SSLL117 – Validation of modelings second gradient

Summary:

This test makes it possible to validate modelings second gradieNT [R5.04.03] while being based on analytical solutions.

1 Problem of reference

1.1 Geometry

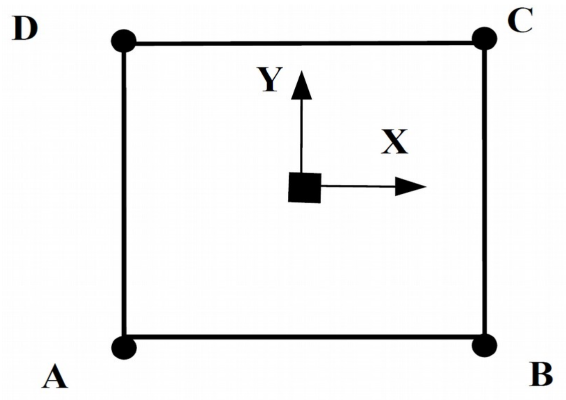

Figure 1.1-a

Coordinates of the points	X	Y
A	-1.0	-1.0
B	1.0	-1.0
C	1.0	1.0
D	-1.0	1.0

The geometry of the structure is a square length of with dimensions 1m .

The test is carried out on only one isoparametric finite element of quadratic form, named group of meshs ROCHE . The various sides of this square, useful for this modeling, are groups of meshs, DA and BC *.* The group of meshs ROCHE *,* contains the groups of meshs in addition SROCHE and MROCHE ; who correspond respectively to the nodes tops and mediums of this same group.

1.2 Properties of materials

In the context of the mediums of the second gradient, it is necessary to define properties materials for the parts attached to the first and second gradients of the field of displacement. The properties of material are elastic for these two parts:

- Young modulus: $\text{E}=1000\,\text{Pa}$;
- Poisson's ratio: $\nu=0$;
- Microscopic module of rigidity: $a_1=10\,\text{Pa.m}^2$.
- Parameter of penalization: $r=1\text{E}8$
- Parameters material of the mediums of the second gradient: $a_2=a_3=a_4=a_5=0\,\text{Pa.m}^2$

1.3 Boundary conditions and loadings

- $Dy=0$ everywhere (modeling is brought back in the case 1D)
- $Dx=0$ on the segment BC
- $Dx=0.1\text{m}$ on the segment AD

2 Reference solution

2.1 Method of calculating

The problem to be solved is:

$$\int_{\Omega} \sigma\left(\frac{\partial u^{\to}}{\partial x}\right)+\sum\left(\frac{\partial \nu^{\to}}{\partial x}\right)-\lambda\left(\frac{\partial u^{\to}}{\partial x}-\nu^{\to}\right)+\lambda^{\to}\left(\frac{\partial u}{\partial x}-\nu\right)+r\left(\frac{\partial u}{\partial x}-\nu\right)\left(\frac{\partial u^{\to}}{\partial x}-\nu^{\to}\right)=0$$

with $\forall\, u^{*}, \nu^{*}, \lambda^{*}$ kinematically acceptable.

We have:

$$\begin{cases} \sigma=E\dfrac{\partial u}{\partial x} \\ \Sigma=F\dfrac{\partial \nu}{\partial x} \end{cases}$$

Let us note ω functions of form of the second order and N functions of first order form on the linear elements,

Let us recall by the same occasion the definitions of `SEG2` and `SEG3`:

`SEG2` : segment with 2 nodes
many nodes: 2
many nodes tops: 2

Figure 2.1-a

`SEG3` : segment with 3 nodes
many nodes: 3
many nodes tops: 2

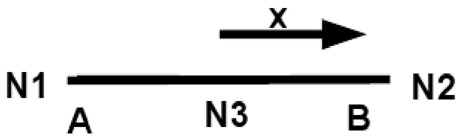

Figure 2.1-b

$N_1=\dfrac{1-x}{2}$ and $N_2=\dfrac{1+x}{2}$ on the element of reference $x\in[-1,+1]$ with N_1 the function of form
with the first node.

The functions of form of the segment to the 3 nodes are then:

$\omega_1=-\dfrac{x(1-x)}{2}$ $\omega_2=\dfrac{x(1+x)}{2}$ and $\omega_3=1-x^2$

While posing $u^*=\nu^*=0$ we show that:

$$\int_{-1}^{+1}\lambda^*\left(\frac{\partial u}{\partial x}-\nu\right)dx=0$$ however $u=\omega_1.u_1+\omega_2.u_2+\omega_3.u_3$ with $u_2=0$ (C.L.)

from where $\frac{\partial u}{\partial x}=(x-\frac{1}{2})u_1-2xu_3$ and $\nu=N_1.\nu_1+N_2.\nu_2$ with $\nu_2=0$ (C.L.)

We thus have $\nu=\frac{(1-x)}{2}\nu_1$ however, it should be noted after integration that: $\boxed{\nu_1=-u_1}$ (1) $\forall u^*,\nu^*,\lambda^*$ kinematically acceptable.

