SSLS101 - Circular plate posed subjected with a uniform pressure

Summary:

One treats the case of a circular plate posed on the edge in linear elasticity under 3 loadings (actual weight, pressure, effort distributed constant) which give the same deformation.

The first two modelings make it possible to evaluate the influence of the grid.

The test gathers 12 modelings (modelS of Coils-Kirchhoff, Mindlin-Reissner and COQUE_3D and SHB) more 2 modelings of “connection” enters of the hulls or between hulls and 3D.

1 Problem of reference

1.1 Geometry

Coordinates of the points:

	O	A	B	C	D	E	F
x	0.	1.	$\sqrt{2}/2$	0.	0.5	0.	0.4
y	0.	0.	$\sqrt{2}/2$	1.	0.	0.5	0.4
z	0.	0.	0.	0.	0.	0.	0.

1.2 Material properties

$E=1.Pa$
$\nu=0.3$
$\rho=1.kg/m^3$

1.3 Boundary conditions and loadings

Simple support on the edge of the plate:
in all the points P such as $OP=R$: $u=v=w=0$

FORCE_COQUE	Uniform pressure	$P=1\,N/m^2$
FORCE_COQUE	Normal distributed load	$F3=-1\,N/m^2$
GRAVITY	$g=10\,m/s^2$ according to Z from where	$FZ=\rho g t=-1\,N/m^2$

These three loadings lead to the same solution.

2 Reference solution

2.1 Method of calculating used for the reference solution

Two reference solutions are usable, for the calculation of the deformation, according to the theory of plate used:

- the theory of Coil-Kirchhoff, usually used for the plates known as “thin”, that one will retain for modelings A, B and E,
- the theory of Reissner, including the effects of shearing for the plates known as “thick”, that one will retain for modelings F, G and H.

In any distant point of r center of the plate $r \leq R$, one has for the calculation of the arrow:

$$w(r) = -P\,\frac{R^4}{64\,D}\left(1-\frac{r^2}{R^2}\right)\left(1+\frac{r^2}{R^2}-\frac{2(3+\nu)}{1+\nu}-\varphi\right) \text{ with } D=\frac{E\,t^3}{12(1-\nu^2)}$$

and $\varphi = 0\,(\text{Love-Kirchhoff})$ or $\varphi = \frac{16}{5}\left(\frac{t}{R}\right)^2\frac{1}{1-\nu}\,(\text{Reissner})$

For the calculation of the moments the two theories lead to the same expressions:

$$M_{rr}(r) = \frac{PR^2}{16}(3+\nu)\left[\left(\frac{r}{R}\right)^2-1\right] \quad M_{\theta\theta}(r) = \frac{PR^2}{16}(3+\nu)\left[1-\frac{1+3\nu}{3+\nu}\left(\frac{r}{R}\right)^2\right]$$

In the center of the plate:

$$w(0) = -\frac{PR^4}{64\,D}\left(\frac{5+\nu}{1+\nu}\right)(\text{Love}-\text{Kirchhoff}) \text{ ou } w(0) = -\frac{PR^4}{64\,D}\left(\frac{5+\nu}{1+\nu}+\varphi\right)(\text{Reissner})$$

$$M_{rr}(0) = M_{\theta\theta}(0) = -\frac{PR^2}{16}(3+\nu)$$

Note:

> *Code_Aster calculates the moments with the nodes of each finite element in the reference mark of reference defined by the external normal and the reference axes defined on the hull (see AFFE_CARA_ELEM in the documentation of use).*
>
> *The value of the moment M_{xx} (or M_{yy}), extracted the field 'EFGE_ELNO' , in a node pertaining to several finite elements can be regarded as being the average of the computed values on the elements which have this joint node. This average can be obtained by the procedure POST_RELEVE [U4.74.03].*

For each node, one a: $(M_{rr}+M_{\theta\theta}) = (M_{xx}+M_{yy}) = Sm$

for the point O	$M_{xx}=M_{yy}=M_{rr}=M_{\theta\theta}$
for the points $A et D$	$M_{xx}=M_{rr} et M_{yy}=M_{\theta\theta}$
for the points $C et E$	$M_{xx}=M_{\theta\theta} et M_{yy}=M_{rr}$
for the points $B et F$	$M_{xx}=M_{yy}=(M_{rr}+M_{\theta\theta})/2$

2.2 Results of reference

Arrow and moments at the points: O, A, BC, DE, F.

2.3 Uncertainty on the solution

Analytical solution

2.4 Bibliographical references

1.TIMOSHENKO and WOINOWSKY-KRIEGER, Plates and hulls, Béranger Edition - (1961).

3 Modeling A

3.1 Characteristics of modeling

Element of hull DKT (modeling of a quarter of plate)

3.1.1 Limiting conditions

in all the nodes of the arc ABC	: DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the segment $]OA[$	: DY: 0. , DRX: 0. , DRZ: 0.
in all the nodes of the segment $]OC[$	: DX: 0. , DRY MARTINI: 0. , DRZ: 0.
with the node O	: DX: 0. , DY: 0. , DRX: 0. , DRY MARTINI: 0. , DRZ: 0.

