

## SSLS127 – Inflection of a reinforced concrete flagstone (model GLRC\_DAMAGE) supported on 4 with dimensions: elastic mode of plate

---

### Summary:

This test represents the calculation of a reinforced concrete flagstone, in inflection, subjected to a pressure. It makes it possible to validate modeling `DKTG` with model `GLRC_DAMAGE` for the linear elastic behavior and modeling `Q4GG` with the model `ELAS`. The flagstone is in simple supports on its four with dimensions.

Four modelings are carried out:

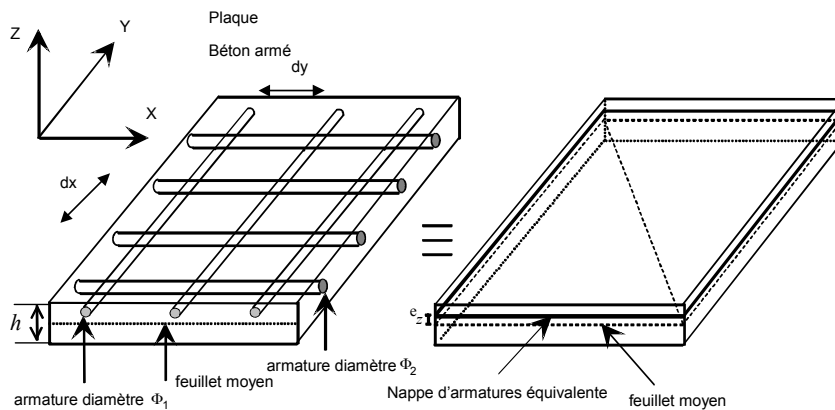
- 1) Modeling A makes it possible to test the model `DKTG` with `TRIA3`
- 2) Modeling B makes it possible to test the model `DKTG` with `QUAD4`
- 3) Modeling C makes it possible to test the model `Q4GG` with `TRIA3`
- 4) Modeling D makes it possible to test the model `Q4GG` with `QUAD4`

## 1 Problem of reference

### 1.1 Geometry

Square flagstone, length  $l=1.8\text{ m}$ , thickness  $h=0.12\text{ m}$ , in simple support on the four edges. The reinforcement of inflection is parallel to the edges; it is identical on each of the two faces and in each of the two directions ( $dx$ ,  $dy$  being spacings of irons in the directions  $x$  and  $y$ ). The coating of the longitudinal irons closest to the faces is of  $22\text{ mm}$ . The coating of irons compared to the side edges of the flagstone of  $2\text{ cm}$  is neglected. The table hereafter recapitulates the data of reinforcement. Geometrical percentage of steel  $\mu$  is given for a face in a direction.

Diameter of the reinforcements	Spacing	Section steel/section of the concrete	distance roasts/average surface of the flagstone
$\Phi=0,01\text{ m}$	$dx=dy=0,1\text{ m}$	$\mu=0,65$	$e_s=\pm 0,038\text{ m}$



One notes  $a_x = \frac{A_x}{d_x}$  and  $a_y = \frac{A_y}{d_y}$  rates of reinforcement (here:  $a_x = a_y = 7,854 \cdot 10^{-4}$ ),  $A_x$  ( $A_y$ )

being the surface of the section of an iron bar in the direction  $x$  ( $y$ );  $e_s$  is the distance from the tablecloths on the average surface.

### 1.2 Material properties

The mechanical properties of steels are the following ones:

Young modulus $E_a$	Poisson's ratio	Yield stress with 0.2% $\sigma_y$	Rupture limit $\sigma_r$	Slope of work hardening	Lengthening with rupture
210000 MPa	0.3	500 MPa	570 MPa	473 MPa	15%

Those of the concrete are the following ones:

Young modulus $E_b$	Poisson's ratio	Resistance in compression $\sigma_c$	Resistance in traction $\sigma_t$
35700 MPa	0.22	52,5 MPa	4,4 MPa

## 1.3 Boundary conditions and loadings

- The boundary conditions are summarized in simple supports: vertical displacement blocked and free rotations on the four edges of the flagstone.
- Uniform pressure  $p = 0,01 \text{ MPa}$

## 1.4 Initial conditions

Without object.

