

## SSLL102 - Fixed beam subjected to efforts unit

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### Summary:

This test allows a simple checking of calculations of right beams and hull 1D in linear mechanics of the structures static. The model is linear.

Modelings A, B, C, D, F, G, I and J make it possible to test the various types of elements of right beams in *Code\_Aster*. For each modeling, one calculates simultaneously 3 beams of different sections: rectangle, circle, angle.

Modeling A makes it possible of more than test the change of reference mark: the beam is directed according to the trisecting one with the total reference mark.

Modeling E tests the loading distributed on voluminal edges of elements.

Modeling F corresponds to a loading distributed varying linearly with modeling `POU_D_E`.

Modeling G corresponds to a loading distributed varying linearly with modeling `POU_D_TG`.

Modeling H makes it possible to test a loading distributed varying linearly with modeling `TUYAU_3M`.

Modeling I takes again the loading of modeling A for `POU_D_EM`.

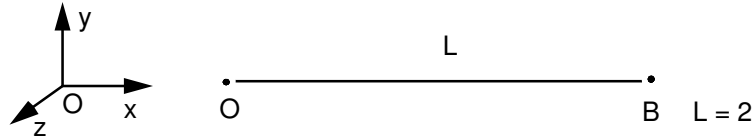
Modeling J corresponds has a loading distributed varying linearly with modeling `POU_D_EM`.

The values tested are the generalized displacements, efforts and the constraints.

## 1 Problem of reference

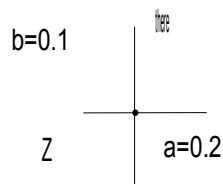
### 1.1 Geometry

Right beam length  $L$ , of direction  $x$ . Dimensions are expressed in meters, [m].



One calculates simultaneously 3 types of different cross sections:

1 rectangular section



1 corner section with equal wings



1 circular section

$R=0.1$

### 1.2 Material properties

Young modulus:  $E = 2 \cdot 10^{11} Pa$

Poisson's ratio:  $\nu = 0.3$

### 1.3 Boundary conditions and loadings

Embedding in  $O$

6 unit loadings in  $B$  :

$$\begin{array}{ll} F_x = 1 & M_x = 1 \\ F_y = 1 & M_y = 1 \\ F_z = 1 & M_z = 1 \end{array}$$

1 loading combined inflection plus traction:  $F_x = 1$  ;  $M_y = 1$  ;  $M_z = 1$  ;

1 loading combined efforts cutting-edges plus torsion:  $F_y = 1$   $F_z = 1$   $M_x = 1$

1 loading distributed linear:  $F_y = 1000 \cdot x$  circular section (modelings F, G, I) (with simple support in  $A$  and  $B$  in this case)

### 1.4 Notation of the characteristics of cross sections

The geometrical characteristics of the cross sections are noted:

$A$	surface of the section
$I_y, I_z$	geometrical moments of inertia compared to the main axes of inertia of the section
$JX$	constant of torsion
$a_y, a_z$	coefficients of shearing in the directions $G_y$ and $G_z$
$A'_y = \frac{A}{a_y}$ and $A'_z = \frac{A}{a_z}$	equivalent reduced surfaces
$e_y, e_z$	eccentricity of the center of torsion
$JG$	constant of warping

## 2 Reference solution

### 2.1 Method of calculating used for the reference solution

Analytical solution [bib1] and [bib2]: displacements in  $B$

Simple traction	$u_x = \frac{F_x L}{E S}$		
Pure bending	$u_y = \frac{F_y L^3 (4 + \phi_y)}{12 E I_z}$	$\theta_z = \frac{L^2 F_y}{2 E I_z}$	$\phi_y = \frac{12 E I_y}{L^2 G A'_y}$
Pure bending	$u_z = \frac{F_z L^3 (4 + \phi_z)}{12 E I_y}$	$\theta_y = \frac{-L^2 F_z}{2 E I_y}$	$\phi_z = \frac{12 E I_z}{L^2 G A'_z}$
Torsion		$\theta_x = \frac{M_x L}{G J_x}$	
Pure inflection	$u_z = -\frac{M_y L^2}{2 E I_y}$	$\theta_y = \frac{M_y L}{E I_y}$	
Pure inflection	$u_y = \frac{M_z L^2}{2 E I_z}$	$\theta_z = \frac{M_z L}{E I_z}$	