Not O	meshs: $M30\ M33$
Not A	meshs: $M76$
Not B	meshs: $M39\ M40\ M51$
Not C	meshs: $M1$
Not D	meshs: $M55\ M56\ M65$
Not E	meshs: $M8\ M17\ M18$
Not F	meshs: $M34\ M35\ M37\ M41\ M46\ M47\ M48$

3.2 Characteristics of the grid

Many nodes: 50
Many meshs and types: 76 TRIA3

3.3 Sizes tested and results

One tests the structural parameters of data results:

Identification	Type of Reference	Value of reference
INST for NUME_ORDRE= 3	'ANALYTICAL'	0,6
INST for NUME_ORDRE= 4	ANALYTICAL	1

Identification	Type of reference	Values of reference
O $w(r)$	'ANALYTICAL'	– 695.6256
D $w(r)$	'ANALYTICAL'	– 489,727
E $w(r)$	'ANALYTICAL'	– 489,727
F $w(r)$	'ANALYTICAL'	– 435.8974

Identification		Type of reference	Values of reference	Tolerance
O	M_{rr}	'ANALYTICAL'	– 0.20625	1.5%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.20625	1.5%
With	M_{rr}	'ANALYTICAL'	0.	0.01
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.0875	6%
B	M_{rr}	'ANALYTICAL'	– 0.04375	13%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.04375	14%
C	M_{rr}	'ANALYTICAL'	– 0.0875	6%
	$M_{\theta\theta}$	'ANALYTICAL'	0.	0.01
D	M_{rr}	'ANALYTICAL'	– 0.15469	0.5%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	0.5%
E	M_{rr}	'ANALYTICAL'	– 0.15469	0.3%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	0.3%
F	M_{rr}	'ANALYTICAL'	– 0.14025	10%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.16825	10%

Elastic energy

Identification				Type of reference	Values of reference	Tolerance %
Size		Node	Mesh			
ENEL_ELNO	*TOTALE*	N2	M1	'NON_DEFINI'	94,237	0.1
	MEMBRANE			'NON_DEFINI'	0.0	0.1
	FLEXION			'NON_DEFINI'	93,112	0.1
	CISAILLE			'NON_DEFINI'	1,125	0.1
	COUPL_MF			'NON_DEFINI'	0.0	0.1

Identification				Type of reference	Values of reference	Tolerance %
Size		Not	Mesh			
ENEL_ELGA	*TOTALE*	1	M1	'NON_DEFINI'	77,080	0.1
	MEMBRANE			'NON_DEFINI'	0.0	0.1
	FLEXION			'NON_DEFINI'	75,955	0.1
	CISAILLE			'NON_DEFINI'	1.1248	0.1
	COUPL_MF			'NON_DEFINI'	0.0	0.1

Identification			Type of reference	Values of reference	Tolerance %
Size		Mesh			
ENEL_ELEM	*TOTALE*	M1	'NON_DEFINI'	0,781	0.1
	MEMBRANE		'NON_DEFINI'	0.0	0.1
	FLEXION		'NON_DEFINI'	0,767	0.1
	CISAILLE		'NON_DEFINI'	0,015	0.1
	COUPL_MF		'NON_DEFINI'	0.0	0.1

One also tests in nonregression:

- various components of elastic energy in the case of a calculation with MECA_STATIQUE,
- component VMIS of SIEQ_ELNO.

4 Modeling B

4.1 Characteristics of modeling

Element of hull DKT (modeling of a quarter of plate)

4.1.1 Limiting conditions

in all the nodes of the arc ABC : DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the segment $]OA[$: DY: 0. , DRX: 0. , DRZ: 0.
in all the nodes of the segment $]OC[$: DX: 0. , DRY MARTINI: 0. , DRZ: 0.
with the node O : DX: 0. , DY: 0. , DRX: 0. , DRY MARTINI: 0. , DRZ: 0.

Not O meshs: $M1\ M2$
Not A meshs: $M248\ M255$
Not B meshs: $M292\ M293\ M296$
Not C meshs: $M74\ M75$
Not D meshs: $M76\ M108\ M109$
Not E meshs: $M34\ M40\ M41$
Not F meshs: $M122\ M123\ M124\ M148\ M152\ M153$

4.2 Characteristics of the grid

Many nodes: 170
Many meshs and types: 296 TRIA3

4.3 Sizes tested and results

Identification	Type of reference	Values of reference	Tolerance
O $w(r)$	'ANALYTICAL'	– 695.6256	0.2%
D $w(r)$	'ANALYTICAL'	– 489,727	0.2%
E $w(r)$	'ANALYTICAL'	– 489,727	0.2%
F $w(r)$	'ANALYTICAL'	– 435.8974	0.2%

Identification		Type of reference	Values of reference	Tolerance
O	M_{rr}	'ANALYTICAL'	– 0.20625	0.1%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.20625	0.2%
With	M_{rr}	'ANALYTICAL'	0.	0.0025
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.0875	3.5%
B	M_{rr}	'ANALYTICAL'	– 0.04375	6%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.04375	6%
C	M_{rr}	'ANALYTICAL'	– 0.0875	3.5%
	$M_{\theta\theta}$	'ANALYTICAL'	0.	0.0025
D	M_{rr}	'ANALYTICAL'	– 0.15469	0.5%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	0.5%
E	M_{rr}	'ANALYTICAL'	– 0.15469	0.5%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	0.5%
F	M_{rr}	'ANALYTICAL'	– 0.14025	0.3%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.16825	0.3%

5 Modeling C

5.1 Characteristics of modeling

Element of hull SHB8 (modeling of a quarter of plate)

5.1.1 Limiting conditions

Boundary conditions kinematics: MOVIES

in all the nodes of the arc ABC	: DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the face $]OA[$	: DY: 0. ,
in all the nodes of the face $]OC[$	: DX: 0. ,
with the node O	: DX: 0. , DY: 0. ,

The grid is built starting from the surface grid of modeling E, by thickening, with the assistance CREA_MAILLAGE/COQU_VOLU. 2 layers of meshs are built HEXA8.

