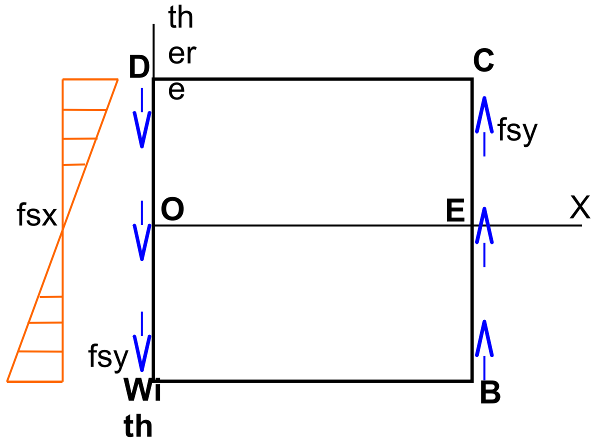
# SSLP106 – Rectangular solid mass in pure inflection (test of elements QUAD4 under integrated)

**Summary:**
One tests the finite elements under integrated into a point of Gauss stabilized by the method *assumed strain* on a calculation of pure inflection in plane deformation.

# 1 Problem of reference

## 1.1 Geometry

The geometry is a square on side $L=100\,\text{mm}$ .




## 1.2 Properties of material

The material is elastic incompressible and has as properties:

$E=100\,\text{MPa}$

$\nu=0.4999$

## 1.3 Boundary conditions and loadings

Taking into account the antisymmetric nature of the problem, one models only half of the solid mass with the boundary conditions following:

On $OE$ :

$DX(OE)=0$

On $OD$ :

$DX(O)=DY(O)=0$

$DX(D)=0$

$$fsx=\frac{8y}{L}.\sigma_d$$

$$fsy=-\left(1-\frac{4y^2}{L^2}\right).\sigma_d$$

On $BC$ :

$$fsy=+\left(1-\frac{4y^2}{L^2}\right).\sigma_d$$

With $\sigma_d$ a given constraint, which one will take equal to 1 in the test. $\sigma_d=1\,\text{MPa}$

# 2 Reference solution

## 2.1 Method of calculating

The reference solution comes from an analytical solution from [Bib1]:

$$u_x(x,y)=\frac{4(1-\nu^2)}{EL^2}.\left(y.(x^2-2.\mathrm{Lx})+\frac{2+\nu(1-\nu)}{3}.y.(\frac{L^2}{4}-y^2)\right).\sigma_d \quad (1)$$

And following $y$ :

$$u_y(x,y)=\frac{4(1-\nu^2)}{EL^2}\left(Lx^2-\frac{x^3}{3}-\nu.(1-\nu).y^2.(x-L)+\frac{4+5\nu.(1-\nu)}{12}.xL^2)\right).\sigma_d \quad (2)$$

## 2.2 Sizes and results of reference

While applying 1, one finds displacement following $x$ at the point $C$ :
$u_x(L,L/2)=-1.5\,\mathrm{mm}$
And while applying 2, one finds displacement following $y$ at the point $C$ :
$u_y(L,L/2)=4.25\,\mathrm{mm}$

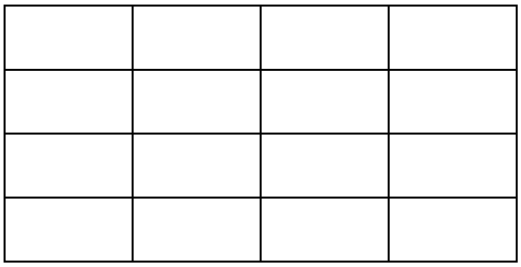
## 2.3 Bibliographical references

[Bib1] Timoshenko & Woinowsky-Krieger, "Theory of punts and shells", McGrawHill, 1964.

# 3 Modeling A

## 3.1 Characteristics of modeling

Modeling in plane deformations on the grid $2D$ according to:



## 3.2 Characteristics of the grid

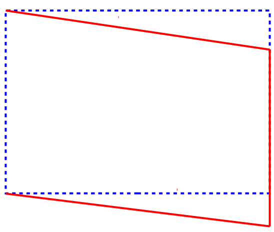
Many nodes: 25
Numbers and types of meshs: 16 `SEG2`, 16 `QUAD4` with the elements `D_PLAN_SI`.

## 3.3 Sizes tested and results

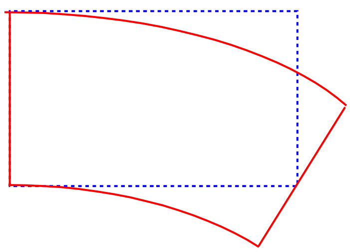
| Value tested | Reference | Type | *Variation* |
|---|---|---|---|
| Displacement $DX$ in $C$ | -1.25 | Analytical | 1.7% |
| Displacement $DY$ in $C$ | 4.25 | Analytical | 0.3% |

## 3.4 Remarks

The request is said to dominant inflection. Through this calculation, one shows the difficulty for `QUAD4` even under-integrated to represent the modes of deformation in inflection in plane deformation and for a coefficient $\nu$ near to 0.5. This results in an excessive rigidity of the element due under the terms of shearing of the operator discretized gradient: it is about a digital blocking.



**QUAD4**
(important shearing)



**QUAD8**

# 4 Modeling B

## 4.1 Characteristics of modeling

One takes again the preceding grid which one passes in quadratic elements with an aim of using modeling D_PLAN_INCO_UPG (elements adapted to the incompressible problems).

## 4.2 Characteristics of the grid

Many nodes: 65
Numbers and types of meshs: 16 SEG3, 16 QUAD8 with the elements D_PLAN_INCO_UPG.

## 4.3 Sizes tested and results

| Value tested | Reference | Type | *Variation* |
|---|---|---|---|
| Displacement $DX$ in $C$ | -1.25 | Analytical | <0.01% |
| Displacement $DY$ in $C$ | 4.25 | Analytical | <0.01% |

## 4.4 Remarks

These elements adapted to the incompressible problems give a result identical to the analytical solution.

# 5 Summary of the results

The poor quality of the result of under-integrated elements QUAD4 is explained by the digital phenomenon of blocking which makes the element very rigid. Moreover, its convergence towards the analytical solution is very slow. This phenomenon appears of course also for the completely integrated element. Calculation using incompressible quadratic elements makes it possible to obtain an exact solution.