#### Notice 1:

For the corner section, as the center of shearing is not confused with the centre of gravity ( $e_y \neq 0$ ), it is necessary to add the torque:  $M_x = F_z e_y$  with the loading  $F_z = 1$ .

This modifies displacement:

$$u_z = \frac{F_z L^3 (4 + \phi_z)}{12 E I_y} + \theta_x e_y \quad \theta_x = \frac{M_x L}{G J_x}$$

In the same way, the loading  $M_x = 1$  involve a displacement  $u_z = \theta_x e_y$ .

Loading distributed linear:

$$u_y(x) = \frac{p x}{360 L E I} (3x^4 - 10L^2 x^2 + 7L^4) \quad u_y^{max} = \frac{0.00652 p L^4}{E I}$$

$en x = 0.519 L$

#### Notice 2:

With regard to modeling A, the beam is carried by the vector  $e_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ . The other vectors of

the local reference mark are:  $e_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$  and  $e_3 = \frac{1}{\sqrt{6}} \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$

The components of the vector displacement in the total reference mark are obtained by:

$$u_G = \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \end{pmatrix} u_{local}$$

Generalized efforts and constraints in  $O$  :

$$N(O) = F_x \quad \sigma_{xx} = \frac{N}{S}$$

$$M_z(O) = T_y L \quad T_y = F_y \quad \sigma_{xx}(y) = \frac{M_z y}{I_z} \quad \sigma_{xy} = \frac{T_y}{k_y S}$$

$$M_y(O) = -T_z L \quad T_z(O) = F_z \quad \sigma_{xx}(y) = \frac{-M_y z}{I_y} \quad \sigma_{xz} = \frac{T_z}{k_z S}$$

$$M_x(0) = M_x(B) \quad \sigma_{xy} = \sigma_{xz} = \frac{M_x R_T}{J_x}$$

$$M_y(0) = M_y(B) \quad \sigma_{xx}(z) = \frac{M_y z}{I_y}$$

$$M_z(0) = M_z(B) \quad \sigma_{xx}(y) = \frac{M_z y}{I_z}$$

Loading distributed linear:

$$M_z(x) = \frac{-1000}{6} (L^2 x - x^3) \quad V_y(x) = \frac{1000 L^2}{6} - \frac{1000 x^2}{2} \quad \sigma_{xx}^{max} = \frac{M_z^{max} R}{I_z}$$

$$en x = \frac{L\sqrt{3}}{3}$$

## 2.2 Results of reference

Displacement of the point  $B$  ,  
Efforts generalized at the point  $O$  ,  
Constraints of the point  $O$  .

## 2.3 Uncertainty on the solution

Analytical solution.

## 2.4 Bibliographical references

- 1.J.L. BATOZ, G. DHATT: "Modeling of the structures by finite elements" - Volume 2 ED. HERMES.
- 2.N.D. PIKLEY: "Formulated for Stress, Stain & Structural Matrices" ED. John Wiley & Sounds.

