with the first order of the required solution), the step of sampling must check:

$$dt<<\frac{1}{\omega_{\mathbf{max}}}$$

At the initial moment (t=0), since one does not have any information a priori on the solution, calculation is carried out by seeking the solution of minimal standard. In order to avoid propagating the error which results from it, it can be necessary to assign a weak confidence to information a priori on the first steps of time (via the parameter α) and to exploit the results only as from the moment when one can consider that the errors sufficiently attenuated. If necessary, complementary studies will be conducted in order to determine the optimal parameters of use of the functionality developed in *Code_Aster*.

In the frequential field, many opportunities are given to determine information a priori. They are based either on a physical knowledge of the solution (put in experimental obviousness of resonances or forced answers), or on a formulation of the displacements generalized according to the frequency (standard: functions profit), in which case minimization finally results in characterizing the dynamic stresses.

6 Implementation in *Code_Aster*

The base of projection is made up is dynamic modes calculated by the order CALC_MODES [U4.52.02] stored in a concept of the mode_meca type, that is to say dynamic modes and static modes calculated by the order DEFI_BASE_MODALE [U4.64.02] stored in a concept of the base_modale type.
The phase of calculation of the generalized coordinates is treated by the order PROJ_MESU_MODAL [U4.73.01]. The data are gathered there under 4 keywords factors.
The relative data with the digital model (projection bases) are gathered under the keyword factor MODELE_CALCUL. The digital model there is specified and projection bases.
The relative data with measurements are gathered under the keyword factor MODELE_MESURE. One specifies there in particular the model associated with the structure and the measurement read by the order LIRE_RESU.
The possible manual space association of the nodes is given under the keyword factor CORR_MANU.
The data concerning the resolution of the problem reverses are gathered under the keyword factor RESOLUTION. One specifies there the method of decomposition employed (LU, SVD) and the taking into account of term of regularization.
The restitution of the results in physical base can then be carried out by the order REST_GENE_PHYS [U4.63.31].

7 Bibliography

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8 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
5		Initial text
6,4	*S. AUDEBERT,* *H. ANDRIAMBOLOLONA* EDF-R&D/AMA	