Boundary conditions in pressure imposed : NEAR
Boundary conditions in gravity with AFFE_CHAR_MECA: CHF3
Boundary conditions in gravity with AFFE_CHAR_MECA_F: CHFC

5.2 Characteristics of the grid

Many nodes: 338
Many meshs and types: 147 HEXA8

5.3 Sizes tested and results

The fields are tested solutions following :
'DEPR' solution with SHB and a loading CINE+PRES
'OFF3'solution with SHB and a loading CINE+CHF3
'OFFC'solution with SHB and a loading CINE+CHFC
'RESU' solution obtained with CREA_RESU starting from DEFC.

One points out the sizes tested by these 4 fields solutions:

Identification	Type of reference	Values of reference	Tolerance
O $w(r)$	'ANALYTICAL'	– 695.6256	0.5%
DEPL DZ in A	'ANALYTICAL'	0	1.E-10
DEPL DZ out of B	'ANALYTICAL'	0	1.E-10
D $w(r)$	'ANALYTICAL'	– 489,727	0.5%
E $w(r)$	'ANALYTICAL'	– 489,727	0.5%
F $w(r)$	'ANALYTICAL'	– 435.8974	0.5%
COOR_ELGA Z POINT 1	'ANALYTICAL'	0.00234550385153	1.E-7 %
COOR_ELGA Z POINT 4	'ANALYTICAL'	0.0384617327526	1.E-7 %
RESU - SIEF_ELGA SIXX NETS MH5 POINT5	'ANALYTICAL'	-117.944877967	0.5 %

6 Modeling D

6.1 Characteristics of modeling

Element of hull SHB20 (modeling of a quarter of plate)

6.1.1 Limiting conditions

in all the nodes of the arc ABC	:	DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the face $]OA[$	:	DY: 0. ,
in all the nodes of the face $]OC[$	:	DX: 0. ,
with the node O	:	DX: 0. , DY: 0. ,

6.2 Characteristics of the grid

Many nodes: 1137
Many meshs and types: 147 HEXA20

6.3 Sizes tested and results

The fields are tested solutions following :
'DEPR' solution with SHB and a loading CINE+PRES
'OFF3'solution with SHB and a loading CINE+CHF3
'OFFC'solution with SHB and a loading CINE+CHFC

One points out the sizes tested by these 4 fields solutions:

Identification	Type of reference	Values of reference	Tolerance
O $w(r)$	'ANALYTICAL'	– 695.6256	1.2%
D $w(r)$	'ANALYTICAL'	– 489,727	1.3%
E $w(r)$	'ANALYTICAL'	– 489,727	1.3%
F $w(r)$	'ANALYTICAL'	– 435.8974	1.5%

7 Modeling E

7.1 Characteristics of modeling

Element of hull DKQ (modeling of a quarter of plate)

7.1.1 Limiting conditions

in all the nodes of the arc ABC : DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the segment $]OA[$: DY: 0. , DRX: 0. , DRZ: 0.
in all the nodes of the segment $]OC[$: DX: 0. , DRY MARTINI: 0. , DRZ: 0.
with the node O : DX: 0. , DY: 0. , DRX: 0. , DRY MARTINI: 0. , DRZ: 0.

Not O meshs: $M1$
Not A meshs: $M147$
Not B meshs: $M98\ M111$
Not C meshs: $M14$
Not D meshs: $M85\ M99$
Not E meshs: $M7\ M8$
Not F meshs: $M91\ M92\ M105$

7.2 Characteristics of the grid

Many nodes: 169
Many meshs and types: 147 QUAD4

7.3 Sizes tested and results

Identification	Type of reference	Values of reference	Tolerance
O $w(r)$	'ANALYTICAL'	– 695.6256	0.1%
D $w(r)$	'ANALYTICAL'	– 489,727	0.15%
E $w(r)$	'ANALYTICAL'	– 489,727	0.15%
F $w(r)$	'ANALYTICAL'	– 435.8974	0.15%

Identification		Type of reference	Values of reference	Tolerance
O	M_{rr}	'ANALYTICAL'	– 0.20625	0.1%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.20625	0.1%
With	M_{rr}	'ANALYTICAL'	0.	0.0004
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.0875	0.5%
B	M_{rr}	'ANALYTICAL'	– 0.04375	1.1%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.04375	1.1%
C	M_{rr}	'ANALYTICAL'	0.	0.0004
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.0875	0.5%
D	M_{rr}	'ANALYTICAL'	– 0.15469	0.1%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	0.1%
E	M_{rr}	'ANALYTICAL'	– 0.15469	0.5%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	0.1%
F	M_{rr}	'ANALYTICAL'	– 0.14025	0.35%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.16825	0.35%

8 Modeling F

8.1 Characteristics of modeling

Element of hull DST (modeling of a quarter of plate)

8.1.1 Limiting conditions

in all the nodes of the arc ABC : DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the segment $]OA[$: DY: 0. , DRX: 0. , DRZ: 0.
in all the nodes of the segment $]OC[$: DX: 0. , DRY MARTINI: 0. , DRZ: 0.
with the node O : DX: 0. , DY: 0. , DRX: 0. , DRY MARTINI: 0. , DRZ: 0.