## 3 Modeling A

### 3.1 Characteristics of modeling

2 elements POU\_D\_E  $k_y = k_z = 1$   $\phi = 0$  by type of section

S1 : Rectangular section modelled by SECTION: 'GENERAL'

$$A = 0.02 \quad I_y = 0.1666E-4 \quad I_z = 0.6666E-4 \quad J_x = 0.45776E-4$$

$$R_y = 0.1 \quad R_z = 0.05 \quad R_T = 0.0892632$$

*Point de calcul des contraintes*

S2 : Corner section

$$A = 1.856E-3 \quad I_y = 4.167339E-4 \quad I_z = 1.045547E-4$$

$$J_x = 0.39595E-8 \quad e_y = 41.012E-3 \quad e_z = 0.0$$

S3 : Rectangular section modelled by SECTION: RECTANGLE

$$H_y = 0.2 \quad H_z = 0.1$$

S4 : Section CIRCLE  $R = 0.1$

$$I_y = I_z = \frac{\pi R^4}{4} = \frac{\pi}{4} 10^{-4}$$

### 3.2 Characteristics of the grid

4x2 elements POU\_D\_E. The beam is directed according to the vector (1,1,1).

### 3.3 Sizes tested and results

Loading case	Beam	Identification	Reference
$F_x = 1$	S1=S3	$u_x(B)$	2.887E-10
		$\theta_{xx}(0)$	50.
	S2	$u_x(B)$	3.11E-9
	S4	$u_x(B)$	1.838E-10
		$\sigma_{xx}$	31.83
	$F_y = 1$	S1=S3	$u_y(B)$
$\theta_z(B)$			1.225E-7
$\sigma_{xx}(0)$			3000
S2		$u_y(B)$	9.017E-8
S4		$\sigma_{xx}(0)$	2546.479
$F_z = 1$		S1=S3	$u_z(B)$
	$\theta_y(B)$		-4.243E-7
	$\sigma_{xx}(0)$		6000
	$\sigma_{xz}(0)$		50
	S2		$u_z(B)$
	S4	$\theta_y(B)$	1.553E-5
		$\theta_x(B)$	1.555E-5
		$u_z(B)$	1.386E-7
		$\theta_y(B)$	9th-8

		$\sigma_{xx}(0)$	2546.479
		$\sigma_{xz}(0)$	31,831
$M_x = 1$	$S1 = S3$	$\theta_x(B)$	3.279E-7
		$\sigma_{xy} = \sigma_{xz}(0)$	1950.0
	$S2$	$\theta_x(B)$	3.791E-4
		$u_z(B)$	2.199E-5
	$S4$	$\theta_x(B)$	9.556E-8
		$\sigma_{xy} = \sigma_{xz}(0)$	636.62
$M_y = 1$	$S1 = S3$	$u_z(B)$	-4.899E-7
		$\theta_y(B)$	4.243E-7
		$\sigma_{xx}(0)$	3000
	$S2$	$u_z(B)$	-1.959E-8
		$\theta_y(B)$	1.697E-8
	$S4$	$u_z(B)$	-1.04E-7
		$\theta_y(B)$	9.0E-8
		$\sigma_{xx}(0)$	1273.2395
$M_z = 1$	$S1 = S3$	$u_y(B)$	1.061E-7
		$\theta_z(B)$	1.225E-7
		$\sigma_{xx}(0)$	1500.0
	$S2$	$u_y(B)$	6.763E-8
		$\theta_z(B)$	7.809E-8
	$S4$	$u_y(B)$	9.0E-7
		$\sigma_z(B)$	1.04E-7
		$\sigma_{xx}(0)$	1273.2395
$M_y = 1$	$S1 = S3$	$\sigma_{xx} \max(0)$	4550.0
$M_z = 1$		$\sigma_{xx} \left( \frac{a}{2}, \frac{b}{2} \right)$	1550.0
$F_x = 1$			
	$S4$	$\sigma_{xx} \max(0)$	1832.4636
$F_y = 1$	$S1, S3$	$\sigma_{xy}(0)$	2000.0
$F_z = 1$		$\sigma_{xz}(0)$	2000.0
$M_x = 1$		$\sigma_{xx} \max(0)$	9000.0
	$S1, S3$	$\sigma_{xx} \left( \frac{a}{2}, \frac{b}{2} \right)$	-9000.0
	$S4$	$\sigma_{xx} \max(0)$	3601.27
		$\sigma_{xy}(0)$	668,451

## 4 Modeling B

### 4.1 Characteristics of modeling

2 elements POU\_D\_T.