While proceeding in the same way and while posing: $\nu^*=\lambda^*=0$ we find that:

$$\boxed{u_3=\frac{2(E-r)}{4(E+r)}.u_1\approx-\frac{u_1}{4}} \quad (2)$$

from where the general formula $S_1=3.a^1.\frac{\partial\nu}{\partial x}=-\frac{3}{2}.a^1.\nu_1$ and after simplification $S_1=\frac{3}{2}.a^1.u_1$.

2.2 Sizes and results of reference

The fundamental and very general formula for the solid is $S_1=\frac{3}{2}.a^1.u_1$.

2.3 Uncertainties on the solution

Analytical solution.

3 Modeling A

3.1 Characteristics of modeling

The characteristics are identical to the reference solution. Modeling relating to the first gradient of the field of displacement is D_PLAN_SI and that bearing on the second gradient of dilation is D_PLAN_DIL with the choice of interpolation P2-P1-P0.

3.2 Characteristics of the grid

The group of meshs ROCHE_REG is obtained by duplication of group of meshs named ROCHE , whose objective is to accommodate modeling second gradient of dilation for the regularization.

Many nodes 8
Number of SEG3 4
Number of QUAD8 2
Number of group of meshs 6

3.3 Sizes tested and results

Value tested	Moment	Node	Reference	Criterion	Aster	Tolerance
Displacement GONF	1.0	N1	'NON_REGRESSION'	RELATIVE	-0.10	0,010
Displacement DX	1.0	N5	'NON_REGRESSION'	RELATIVE	0.0250	0,010
Force of reaction SIG1	1.0		'NON_REGRESSION'	RELATIVE	1.5	0,010
Force of reaction DEPV	1.0		'NON_REGRESSION'	ABSOLUTE	1.E-6	1.0E-04

3.4 Remarks

With an aim of validating the option of resolution RIGI_MECA_ELAS of STAT_NON_LINE , a calculation identical to the first (option MATRICE=TANGENTE) is realized by imposing this option of resolution. The results are rigorously identical to those obtained with the first option of calculation which are presented in the table of results above.

4 Modeling B

4.1 Characteristics of modeling

In this case, the parameters materials resulting from Commandes tests of nonregression generated with code source are identical in two modelings A and B, only differs the quadratic grid. The objective of this modeling is to have a validation on a structure with a grid, the analytical solution being difficult to obtain and consequently the solution in not-regression, is privileged here.
Modeling relating to the first gradient of the field of displacement is D_PLAN_SI and that bearing on the second gradient of dilation is D_PLAN_DIL with the choice of interpolation P2-P1-P0.

4.2 Characteristics of the grid

The group of meshs ROCHE_REG is obtained by duplication of group of meshs named ROCHE , whose objective is to accommodate modeling second gradient of dilation for the regularization.

Many nodes	341
Number of SEG3	40
Number of QUAD8	200
Number of groupE of meshs	6

4.3 Sizes tested and results

Value tested	Moment	Node	Type	Criterion	Aster	Tolerance
Displacement GONF	1.0	*N1*	'NON-REGRESSION'	RELATIVE	-0.05500	1.0
Displacement DX	1.0	*N5*	'NON-REGRESSION'	RELATIVE	0.09450	1.0
Force of reaction DEPV	1.0		'NON-REGRESSION'	ABSOLUTE	1.E-6	1.0E-04

5 Modeling C

5.1 Characteristics of modeling

This modeling is identical to modeling A. modeling relating to the first gradient of the field of displacement is D_PLAN_SI and that bearing on the second gradient of dilation is D_PLAN_DIL with the choice of interpolation P2-P1-P0.

5.2 Characteristics of the grid

The group of meshs $ROCHE\_REG$ ESt obtained by duplication of group of meshs named $ROCHE$, whose objective is to accommodate modeling second gradient of dilation for the regularization.

Many nodes 9
Number of SEG3 4
Number of TRIA6 4
Number of group of meshs 6

5.3 Sizes tested and results

Value tested	Moment	Node	Type	Criterion	Aster	Tolerance
Displacement GONF	1.0	$N1$	'ANALYTICAL'	RELATIVE	-0.10	0,010
Displacement DX	1.0	$N5$	'ANALYTICAL'	RELATIVE	0.0250	0,010
Force of reaction SIG1	1.0		'ANALYTICAL'	RELATIVE	1.5	0,010
Force of reaction DEPV	1.0		'ANALYTICAL'	ABSOLUTE	1.E-6	1.0E-04

6 Modeling D

6.1 Characteristics of modeling

Modeling is identical to modeling B with a mapping composed of triangles. Modeling relating to the first gradient of the field of displacement is D_PLAN_SI and that bearing on the second gradient of dilation is D_PLAN_DIL with the choice of interpolation P2-P1-P0.

6.2 Characteristics of the grid

The group of meshs ROCHE_REG ESt obtained by duplication of group of meshs named ROCHE, whose objective is to accommodate modeling second gradient of dilation for the regularization.