Not O meshs: $M1\ M2$
Not A meshs: $M248\ M255$
Not B meshs: $M3292\ M293\ M296$
Not C meshs: $M74\ M75$
Not D meshs: $M76\ M108\ M109$
Not E meshs: $M34\ M40\ M41$
Not F meshs: $M122\ M123\ M124\ M148\ M152\ M153$

8.2 Characteristics of the grid

Many nodes: 170
Many meshs and types: 296 TRIA3

8.3 Sizes tested and results

Identification	Type of reference	Values of reference Reissner	Tolerance
O $w(r)$	'ANALYTICAL'	– 703.40	0.2%
D $w(r)$	'ANALYTICAL'	– 495.56	0.1%
E $w(r)$	'ANALYTICAL'	– 495.56	0.1%
F $w(r)$	'ANALYTICAL'	– 441.18	0.1%

Identification		Type of reference	Values of reference	Tolerance
O	M_{rr}	'ANALYTICAL'	– 0.20625	0.5%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.20625	0.55%
With	M_{rr}	'ANALYTICAL'	0.	0.02
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.0875	9%
B	M_{rr}	'ANALYTICAL'	-0.04375	9.5%
	$M_{\theta\theta}$	'ANALYTICAL'	-0.04375	9.5%
C	M_{rr}	'ANALYTICAL'	– 0.0875	9%
	$M_{\theta\theta}$	'ANALYTICAL'	0.	0.02
D	M_{rr}	'ANALYTICAL'	– 0.15469	0.9%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	0.9%
E	M_{rr}	'ANALYTICAL'	– 0.15469	0.9%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	0.9%
F	M_{rr}	'ANALYTICAL'	– 0.14025	0.2%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.16825	0.2%

9 Modeling G

9.1 Characteristics of modeling

Element of hull DSQ (modeling of a quarter of plate)

9.1.1 Limiting conditions

in all the nodes of the arc ABC : DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the segment $]OA[$: DY: 0. , DRX: 0. , DRZ: 0.
in all the nodes of the segment $]OC[$: DX: 0. , DRY MARTINI: 0. , DRZ: 0.
with the node O : DX: 0. , DY: 0. , DRX: 0. , DRY MARTINI: 0. , DRZ: 0.

Not O meshs: $M1$
Not A meshs: $M147$
Not B meshs: $M98\ M111$
Not C meshs: $M14$
Not D meshs: $M85\ M99$
Not E meshs: $M7\ M8$
Not F meshs: $M91\ M92\ M105$

9.2 Characteristics of the grid

Many nodes: 169
Many meshs and types: 147 QUAD4

9.3 Sizes tested and results

Identification	Type of reference	Values of reference Reissner	Tolerance
O $w(r)$	'ANALYTICAL'	– 703.40	0.15%
D $w(r)$	'ANALYTICAL'	– 495.56	0.15%
E $w(r)$	'ANALYTICAL'	– 495.56	0.15%
F $w(r)$	'ANALYTICAL'	– 441.18	0.2%

Identification		Type of reference	Values of reference	Tolerance
O	M_{rr}	'ANALYTICAL'	– 0.20625	1%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.20625	1%
With	M_{rr}	'ANALYTICAL'	0.	0.01
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.0875	2.5%
B	M_{rr}	'ANALYTICAL'	-0.04375	7.5%
	$M_{\theta\theta}$	'ANALYTICAL'	-0.04375	7.5%
C	M_{rr}	'ANALYTICAL'	– 0.0875	2.5%
	$M_{\theta\theta}$	'ANALYTICAL'	0.	0.01
D	M_{rr}	'ANALYTICAL'	– 0.15469	0.6%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	0.6%
E	M_{rr}	'ANALYTICAL'	– 0.15469	0.7%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	0.7%

F	M_{rr}	'ANALYTICAL'	– 0.14025	19%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.16825	19%

Identification				Type of reference	Values of reference	Tolerance
Size		Not	Mesh			
ENEL_ELNO	*TOTALE*	C	M14	'NON_REGRESSION'	45.79374	0.1%
	MEMBRANE			'NON_REGRESSION'	0.0	0.1%
	FLEXION			'NON_REGRESSION'	44.78267	0.1%
	CISAILLE			'NON_REGRESSION'	1.01107	0.1%
	COUPL_MF			'NON_REGRESSION'	0.0	0.1%

Identification				Type of reference	Values of reference	Tolerance
Size		Not	Mesh			
ENEL_ELGA	*TOTALE*	1	M14	'NON_REGRESSION'	57.36959	0.1%
	MEMBRANE			'NON_REGRESSION'	0.0	0.1%
	FLEXION			'NON_REGRESSION'	56.47085	0.1%
	CISAILLE			'NON_REGRESSION'	0.89874	0.1%
	COUPL_MF			'NON_REGRESSION'	0.0	0.1%