The coefficients of shearing are:

S1 : Rectangular section

$$AY = AZ = 1.2 = \frac{1}{k_y}$$

S2 : Corner section

$$AY = AZ = \frac{1}{0.358}$$

S4 : Section CIRCLE

$$AY = AZ = \frac{10}{9}$$

### 4.2 Characteristics of the grid

4×2 elements POU\_D\_T

### 4.3 Sizes tested and results

One gives only the values which differ from modeling A (because of the taking into account of transverse shearing).

Loading	Section	Identification	Reference
$F_y = 1$	S1, S3	$u_y(B)$	2.0156E-7
		$\sigma_{xy}(0)$	60.
	S2	$u_y(B)$	1.666552E-7
	S4	$u_y(B)$	1.707308E-7
		$\sigma_{xy}(0)$	37.13615
$F_z = 1$	S1, S3	$u_z(B)$	8.0156E-7
		$\sigma_{xz}(0)$	60.
	S2	$u_z(B)$	1.17559754E-6
	S4	$u_z(B)$	1.707308E-7
		$\sigma_{xz}(0)$	37.13615
$F_y = 1$	S4	$\sigma_{xz}(0)$	673.75592
$F_z = 1$		$\sigma_{xy}(0)$	673.75592
$M_x = 1$			



## 5 Modeling C

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### 5.1 Characteristics of modeling

2 elements POU\_D\_TG.

Warping is not constrained.

The coefficients of shearing are identical to those of modeling B.

### 5.2 Characteristics of the grid

4×2 elements POU\_D\_TG

### 5.3 Sizes tested and results

Loading	Section	Identification	Reference
$F_y=1$	S1=S3	$u_y(B)$	2.0156E-7
		$\theta_{xy}(0)$	60.
	S2	$u_y(B)$	1.666552E-7
	S4	$u_y(B)$	1.70684E-7
		$\theta_{xy}(0)$	35.367765
$F_z=1$	S1, S3	$u_z(B)$	8.0156E-7
		$\theta_{xz}(0)$	60.
	S2	$u_z(B)$	1.17559754E-6
	S4	$u_z(B)$	1.70684E-7
		$\theta_{xz}(0)$	35.367765

### 5.4 Notice

Warping is not constrained. The results are thus identical to those of modeling B.

## 6 Modeling D

### 6.1 Characteristics of modeling

Elements  $POU\_D\_TG$ , constrained torsion

$$JG = \begin{cases} 5.5556E-8 & \text{pour } S_1 \\ 4.439822E-11 & \text{pour } S_2 \end{cases}$$

in 0  $GRX = 0$

### 6.2 Characteristics of the grid

- 10 elements,
- refinement towards embedding.

### 6.3 Sizes tested and results

Same results as for modeling C, except those which relate to the effects of warping.

Loading	Section	Identification	Reference
$F_z = 1$	S2	$\theta_x = DRX$	2.62034E-5
		$u_z = DZ$	1.14578E-6
		$GRX$	1.34652E-5
$M_x = 1$	S1	$u_z = DZ$	5.52E-7
		$GRX$	2.84E-7
	S2	$u_z$	2.6203E-5
		$\theta_x$	6.3892E-4
		$GRX$	3.28324E-4