## 2 Reference solution

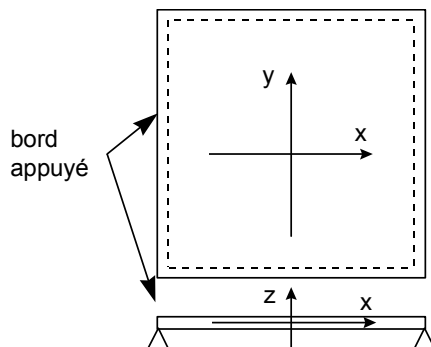
### 2.1 Method of calculating used for the reference solution

Elastic relations, connecting the membrane efforts  $N$  and of inflection  $M$  with the membrane deformations  $\epsilon$  and curves  $\kappa$  and taking account of two symmetrical grids, are written:

$$N = \left( \frac{E_b h}{1 - \nu_b^2} \begin{bmatrix} 1 & \nu_b & 0 \\ \nu_b & 1 & 0 \\ 0 & 0 & \frac{1 - \nu_b}{2} \end{bmatrix} + 2 E_a \begin{bmatrix} a_x & 0 & 0 \\ 0 & a_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) \epsilon$$

$$M = \left( \frac{E_b h^3}{12(1 - \nu_b^2)} \begin{bmatrix} 1 & \nu_b & 0 \\ \nu_b & 1 & 0 \\ 0 & 0 & \frac{1 - \nu_b}{2} \end{bmatrix} + 2 E_a e_s^2 \begin{bmatrix} a_x & 0 & 0 \\ 0 & a_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) \kappa$$

As regards a configuration paves, one assigns to the concrete the usual Poisson's ratio  $\nu_b = 0,22$ . The flagstone is simply pressed on the four edges:



The stiffness of equivalent inflection is:

$$D_{\acute{e}q} = \frac{E_b h^3}{12(1 - \nu_b^2)} + 2 E_a e_s^2 a_x,$$

that is to say here:  $D_{\acute{e}q} = 5,8786 \text{ MNm}$  ;

$$\text{Moreover: } \nu_{\acute{e}q} = \frac{\nu_b E_b h^3}{12(1 - \nu_b^2) D_{\acute{e}q}},$$

that is to say:  $\nu_{\acute{e}q} = 0,2022$

Size	Expression
Arrow in the center under pressure [2]	$w = \frac{0,0464 pl^4}{12(1 - \nu_{\acute{e}q}^2) D_{\acute{e}q}}$
Curve in the center [2]	$\kappa_{xx} = \kappa_{yy} = \frac{0,04784 pl^2}{(1 + \nu_{\acute{e}q}) D_{\acute{e}q}}$
Total moment in the center [2]	$M_{xx} = M_{yy} = 0,04784 pl^2$

### 2.2 Results of reference

For modelings A and B in which one validates law GLRC\_DAMA with elements DKTG:

- Arrow in the center under pressure:  $w = 6,926 \cdot 10^{-5} \text{ m}$
- Curve in the center:  $\kappa_{xx} = \kappa_{yy} = 2,193 \cdot 10^{-4} \text{ m}^{-1}$
- Total moment in the center:  $M_{xx} = M_{yy} = 1550 \text{ Nm/ml}$

For modelings C and D in which one validates law ELAS with elements DKTG:

- Arrow in the center under pressure:  $w = 7,895 \cdot 10^{-5} \text{ m}$

- Curve in the center:  $\kappa_{xx} = \kappa_{yy} = 2,351 \cdot 10^{-4} \text{ m}^{-1}$
- Total moment in the center:  $M_{xx} = M_{yy} = 1550 \text{ Nm/ml}$

## 2.3 Uncertainty on the solution

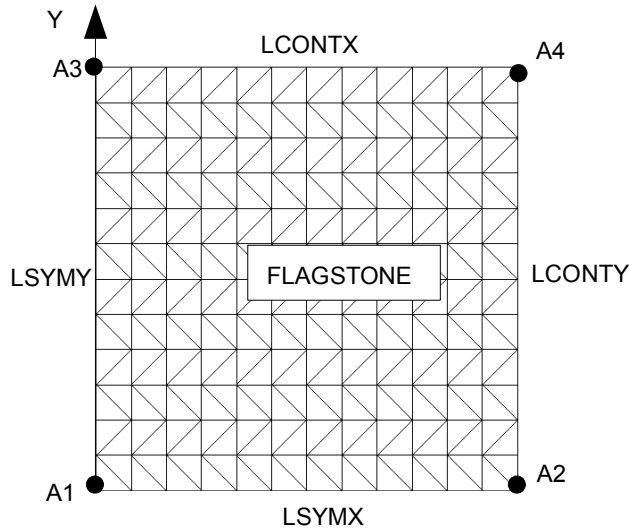
Analytical solution.