Identification			Type of reference	Values of reference	Tolerance
Size		Mesh			
ENEL_ELEM	*TOTALE*	M14	'NON_REGRESSION'	0.42940	0.1%
	MEMBRANE		'NON_REGRESSION'	0.0	0.1%
	FLEXION		'NON_REGRESSION'	0.42229	0.1%
	CISAILLE		'NON_REGRESSION'	7.11225E-03	0.1%
	COUPL_MF		'NON_REGRESSION'	0.0	0.1%

10 Modeling H

10.1 Characteristics of modeling

Element of hull Q4G (modeling of a quarter of plate)

10.1.1 Limiting conditions

in all the nodes of the arc ABC : DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the segment $]OA[$: DY: 0. , DRX: 0. , DRZ: 0.
in all the nodes of the segment $]OC[$: DX: 0. , DRY MARTINI: 0. , DRZ: 0.
with the node O : DX: 0. , DY: 0. , DRX: 0. , DRY MARTINI: 0. , DRZ: 0.

Not O meshs: $M1$
Not A meshs: $M147$
Not B meshs: $M98\ M111$
Not C meshs: $M14$
Not D meshs: $M85\ M99$
Not E meshs: $M7\ M8$
Not F meshs: $M91\ M92\ M105$

10.2 Characteristics of the grid

Many nodes: 169
Many meshs and types: 147 QUAD4

10.3 Sizes tested and results

Identification	Type of reference	Values of reference Reissner	Tolerance
O $w(r)$	'ANALYTICAL'	– 703.40	0.2%
D $w(r)$	'ANALYTICAL'	– 495.56	0.3%
E $w(r)$	'ANALYTICAL'	– 495.56	0.3%
F $w(r)$	'ANALYTICAL'	– 441.18	0.3%

Identification		Type of reference	Values of reference	Tolerance
O	M_{rr}	'ANALYTICAL'	– 0.20625	0.2%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.20625	0.2%
With	M_{rr}	'ANALYTICAL'	0.	0.02
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.0875	4.5%
B	M_{rr}	'ANALYTICAL'	-0.04375	20%
	$M_{\theta\theta}$	'ANALYTICAL'	-0.04375	20%
C	M_{rr}	'ANALYTICAL'	– 0.0875	0.02
	$M_{\theta\theta}$	'ANALYTICAL'	0.	4.5%
D	M_{rr}	'ANALYTICAL'	– 0.15469	0.25%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	0.25%
E	M_{rr}	'ANALYTICAL'	– 0.15469	0.25%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	0.25%
F	M_{rr}	'ANALYTICAL'	– 0.14025	0.35%

$M_{\theta\theta}$	'ANALYTICAL'	– 0.16825	0.35%

11 Modeling I

11.1 Characteristics of modeling

Modeling: Element of COQUE_3D MEC3QU9H

11.1.1 Limiting conditions

in all the nodes of the arc ABC	DX: 0. , DY: 0. , DZ: 0. DRX: 0. , DRY MARTINI: 0. , DRZ: 0.
segment $\left]OA\right]$	DY: 0. , DRX: 0. , DRZ: 0.
segment $\left]OC\right]$	DX: 0. , DRY MARTINI: 0. , DRZ: 0.
with the node O	DX: 0. , DY: 0. , DRX: 0. , DRY MARTINI: 0. , DRZ: 0.

Names of the nodes:

Not 0	meshs: $M1$	Not A	meshs: $M21$
Not B	meshs: $M25$	Not C	meshs: $M5$
Not D	meshs: $M11$	Not E	meshs: $M3$

11.2 Characteristics of the grid

Many nodes: 96
Many meshs and types: 25 QUAD9

11.3 Sizes tested and results

Identification	Type of reference	Values of reference Reissner	Tolerance
O $w(r)$	'ANALYTICAL'	– 703.40	0.5%
With $w(r)$	'ANALYTICAL'	0.	10^{-10}
B $w(r)$	'ANALYTICAL'	0.	10^{-10}
C $w(r)$	'ANALYTICAL'	0.	10^{-10}
D $w(r)$	'ANALYTICAL'	– 495.56	0.5%
E $w(r)$	'ANALYTICAL'	– 495.56	0.5%

Identification		Type of reference	Values of reference	Tolerance
O	M_{rr}	'ANALYTICAL'	– 0.20625	1.3%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.20625	1.3%
With	M_{rr}	'ANALYTICAL'	0.	0,002
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.0875	1.3%
C	M_{rr}	'ANALYTICAL'	0.	0,002
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.0875	2.3%
D	M_{rr}	'ANALYTICAL'	– 0.15469	1%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	1%
E	M_{rr}	'ANALYTICAL'	– 0.15469	1%

$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	1%

11.4 Remarks

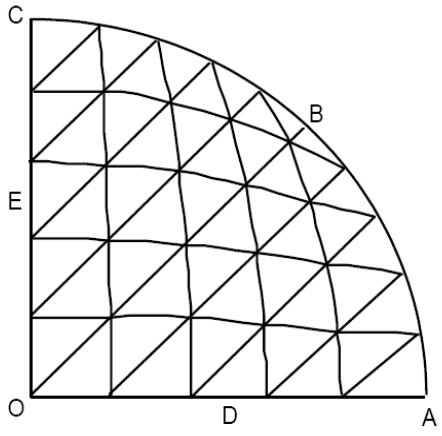
The test of the values is carried out automatically using the features offered by the procedure POST_RELEVE :

- extraction on the nodes corresponding to the points observed of the median values of the components M_{xx} and M_{yy} ; these values are extracted from the field 'EFGE_ELNO' , and the average is calculated for all the liquid assets on the meshs which contain the node observed,
- calculation of the variation compared to the value of reference provided by observing the rules of correspondence enters M_{xx} , M_{yy} and M_{rr} , $M_{\theta\theta}$ data page 3.