### 6.4 Remarks

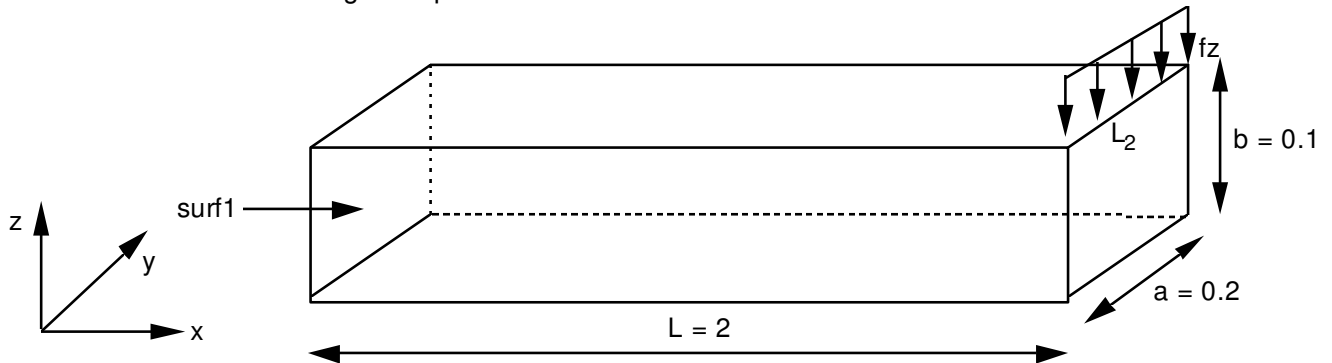
For  $\theta_x$  the solution is (cf [bib1]):

$$\theta_x = \frac{M_x L}{G J_x} + \frac{M_x (1 - e^{2\alpha L} - 2e^{\alpha L})}{\alpha^3 E J G (1 + e^{2\alpha L})} \quad \alpha^2 = \frac{G J}{E J G}$$

## 7 Modeling E

### 7.1 Characteristics of modeling

The beam is with a grid in quadratic solid elements HEXA20.



The beam is embedded on the level of the section *surf1*. It is subjected to a unit shearing action which is modelled by a linear density of load  $fz$  applying to the 4 meshes SEG3 constituting the higher edge  $L2$ .

### 7.2 Characteristics of the grid

The beam is with a grid with 640 quadratic solid elements HEXA20.  
The model comprises 3665 nodes.

### 7.3 Sizes tested and results

One tests the value of the arrow according to  $z$  node medium of the section where one applies the loading (node  $N62$ ).

Identification	Reference	Aster	% difference
$dz$ node $N62$	-8.0E-7	-7.9523E-7	-0,596

### 7.4 Remarks

The value of reference corresponds to the value given by the Resistance Of Materials.

## 8 Modeling F

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### 8.1 Characteristics of modeling

The model is composed of 10 elements right beam of Euler. The section is circular full, of ray 0.1m .

### 8.2 Characteristics of the grid

It consists of 10 elements `POU_D_E`. The length of the beam is  $L=6m$

### 8.3 Sizes tested and results

#### 8.3.1 Interior efforts

	Analytical results
$V_y(0)$	6.0000E+03
$V_y(6)$	-1.2000E+04
$MFZ(2\sqrt{3})$	-1.3856E+04

#### 8.3.2 Constraint

	Analytical results
$SIXX(2\sqrt{3})$	1.7642E+07

## 9 Modeling G

### 9.1 Characteristics of modeling

The model is composed of 10 elements right beam of Timoshenko with warping. The section is circular full, of ray 0.1m .

### 9.2 Characteristics of the grid

It consists of 10 elements POU\_D\_TG. The length of the beam is  $L=6m$

### 9.3 Sizes tested and results

#### 9.3.1 Interior efforts

	Analytical results
$V_y(0)$	6.0000E+03
$V_y(6)$	-1.2000E+04
$MFZ(2\sqrt{3})$	-1.3856E+04

#### 9.3.2 Displacement (marks with arrows near to the medium of the beam)

	Results Aster (not regression)	Tolerance (%)
$DY(3.115977734)$	3.23499E-03	1.E-4

## 10 Modeling H

### 10.1 Characteristics of modeling

The model is composed of 21 elements TUYAU\_3M being pressed on meshes SEG4.  
The effort distributed is imposed along the axis  $y$ . the inflection thus takes place around  $z$ .