## 2.4 Bibliographical references

- [1] KOEHLIN P., MILL S., "Model total behavior of the reinforced concrete plates under dynamic loading of inflection: Law GLRC", Notes EDF/R & D /AMA HT-62/01/028A.
- [2] J.Dulac, "elastoplastic dynamic Behavior of the reinforced concrete flagstones. Tests CEMETE – December 1979 – Flagstones 8 to 12": EDF note: ESE/GC/82/13/A

## 3 Modeling A

### 3.1 Characteristics of modeling



Modeling Q4GG (TRIA3)

- Boundary conditions:
  - . Dimensioned *A2A4* :  $DZ = 0$
- Conditions of symmetry
  - . Dimensioned *A1A2* :  
 $DY = DRX = 0$
  - . Dimensioned *A1A3* :  
 $DX = DRY = 0$

The flagstone is symmetrical compared to the plans ( $X=0$ ) and ( $Y=0$ ), calculations are carried out on a quarter of the flagstone.

### 3.2 Characteristics of the grid

Many nodes: 169  
Number of meshes and type: 288 TRIA3

### 3.3 Sizes tested and results

Identification	Type of reference	Reference	Tolerance (%)
$DZ(AI)$	'ANALYTICAL'	$6,926 \cdot 10^{-5}$	5 %
$MXX(AI)$	'ANALYTICAL'	1550	8 %
$MYY(AI)$	'ANALYTICAL'	1550	8 %
$KXX(AI)$	'ANALYTICAL'	$2,193 \cdot 10^{-4}$	8 %
$KYY(AI)$	'ANALYTICAL'	$2,193 \cdot 10^{-4}$	8 %

The sizes are expressed in the reference mark defined by the nautical angles  $\alpha = 33^\circ$  and  $\beta = 12^\circ$

Identification	Type of reference	Reference	Tolerance
$DZ(AI)$	'ANALYTICAL'	$6,926 \cdot 10^{-5}$	5 %
$MXX(AI)$	'ANALYTICAL'	1550.0	8 %
$MYY(AI)$	'ANALYTICAL'	1550.0	8 %
$MXY(AI)$	'ANALYTICAL'	0.	2.
$KXX(AI)$	'ANALYTICAL'	$2,193 \cdot 10^{-4}$	8 %
$KYY(AI)$	'ANALYTICAL'	$2,193 \cdot 10^{-4}$	8 %

$KXY(AI)$	'ANALYTICAL'	0.	0,001
-----------	--------------	----	-------

Identification			Type of reference	Reference	Tolérance%
$MXX$	$M266$	Point 3	'NON_REGRESSION'	1445.794	1.e-6
$MYX$	$M266$	Point 3	'NON_REGRESSION'	1447.847	1.e-6
$MXY$	$M266$	Point 3	'NON_REGRESSION'	0,526	1.e-6
$KXX$	$M266$	Point 3	'NON_REGRESSION'	$2.14096 \cdot 10^{-4}$	1.e-6
$KYY$	$M266$	Point 3	'NON_REGRESSION'	$2.14565 \cdot 10^{-4}$	1.e-6
$KXY$	$M266$	Point 3	'NON_REGRESSION'	$1.2018 \cdot 10^{-7}$	1.e-6

### 3.4 Remarks

The coefficients of the following matrices of elasticity, used during calculations, were calculated with  $\nu_b=0,22$  :

- 1) Matrix of elasticity out of membrane: 
$$\begin{bmatrix} 4832 & 990,4 & 0 \\ 990,4 & 4832 & 0 \\ 0 & 0 & 1756 \end{bmatrix} 10^6 \text{ N/m}$$
- 2) Matrix of elasticity in inflection: 
$$\begin{bmatrix} 5,879 & 1,188 & 0 \\ 1,188 & 5,879 & 0 \\ 0 & 0 & 2,107 \end{bmatrix} 10^6 \text{ N/m}$$