12 Modeling J

12.1 Characteristics of modeling

Modeling: Element of COQUE_3D MEC3TR7H



12.1.1 Limiting conditions

in all the nodes of the arc ABC	DX: 0. , DY: 0. , DZ: 0. DRX: 0. , DRY MARTINI: 0. , DRZ: 0.
segment $]OA]$	DY: 0. , DRX: 0. , DRZ: 0.
segment $]OC]$	DX: 0. , DRY MARTINI: 0. , DRZ: 0.
with the node O	DX: 0. , DY: 0. , DRX: 0. , DRY MARTINI: 0. , DRZ: 0.

Names of the nodes:

Not 0	meshs: $M1$ and $M2$	Not A	meshs: $M41$
Not B	meshs: $M49$ and $M50$	Not C	meshs: $M10$
Not D	meshs: $M21$	Not E	meshs: $M6$

12.2 Characteristics of the grid

Many nodes: 121
Many meshs and types: 50 TRIA7

12.3 Sizes tested and results

Identification	Type of reference	Values of reference Reissner	Tolerance
O $w(r)$	'ANALYTICAL'	– 703.40	0.5%
With $w(r)$	'ANALYTICAL'	0.	10^{-10}
B $w(r)$	'ANALYTICAL'	0.	10^{-10}
C $w(r)$	'ANALYTICAL'	0.	10^{-10}
D $w(r)$	'ANALYTICAL'	– 495.56	0.5%
E $w(r)$	'ANALYTICAL'	– 495.56	0.5%

Identification		Type of reference	Values of reference	Tolerance
O	M_{rr}	'ANALYTICAL'	– 0.20625	1.3%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.20625	1.3%
With	M_{rr}	'ANALYTICAL'	0.	0,002
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.0875	1.5%
C	M_{rr}	'ANALYTICAL'	– 0.0875	2.3%
	$M_{\theta\theta}$	'ANALYTICAL'	0.	0,002
D	M_{rr}	'ANALYTICAL'	– 0.15469	1%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	1%
E	M_{rr}	'ANALYTICAL'	– 0.15469	1%
	$M_{\theta\theta}$	'ANALYTICAL'	– 0.17656	1%

12.4 Remarks

The test of the values is carried out automatically using the features offered by the procedure POST_RELEVE :

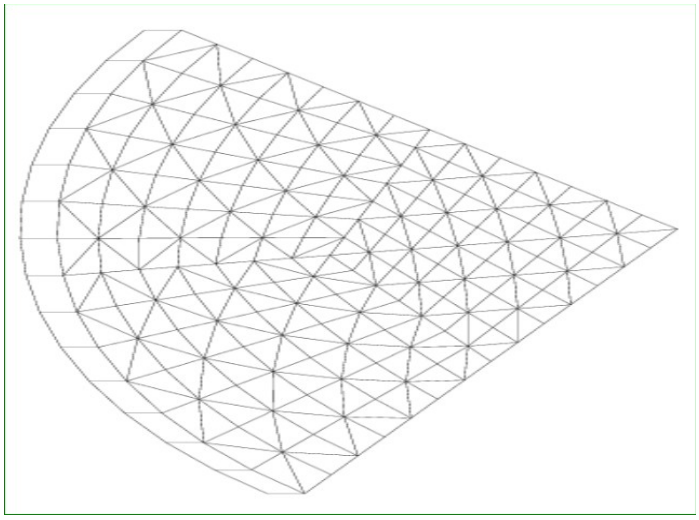
• extraction on the nodes corresponding to the points observed of the median values of the components M_{xx} and M_{yy} ; these values are extracted from the field 'EFGE_ELNO' , and the average is calculated for all the liquid assets on the meshs which contain the node observed,
• calculation of the variation compared to the value of reference provided by observing the rules of correspondence enters M_{xx} , M_{yy} and M_{rr} , $M_{\theta\theta}$ data page 3.

13 Modeling K

13.1 Characteristics of modeling

Element of hull SHB6 (modeling of a quarter of plate)

13.1.1 Limiting conditions



in all the nodes of the arc ABC : DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the face $]OA[$: DY: 0. ,
in all the nodes of the face $]OC[$: DX: 0. ,
with the node O : DX: 0. , DY: 0. ,

13.2 Characteristics of the grid

Many nodes: 338
Many meshs and types: 294 SHB6

13.3 Sizes tested and results

The fields are tested solutions following :
'DEPR' solution with SHB and a loading CINE+PRES
'OFF3'solution with SHB and a loading CINE+CHF3
'OFFC'solution with SHB and a loading CINE+CHFC

One points out the sizes tested by these 4 fields solutions:

Identification	Type of reference	Values of reference	Tolerance
O $w(r)$	'ANALYTICAL'	– 695.6256	0.1%
D $w(r)$	'ANALYTICAL'	– 489,727	0.1%
E $w(r)$	'ANALYTICAL'	– 489,727	0.1%
F $w(r)$	'ANALYTICAL'	– 435.8974	0.1%