### 10.2 Characteristics of the grid

It consists of 21 meshes SEG3. The length of the pipe is  $L=6m$

### 10.3 Sizes tested and results

#### 10.3.1 Displacements

	Analytical results
$D_y \text{ maxi}$	9.38888E-03

#### 10.3.2 Interior efforts

	Analytical results
$V_y(x=0)$	6.0000E+03
$V_y(x=L=6)$	-1.2000E+04
$MFZ 2\sqrt{3}$	-1.3856E+04

#### 10.3.3 Constraints

It are calculated at the point of X-coordinate  $x = \frac{L\sqrt{3}}{3}$  who corresponds to the maximum moment:

$$M_z(x) = \frac{-1000}{9\sqrt{3}} L^3 = -13856.41 \text{ N.m}$$

For the angle 0 on the circumference of the pipe (the origin of the angles being the axis  $z$ ), the constraints are worthless, and for angle 90, they are maximum:

$$\sigma_{xx}^{max} = \frac{M_z^{max}(R-e/2)}{I_z} = -4.87363E+07 \text{ Pa}$$

	Reference	Tolerance
$\sigma_{xx}(\alpha=0)$	0	0,10%
$\sigma_{xx}(\alpha=90)$	-4.87363E+07	1,00%
$MFZ$	-1.3856E+04	1.0%

## 11 Modeling I

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### 11.1 Characteristics of modeling

The model is composed of 2 elements `POU_D_EM`.

The loading is similar to that of modeling `A` (torque only)

### 11.2 Characteristics of the grid

It consists of 2 meshes `SEG2`. The length of the beam is  $L = 2\text{ m}$

The beam is directed according to the vector  $(1, 1, 1)$ .

The section is rectangular, identical to that of modeling `A`.

### 11.3 Sizes tested and results

#### 11.3.1 Displacement (rotation due to the torque)

	Results Aster (not regression)	Tolerance (%)
$DX = DY = DZ$	3.2792525E-07	1.E-6

## 12 Modeling J

### 12.1 Characteristics of modeling

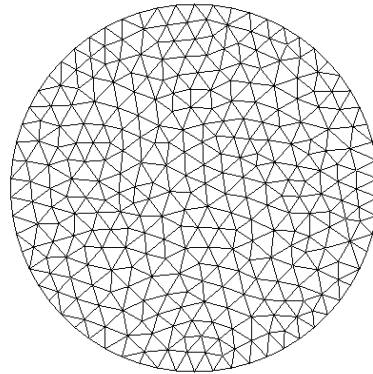
The model is composed of 10 elements `POU_D_EM`.  
One applies a force distributed of  $6000\text{N}/m$  on all the beam.

### 12.2 Characteristics of the grid

It consists of 10 meshes `SEG2`. The length of the beam is  $L=6m$

The grid of the section consists of:

- 373 nodes
- 62 `SEG2`
- 682 `TRIA3`



### 12.3 Sizes tested and results

#### 12.3.1 Interior efforts

Interior efforts	Analytical results
$V_y(0)$	6.0000E+03
$V_y(6)$	-1.2000E+04
$MFZ$	-1.3856E+04

#### 12.3.2 Displacement (marks with arrows near to the medium of the beam)

	Results Aster (not regression)	Tolerance (%)
$DY(3.115977734)$	3.23499E-03	1.E-4



## 13 Summary of the results

This test makes it possible to check the good performance of the elements simultaneously `POU_D_E`, `POU_D_T` and `POU_D_TG` on 3 types of different sections. The perfect coincidence of the results with the analytical solutions ( RDM ) is normal, and must always be observed, since the solution is contained in the functions of form of the elements.

Moreover, modeling E makes it possible to test the loading distributed on voluminal edges of elements. The variation with the analytical solution ( RDM ) is lower than 0.6% .

Modelings F, G, H and J make it possible to test the loading distributed (linear variation) for the elements of beam `POU_D_E`, `POU_D_TG`, `POU_D_EM` and elements of `PIPE`. The variation with the analytical solution (Resistance of Materials) is lower than 0.6% .