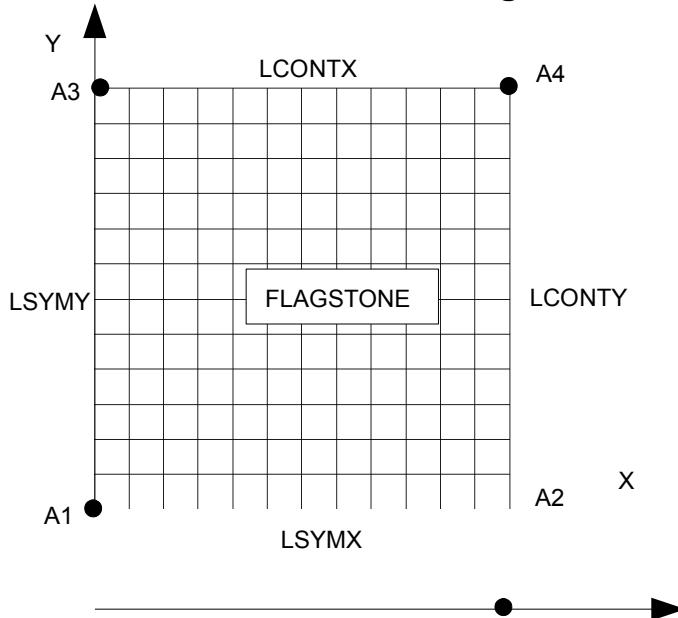
To be certain to remain in the elastic range, the yield stresses expressed in the reference mark of orthotropism, are fixed arbitrarily at a very high value:

1. Yield stresses in positive inflection:  
 Directorate X:  $1 \cdot 10^{10}$  MNm/ml  
 Direction there:  $1 \cdot 10^{10}$  MNm/ml
2. Yield stresses in negative inflection:  
 Directorate X:  $-1 \cdot 10^{10}$  MNm/ml  
 Direction there:  $-1 \cdot 10^{10}$  MNm/ml

As the structure remains in the elastic range, the kinematic coefficient of recall (constant of Prager) can take an unspecified value.

## 4 Modeling B

### 4.1 Characteristics of modeling



Modeling Q4GG (QUAD4)

- Boundary conditions:

- . Dimensioned A2A4 :  $DZ=0$
- . Dimensioned A3A4 :  $DZ=0$

- Conditions of symmetry

- . Dimensioned A1A2 :  
 $DY = DRX = 0$
- . Dimensioned A1A3 :  
 $DX = DRY = 0$

The flagstone is symmetrical compared to the plans ( $X=0$ ) and ( $Y=0$ ), calculations are carried out on a quarter of the flagstone.

### 4.2 Characteristics of the grid

Many nodes: 169

Number of meshes and type: 144 QUAD4

### 4.3 Sizes tested and results

Identification	Type of reference	Reference	Tolerance (%)
$DZ(AI)$	'ANALYTICAL'	$6,926.10^{-5}$	5 %
$MXX(AI)$	'ANALYTICAL'	1550	8 %
$MYY(AI)$	'ANALYTICAL'	1550	8 %
$KXX(AI)$	'ANALYTICAL'	$2,193.10^{-4}$	8 %
$KYY(AI)$	'ANALYTICAL'	$2,193.10^{-4}$	8 %

The sizes are expressed in the reference mark defined by the nautical angles  $\alpha=33^\circ$  and  $\beta=12^\circ$ .

Identification	Type of reference	Reference	Tolerance
$DZ(AI)$	'ANALYTICAL'	$6,926.10^{-5}$	5 %
$MXX(AI)$	'ANALYTICAL'	1550.0	8 %
$MYY(AI)$	'ANALYTICAL'	1550.0	8 %
$MXY(AI)$	'ANALYTICAL'	0.	2.
$KXX(AI)$	'ANALYTICAL'	$2,193 \cdot 10^{-4}$	8 %
$KYY(AI)$	'ANALYTICAL'	$2,193 \cdot 10^{-4}$	8 %
$KXY(AI)$	'ANALYTICAL'	0.	0,001