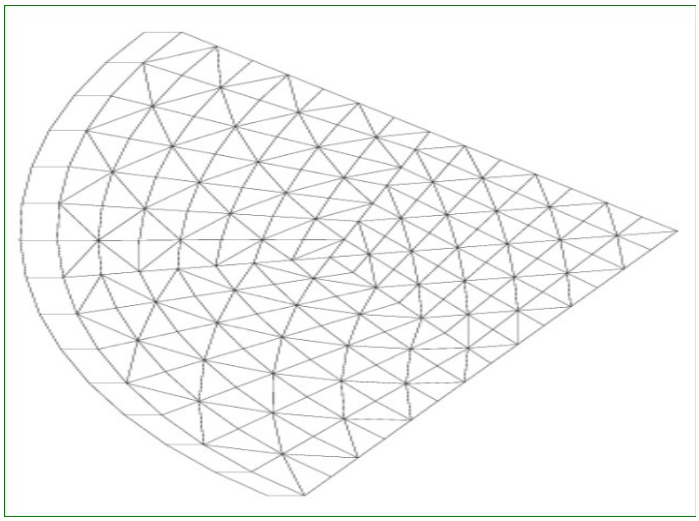
13.4 Notice

When the grid is refined, the solution does not move practically any more but tends towards an arrow higher from about 5% than the analytical solution. The element thus presents on this test a light digital blocking, much lower however than that of its counterpart 3D PENTA6 (30% of error).

14 Modeling L

14.1 Characteristics of modeling

Element of hull SHB15 (modeling of a quarter of plate)



14.1.1 Limiting conditions

in all the nodes of the arc ABC	:	DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the face $]OA[$	:	DY: 0. ,
in all the nodes of the face $]OC[$	:	DX: 0. ,
with the node O	:	DX: 0. , DY: 0. ,

14.2 Characteristics of the grid

Many nodes: 1431
Many meshs and types: 294 SHB15

14.3 Sizes tested and results

The fields are tested solutions following :
'DEPR' solution with SHB and a loading CINE+PRES
'OFF3'solution with SHB and a loading CINE+CHF3
'OFFC'solution with SHB and a loading CINE+CHFC

One points out the sizes tested by these 4 fields solutions:

Identification	Type of reference	Values of reference	Tolerance
O $w(r)$	'ANALYTICAL'	– 695.6256	1.2%
D $w(r)$	'ANALYTICAL'	– 489,727	1.3%
E $w(r)$	'ANALYTICAL'	– 489,727	1.3%
F $w(r)$	'ANALYTICAL'	– 435.8974	1.5%

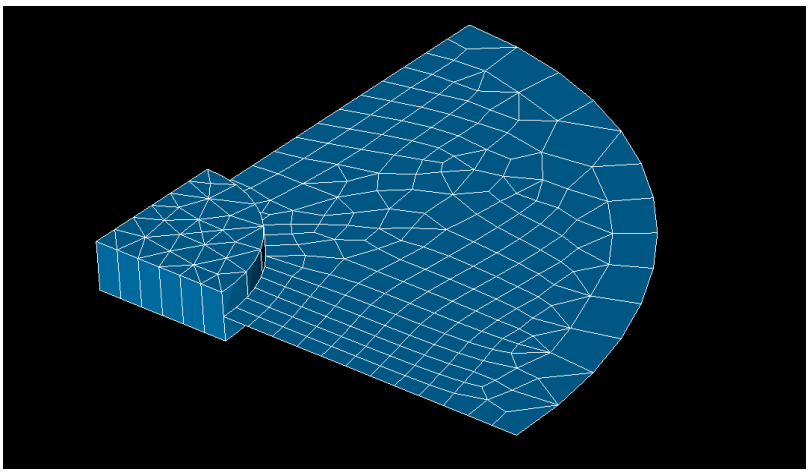
14.4 Notice

This element does not present digital blocking and converges well towards the analytical solution.

15 Modeling M

15.1 Characteristics of modeling

Connection between elements 3D and elements of hull (DKT)