Many nodes 441
Number of SEG3 40
Number of TRIA6 400
Number of group of meshs 6

6.3 Sizes tested and results

Value tested	Moment	Node	Type	Criterion	Aster	Tolerance
Displacement GONF	1.0	N1	'NON-REGRESSION'	RELATIVE	-5.5E-2	1.0
Displacement DX	1.0	N5	'NON-REGRESSION'	RELATIVE	0.0945	1.0
Force of reaction DEPV	1.0		'NON-REGRESSION'	ABSOLUTE	1.E-6	1.0E-04

7 Modeling E

7.1 Characteristics of modeling

Modeling relating to the first gradient of the field of displacement is `D_PLAN_SI` and that bearing on the second gradient of dilation is `D_PLAN_DIL` with the choice of interpolation P2-P1-P0.
The case present is identical to modeling A with $r=0$ we have then

$$\begin{cases} \nu_1=-u_1 \\ u_3=\frac{2(E)}{4(E)}.u_1\approx\frac{u_1}{2} \\ S_1=\frac{3}{2}.a^1.u_1 \end{cases}$$

7.2 Characteristics of the grid

The group of meshs ROCHE_REG ESt obtained by duplication of group of meshs named ROCHE, whose objective is to accommodate modeling second gradient of dilation for the regularization.

Many nodes 8
Number of `SEG3` 4
Number of `QUAD8` 2
Number of group of meshs 6

7.3 Sizes tested and results

Value tested	Moment	Node	Type	Criterion	Aster	Tolerance
Displacement `GONF`	1.0	*N1*	`'ANALYTICAL'`	RELATIVE	-0.10	0,010
Displacement `DX`	1.0	*N5*	`'ANALYTICAL'`	RELATIVE	0.0550	0,010
Force of reaction `SIG1`	1.0		`'ANALYTICAL'`	RELATIVE	1.5	0,010
Force of reaction `DEPV`	1.0		`'ANALYTICAL'`	RELATIVE	0.0288675	0,010

8 Modeling F

8.1 Characteristics of modeling

The case present is identical to modeling A with a generalization of the behavior second complete gradient [R5.04.03]. Modeling relating to the first gradient of the field of displacement is `D_PLAN_SI` and that bearing on the second gradient of dilation is `D_PLAN_2DG` with the choice of interpolation P2-P1-P0.

We show as for modeling E that:

$$u_1=-v_1 \text{ in the same way } u_3=\frac{u_1}{2}$$

For the calculation of Σ_{111} and Σ_{221} for the definition of the field of double constraints Σ

we thus have:

$$\Sigma_{111}=a^{12345}\chi_{111}+a^{23}\chi_{122}+a^{12}(\chi_{212}+\chi_{221})$$

with:

$$\begin{cases} a^{12345}=2(a^1+a^2+a^3+a^4+a^5) \\ a^{23}=a^2+2a^3 \\ a^{12}=a^1+\frac{a^2}{2} \end{cases}$$

Being given the values of the parameters material and limiting conditions we find:

$$\Sigma_{111}=a^1.u_1$$

in the same way we end to:

$$\Sigma_{221}=\frac{a^1}{2}.u_1$$

8.2 Characteristics of the grid

The group of meshs ROCHE_REG ESt obtained by duplication of group of meshs named ROCHE, whose objective is to accommodate modeling second complete gradient for the regularization.

Many nodes	8
Number of `SEG3`	4
Number of `QUAD8`	2
Number of group of meshs	6

8.3 Sizes tested and results

Value tested	Moment	Node	Type	Criterion	Aster	Tolerance
Displacement `V11`	1.0	*N1*	`'ANALYTICAL'`	RELATIVE	-0.10	0,010

Displacement DX	1.0	$N5$	'ANALYTICAL'	RELATIVE	0.050	0,010
Force of reaction SIG111	1.0		'ANALYTICAL'	RELATIVE	1	0,010
Force of reaction SIG221	1.0		'ANALYTICAL'	RELATIVE	0.5	0,010
Force of reaction DEPV11	1.0		'ANALYTICAL'	RELATIVE	0.0288675	0,010

9 Modeling G

9.1 Characteristics of modeling

Modeling G. is identical to modeling A but with one interpolation without multipliers of Lagrange (P2-P1) – to see R5.04.03.

9.2 Characteristics of the grid

The group of meshs $\mathrm{ROCHE\_REG}$ ESt obtained by duplication of group of meshs named ROCHE, whose objective is to accommodate modeling second gradient of dilation for the regularization.

Many nodes 8
Number of SEG3 4
Number of QUAD8 2
Number of group of meshs 6

9.3 Sizes tested and results

Value tested	Moment	Node	Type	Criterion	Aster	Tolerance
Displacement GONF	1.0	N1	'NON-REGRESSION'	RELATIVE	-0,1	1.0E-04
Displacement DX	1.0	N5	'NON-REGRESSION'	RELATIVE	0,03	1.0E-04
Constraint SIG1	1.0		'NON-REGRESSION'	RELATIVE	1,5	1.0E-04

10 Summary of the results

This test makes it possible to check in a very simple way the good performance of the modeling second gradient, which also coincides with the results with the analytical solution.