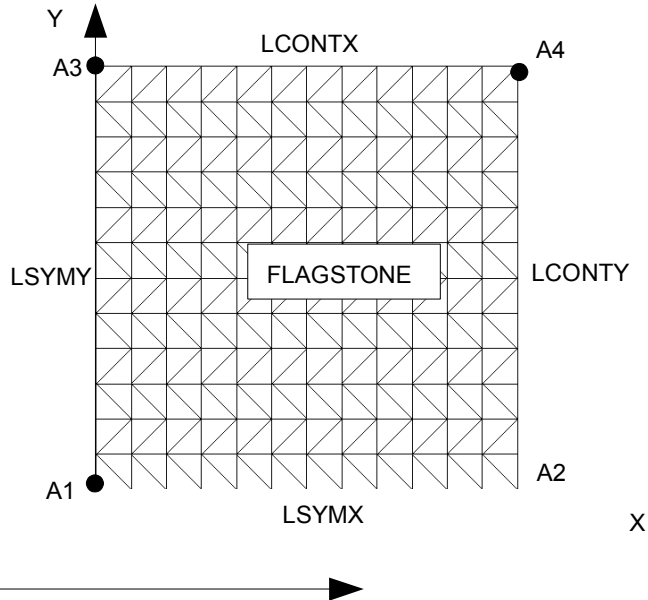
Identification			Type of reference	Reference	Tolérance %
<i>MXX</i>	<i>MI33</i>	<i>Point 4</i>	'NON_REGRESSION'	1444.999	1.e-6
<i>MYY</i>	<i>MI33</i>	<i>Point 4</i>	'NON_REGRESSION'	1447.976	1.e-6
<i>MXY</i>	<i>MI33</i>	<i>Point 4</i>	'NON_REGRESSION'	-0.6626	1.e-6
<i>KXX</i>	<i>MI33</i>	<i>Point 4</i>	'NON_REGRESSION'	2.1394 10 <sup>-4</sup>	1.e-6
<i>KYY</i>	<i>MI33</i>	<i>Point 4</i>	'NON_REGRESSION'	2.1462 10 <sup>-4</sup>	1.e-6
<i>KXY</i>	<i>MI33</i>	<i>Point 4</i>	'NON_REGRESSION'	-1.5151 10 <sup>-7</sup>	1.e-6

## 4.4 Remarks

See remarks of modeling A

## 5 Modeling C

### 5.1 Characteristics of modeling



Modeling Q4GG (TRIA3)

- Boundary conditions:
  - . Dimensioned  $A2A4$  :  $DZ=0$
- Conditions of symmetry
  - . Dimensioned  $A1A2$  :  
 $DY = DRX = 0$
  - . Dimensioned  $A1A3$  :  
 $DX = DRY = 0$

The flagstone is symmetrical compared to the plans  $(X=0)$  and  $(Y=0)$ , calculations are carried out on a quarter of the flagstone.

### 5.2 Characteristics of the grid

Many nodes: 169  
Number of meshes and type: 288 TRIA3

### 5.3 Sizes tested and results

Identification	Type of Reference	Reference	% Tolerance
$DZ(AI)$	'ANALYTICAL'	$7,895.10^{-5}$	8 %
$MXX(AI)$	'ANALYTICAL'	1550	3 %
$MYY(AI)$	'ANALYTICAL'	1550	3 %
$KXX(AI)$	'ANALYTICAL'	$2,351.10^{-4}$	3 %
$KYY(AI)$	'ANALYTICAL'	$2,351.10^{-4}$	3 %

The sizes are expressed in the reference mark defined by the nautical angles  $\alpha=33^\circ$  and  $\beta=12^\circ$

Identification	Type of reference	Reference	Tolerance
$DZ(AI)$	'ANALYTICAL'	$7,895.10^{-5}$	8 %
$MXX(AI)$	'ANALYTICAL'	1550.0	3 %
$MYY(AI)$	'ANALYTICAL'	1550.0	3 %
$MXZ(AI)$	'ANALYTICAL'	0.	0.1
$KXX(AI)$	'ANALYTICAL'	$2,351.10^{-4}$	3 %
$KYY(AI)$	'ANALYTICAL'	$2,351.10^{-4}$	3 %
$KXZ(AI)$	'ANALYTICAL'	0.	0,001