15.1.1 Limiting conditions

Circular edge : simple support
right sides : conditions of symmetry

15.1.2 Characteristics of the grid

MODELING	FINITE ELEMENT	TYPE NETS	NUMBER
DKT	MEDKTR3	TRIA3	32
DKT	MEDKQU4	QUAD4	196
3D	MECA_PENTA15	PENTA15	48

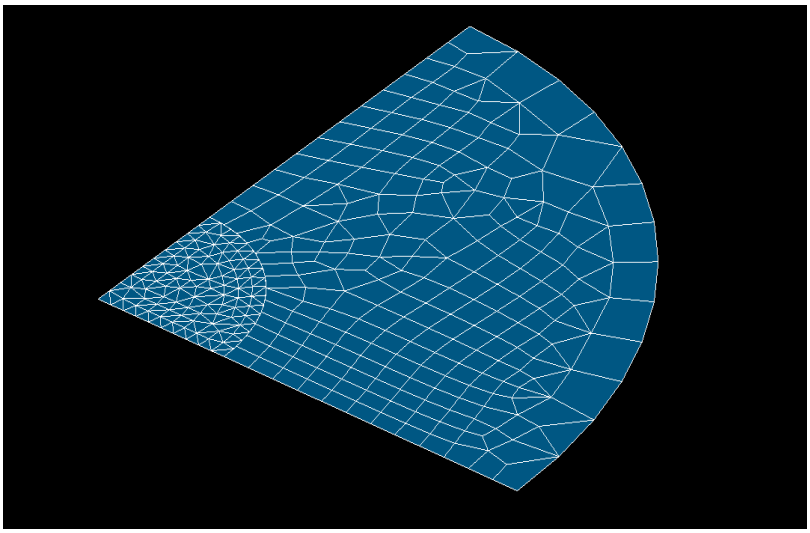
15.2 Sizes tested and results

Identification		Type of reference	Values of reference	Tolerance
Not	TYPE_RACCORD			
O $w(r)$	COQUE_MASSIF	'ANALYTICAL'	– 695.6256	0.3%
O $w(r)$	MASSIF_COQUE	'ANALYTICAL'	– 695.6256	1%

16 Modeling NR

16.1 Characteristics of modeling

Connection between 2 grids of hull (DKT) incompatible



16.1.1 Limiting conditions

Circular edge : simple support
right sides : conditions of symmetry

16.1.2 Characteristics of the grid

MODELING	FINITE ELEMENT	TYPE NETS	NUMBER
DKT	MEDKTR3	TRIA3	213
DKT	MEDKQU4	QUAD4	196

16.2 Sizes tested and results

Identification	Type of reference	Values of reference	Tolerance
○ $w(r)$	'ANALYTICAL'	– 695.6256	0.2%

17 Summary of the results

% of the differences compared to the reference solutions

	DKT With Coil-Kirchhoff 50 nodes 76 TRIA3	B Coil-Kirchhoff 170 nodes 296 TRIA3	**DKQ** E Coil-Kirchhoff 169 nodes 147 QUAD4	**DST** F Reissner 170 nodes 296 TRIA3	**DSQ** G Reissner 169 nodes 147 QUAD4	**Q4G** H Reissner 169 nodes 147 QUAD4
O $w(r)$	– 1.10	– 0.09	– 0.09	+0.12	– 0.11	– 0.15
D $w(r)$	– 1.01	– 0.1	– 0.11	+0.08	– 0.13	– 0.20
E $w(r)$	– 1.03	– 0.09	– 0.12	+0.09	– 0.13	– 0.20
F $w(r)$	– 1.05	– 0.09	– 0.09	+0.07	– 0.15	– 0.21
	MEC3QU9H I 96 nodes 25 QUAD9	**MEC3TR7H** J 121 nodes 50 TRIA7	**SHB8** C 338 nodes 147 HEXA8	**SHB20** D 1137 nodes 147 HEXA20	**SHB6** K 338 nodes 294 PENTA6	**SHB15** L 1431 nodes 294 PENTA15
O $w(r)$	$1.42\ 10^{-3}$	– 0.03	0.4	1.1	3.7	1.1
D $w(r)$	$2.\ 10^{-3}$	– 0.07	0.3	1.3	4.1	1.2
E $w(r)$	$2.\ 10^{-3}$	– 0.07	0.3	1.3	4.0	1.2
F $w(r)$	-	-	0.2	1.3	3.8	1.3

Concerning displacements:
The elements plates and hulls give good performances on rather coarse grids.
Elements 3D- hulls SHB less good performances give, especially it SHB6 who presents a light digital blocking.

	DKT With Coil-Kirchhoff 50 nodes 76 TRIA3	B Coil-Kirchhoff 170 nodes 296 TRIA3	**DKQ** E Coil-Kirchhoff 169 nodes 147 QUAD4	**DST** F Reissner 170 nodes 296 TRIA3	**DSQ** G Reissner 169 nodes 147 QUAD4	**Q4G** H Reissner 169 nodes 147 QUAD4
O Sm/2	– 1.19	+0.02	+0.07	+0.07	– 0.76	– 0.14
In Sm/2	+5.79	– 0.06	– 0.49	– 4.40	– 9.80	+17.80
B Sm/2	– 13,100	– 5.53	+1.00	– 9.10	– 7.12	+19.70
C Sm/2	+5.73	– 0.06	– 0.46	– 4.41	– 9.44	+17.90
D Sm/2	+0.20	+0.35	+0.50	+0.43	+0.49	+0.05
E Sm/2	+0.19	+0.42	+0.50	+0.49	+0.50	+0.05
F Sm/2	– 0.66	+0.25	– 0.30	+0.15	+19.00	– 0.33
	MEC3QU9H I 96 nodes 25 QUAD9	**MEC3TR7H** J 121 nodes 50 TRIA7				
O Sm/2	1.05	1.14				
In Sm/2	2.9	0.25				
B Sm/2	-	-				
C Sm/2	2.9	0.25				
D Sm/2	0.28	– 0.28				
E Sm/2	0.28	– 0.28				
F Sm/2	-	-				

Concerning the efforts:

- on the supported edge, one notes important errors (going up to 20%) compared to the analytical solutions. The error is marked on modeling H (Q4G).
- by refining the grid of each modeling one observes the convergence of the efforts, i.e. that the error tends towards 0. Nevertheless the order of convergence is lower for modeling H : the element Q4G request

indeed to net very finely in the directions requested in inflection (it uses a bilinear approximation of rotations whereas modeling DST be based on a quadratic approximation).