---

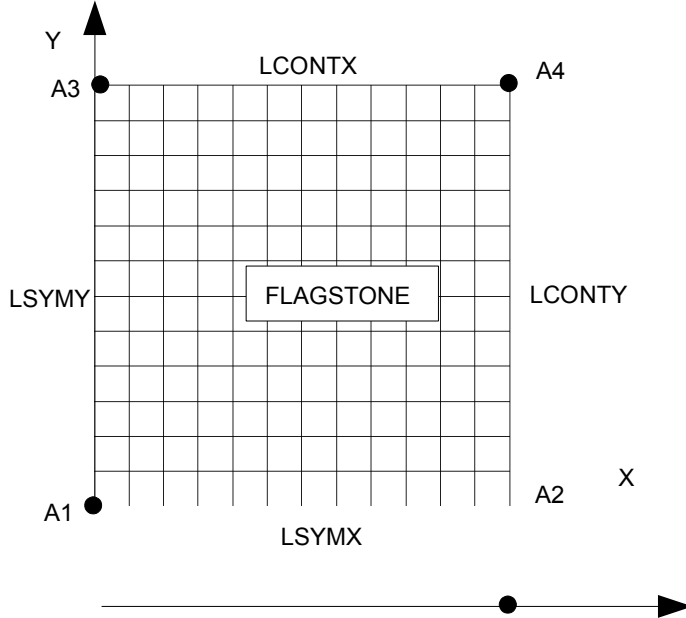
---

Identification			Type of reference	Reference	Tolérance%
<i>MXX</i>	<i>M266</i>	<i>Point 1</i>	'NON_REGRESSION'	1506.61	1.e-6
<i>MY Y</i>	<i>M266</i>	<i>Point 1</i>	'NON_REGRESSION'	1506.70	1.e-6
<i>KXX</i>	<i>M266</i>	<i>Point 1</i>	'NON_REGRESSION'	2,228 10 <sup>-4</sup>	1.e-6
<i>KYY</i>	<i>M266</i>	<i>Point 1</i>	'NON_REGRESSION'	2,228 10 <sup>-4</sup>	1.e-6
<i>KXY</i>	<i>M266</i>	<i>Point 1</i>	'NON_REGRESSION'	2.31 10 <sup>-8</sup>	1.e-6

## 5.4 Remarks

## 6 Modeling D

### 6.1 Characteristics of modeling



Modeling Q4GG (QUAD4)

- Boundary conditions:

- . Dimensioned A2A4 :  $DZ=0$
- . Dimensioned A3A4 :  $DZ=0$

- Conditions of symmetry

- . Dimensioned A1A2 :  
 $DY = DRX = 0$
- . Dimensioned A1A3 :  
 $DX = DRY = 0$

The flagstone is symmetrical compared to the plans ( $X=0$ ) and ( $Y=0$ ), calculations are carried out on a quarter of the flagstone.

### 6.2 Characteristics of the grid

Many nodes: 169

Number of meshes and type: 144 QUAD4

### 6.3 Sizes tested and results

Identification	Type of Reference	Reference	% Tolerance
$DZ(AI)$	'ANALYTICAL'	$7,895.10^{-5}$	8 %
$MXX(AI)$	'ANALYTICAL'	1550	3 %
$MYY(AI)$	'ANALYTICAL'	1550	3 %
$KXX(AI)$	'ANALYTICAL'	$2,351.10^{-4}$	3 %
$KYY(AI)$	'ANALYTICAL'	$2,351.10^{-4}$	3 %

The sizes are expressed in the reference mark defined by the nautical angles  $\alpha=33^\circ$  and  $\beta=12^\circ$ .

Identification	Type of reference	Reference	Tolerance
$DZ(AI)$	'ANALYTICAL'	$7,895.10^{-5}$	8 %
$MXX(AI)$	'ANALYTICAL'	1550.0	3 %
$MYY(AI)$	'ANALYTICAL'	1550.0	3 %
$MXY(AI)$	'ANALYTICAL'	0.	0.1
$KXX(AI)$	'ANALYTICAL'	$2,351.10^{-4}$	3 %
$KYY(AI)$	'ANALYTICAL'	$2,351.10^{-4}$	3 %
$KXY(AI)$	'ANALYTICAL'	0.	0,001

Identification			Type of reference	Reference	Tolérance%
<i>MXX</i>	<i>M266</i>	<i>Point 1</i>	'NON_REGRESSION'	1512.79	1.e-6
<i>MYY</i>	<i>M266</i>	<i>Point 1</i>	'NON_REGRESSION'	1515.82	1.e-6
<i>MXY</i>	<i>M266</i>	<i>Point 1</i>	'NON_REGRESSION'	-0.6749	1.e-6
<i>KXX</i>	<i>M266</i>	<i>Point 1</i>	'NON_REGRESSION'	2,294 10 <sup>-4</sup>	1.e-6
<i>KYY</i>	<i>M266</i>	<i>Point 1</i>	'NON_REGRESSION'	2,301 10 <sup>-4</sup>	1.e-6
<i>KXY</i>	<i>M266</i>	<i>Point 1</i>	'NON_REGRESSION'	-1,601 10 <sup>-7</sup>	1.e-6

## 6.4 Remarks

## 7 Summary of the results

---

By comparing the results of four modelings with the analytical solution, one observes:

- DKTG : to the maximum 5 % of variation for displacements, and 8% for the moment and the curve.
- Q4GG : to the maximum 8% of variation for displacements, and 3% for the moment and the curve.

One can thus estimate that these modelings validate modeling DKTG and the model GLRC in elastic behavior, modeling Q4GG and the model ELAS.